



HAL
open science

Coupled finite Element-Boundary element formulation for noise and vibration attenuation using shunt piezoelectric materials

Walid Larbi, Jean-François Deü, Roger Ohayon

► **To cite this version:**

Walid Larbi, Jean-François Deü, Roger Ohayon. Coupled finite Element-Boundary element formulation for noise and vibration attenuation using shunt piezoelectric materials. 5th International Congress Design and Modelling of Mechanical Systems, CMSM' 2013, Mar 2013, Djerba, Tunisia. pp.127-134, 10.1007/978-3-642-37143-1_16 . hal-03179189

HAL Id: hal-03179189

<https://hal.science/hal-03179189v1>

Submitted on 8 Nov 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Coupled Finite Element-Boundary Element Formulation for Noise and Vibration Attenuation Using Shunt Piezoelectric Materials

Walid Larbi, Jean-François Deü, and Roger Ohayon

Structural Mechanics and Coupled Systems Laboratory,
Conservatoire National des Arts et Métiers,
292, rue Saint-Martin,
75141 Paris Cedex 03, France
{walid.larbi, jean-francois.deu, roger.ohayon}@cnam.fr

Abstract. In this paper, we present a coupled finite element/boundary element method (FE/BE) for control of noise radiation and sound transmission of vibrating structure by active piezoelectric techniques. The system consists of an elastic structure (with surface mounted piezoelectric patches) coupled to external/internal acoustic domains. The passive shunt damping strategy is employed for vibration attenuation in the low frequency range.

Keywords: finite element method, boundary element method, vibroacoustics, piezoelectric patches, shunt damping technique, noise and vibration attenuation.

1 Introduction

During the last two decades there has been an accelerating level of interest in the control of noise radiation and sound transmission from vibrating structures by active piezoelectric techniques in the low frequency range. In this context, resonant shunt damping techniques have been recently used for interior structural-acoustic problems [1, 2]. The present work concerns the extension of this technique to internal/external vibroacoustic problems using a finite-element/boundary-element method (FEM/BEM) for the numerical resolution of the fully coupled electro-mechanical-acoustic system.

First, a finite element formulation of an elastic structure with surface-mounted piezoelectric patches and subjected to pressure load due to the presence of an external fluid is derived from a variational principle involving structural displacement, electrical voltage of piezoelectric elements and acoustic pressure at the fluid-structure interface. This formulation, with only one couple of electric variables per patch, is well adapted to practical applications since realistic electrical boundary conditions, such that equipotentiality on the electrodes and prescribed global electric charges, naturally appear. The global charge/voltage variables are intrinsically adapted to include any external electrical circuit into the electromechanical problem and to simulate the effect of resistive or resonant shunt techniques.

In the second part of this work, the direct boundary element method is used for modeling the scattering/radiation of sound by the structure coupled to an acoustic domain. The BEM is derived from Helmholtz integral equation involving the surface pressure and normal acoustic velocity at the boundary of the acoustic domain. The coupled FE-BE model is obtained by using a compatible mesh at the fluid-structure interface. The present coupling procedure is quite general and suitable for modeling any three-dimensional geometry for bounded or unbounded structural-acoustic radiation problems.

2 Finite Element Formulation of Elastic Structure with Piezoelectric Shunt Systems

2.1 Harmonic Equations

An elastic structure occupying the domain Ω_E is equipped with P piezoelectric patches and coupled to an inviscid linear acoustic fluid occupying the domain Ω_F (figure 1). Each piezoelectric patch has the shape of a plate with its upper and lower surfaces covered with a very thin layer electrodes. The p th patch, $p \in \{1, \dots, P\}$, occupies a domain $\Omega^{(p)}$ such that $(\Omega_E, \Omega^{(1)}, \dots, \Omega^{(P)})$ is a partition of the all structure domain Ω_S . In order to reduce the vibration amplitudes of the coupled problem, a resonant shunt circuit made up of a resistance $R^{(p)}$ and an inductance $L^{(p)}$ in series is connected to each patch [3, 1, 4].

We denote by Σ the fluid-structure interface and by \mathbf{n}^S and \mathbf{n}^F the unit normal external to Ω_S and Ω_F , respectively. Moreover, the structure is clamped on a part Γ_u and subjected (i) to a given surface force density \mathbf{F}^d on the complementary part Γ_σ of its external boundary and (ii) to a pressure field p due to the presence of the fluid on its boundary Σ . The electric boundary condition for the p th patch is defined by a prescribed surface density of electric charge Q^d on $\Gamma_D^{(p)}$.

The linearized deformation tensor is $\varepsilon = \frac{1}{2}(\nabla \mathbf{u} + \nabla^T \mathbf{u})$ and the stress tensor is denoted by σ . Moreover, \mathbf{D} denotes the electric displacement and \mathbf{E} the electric

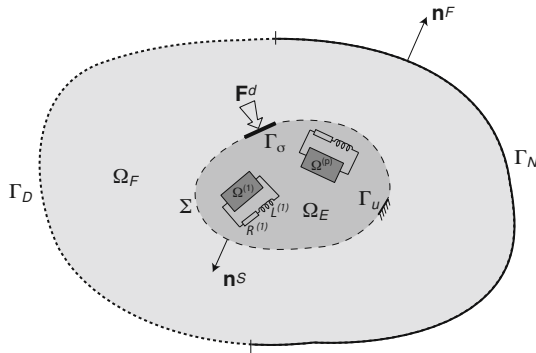


Fig. 1 Vibrating structure with piezoelectric shunt systems coupled to an acoustic domain

field such that $\mathbf{E} = -\nabla\psi$ where ψ is the electric potential. ρ_S is the mass density of the structure. The linear piezoelectric constitutive equations write:

$$\boldsymbol{\sigma} = \mathbf{c}\boldsymbol{\varepsilon} - \mathbf{e}^T \mathbf{E} \quad (1)$$

$$\mathbf{D} = \mathbf{e}\boldsymbol{\varepsilon} + \boldsymbol{\epsilon}\mathbf{E} \quad (2)$$

where \mathbf{c} denotes the elastic moduli at constant electric field, \mathbf{e} denotes the piezoelectric constants, and $\boldsymbol{\epsilon}$ denotes the dielectric permittivities at constant strain.

The local equations of elastic structure with piezoelectric patches and submitted to an acoustic pressure are [5]

$$\operatorname{div} \boldsymbol{\sigma} + \omega^2 \rho_S \mathbf{u} = \mathbf{0} \quad \text{in } \Omega_S \quad (3a)$$

$$\boldsymbol{\sigma} \mathbf{n}^S = \mathbf{F}^d \quad \text{on } \Gamma_\sigma \quad (3b)$$

$$\mathbf{u} = \mathbf{0} \quad \text{on } \Gamma_u \quad (3c)$$

$$\boldsymbol{\sigma} \mathbf{n}^S = p \mathbf{n} \quad \text{on } \Sigma \quad (3d)$$

$$\operatorname{div} \mathbf{D} = 0 \quad \text{in } \Omega^{(p)} \quad (4a)$$

$$\mathbf{D} \cdot \mathbf{n}^S = Q^d \quad \text{on } \Gamma_D^{(p)} \quad (4b)$$

where ω is the angular frequency.

For each piezoelectric patch, a set of hypotheses, which can be applied to a wide spectrum of practical applications, are formulated [1, 4].

Under those assumptions, the electric field vector $\mathbf{E}^{(p)}$ can be considered normal to the electrodes and uniform in the piezoelectric patch [6], so that for all $p \in \{1, \dots, P\}$:

$$\mathbf{E}^{(p)} = -\frac{V^{(p)}}{h^{(p)}} \mathbf{n}^{(p)} \quad \text{in } \Omega^{(p)} \quad (5)$$

where $V^{(p)}$ is the potential difference between the upper and the lower electrode surfaces of the p th patch which is constant over $\Omega^{(p)}$ and where $\mathbf{n}^{(p)}$ is the normal unit vector to the surface of the electrodes.

2.2 Finite Element Formulation

After applying variational formulation to Eqs. (3) and (4) (see [1]) and discretization by finite element method and using the following additional relation between electrical potential differences and electric charges due to the shunt circuits:

$$-\omega^2 \mathbf{LQ} - i\omega \mathbf{RQ} + \mathbf{V} = \mathbf{0} \quad (6)$$

we find the following matrix equation:

$$\begin{aligned}
& \begin{bmatrix} \mathbf{K}_u + \mathbf{C}_{uV} \mathbf{K}_V^{-1} \mathbf{C}_{uV}^T & \mathbf{C}_{uV} \mathbf{K}_V^{-1} & -\mathbf{C}_{up} \\ \mathbf{K}_V^{-1} \mathbf{C}_{uV}^T & \mathbf{K}_V^{-1} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \mathbf{Q} \\ \mathbf{P}_\Sigma \end{bmatrix} - i\omega \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \mathbf{Q} \\ \mathbf{P}_\Sigma \end{bmatrix} + \\
& - \omega^2 \begin{bmatrix} \mathbf{M}_u & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \mathbf{Q} \\ \mathbf{P}_\Sigma \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \mathbf{0} \end{bmatrix} \quad (7)
\end{aligned}$$

where $\mathbf{Q} = (Q^{(1)} Q^{(2)} \dots Q^{(P)})^T$ and $\mathbf{V} = (V^{(1)} V^{(2)} \dots V^{(P)})^T$ are the column vectors of electric charges and potential differences; $\mathbf{R} = \text{diag}(R^{(1)} R^{(2)} \dots R^{(P)})$ and $\mathbf{L} = \text{diag}(L^{(1)} L^{(2)} \dots L^{(P)})$ are the diagonal matrices of the resistances and inductances of the patches; \mathbf{U} and \mathbf{P}_Σ are the vectors of nodal values of \mathbf{u} and p ; \mathbf{M}_u and \mathbf{K}_u are the mass and stiffness matrices of the structure (elastic structure with piezoelectric patches); \mathbf{C}_{uV} is the electric mechanical coupled stiffness matrix; $\mathbf{K}_V = \text{diag}(C^{(1)} C^{(2)} \dots C^{(P)})$ is a diagonal matrix filled with the P capacitances of the piezoelectric patches; \mathbf{C}_{up} is the fluid-structure coupled matrix; \mathbf{F} is the applied mechanical force vector.

3 Boundary Element Formulation for External/Internal Acoustic Fluid

3.1 Harmonic Equations

In this section, the direct boundary element method for exterior/interior acoustic domain is presented. The governing equations of the acoustic fluid are [7, 8]

$$\Delta p + k^2 p = 0 \quad \text{in } \Omega_F \quad (8a)$$

$$\frac{\partial p}{\partial n} = 0 \quad \text{on } \Gamma_D \quad (8b)$$

$$\frac{\partial p}{\partial n} = \rho_F \omega^2 \mathbf{u} \cdot \mathbf{n} \quad \text{on } \Sigma \quad (8c)$$

$$\frac{\partial p}{\partial r} + ikp = \theta \left(\frac{1}{r} \right) \quad \text{for } r \rightarrow \infty \quad (8d)$$

Eq. (8a) represents the Helmholtz equation where $k = \omega/c$ is the wave number, i.e. the ration of the circular frequency ω and the sound velocity c ; Eq. (8b) is the rigid boundary condition on Γ_D ; Eq. (8c) is the kinematic interface fluid-structure condition on Σ ; Eq. (8d) represents the Sommerfield condition at infinity.

3.2 Boundary Element Formulation

The boundary element formulation for acoustic problems can be used for the interior and exterior problems. The Helmholtz equation is valid for the pressure p at the arbitrary collocation point \mathbf{x} within the acoustic domain Ω_F . A weak form of this equation is obtained by weighting with the fundamental solution:

$$G(\mathbf{x}, \mathbf{y}) = \frac{e^{ik|\mathbf{x}-\mathbf{y}|}}{4\pi|\mathbf{x}-\mathbf{y}|} \quad (9)$$

where $|\mathbf{x}-\mathbf{y}|$ denotes the distance between the arbitrary point \mathbf{x} and the load source point \mathbf{y} .

Applying Green's second theorem, the Helmholtz equation can be transformed into a boundary integral equation, which can be expressed as follows

$$c(\mathbf{x})p(\mathbf{x}) = \int_{\partial\Omega_F} p(\mathbf{y}) \frac{\partial G(\mathbf{x}, \mathbf{y})}{\partial n_y} dS - \int_{\partial\Omega_F} \frac{\partial p(\mathbf{y})}{\partial n_y} G(\mathbf{x}, \mathbf{y}) dS \quad (10)$$

where $c(\mathbf{x})$ is the corner coefficient.

The fluid boundary is divided into N quadrilateral elements ($\partial\Omega_F = \sum_{j=1}^N S_j$) and Eq. (10) is discretized. After using the relation between the acoustic pressure and the fluid normal velocity $\frac{\partial p}{\partial n} = -i\rho_F\omega v$ (where $v = \mathbf{v}^F \cdot \mathbf{n}$) and numerical evaluation, the discrete Helmholtz equation can be written in the following matrix form:

$$\mathbf{HP} = i\rho_F\omega\mathbf{Gv} \quad (11)$$

where \mathbf{P} and \mathbf{v} are the vectors with sound pressure and velocity in the normal direction to the boundary surface at the nodal position of the boundary element mesh.

4 FE/BE Formulation for the Fluid-Structure with Shunt Systems Coupled Problem

The fluid boundary domain $\partial\Omega_F$ is divided into two parts including Γ_D (where the rigid boundary condition is applied) and the interface Σ (for the fluid-structure interface) such as $\partial\Omega_F = \Gamma_D \cup \Sigma$ and $\Gamma_D \cap \Sigma = \emptyset$. The boundary conditions given in Eqs. (8b) and (8c) can be expressed in discretized form

$$\mathbf{v}_D = \mathbf{0} \quad \text{on } \Gamma_D \quad (12a)$$

$$\mathbf{v}_\Sigma = i\omega\mathbf{TU} \quad \text{on } \Sigma \quad (12b)$$

where \mathbf{T} is the global coupling matrix that transforms the nodal normal displacement of the structure to the normal velocity of the acoustic fluid at the interface. Substituting Eqs. (12) into the BE matrix expression (Eq. (11)) yields

$$\begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{P}_\Sigma \\ \mathbf{P}_D \end{bmatrix} = \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & \mathbf{G}_{22} \end{bmatrix} \begin{bmatrix} i\rho_F\omega^2\mathbf{TU} \\ \mathbf{0} \end{bmatrix} \quad (13)$$

By combining Eq. (7) with Eq. (13), we find the following coupled FE/BE matrix equation

$$\begin{bmatrix}
\mathbf{K}_u + \mathbf{C}_{uV} \mathbf{K}_V^{-1} \mathbf{C}_{uV}^T - \omega^2 \mathbf{M}_u & \mathbf{C}_{uV} \mathbf{K}_V^{-1} & -\mathbf{C}_{up} & \mathbf{0} \\
\mathbf{K}_V^{-1} \mathbf{C}_{uV}^T & \mathbf{K}_V^{-1} - i\omega \mathbf{R} - \omega^2 \mathbf{L} & \mathbf{0} & \mathbf{0} \\
i\rho_F \omega^2 \mathbf{G}_{11}^T \mathbf{T} & \mathbf{0} & \mathbf{H}_{11} & \mathbf{H}_{12} \\
i\rho_F \omega^2 \mathbf{G}_{21}^T \mathbf{T} & \mathbf{0} & \mathbf{H}_{21} & \mathbf{H}_{22}
\end{bmatrix}
\begin{bmatrix}
\mathbf{U} \\
\mathbf{Q} \\
\mathbf{P}_\Sigma \\
\mathbf{P}_D
\end{bmatrix}
=
\begin{bmatrix}
\mathbf{F} \\
\mathbf{0} \\
\mathbf{0} \\
\mathbf{0}
\end{bmatrix} \quad (14)$$

5 Numerical Example

We consider a 3D hexaedric acoustic cavity of size $A=0.3048$ m, $B=0.1524$ m and $C=0.1524$ m along the directions x , y , and z , respectively. The cavity is completely filled with air (density $\rho=1.2$ kg/m³ and speed of sound $c=340$ m/s). The cavity walls are rigid except the top one, which is a flexible aluminum plate of thickness 1 mm. The density of the plate is 2690 kg/m³, the Youngs modulus is 70 GPa and Poisson ratio 0.3. On the top surface of the plate, a PIC 151 patch is bonded, whose in plane dimensions are 0.0762×0.0508 m² along x and y and 0.5 mm thick (see figure 2). The mechanical characteristics of the piezoelectric material PIC 151 are given in [2].

The plate is excited by an unit distributed time harmonic pressure load. The coupled FEM/BEM results are compared with those obtained from a FEM/FEM analysis. As can be seen in figure 3, the sound pressure level is calculated on the plate center. The results for the two methods are very similar at sound level peaks

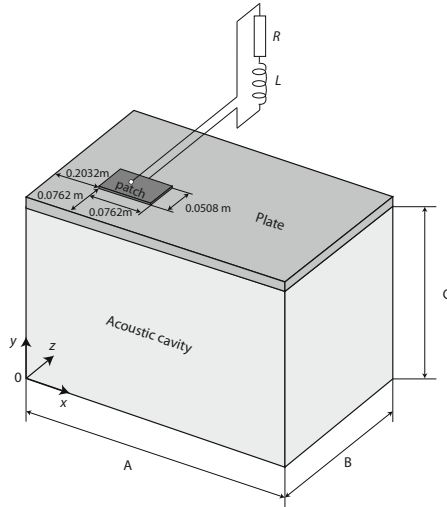


Fig. 2 Electromechanical-acoustic coupled system: geometrical data

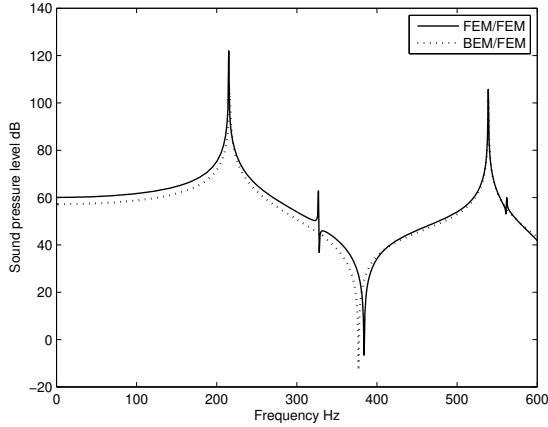


Fig. 3 Sound pressure level on plate center: comparison between FEM/FEM and FEM/BEM approaches

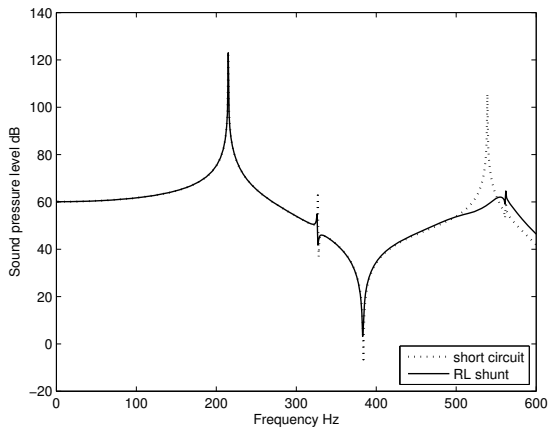


Fig. 4 Sound pressure level on plate center with and without shunt system

(resonance frequencies) which enable us to check the validity of the proposed FEM/BEM coupled formulation.

In order to achieve maximum vibration dissipation of the third coupled mode, the patch is tuned now to an RL shunt circuit. The optimal values of the shunt electrical circuit are taken $R=348 \Omega$ and $L=0.61$ H. The system vibratory response is obtained with the proposed BEM/FEM approach. Figure 4 presents the sound pressure level on plate center with and without shunt system. This figure shows that the resonant magnitude for the third mode has been significantly reduced due to the shunt effect. In fact, the strain energy present in the piezoelectric material is

converted into electrical energy and hence dissipated into heat using the RL shunt device.

6 Conclusions

In this work, a coupled finite element/boundary element method (FEM/BEM) for control of noise radiation and sound transmission of vibrating structure by active piezoelectric techniques is presented. The passive shunt damping strategy is employed for vibration attenuation in the low frequency range. Work in progress concerns the optimization of piezoelectric patches positioning and form.

References

1. Larbi, W., Deü, J.-F., Ciminello, M., Ohayon, R.: Structural-acoustic vibration reduction using switched shunt piezoelectric patches: A finite element analysis. *Journal of Vibration and Acoustics* 132(5), 051006 (9 pages) (2010)
2. Larbi, W., Deü, J.-F., Ohayon, R.: Finite element formulation of smart piezoelectric composite plates coupled with acoustic fluid. *Composite Structures* 94(2), 501–509 (2012)
3. Hagood, N., Von Flotow, A.: Damping of structural vibrations with piezoelectric materials and passive electrical network. *Journal of Sound and Vibration* 146(2), 243–268 (1991)
4. Deü, J.-F., Larbi, W., Ohayon, R.: Structural-acoustic vibration reduction using piezoelectric shunt techniques: FE formulation and reduced order model. In: *Proc. of the 3rd International Conference on Computational Methods in Structural Dynamics and Earthquake Engineerin, COMPDYN 2011, Corfu, Greece, May 25-28 (2011)*
5. Deü, J.-F., Larbi, W., Ohayon, R.: Piezoelectric structural acoustic problems: Symmetric variational formulations and finite element results. *Computer Methods in Applied Mechanics and Engineering* 197(19-20), 1715–1724 (2008)
6. Thomas, O., Deü, J.-F., Ducarne, J.: Vibrations of an elastic structure with shunted piezoelectric patches: Efficient finite element formulation and electromechanical coupling coefficients. *International Journal for Numerical Methods in Engineering* 80(2), 235–268 (2009)
7. Walker, S.: Boundary elements in fluid-structure interaction problems rotational shells. *Applied Mathematical Modelling* 4, 345–350 (1980)
8. Rajakumar, C., Ali, A.: Boundary Element-Finite Element Coupled Eigenanalysis of Fluid-Structure Systems. *International Journal for Numerical Methods in Engineering* 39(10), 1625–1634 (1996)