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ACTIVE VIBRATION CONTROL OF A RECTANGULAR FLEXIBLE WALL OF AN EMPTY AND FLUID-FILLED TANK

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ABSTRACT

A rectangular plate bolted to a thick Plexiglas rectangular container is investigated in the case of empty and water-filled tank.

A modal analysis is firstly realized in order to verify the effects of different water levels and of the free surface waves on the modal parameters and on the modal shapes.

A filtered-x LMS (least mean square) adaptive feedforward algorithm is then applied to the perturbed system realizing structural vibration control in linear field with a SISO approach. Five piezoelectric PZT actuators apply the secondary control input in a nearly-located configuration. Their positioning is based on the knowledge on the deformation energy of the plate. The present study investigates primarily the control of the first vibration mode, but second and third modes are also experimentally studied.

Satisfactory reductions (up to about 45 dB on the second mode measured on Channel 4) are reached for vibration amplitude of the three modes investigated in absence of water. For each modal shape, a particular effectiveness of the optimally placed actuators clearly appears. The introduction of water in the tank reduces the effectiveness of control of the first mode (maximum 5.5 dB of reduction of the vibration amplitude), but the five control channels show a more global uniform effectiveness.

Keywords: active control, vibrations, fluid-structure interaction, feedforward, PZT

1. INTRODUCTION

The use of active control techniques for the suppression of vibrations of very light structures is a very important target in a lot of applications. Particularly, it allows to avoid the increment of the mass of the system due to stiffeners or dampers. Moreover, aeronautical and space structures often present thin walls containing fuel, where the fluid-structure interaction is fundamental. For these reasons, the study of active vibration control of a panel in air and partially wet on one side by a liquid is of great interest.

In this study, a rectangular plate representing a flexible wall of a Plexiglas rectangular container is investigated, both with empty and water-filled tank. The use of water typically allows realizing dynamic testing in aerospace structures avoiding the possible dangers due to the use of fuel.

The plate is bolted to the rectangular container, which can be considered practically rigid; this constraint approximates clamped boundaries.

The presence of water increases the modal mass of the system. Moreover, free surface waves complicate the system dynamics, as reported by Amabili [1].

Morand and Ohayon [2] deeply explain, using computational finite elements and/or dynamic substructuring models, the effects of fluid-structure interactions on the dynamics of structure, underlining the problems of free-surface sloshing and their importance in the overall fluid-structure interaction problem.

Habault and Filippi [3] recently proposed a numerical method for the computation of the resonance frequencies and modes of a fluid-loaded rectangular clamped plate, following an approach similar to the one proposed by Amabili, Frosali and Kwak [4] in the case of annular plates.

In the present study, an experimental modal analysis is firstly performed on the plate for different levels of fluid, in order to verify the effects of the position of the free-surface on the modal parameters and on the modal shapes. Then, the influence of fluid-structure interaction on the effectiveness of active control is experimentally investigated and compared with the case of absence of water.

The secondary control input is generated by some piezoelectric PZT actuators, which have been already used in several experimental applications of active control of thin-walled structures, as reported in the literature.

Crawley and Louis [5] firstly showed the effectiveness of piezoelectric actuators for vibration suppression on a mono-dimensional structure. Dimitridis et al [6] extended this application to two-dimensional structures. Tzou and Fu [7] studied the vibration control of plates with laminated piezoelectric layers.

The positioning of the PZT patches is here based on the knowledge of the natural modes of the plate and, particularly, of their deformation energy.

The accelerometers are positioned very closed to the piezoelectric patches, in order to obtain a nearly-collocated system. Sensors and actuators are placed on the same side of the plate surface (air side).

The optimization of the actuators position is in fact of fundamental significance for the success of the control, as already investigated in literature. The differences between collocated and non-collocated control are deeply explained by Preumont [8]. Young-Sup Lee et al. [9] investigated the problem of the optimal relative position of piezoactuators and sensors. They evaluated different possible configurations and tested a solution with two matched pairs of piezoelectric transducers bonded on a cantilever beam.

In this study, a classical feedforward algorithm is applied to the system to be controlled. Such traditional techniques of active control on mechanical systems and their possible applications in active control of vibrations are explained by Fuller et al [10].

In the last years, several studies have been dedicated to the case of control of fluid-loaded structures. Some of them are specifically dedicated to the active control of sound radiation. Gu and Fuller [11] used the optimal solution of a quadratic cost function to determine the control forces to be applied on a fluid-loaded rectangular plate, while Meirovitch [12] used the

non-linear Riccati equation to get optimal control and reduce the sound radiation pressure in submerged structures. Ruckman and Fuller [13] investigated the active control of acoustic radiation from a fluid-loaded spherical shell.

Other studies have been dedicated also to the application of active control on structural vibrations of fluid-loaded structures.

Lee and Park [14] used a near-field approach to simulate the active control of sound radiation from a fluid-loaded rectangular plate, obtaining high global reductions both in radiated pressure and plate response. Sheng Li and Deyou Zhao [15] numerically studied the active control of both structural vibration and acoustic radiation of a fluid-loaded laminated plate with active damping control and active constrained layer damping (ACLD), showing a better effectiveness of the ACLD approach.

Control experiments have been realized by Ma [16] on a rectangular clamped plate, obtaining good vibration reductions with both a classic adaptive feedforward control and an adaptive non-linear method in the case of sinusoidal excitations and of swept-frequency harmonic excitation. The effect of fluid-structure interaction on the effectiveness of feedforward control is not evaluated in [16].

The aim of the present research is to realize tests of experimental control on a plate bolted to a wall of a thick Plexiglas rectangular container in the cases of (i) empty tank and (ii) water-filled tank.

The main target is the control of the first natural mode, but also the second and third modes have been investigated.

A filtered-x LMS adaptive feedforward algorithm is used in order to control plate vibrations in linear regime, by using a SISO approach. Both the cases of empty and completely water-filled tank are investigated.

2. MODAL ANALYSIS

2.1 Experimental set-up

The system to be controlled is constituted by a rectangular aluminium plate ($400 \times 283 \times 0.8$ mm), which is connected by bolts to the thick wall of a Plexiglass container (internal dimensions: $440 \times 350 \times 500$ mm). The plate has boundary conditions close to those of clamped edges. The Plexiglas walls are quite thick (30 mm), so they can be considered rigid in the low-frequency range.

The plate is positioned on the wall in correspondence of an opening of dimensions 350×233 mm at a distance of 66 mm from the bottom of the tank. The long edges of the plate are parallel to the tank bottom. The tank is studied empty or partially filled with water.

An electrodynamic shaker LDS V406, with LDS PA100E power amplifier, excites the plate through a thin stinger, which is connected to a piezoelectric force transducer B&K 8200 glued to the plate. The load cell is in eccentric

position, in order to excite a lot of modes (Fig. 1). During the control tests, a B&K shaker Type 4809 is used.

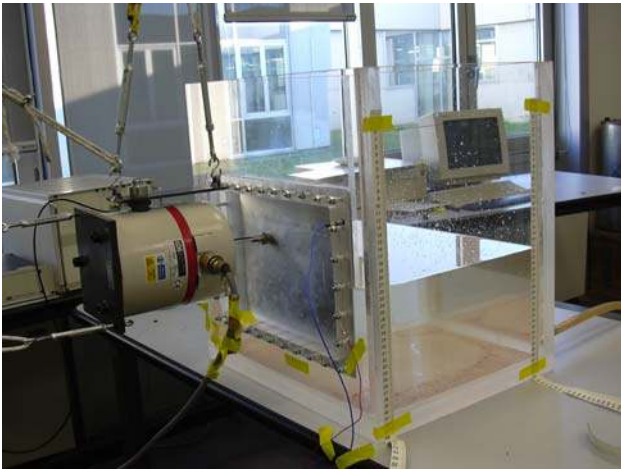


FIGURE 1. Experimental set-up for modal analysis; tank partially filled with water.

The system response has been measured by using a sub-miniature accelerometer, model Endevco 22, of mass 0.14 g, in order to minimize the presence of additional masses.

The plate has been subjected to random excitation in order to identify the natural frequencies.

Modal analysis has been performed by measuring the panel response on a grid of 9×7 points, both in absence of water and with different levels of fluid.

The time responses have been measured by using the Difa Scadas II front-end, connected to a HP c3000 workstation, and the software CADA-X of LMS for signal processing and data analysis; the same front-end has been used to generate the excitation signal.

The frequency response functions (FRFs) have been estimated using the H_V technique.

2.2 Identification results

Natural frequencies and damping ratios have been estimated by using the Frequency domain Direct Parameter Identification (FDPI) technique of LMS. The model has been validated by using the Modal Assurance Criterion and the Modal Phase Collinearity (MPC).

Three modal analyses have been performed on the plate before applying the PZT patches. These are for empty tank, tank filled to half-plate and tank full of water (to 410 mm from the bottom).

Natural frequencies identified for the three cases are given in Fig. 2. The FRF measured at the driving point for the half-wet plate is given in Fig. 3; arrows identify modes.

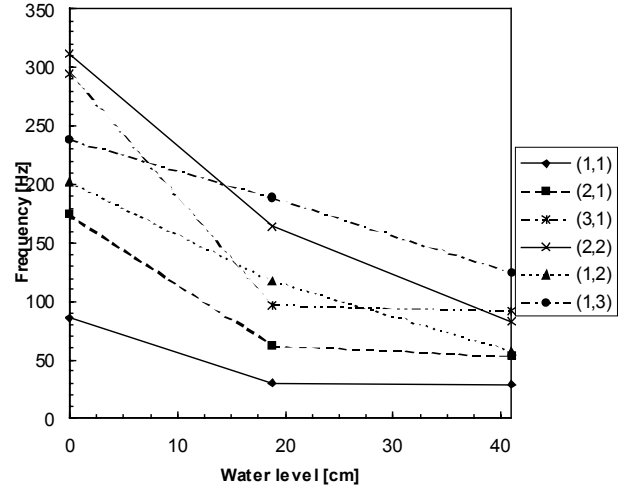


FIGURE 2. Natural frequencies of the first six modes of the plate for different levels of water in the tank. Integers (m, n) identify the mode shape. m = number of half-waves in horizontal direction; n = number of half-waves in vertical direction.

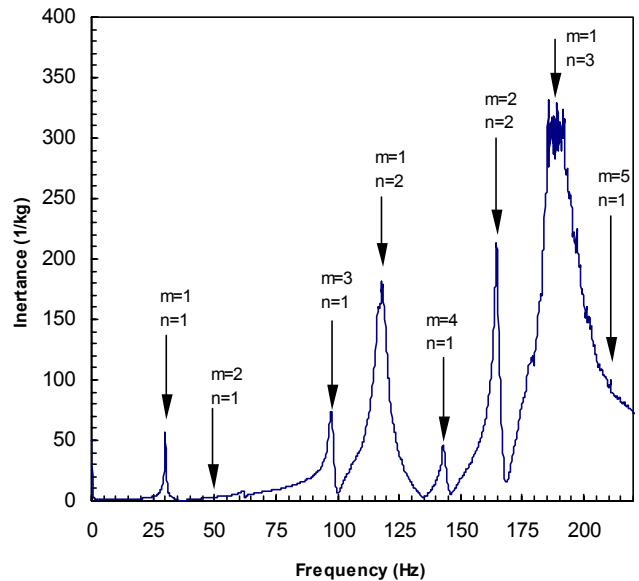


FIGURE 3. Measured FRF (driving point) for the half-wet plate; arrows identify natural frequencies.

3. THE CONTROL SYSTEM

Five piezoelectric PZT patches (25mm x 25mm x 0.2mm, PIC 255 material, from Physik Instrumente) have been glued (conductive glue) on the dry surface of the aluminium plate.

Their position has been optimised by using numerical calculation of the deformation energy of the plate for the first natural modes.

Calculations have been performed with a self-developed computer program based on Rayleigh-Ritz method.

The PZT actuators (capacity 50 nF) are subjected to a contraction of $180\mu\epsilon$ at 200V, that is enough to realize control in linear field. Every actuator receives the command signal from a high-voltage power amplifier (model E-507.00, from Physik Instrumente).

A PCB accelerometer (model M352C66) has been positioned close to every PZT actuator, realizing a nearly-collocated control system. Every couple accelerometer-piezoactuator represents a Channel of the control system and it is numbered as shown in Fig. 4.

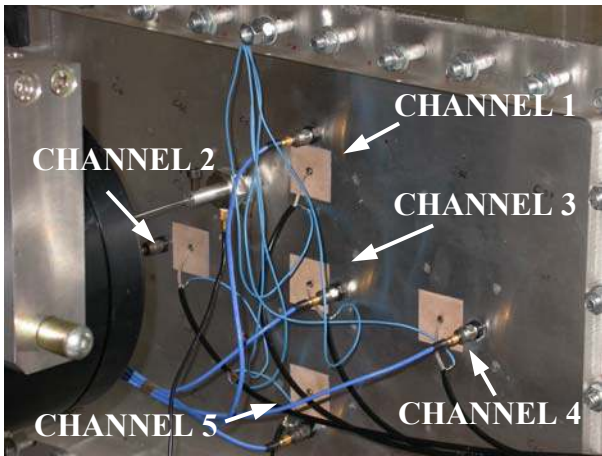


FIGURE 4. Positions of PZT patches and sensors

A modal analysis has been performed on the plate after applying the piezoelectric PZT patches, in the cases of empty tank, tank filled to half-plate and tank full of water, in order to identify the natural frequencies and damping ratios of the system modified with the PZT patches. The same sequence of modes has been observed.

The PZT patches are subjected to a constant negative voltage of $-200V$ when they are connected to the power amplifiers, so they are characterized by a fixed initial precompression.

The frequency response functions have been measured in absence and in presence of such precompression, but no important changes in the resonance peaks have emerged.

The complete control system for a single SISO control channel is shown in Fig. 5, where the acceleration signal e is firstly analogically filtered in order to avoid aliasing problems and then it is sent to a dSPACE real-time digital controller (made of one DS1104 processor Board at 250 MHz) passing through a 12-bit A/D converter.

The controller generates the command input u for the correspondent collocated PZT actuator: such signal is reconstructed by a 16-bit D/A converter.

The high frequencies due to digitalization are cut by a low-pass analogical smoothing filter acting on signal u .

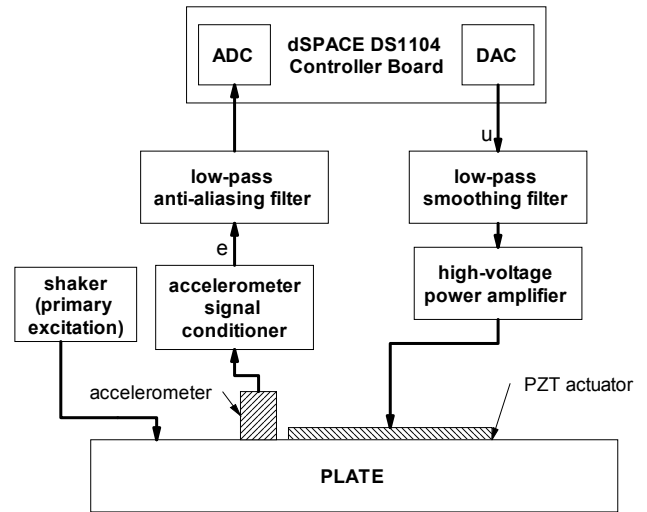


FIGURE 5. Schematization of the control system hardware for a single SISO channel.

The control algorithm has been implemented on the processor board working in a Matlab-Simulink environment and choosing a sampling frequency of 1000 Hz. Some amplification gains have been opportunely introduced into the control-loop in order to increase the ratio signal/noise.

4. THE CONTROL ALGORITHM

The experimental control tests are performed using a classical feedforward control based on a filtered-x last mean squared (LMS) technique, which belongs to the class of adaptive algorithms [10].

The concept of feedforward control is to derive a reference signal from the primary excitation and to use it directly to drive the control secondary input.

Theoretically, in absence of noise, this technique could bring to a perfect cancellation of the primary disturbance.

In real cases, the continuous changes in the excitation signal and in the primary path make necessary that the frequency response of the controller is adjusted with time, using an adaptive technique.

The model of a practical implementation of a single-channel feedforward control with filtered-x last mean squared (LMS) adaptive algorithm is showed in Fig. 6, where x is the primary excitation, d is the response of the mechanical system in absence of control, u is the secondary control input and e is the error signal (that is the measured acceleration signal).

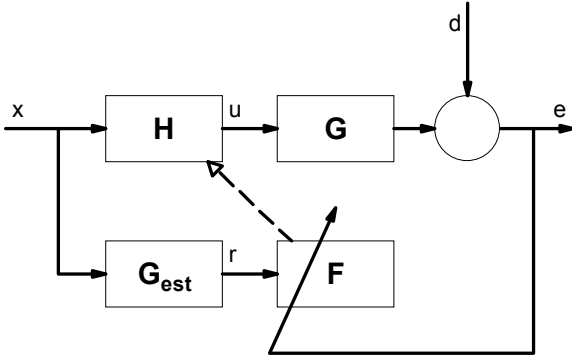


FIGURE 6: Block diagram of a practical implementation of the filtered-x LMS algorithm.

The controller is represented by H , that is a Finite Impulse Response (FIR) filter that acts on the primary disturbance x in order to generate the control secondary input u .

The coefficients of such filter must be continuously changed with time, in order to have optimal control performances. For this reason, the cost function J must be minimized at every sample time n

$$J = e^2(n). \quad (1)$$

Such minimization is realized by the LMS adaptive filter F , that calculates the coefficients w_i of the H filter

$$w_i(n+1) = w_i(n) - \mu \frac{\partial J}{\partial w_i(n)} \quad (2)$$

where $w_i(n)$ represents the i th controller coefficient at the n th sample time and μ is a convergence parameter.

Following [10], Equation (2) can be rewritten as

$$w_i(n+1) = w_i(n) - 2\mu e(n)r(n-i). \quad (3)$$

The so-called *filtered reference signal* r is obtained filtering the primary disturbance x with an appropriate filter G_{est} and it is then sent to the adaptive block F . G_{est} represents a model of the system secondary path and it is identified as FIR of the transfer function G between the sensors and the PZT patches in absence of any other external perturbation. Therefore the identification of G_{est} must be realized before applying any control, in order to calculate its coefficients.

5. EXPERIMENTAL RESULTS OF ACTIVE CONTROL

In this study, a single-channel feedforward control is applied to the system by using a classical SISO approach: only the effect of each single piezoelectric actuator on the corresponding sensor is evaluated.

The necessary identification of G_{est} (see Section 4) has been firstly performed for every control channel, in absence of

any external perturbation, using in sequence every different PZT patch as actuator to produce the excitation.

The filtered-x LMS algorithm has been then applied to the structure in presence of a sinusoidal perturbation generated by the shaker described in Section 2.1.

Several experimental tests have been carried out on the Plexiglas tank both in absence of fluid and in the case of completely water-filled tank (Fig. 7). Particularly, the effectiveness of active control on the first three modes has been investigated.



FIGURE 7. Control set-up for water-filled tank.

5.1 EMPTY TANK: ACTIVE CONTROL OF THE FIRST THREE MODES

The studied structure has been firstly subjected to sinusoidal excitation at its first three resonant frequencies, that have already been previously identified (see Section 2.2), in the case of absence of fluid-structure interaction.

Table 1 synthesizes the results obtained on all the control channels for the first mode at 85 Hz using the filtered-x LMS algorithm with a SISO approach. The reported values refer to the reduction (sign -) of the peak of the FFT spectrum of the acceleration signals, expressed in dB.

	<i>SISO effect [dB]</i>
Channel 1	-17.77
Channel 2	-14.22
Channel 3	-38.15
Channel 4	-23.28
Channel 5	-23.28

TABLE 1. Effects of SISO approach on the first mode at 85 Hz in presence of a sinusoidal excitation (case of empty tank).

During each test, the acceleration signal is influenced only by the control action of the corresponding collocated piezoelectric actuator. The other patches are connected to the

power amplifiers, in order to keep into account the effect of their constant pre-compression (see Section 3), but they are not transmitting any additional secondary control input to the plate.

The SISO approach seems to have a global good effect on all the channels, with a particularly important reduction of 38.15 dB on Channel 3 (central patch), for the theoretical optimal position to control mode (1,1). Figure 8 shows the spectral components of the acceleration signal on channel 3 in presence and in absence of control.

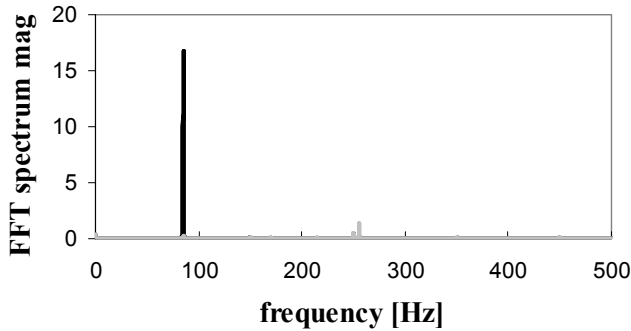


FIGURE 8. FFT spectrum of the acceleration signal on Channel 3 for the first mode at 85 Hz (case of empty tank):—no control,— SISO control

Some little additional components can be observed in Figure 8, but their magnitude is very small if compared with the main component at 85Hz; the system can therefore be considered linear, as initially supposed during the phase of modal analysis. Particularly, the presence of a harmonic component at a frequency three times the fundamental one (i.e. 255 Hz) is typical of plates, which present hardening type nonlinearity.

Also other channels show a good effectiveness, even if actuators do not seem to have identical behaviour. This is probably due to the small asymmetry given to the plate system by the presence of the additional mass of the load cell, which is in a non-central position.

A global positive reduction of the acceleration signal can also be observed on the second resonance peak at 163 Hz, as reported in Table 2 for all the control channels.

	<i>SISO effect [dB]</i>
Channel 1	-27.40
Channel 2	-39.05
Channel 3	-20.55
Channel 4	-45.38
Channel 5	-29.67

TABLE 2. Effects of SISO approach on the second mode at 163 Hz in presence of a sinusoidal excitation (case of empty tank).

In this case, the second mode (2,1) is theoretically better controlled by channels 2 and 4, which have the optimal positions to control it. The showed experimental results confirm this assumption. Particularly, Figure 9 shows the satisfactory results obtained on Channel 2.

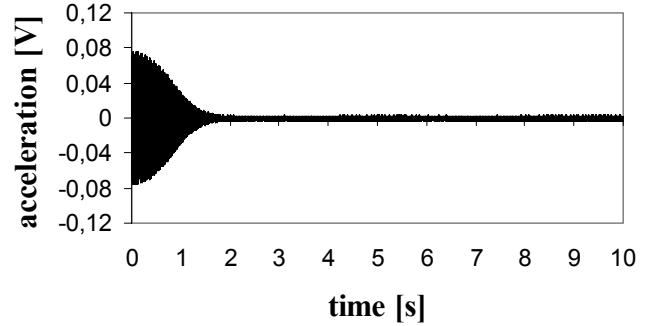


FIGURE 9. Transient acceleration signal on Channel 2 during the first 10 seconds of control, for the second mode at 163 Hz (case of empty tank).

Table 3 finally shows the results obtained for the third resonant peak (1,2), at 191 Hz. In this case, the results indicate that the actuators 1 and 5 have the optimal position to control the modal shape (1,2).

	<i>SISO effect [dB]</i>
Channel 1	-17.07
Channel 2	+0.96
Channel 3	-18.24
Channel 4	+3.03
Channel 5	-14.15

TABLE 3. Effects of SISO approach on the third mode at 191 Hz in presence of a sinusoidal excitation (case of empty tank).

On Channel 2 and 4 the control action is very difficult and it brings to a little amplification of the acceleration signal. The PZT actuators 2 and 4 are in fact positioned on nodal lines of mode (1,2).

5.2 WATER-FILLED TANK: ACTIVE CONTROL OF THE FIRST THREE MODES

Active control has then be applied to the studied plate in the case of completely water-filled tank, in order to verify the effects of the presence of the fluid on the effectiveness of the control of the plate.

It is assumed that gravity and sloshing effects, as well as compressibility of the fluid, do not give significant effects on the structural modes in this case (see [1,2]). In fact, sloshing modes of the free surface and structural modes arise in very different frequency ranges.

The sloshing resonances f_{mn} for a rigid rectangular container (only slightly changed by a flexible wall) are given by [17]:

$$f_{mn} = \sqrt{g k \tanh(k h)} / (2\pi), \quad (4)$$

where $k^2 = \pi^2 \left(\frac{m^2}{a^2} + \frac{n^2}{b^2} \right)$, g is the gravity acceleration, h

is the liquid level, a and b are the dimensions of the base of the tank and m and n are integers indicating the number of half-waves on the free surface in the two directions.

In the case of the studied system, a first sloshing mode (1,0) at 1.33 Hz, a second mode (0,1) at 1.49 Hz and a third mode (1,1) at 1.69 Hz can be calculated. The sloshing frequencies are therefore at much lower frequencies than the first structural mode to be controlled (23 Hz).

Table 4 shows the results obtained for the first resonant peak.

	<i>SISO effect [dB]</i>
Channel 1	-5.50
Channel 2	-4.86
Channel 3	-3.51
Channel 4	-4.43
Channel 5	-4.86

TABLE 4. Effects of SISO approach on the first mode at 23 Hz in presence of a sinusoidal excitation (case of water-filled tank).

The effectiveness of control seems to be uniformly distributed on all the five control channels, which show similar values of reduction.

The results obtained for the second resonant peak, at 51 Hz, are showed in Table 5.

	<i>SISO effect [dB]</i>
Channel 1	-10.45
Channel 2	-9.19
Channel 3	-3.58
Channel 4	-19.68
Channel 5	-3.62

TABLE 5. Effects of SISO approach on the second mode at 51 Hz in presence of a sinusoidal excitation (case of water-filled tank).

In this case, the optimally-located actuators (number 2 and 4) for the mode shape (2,1) presents different values of reduction: particularly, Channel 4 shows better results than Channel 2.

A similar effect can be observed in Tab. 6 for the case of control of the third resonant peak, at 60.4 Hz, characterized by the modal shape (1,2).

	<i>SISO effect [dB]</i>
Channel 1	-3.34
Channel 2	-0.83
Channel 3	-1.76
Channel 4	-1.74
Channel 5	-1.31

TABLE 6. Effects of SISO approach on the third mode at 60.4 Hz in presence of a sinusoidal excitation (case of water-filled tank).

In this case, the optimally-located actuators (number 1 and 5) for the mode shape (1,2) present different values of reduction: particularly, Channel 1 shows better results than Channel 5.

A possible reason is related to mode shapes, which are deformed by the presence of water, as showed by modal analysis. The position of the PZT patches has been in fact optimized in the case of empty tank. Moreover, the used PZT actuators have small dimensions; therefore, even a small change of mode shapes has a large effect on the control.

Globally, lower reduction values can be observed for the third mode, if compared with the first and the second ones.

Further investigation should be conducted by using a white-noise excitation, in order to verify the effectiveness of the chosen control algorithm on a wide frequency range.

6. CONCLUSIONS

The modal analysis of the aluminium rectangular plate in contact with different water levels shows that the mass of the fluid gives a significant reduction of the natural frequencies. Also mode shapes are changed, but less significantly.

A feedforward control, based on a filtered-x last mean squared (LMS) algorithm, has been applied in case of empty and completely filled tank by using 5 PZT patches. In the case of empty tank, good reductions (up to about 38 dB for Channel 3) have been obtained in controlling the first mode (1,1) at 85 Hz for sinusoidal excitation. Introducing water, the same mode goes down to 23 Hz and the control action gives lower effects (from 3.5 dB to 5.5 dB), but such effectiveness seems to be more uniformly distributed on the five channels.

The control of second and third modes has been experimentally tested, still obtaining good reduction values (up to about 45 dB on Channel 4 for the second mode with empty tank).

In the case of empty tank, a particular effectiveness of the actuators optimally positioned for such modal shapes can be observed for all the three modes investigated. The introduction of water in the tank reduces the effectiveness of the control.

This may be due to: (i) small changes in the modes shapes, that can influence the effect of the control, especially in the present case, as a consequence that the applied PZT actuators have small dimensions; (ii) larger force required to the small PZT patches, with negative consequence.

Further investigations will be conducted starting from the obtained results. A model will be used to optimize the locations of PZT actuators in the case of fluid-structure interaction and to verify the possible effects of gravity (sloshing).

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