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STRUCTURAL-ACOUSTIC VIBRATION REDUCTION USING PIEZOELECTRIC SHUNT TECHNIQUES: FE FORMULATION AND REDUCED ORDER MODEL

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Abstract. *For noise and vibration attenuation, various approaches can be employed depending on the frequency range to attenuate. Generally, active or passive piezoelectric techniques are effective in the low frequency range, while dissipative materials, such as viscoelastic or porous treatments, are efficient for higher frequency domain. In this work, a reduced order model is developed for the resolution of a fully coupled electro-mechanical-acoustic system using modal projection techniques. The problem consists of an elastic structure with surface-mounted piezoelectric patches coupled with an inviscid and compressible fluid. The piezoelectric elements, connected with a resistive or resonant shunt circuit, are used for the vibration damping of the coupled system. Numerical examples are presented in order to illustrate the accuracy and the versatility of the proposed reduced order models, notably in terms of prediction of attenuation..*

1 INTRODUCTION

For noise and vibration reduction, various approaches can be employed depending on the frequency range to attenuate. Generally, active or passive piezoelectric techniques are effective in the low frequency range, while dissipative materials (such as viscoelastic treatments or porous insulations) are efficient for higher frequency domain. In this work, we propose a reduced order model to describe a fully coupled electro-mechanical-acoustic problem in the low frequency range. The system consists of an elastic structure with surface-mounted piezoelectric patches coupled with an inviscid, compressible and barotropic fluid, gravity effects being neglected. The piezoelectric elements, connected with a resonant shunt circuit, are used for the vibration damping of the coupled system.

First, a non-symmetric finite element formulation of the coupled system is derived from a variational principle involving structural displacement, electrical voltage of piezoelectric elements, and acoustic pressure inside the fluid cavity. This formulation, with only one couple of electric variables per patch, is well adapted to practical applications since realistic electrical boundary conditions, such that equipotentiality on the electrodes and prescribed global electric charges, naturally appear. The global charge/voltage variables are intrinsically adapted to include any external electrical circuit into the electromechanical problem and to simulate the effect of resistive or resonant shunt damping techniques.

The second part of this work is devoted to the introduction of a reduced-order model of the coupled problem. The proposed methodology, based on a normal mode expansion, requires the computation of the eigenmodes of the structure with short-circuited piezoelectric patches, and the rigid acoustic cavity. It is shown that the projection of the full-order coupled finite element model on the uncoupled bases, leads to a reduced order model in which the main parameters are the classical fluid-structure and electromechanical modal coupling factors. Despite its reduced size, this model is proved to be very efficient for simulations of steady-state and transient analyses of the coupled structural-acoustic system with shunt damping.

In the last part of the paper, a three-dimensional numerical example is investigated. The problem consists of an elastic plate equipped with piezoelectric elements and coupled with an acoustic cavity. This example is analyzed in order to show that the reduced order model is capable of capturing the main characteristics of the system dynamic behavior, notably in terms of attenuation.

2 FINITE ELEMENT FORMULATION OF THE STRUCTURAL ACOUSTIC PROBLEM WITH PIEZOELECTRIC PATCHES

We briefly recall in this section the variational formulation of a fluid/piezoelectric-structure interaction problem in terms of structural mechanical displacement \mathbf{u} , electric potential in the structure ψ and fluid pressure p of the inviscid acoustic fluid (for more details, we refer the reader to [1, 2]). Secondly, this coupled formulation is adapted to the general case of an elastic structure equipped with P piezoelectric patches (see Fig. 1) as done for structural vibrations in [3]. This modified formulation allows taking into account realistic electrical boundary conditions such as equipotentiality on patches electrodes and prescribed global charges. Finally, the resulting finite element formulation is applied to a structural acoustic problem with one piezoelectric patch connected to a RL series shunt circuit.

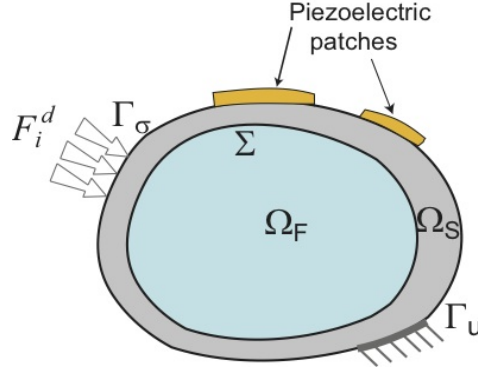


Figure 1: Fluid/piezoelectric-structure coupled system.

It should be noted that standard indicial notations are adopted throughout the paper: subscripts i, j, k and l denote the three-dimensional vectors and tensor components and repeated subscripts imply summation. In addition, a comma indicates a partial derivative.

2.1 Variational formulation of the fluid/structure/piezo-patches coupled system

We consider a piezoelectric structure occupying the domain Ω_S filled with an inviscid linear acoustic fluid occupying the domain Ω_F . We denote by Σ the fluid-structure interface and by n_i^S and n_i^F the unit normal external to Ω_S and Ω_F , respectively.

The structure is clamped on a part Γ_u and subjected (i) to a given surface force density F_i^d on the complementary part Γ_σ of its external boundary and (ii) to a pressure field p due to the presence of the fluid on its internal boundary Σ . The electric boundary conditions are defined by a prescribed electric potential ψ^d on Γ_ψ and a surface density of electric charge q^d on the remaining part Γ_D . Thus, the total structure boundary, denoted $\partial\Omega_S$, is such that $\partial\Omega_S = \Gamma_u \cup \Gamma_\sigma \cup \Sigma = \Gamma_D \cup \Gamma_\psi$ with $\Gamma_u \cap \Gamma_\sigma \cap \Sigma = \Gamma_\psi \cap \Gamma_D = \emptyset$.

The linearized deformation tensor is $\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$ and the stress tensor is denoted by σ_{ij} . Concerning the electric field variables, D_i is the electric displacement verifying the electric charge equation for a dielectric medium $D_{i,i} = 0$ in Ω_S and the electric boundary conditions $D_i n_i^S = -q^d$ on Γ_D ; E_i denotes the electric field vector such that $E_i = -\psi_{,i}$.

The linear piezoelectric constitutive equations write:

$$\sigma_{ij}(u, \psi) = c_{ijkl} \varepsilon_{kl}(u) - e_{kij} E_k(\psi) \quad (1)$$

$$D_i(u, \psi) = e_{ikl} \varepsilon_{kl}(u) + \varepsilon_{ik} E_k(\psi) \quad (2)$$

where c_{ijkl} denotes the elastic moduli at constant electric field, e_{kij} the piezoelectric constants and ε_{ik} the dielectric permittivity at constant strain. Moreover, we denote by ρ_S the mass density of the structure.

Let us introduce the admissible spaces C_u and C_ψ of regular functions u_i and ψ defined in Ω_S . We then consider the subspaces $C_u^* = \{u_i \in C_u \mid u_i = 0 \text{ on } \Gamma_u\}$, $C_\psi^d = \{\psi \in C_\psi \mid \psi = \psi^d \text{ on } \Gamma_\psi\}$ and $C_\psi^* = \{\psi \in C_\psi \mid \psi = 0 \text{ on } \Gamma_\psi\}$.

The variational formulation, corresponding to the response of the piezoelectric structure subjected to the prescribed boundary conditions and to the pressure field p on the interface Σ writes:

Find $u_i \in C_u^*$ and $\psi \in C_\psi^d$ such that:

$\forall \delta u_i \in C_u^*$:

$$\int_{\Omega_S} c_{ijkl} \varepsilon_{kl} \delta \varepsilon_{ij} \, dv - \int_{\Omega_S} e_{kij} E_k \delta \varepsilon_{ij} \, dv + \rho_S \int_{\Omega_S} \frac{\partial^2 u_i}{\partial t^2} \delta u_i \, dv = \int_{\Gamma_\sigma} F_i^d \delta u_i \, ds + \int_{\Sigma} p n_i^F \delta u_i \, ds \quad (3)$$

where $\delta \varepsilon_{ij} = \frac{1}{2}(\delta u_{i,j} + \delta u_{j,i})$, ε_{kl} and E_k being functions of u_i and ψ , and

$\forall \delta \psi \in C_\psi^*$:

$$\int_{\Omega_S} e_{ikl} \varepsilon_{kl} \delta E_i \, dv + \int_{\Omega_S} \varepsilon_{ik} E_k \delta E_i \, dv = \int_{\Gamma_D} q^d \delta \psi \, ds \quad (4)$$

where $\delta E_i = -\delta \psi_{,i}$, ε_{kl} and E_k being functions of u_i and ψ .

This formulation must be completed by appropriate initial conditions.

We consider now the special case of an elastic structure (domain Ω_E) equipped with P piezo-electric patches and completely filled with an internal fluid (domain Ω_F). Each piezo-electric patch has the shape of a plate with its upper and lower surfaces covered with a very thin layer of conducting material to obtain electrodes. The p -th patch, $p \in \{1, \dots, P\}$, occupies a domain $\Omega^{(p)}$ such that $(\Omega_E, \Omega^{(1)}, \dots, \Omega^{(P)})$ is a partition of the all structure domain Ω_S . A set of hypotheses, which can be applied to a wide spectrum of practical applications, are now formulated:

- The piezoelectric patches are thin, with a constant thickness $h^{(p)}$ for the p -th patch;
- The thickness of the electrodes is much smaller than $h^{(p)}$ and is thus neglected;
- The piezoelectric patches are polarized in their transverse direction (i.e. the direction normal to the electrodes).
- The electric field vector, of components E_k , is normal to the electrodes and uniform in the piezoelectric patch, so that for all $p \in \{1, \dots, P\}$:

$$E_k = -\frac{V^{(p)}}{h^{(p)}} n_k \quad \text{in } \Omega^{(p)} \quad (5)$$

where $V^{(p)} = \psi_+^{(p)} - \psi_-^{(p)}$ is the potential difference between the upper and the lower electrode surfaces of the p -th patch which is constant over $\Omega^{(p)}$ and n_k is the k -th component of the normal unit vector to the surface of the electrodes.

Under those assumptions and by considering successively each of the $P+2$ subdomains $(\Omega_F, \Omega_E, \Omega^{(1)}, \dots, \Omega^{(P)})$, the variational formulation of the fluid/structure/piezoelectric-patches coupled system can be written in terms of the structural mechanical displacement u_i , the electric potential difference $V^{(p)}$ constant in each piezoelectric patch, and the fluid pressure p :

- *Mechanical equation* : $\forall \delta u_i \in C_u^*$,

$$\int_{\Omega_s} c_{ijkl} \varepsilon_{kl} \delta \varepsilon_{ij} dv + \sum_{p=1}^P \frac{V^{(p)}}{h^{(p)}} \int_{\Omega^{(p)}} e_{kij} n_k \delta \varepsilon_{ij} d\Omega - \int_{\Sigma} p n_i^F \delta u_i ds + \rho_s \int_{\Omega_s} \frac{\partial^2 u_i}{\partial t^2} \delta u_i dv = \int_{\Gamma_\sigma} F_i^d \delta u_i ds \quad (6)$$

- *Electrical equation* : $\forall \delta V^{(p)} \in R$,

$$\sum_{p=1}^P \delta V^{(p)} C^{(p)} V^{(p)} - \sum_{p=1}^P \frac{\delta V^{(p)}}{h^{(p)}} \int_{\Omega^{(p)}} e_{ikl} \varepsilon_{kl} n_i d\Omega = \sum_{p=1}^P \delta V^{(p)} Q^{(p)} \quad (7)$$

where $C^{(p)} = \varepsilon_{33} S^{(p)} / h^{(p)}$ defines the capacitance of the p -th piezoelectric patch ($S^{(p)}$ being the area of the patch and $\varepsilon_{33} = \varepsilon_{ik} n_i n_k$ the piezoelectric material permittivity in the direction normal to the electrodes) and $Q^{(p)}$ is the global charge in one of the electrodes (see [3]).

- *Acoustic equation* : $\forall \delta p \in C_p$,

$$\frac{1}{\rho_F} \int_{\Omega_F} p_{,i} \delta p_{,i} dv + \frac{1}{\rho_F c_F^2} \int_{\Omega_F} \frac{\partial^2 p}{\partial t^2} \delta p dv + \int_{\Sigma} \frac{\partial^2 u_i}{\partial t^2} n_i^F \delta p ds = 0 \quad (8)$$

The first two equations are directly derived from Eqs. (3) and (4) using the procedure described in [3]. The last equation corresponds to the variational formulation of the Helmholtz equation in the acoustic cavity $p_{,ii} = \ddot{p} / c_F^2$ in Ω_F together with the boundary condition $p_{,i} n_i^F = -\rho_F \ddot{u}_i n_i^F$ on Σ . This relation expresses the continuity of the normal displacements of the fluid and the structure on Σ . c_F is the constant speed of sound in the fluid and ρ_F the mass density of the fluid. C_p is the admissible space of regular functions p defined in Ω_F .

Thus, the variational formulation of the fluid/structure/piezo-patches coupled problem writes as follows: given (F^d, ψ^d, q^d) , find $(u_i \in C_u^*, \psi \in C_\psi^d, p \in C_p)$ such that Eqs. (6) to (8) are satisfied. The formulation must be completed by appropriate initial conditions.

This formulation, with only a couple of electric variables per patches, is well adapted to practical applications since (i) realistic electrical boundary conditions such that equipotentiality on the electrodes and prescribed global charges naturally appear, (ii) the global charge/voltage variables are intrinsically adapted to include any external electrical circuit into the electromechanical problem and to simulate shunted piezoelectric patches.

2.2 Finite element formulation of the fluid/structure/piezo-patches coupled system

Let us introduce \mathbf{U} (of length N_s) and \mathbf{P} (of length N_f) corresponding to the vectors of nodal values of u_i and p respectively, and $\mathbf{Q} = (Q^{(1)} Q^{(2)} \dots Q^{(P)})^T$ and $\mathbf{V} = (V^{(1)} V^{(2)} \dots V^{(P)})^T$ the column vectors of electric charges and potential differences. The submatrices corresponding to the various linear and bilinear forms involved in Eqs. (6) to (8) are defined by

$$\begin{aligned}
 \int_{\Omega_S} c_{ijkl} \varepsilon_{kl} \delta \varepsilon_{ij} \, dv &\Rightarrow \delta \mathbf{U}^T \mathbf{K}_u \mathbf{U} & \sum_{p=1}^P \frac{V^{(p)}}{h^{(p)}} \int_{\Omega^{(p)}} e_{kij} n_k \delta \varepsilon_{ij} \, d\Omega &\Rightarrow \delta \mathbf{U}^T \mathbf{C}_{uV} \mathbf{V} \\
 \int_{\Omega_S} \rho_S u_i \delta u_i \, dv &\Rightarrow \delta \mathbf{U}^T \mathbf{M}_u \mathbf{U} & \sum_{p=1}^P \frac{\delta V^{(p)}}{h^{(p)}} \int_{\Omega^{(p)}} e_{ikl} \varepsilon_{kl} n_i \, d\Omega &\Rightarrow \delta \mathbf{V}^T \mathbf{C}_{uV}^T \mathbf{U} \\
 \frac{1}{\rho_F} \int_{\Omega_F} p_{,i} \delta p_{,i} \, dv &\Rightarrow \delta \mathbf{P}^T \mathbf{K}_p \mathbf{P} & \int_{\Sigma} p n_i^F \delta u_i \, ds &\Rightarrow \delta \mathbf{U}^T \mathbf{C}_{up} \mathbf{P} \\
 \frac{1}{\rho_F c_F^2} \int_{\Omega_F} p \delta p \, dv &\Rightarrow \delta \mathbf{P}^T \mathbf{M}_p \mathbf{P} & \int_{\Sigma} u_i n_i^F \delta p \, ds &\Rightarrow \delta \mathbf{P}^T \mathbf{C}_{up}^T \mathbf{U} \\
 \sum_{p=1}^P \delta V^{(p)} C^{(p)} V^{(p)} &\Rightarrow \delta \mathbf{V}^T \mathbf{K}_V \mathbf{V} & \sum_{p=1}^P \delta V^{(p)} Q^{(p)} &\Rightarrow \delta \mathbf{V}^T \mathbf{Q} \\
 & & \int_{\Gamma_\sigma} F_i^d \delta u_i \, ds &\Rightarrow \delta \mathbf{U}^T \mathbf{F}
 \end{aligned} \tag{9}$$

where \mathbf{M}_u and \mathbf{K}_u are the mass and stiffness matrices of the structure; \mathbf{C}_{uV} is the electric mechanical coupled stiffness matrix; $\mathbf{K}_V = \text{diag}(C^{(1)} C^{(2)} \dots C^{(P)})$ is a diagonal matrix filled with the P capacitances of the piezoelectric patches; \mathbf{M}_p and \mathbf{K}_p are the mass and stiffness matrices of the fluid; \mathbf{C}_{up} is the fluid-structure coupled matrix; \mathbf{F} is the applied mechanical force vector.

Thus, the variational equations (6) to (8) for the fluid/structure/piezo-patches coupled problem can be written, in discretized form, as the following unsymmetrical matrix system:

$$\begin{bmatrix} \mathbf{M}_u & 0 & 0 \\ 0 & 0 & 0 \\ \mathbf{C}_{up}^T & 0 & \mathbf{M}_p \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{U}} \\ \ddot{\mathbf{V}} \\ \ddot{\mathbf{P}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_u & \mathbf{C}_{uV} & -\mathbf{C}_{up} \\ -\mathbf{C}_{uV}^T & \mathbf{K}_V & 0 \\ 0 & 0 & \mathbf{K}_p \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \mathbf{V} \\ \mathbf{P} \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \mathbf{Q} \\ 0 \end{bmatrix} \tag{10}$$

with appropriate initial conditions.

2.3 Structural acoustic problem with piezo patches connected to RL shunt circuit

The above discretized formulation (Eq. (10)) can be used for a wide range of applications of mechanicals structure coupled with acoustic domain and associated with piezoelectric patches. It is particularly adapted to the case where the piezoelectric patches are shunted, i.e. connected to a passive electrical network [3]. In this case, neither \mathbf{V} nor \mathbf{Q} are prescribed by the electrical network but the latter imposes only a relation between them. In the case of a resonant shunt connected to the p -th patch and composed of a resistance $R^{(p)}$ and an inductance $L^{(p)}$ in series (Fig. 1), we have the following relation between electrical potential difference $V^{(p)}$ and the electric charge $Q^{(p)}$:

$$\mathbf{L}\ddot{\mathbf{Q}} + \mathbf{R}\dot{\mathbf{Q}} + \mathbf{V} = 0 \tag{11}$$

where $\mathbf{R} = \text{diag}(R^{(1)} \dots R^{(P)})$ and $\mathbf{L} = \text{diag}(L^{(1)} \dots L^{(P)})$ are diagonal matrices filled with the P resistances and inductances of the shunt circuits.

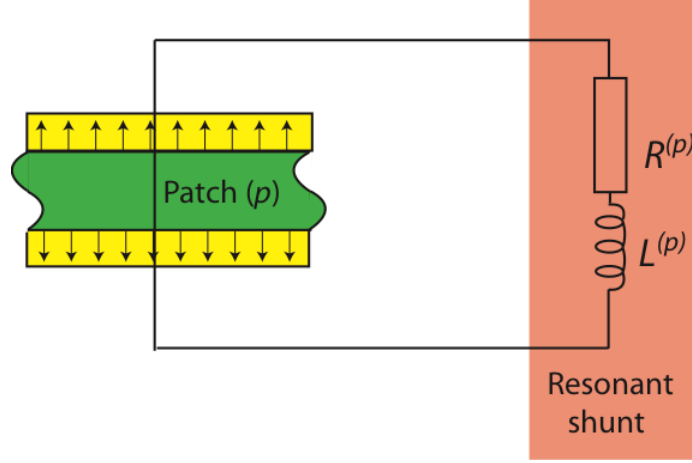


Figure 2: Piezoelectric patch connected to RL shunt circuit.

Due to the direct piezoelectric effect, the piezoelectric patch converts a fraction of the mechanical energy of the vibrating structure into electrical energy which can be dissipated through the resistive components of the RL circuit. It is well known that the damping effect due to this circuit is maximal when the resonance circular frequency $1/\sqrt{LC}$ of the shunt circuit is tuned on the circular frequency of the structural-acoustic eigenmode to be controlled. The resistance R and the inductance L can be adjusted and properly chosen so as to maximize the damping effect. Therefore, the optimal resistance and inductance for a series resonant shunt can be calculated by [4, 5]

$$R^{\text{opt}} = \frac{\sqrt{2k_{\text{eff},i}^2}}{C\omega_i(1+k_{\text{eff},i}^2)} \quad (12)$$

$$L^{\text{opt}} = \frac{1}{C\omega_i^2(1+k_{\text{eff},i}^2)} \quad (13)$$

where ω_i is the short circuit natural frequency of the i -th mode and $k_{\text{eff},i}$ is the effective electromechanical coupling coefficient [3].

Using the second row of Eq. (10), the degrees-of-freedom associated with the electrical potential difference \mathbf{V} can be expressed in terms of structural displacements \mathbf{U} and electric charge \mathbf{Q} as

$$\mathbf{V} = \mathbf{K}_V^{-1} \mathbf{C}_{uV}^T \mathbf{U} + \mathbf{K}_V^{-1} \mathbf{Q} \quad (14)$$

Thus, after substitution of \mathbf{V} into Eq. (11) and using Eq. (10), we get the following electro-mechanical-acoustic system:

$$\begin{bmatrix} \mathbf{M}_u & 0 & 0 \\ 0 & \mathbf{L} & 0 \\ \mathbf{C}_{up}^T & 0 & \mathbf{M}_p \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{U}} \\ \ddot{\mathbf{Q}} \\ \ddot{\mathbf{P}} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mathbf{R} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{U}} \\ \dot{\mathbf{Q}} \\ \dot{\mathbf{P}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_u + \mathbf{C}_{uV} \mathbf{K}_V^{-1} \mathbf{C}_{uV}^T & \mathbf{C}_{uV} \mathbf{K}_V^{-1} & -\mathbf{C}_{up} \\ \mathbf{K}_V^{-1} \mathbf{C}_{uV}^T & \mathbf{K}_V^{-1} & 0 \\ 0 & 0 & \mathbf{K}_p \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \mathbf{Q} \\ \mathbf{P} \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ 0 \\ 0 \end{bmatrix} \quad (15)$$

Note that this $(\mathbf{U}, \mathbf{Q}, \mathbf{P})$ formulation is well suited for switch shunting applications.

3 REDUCED ORDER MODEL

In this section, we introduce a reduced-order formulation of the discretized problem obtained in the previous section. This formulation, based on a modal superposition method, consists in expanding (i) the structural displacement over the *in vacuo* structure modes in short circuited configuration, (ii) the acoustic pressure over the acoustic modes of the fluid in rigid cavity. This approach is widely used and effective for structural acoustic systems with weak coupling. It can be noted that the acoustic basis is enriched by a static mode ([6]) and that the main motivation of choosing the structural modes with short-circuited patches is that they can be computed using a classical elastic formulation.

3.1 Eigenmodes of the structure in vacuo with short-circuited patches

In a first phase, the first M_s eigenmodes of the structure *in vacuo* with all patches short-circuited are obtained from the following equation

$$\left(\mathbf{K}_u - \omega_{si}^2 \mathbf{M}_u\right) \Phi_{si} = 0 \quad \text{for } i \in \{1, \dots, M_s\} \quad (16)$$

where (ω_{si}, Φ_{si}) are the natural frequency and eigenvector for the i -th structural mode. These modes verify the following orthogonality properties

$$\Phi_{si}^T \mathbf{M}_u \Phi_{sj} = \delta_{ij} \quad \text{and} \quad \Phi_{si}^T \mathbf{K}_u \Phi_{sj} = \omega_{si}^2 \delta_{ij} \quad (17)$$

where Φ_{si} have been normalized with respect to the structure mass matrix.

3.2 Eigenmodes of the internal acoustic cavity with rigid walls

In this second phase, the first M_f eigenmodes of the acoustic cavity with rigid boundary conditions are obtained from the following equation

$$\left(\mathbf{K}_p - \omega_{fi}^2 \mathbf{M}_p\right) \Phi_{fi} = 0 \quad \text{for } i \in \{1, \dots, M_f\} \quad (18)$$

where (ω_{fi}, Φ_{fi}) are the natural frequency and eigenvector for the i -th acoustic mode. These modes verify the following orthogonality properties

$$\Phi_{fi}^T \mathbf{M}_p \Phi_{fj} = \delta_{ij} \quad \text{and} \quad \Phi_{fi}^T \mathbf{K}_p \Phi_{fj} = \omega_{fi}^2 \delta_{ij} \quad (19)$$

where Φ_{fi} have been normalized with respect to the fluid mass matrix.

3.3 Modal expansion of the general problem

By introducing the matrices $\Phi_s = [\Phi_{s1} \dots \Phi_{sN_s}]$ of size $N_s \times M_s$ and $\Phi_f = [\Phi_{f1} \dots \Phi_{fN_f}]$ of size $N_f \times M_f$ corresponding to the uncoupled bases, the displacement and pressure are sought as

$$\mathbf{U} = \Phi_s \mathbf{q}_s(t) \quad \text{and} \quad \mathbf{P} = \Phi_f \mathbf{q}_f(t) \quad (20)$$

where the vectors $\mathbf{q}_s = [q_{s1} \dots q_{sN_s}]^T$ and $\mathbf{q}_f = [q_{f1} \dots q_{fN_f}]^T$ are the modal amplitudes of the structure displacement and the fluid pressure respectively.

Substituting these relations into Equations (15) and pre-multiplying the first row by Φ_s^T and the third one by Φ_f^T , we obtain the equation

$$\begin{aligned}
 & \begin{bmatrix} \Phi_s^T (\mathbf{K}_u + \mathbf{C}_{uv} \mathbf{K}_V^{-1} \mathbf{C}_{uv}^T) \Phi_s & \Phi_s^T \mathbf{C}_{uv} \mathbf{K}_V^{-1} & -\Phi_s^T \mathbf{C}_{up} \Phi_f \\ \mathbf{K}_V^{-1} \mathbf{C}_{uv}^T \Phi_s & \mathbf{K}_V^{-1} & 0 \\ 0 & 0 & \Phi_f^T \mathbf{K}_p \Phi_f \end{bmatrix} \begin{bmatrix} \mathbf{q}_s \\ \mathbf{Q} \\ \mathbf{q}_f \end{bmatrix} + \\
 & \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mathbf{R} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\mathbf{q}}_s \\ \dot{\mathbf{Q}} \\ \dot{\mathbf{q}}_f \end{bmatrix} + \begin{bmatrix} \Phi_s^T \mathbf{M}_u \Phi_s & 0 & 0 \\ 0 & \mathbf{L} & 0 \\ \Phi_f^T \mathbf{C}_{up}^T \Phi_s & 0 & \Phi_f^T \mathbf{M}_p \Phi_f \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{q}}_s \\ \ddot{\mathbf{Q}} \\ \ddot{\mathbf{q}}_f \end{bmatrix} = \begin{bmatrix} \Phi_s^T \mathbf{F} \\ 0 \\ 0 \end{bmatrix} \quad (21)
 \end{aligned}$$

This matrix equation represents the reduced order model of the structural acoustic problem with piezoelectric shunt damping treatments. If only few modes are kept for the projection, the size of this reduced order model ($M_s \times P \times M_f$) is much smaller than the initial one ($N_s \times P \times N_f$) (see examples below).

Equation (21) can be also written in the following form of coupled differential equations

- N_s mechanical oscillators

$$\ddot{q}_{si} + 2\xi_i \omega_{si} \dot{q}_{si} + \omega_{si}^2 q_{si} + \sum_{p=1}^P \sum_{k=1}^{N_s} \frac{\gamma_i^{(p)} \gamma_k^{(p)}}{C^{(p)}} q_{sk} + \sum_{p=1}^P \frac{\gamma_i^{(p)}}{C^{(p)}} Q^{(p)} - \sum_{j=1}^{N_f} \beta_{ij} q_{fj} = F_i \quad (22)$$

- N_f acoustic equations

$$\ddot{q}_{fi} + \omega_{fi}^2 q_{fi} + \sum_{j=1}^{N_s} \beta_{ij} q_{sj} = 0 \quad (23)$$

- P electric equations

$$L^{(p)} \ddot{Q}^{(p)} + R^{(p)} \dot{Q}^{(p)} + \frac{Q^{(p)}}{C^{(p)}} + \sum_{i=1}^{N_s} \frac{\gamma_i}{C^{(p)}} q_{si} = 0 \quad (24)$$

where $F_i(t) = \Phi_{si}^T \mathbf{F}$ is the mechanical excitation of the i -th mode; $\beta_{ij} = \Phi_{si}^T \mathbf{C}_{up} \Phi_{fj}$ are the fluid structure coupling coefficients and $\gamma_i = \Phi_{si}^T \mathbf{C}_{uv}$ the electromechanical coupling factors.

We can note that modal damping coefficients ξ_i have been added in Equation (22) in order to take into account the structural damping which can be measured experimentally.

The initial finite element formulation previously introduced has been replaced by the modal formulation of Equations (22), (23), (24) whose unknowns are the N_s structure modal coordinates q_{si} , the N_f fluid modal coordinates q_{fi} and the P charges $Q^{(p)}$ associated with the piezoelectric patch. Its major interest, and especially the choice of the short-circuit eigenmodes as the expansion basis, is that the above computations of the parameters necessitate only a modal analysis of an elastic problem. This operation can thus be done by any standard finite elements code.

4 NUMERICTLA EXAMPLES

We consider a 3D acoustic cavity completely filled with air (density = 1.2 kg/m³; speed of sound = 340 m/s). The cavity walls are rigid except the top one which is a flexible aluminium plate of thickness 1 mm clamped at its four edges. The density of the plate is 2700 kg/m³ the Young modulus 72 GPa and the Poisson ratio 0.34. On the top surface of the plate, two identical piezoelectric patches of thickness 0.5 mm are bounded. For the mechanical characteristics of the piezoelectric material PIC151, the reader can be referred to [7]. The geometrical data and mesh are presented in Fig. 3.

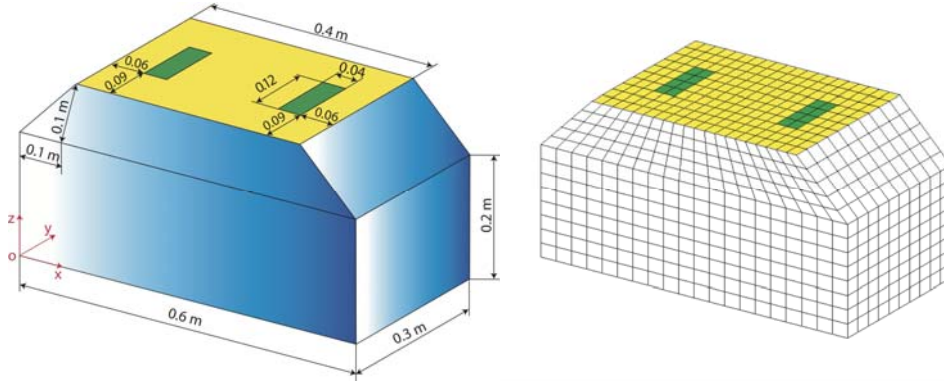


Figure 3: Acoustic/structure/piezo-patches coupled system: (a) geometrical data (in m) and (b) mesh.

Concerning the finite element discretization, we have used, for the structural part, 200 four-node plate elements based on Mindlin theory with five degrees-of-freedom per node ($N_s=1155$). The portions of the plate covered by the PZT patches have been modelled according to the first-order shear deformation laminated theory [8]. As discussed in the previous sections, only one electrical degree of freedom is used to represent the electrical charge Q in each patch. The acoustic cavity is discretized using 2600 hexahedric elements with one degree-of-freedom per node corresponding to the acoustic pressure ($N_f=3234$). Note that the structural and acoustic meshes are compatible at the interface.

4.1 Modal analysis of the acoustic/structure/piezo-patches coupled problem

Table 1 presents the eigenfrequencies in three following cases: (i) the 3D rigid acoustic cavity; (ii) the clamped plate with the two patches short circuited; and (iii) the plate/acoustic-cavity coupled system in the short circuit case. All coupled frequencies, except the sixth, are associated with the first vibration modes of the structure (lower than 450 Hz), and the sixth coupled frequency corresponds to the first acoustic mode in rigid cavity. It can be confirmed by comparing the mode shapes in case (iii) with those obtained in cases (i) or (ii) which are not shown here for sake of brevity. Moreover, as expected, the natural frequencies of the coupled modes (structure dominated) are lower than those for the structure in vacuum (except for the first mode) due to the "added-mass effect" of the fluid.

Fluid in rigid cavity	Structure without fluid	Coupled problem	Type of coupled mode S: Structure / F: Fluid
297.94	77.93	85.40	S
561.56	123.64	122.53	S
569.00	194.29	192.79	S
614.82	212.29	211.94	S
642.29	237.12	236.14	S
681.94	316.11	298.31	F
799.44	346.32	315.59	S
837.71	383.89	346.24	S
875.22	434.73	382.23	S
881.62	451.03	433.61	S

Table 1: Computed frequencies (in Hz) of the structural-acoustic coupled system.

For illustration purpose, Fig. 4 shows the deformed plate and the pressure field for the first ten vibration modes in the coupled case.

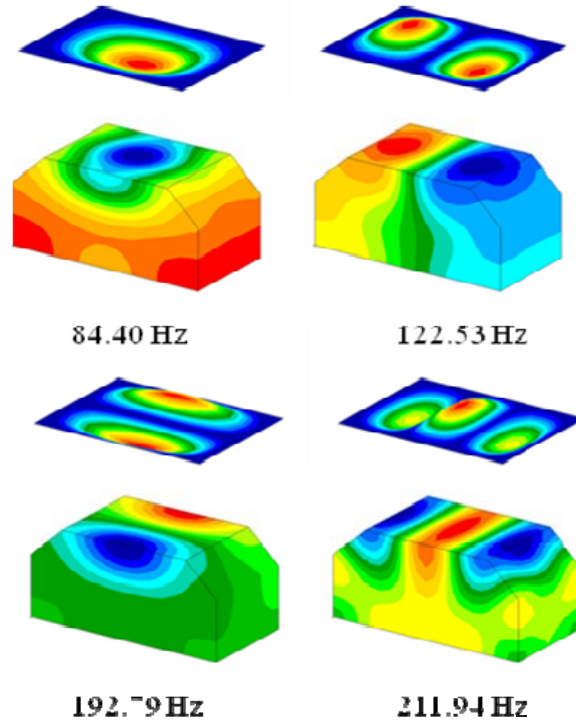


Figure 4: First 4 fluid-structure coupled modes: pressure level in the cavity and plate displacement.

4.2 Transient analysis of the acoustic/structure/piezoelectric-patch coupled problem

In this part, a multiple-mode shunt system with two piezoelectric patches as shown in Fig.5 is used in order to get an optimal multi-modal damping of the coupled system. The patches are tuned simultaneously on the second and the fourth modes of the plate. For each mode, the optimal values of the shunt electrical parameters are computed using Equations (27) and (28) (with $R_1 = 632.73 \Omega$ $L_1 = 9.47$ H for the second mode and $R_2 = 735.56 \Omega$ and $L_2 = 3.17$ H for the fourth mode). The plate is excited by a mechanical force of intensity 1 N located at $(x = 0.14$ m, $y = 0.06$ m, $z = 0.3$ m). The system vibratory response is obtained with the modal reduction approach defined by Equation (36) with a truncation on the first $M_s = 10$ structural modes and the first $M_f = 10$ acoustic modes.

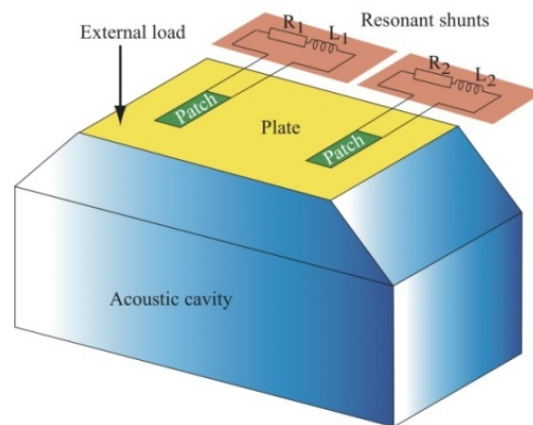


Figure 5: Vibration plate coupled with an acoustic cavity and connected to RL shunt circuit.

Figures 6 and 7 present the frequency response of the system with and without shunt. The mechanical transverse displacement in the plate at the point of coordinates $(x = 0.14 \text{ m}, y = 0.06 \text{ m}, z = 0.3 \text{ m})$ is presented in Fig. 6 and the sound pressure level in the acoustic cavity at the point of coordinates $(x = 0.15 \text{ m}, y = 0.09 \text{ m}, z = 0.1 \text{ m})$ in Fig. 7. These Figures show that the modal resonant magnitude for each considered mode have been significantly reduced simultaneously. In fact, the strain energy present in the piezoelectric material is converted into electrical energy and hence dissipated into heat using the RL shunt device.

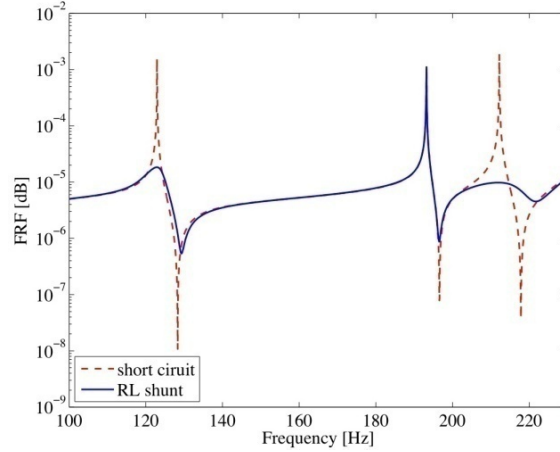


Figure 6: Frequency response function: transverse displacement amplitude in dB at the excited point.

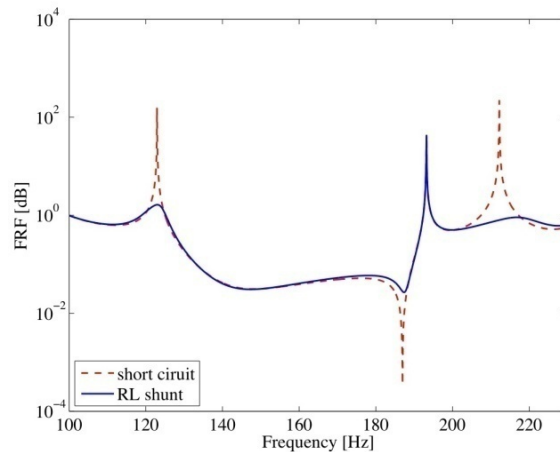


Figure 7: Frequency response function: pressure level in dB at $x = 0.15 \text{ m}, y = 0.09 \text{ m}, z = 0.1 \text{ m}$.

5 CONCLUSIONS

In this work, an original finite element formulation of structural acoustic problems with piezoelectric patches is presented. This formulation, involving only a couple of electric variables by patch, allows take into account naturally realistic electric boundary conditions. A reduced-order model, based on a normal mode expansion, is then developed. The proposed methodology requires the computation of the eigenmodes of the structure with short-circuited piezoelectric patches, and the rigid acoustic cavity. It is shown that the projection of the full-order coupled finite element model on the uncoupled bases, leads to a reduced order model in which the main parameters are the classical fluid-structure and electromechanical modal coupling factors. Despite its reduced size, this model is proved to be very efficient for simulations of steady-state analyses of structural-acoustic coupled systems with shunt damping.

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