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# FINITE ELEMENT REDUCED ORDER MODELING OF PIEZOELECTRIC ENERGY HARVESTERS

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**Abstract.** In the last few years, piezoelectric materials have been widely used for vibration energy harvesting due to its efficiency for converting mechanical energy into electrical one. In the literature, various approaches have been devoted for modeling the piezoelectric harvesters in order to predict its electromechanical coupling responses. In this paper, we present a finite element modeling of a bimorph piezoelectric harvester as typical cantilever beam which is composed of an elastic substrate covered by two piezoceramic layers. The generated electricity is due to the vibration of the host structure. The main purpose of this work is to develop a reduced order model able to predict the responses of the harvester and to improve simulation efficiency with a low computation cost. In order to reach this goal, the electromechanical problem is projected on a truncated eigenvectors basis with short-circuited boundary conditions. Furthermore, a numerical example of a bimorph harvester as case study is presented and analyzed. Through this example, the effect of the basis truncation on the electromechanical outputs is discussed in term of computation error and time. Results show that for an optimal truncated basis, the reduced order model gives good results in comparison to the direct methods with lower cost of calculation.

## 1 INTRODUCTION

Piezoelectric has received a considerable attention for vibration energy scavenging over the last years. This material has been extensively used in vast engineering areas due to its specific characteristics. In fact, piezoelectric material enables the transformation of mechanical strain into electricity and vice-versa. These two conversion ways allow direct connection to electronic components and yield well to two applications: (1) the sensing using the direct effect piezoelectric (e.g. vibration energy harvesting) and (2) the actuating using the inverse effect piezoelectric (e.g. structural vibration reduction). This work focus on the energy harvesting using piezoelectric elements attached to an elastic structure and excited by the base motion.

Recently, various techniques have been appeared in the literature to investigate the electromechanical coupling of a composite cantilevered beam with piezoelectric elements, which is used often as a vibration energy harvester. An analytical solution based on a single degree of freedom model, considers that the composite cantilever beam is similar to a mass-spring-damper system. In this approach, the piezoelectric composite beam is modeled by a second order differential equation of the tip displacement of the harvester as variable [1]. Lately, Erturk and Inman [1, 2, 3] introduced an analytical distributed parameter solutions for bimorph and unimorph piezoelectric energy harvesters with closed-form expressions and showed that for sinusoidal base excitation, the model might yield highly agreement with physical experiments. Whereas, several studies used the finite element method as an alternative sophisticated technique to solve approximately the most complicated electromechanical problem. Bendary [4] developed a finite element model for sensors and actuators based on the classical laminate theory. Results show that as expected for thin plates, the classical theory gives acceptable accuracy with minimal computational effort. De Marqui [5] investigated an electromechanical coupled finite element plate to estimate the electrical output responses of unimorph and bimorph piezoelectric plate harvester devices. This model was derived based on the classic Kirchhoff plate theory, which neglects shear deformation. Amini [6] presented a finite element modeling for functionally graded piezoelectric harvesters based-on the Euler Bernoulli beam theory. The model is validated by comparison to the analytical solution of a similar bimorph piezoelectric energy harvester.

Reduced order modeling of mechatronic systems has become an interesting issue for improving simulation efficiency. For instance, many optimization algorithms require a high number of model evaluations which cause a high computational cost, hence the necessity for such reduced order model. In recent published research, H.-J. Xiang [7] introduced a reduced order model of piezoelectric energy harvesters using a Krylov subspace-based scheme. An implementation of this model with nonlinear energy harvesting circuits is made in a system level. This approach is validated with various harmonic response and transient response analysis. M. Kudryavtsev et al. [8] introduced a novel reduced order model techniques for a large-scale multiport model of a micro electromechanical system based piezoelectric energy harvester. Results show an excellent concordance between the full-scale and the reduced order models of the harvester for harmonic simulation.

This paper proposed an efficient finite element reduced order model of piezoelectric energy harvester able to predict its electrical outputs recovered in an efficient way. A bimorph cantilever beam is chosen as an energy harvester case study due to its widespread usage. A gait is proposed to determinate the reduced order model: (1) an electromechanical finite element formulation for the dynamic analysis of a general piezoelectric structure is first proposed, (2) the development of an appropriate reduced order model of the coupled electromechanical problem is then presented. The proposed methodology based on a normal mode expansion which requires the computation of the eigenmodes of the structure in short circuit condition. Despite its reduced size, this model is proven to be very efficient for simulations of harmonic vibration analyses of the harvester. In the last part of this research, a numerical example of a piezoelectric bimorph cantilever beam, is presented and analyzed.

## 2 FINITE ELEMENT REDUCED ORDER MODEL OF ELECTROMECHANICAL PROBLEM

The finite element formulation of elastic structure with piezoelectric elements proposed in [9, 10] is used. This formulation uses a standard discretization of the mechanical degrees of freedom and provides less restrictive assumptions for the electrical state which is fully described by very few global discrete unknowns: (i) the electric charge contained in the electrodes and (ii) the voltage between the electrodes. It is well adapted to practical applications since realistic electrical boundary conditions, such that equipotentiality on the electrodes and prescribed global electric charges, naturally appear [12]. Therefore, the governing undamped electromechanical problem of piezoelectric structure is thus obtained:

$$\mathbf{M}_m \ddot{\mathbf{U}}(t) + \mathbf{K}_m \mathbf{U}(t) + \mathbf{K}_c \mathbf{V}(t) = \mathbf{F}(t) \quad (1)$$

$$\mathbf{K}_e \mathbf{V}(t) + \mathbf{Q}(t) - \mathbf{K}_c^T \mathbf{U}(t) = \mathbf{0} \quad (2)$$

where  $\mathbf{M}_m$  is the global ( $N \times N$ ) mass matrix,  $\mathbf{K}_m$  is the global ( $N \times N$ ) stiffness matrix and  $\mathbf{K}_c$  is the global ( $N \times P$ ) electromechanical coupling matrix,  $\mathbf{K}_e$  is the diagonal global ( $P \times P$ ) capacitance matrix,  $\mathbf{F}(t)$  is the global ( $N \times 1$ ) vector of mechanical forces,  $\mathbf{Q}(t)$  is the global ( $P \times 1$ ) vector of electric charge outputs,  $\mathbf{U}(t)$  is the global ( $N \times 1$ ) vector of mechanical coordinates and  $\mathbf{V}(t)$  is the global ( $P \times 1$ ) vector of output voltages across the piezoelectric elements. Here,  $N$  and  $P$  respectively, are the number of mechanical degrees of freedom and the number of piezoelectric elements.

The Equation (1) corresponds to the mechanical equation of motion including the force induced by electromechanical coupling. Whereas, the Equation (2) corresponds to the electrical circuit equation with mechanical coupling.

The global mechanical damping matrix  $\mathbf{C}_m$  is often taken to be a linear combination of the mass and stiffness matrices as follow (Rayleigh damping):

$$\mathbf{C}_m = \alpha \mathbf{M}_m + \beta \mathbf{K}_m \quad (3)$$

where  $\alpha$  and  $\beta$  are the constant of proportionality which are determined in physical experiments.

The charge-voltage global variables are also intrinsically adapted to include any external electrical circuit into the electromechanical problem. For the purpose to simulate the piezoelectric energy harvesting, the structure is dissipated through a resistive load. Using Ohm's law, we obtains the following additional equations:

$$\mathbf{V}(t) = R \dot{\mathbf{Q}}(t) \quad (4)$$

### 2.1 Normal modes of short-circuit condition

The system short-circuit normal modes present the solutions of Equation (1) with  $\mathbf{V} = \mathbf{0}$  and  $\mathbf{F} = \mathbf{0}$ . In addition, these modes depend only on the mechanical properties of the system, thus:

$$\left[ \mathbf{M}_m - \omega_i^2 \mathbf{K}_m \right] \Phi_i = \mathbf{0} \quad (5)$$

where  $\omega_i$  is the natural frequency and  $\Phi_i$  is the ( $N \times 1$ ) mode shape vector.

For orthogonality properties, these modes verify the following conditions:

$$\Phi_i^T \mathbf{M}_m \Phi_j = \delta_{ij} \quad \text{and} \quad \Phi_i^T \mathbf{K}_m \Phi_j = \omega_i^2 \delta_{ij} \quad (6)$$

where  $\delta_{ij}$  is the Kronecker symbol.

## 2.2 Projection on the short-circuited basis

Equations (1) and (2) describe the electromechanical behavior of the piezoelectric structure with the dimension of matrices are usually very large, which requires a considerable computational effort. The aim of this section is to construct a reduced system with much smaller dimension [7] by introducing the modal matrix  $\Phi = [\Phi_1, \Phi_2, \dots, \Phi_N]$  of size  $(N \times N)$ ,  $N$  is the total number of degrees of freedom in the finite elements model associated to the structure. In fact, the displacement vector is sought as [11]:

$$\mathbf{U}(t) = \Phi \mathbf{q}(t) \quad (7)$$

where the vector  $\mathbf{q} = [q_1 \ q_2 \ \dots \ q_N]^T$  is the unknown modal amplitudes.

By applying the Ritz-Galerkin projection method, which consists in inserting the above equation in Equations (1) and (2) including the damping term, and multiplying the first obtained equation by  $\Phi^T$  and using the orthogonality properties of Equations (6), the problem writes including the damping term, for all  $i \in \{1, \dots, N\}$  and  $p \in \{1, \dots, P\}$  is:

$$\ddot{q}_i(t) + 2\zeta_i \omega_i \dot{q}_i(t) + \omega_i^2 q_i(t) + \sum_{p=1}^P \chi_i^{(p)} V^{(p)}(t) = F_i(t) \quad (8)$$

$$C^{(p)} V^{(p)}(t) + Q^{(p)}(t) - \sum_{k=1}^N \chi_k^{(p)} q_k(t) = 0 \quad (9)$$

where  $\zeta_i$  is the modal damping coefficient defined as follows:

$$\zeta_i = \frac{\alpha}{2\omega_i} + \frac{\beta\omega_i}{2} \quad (10)$$

$\chi_i^{(p)}$  is the modal coupling coefficient of the  $p^{th}$  piezoelectric element, which is defined for all  $i \in \{1, \dots, N\}$  by:

$$\chi_i = \Phi_i^T \mathbf{K}_c = (\chi_i^{(1)} \chi_i^{(2)} \dots \chi_i^{(p)}) \quad (11)$$

and  $F_i(t) = \Phi_i^T \mathbf{F}(t)$  is the forcing term of the  $i^{th}$  mode.

For harmonic responses in the frequency domain and taking into account the connected resistive load introduced in Equation (4), the voltage of the  $p^{th}$  piezoelectric element is written as follows:

$$V^{(p)} = \frac{j\omega \sum_{i=1}^N \chi_i^{(p)} q_i}{\left(\frac{1}{R} + j\omega C^{(p)}\right)} \quad (12)$$

Substituting the expression of the voltage in Equation (12),  $N$  mechanical oscillators are given as follows:

$$-\omega^2 q_i + 2j\omega \zeta_i \omega_i q_i + \omega_i^2 q_i + j\omega \sum_{p=1}^P \frac{\left(\chi_i^{(p)}\right)^2}{\left(\frac{1}{R} + j\omega C^{(p)}\right)} q_i + j\omega \sum_{p=1}^P \sum_{\substack{k=1 \\ k \neq i}}^N \frac{\chi_i^{(p)} \chi_k^{(p)} q_k}{\left(\frac{1}{R} + j\omega C^{(p)}\right)} = F_i \quad (13)$$

The idea behind the reduced order modeling is the projection of the general electromechanical problem on a truncated short-circuited basis  $\Phi^{trun} = [\Phi_1, \Phi_2, \dots, \Phi_{N_s}]$ ; where  $N_s \in \{1, \dots, N\}$  is

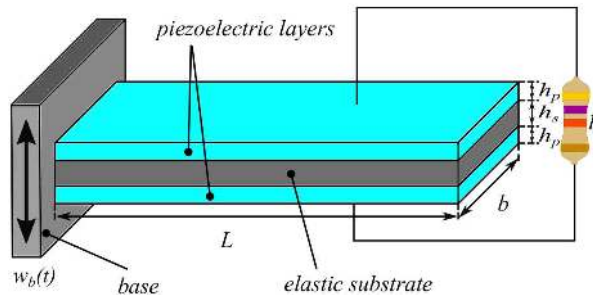
the number of the projection modes and  $N_s \ll N$ . Consequently, the matrix system represents a reduced-order model using the short-circuited basis. If only few modes are kept for the projection, the size of this reduced-order model is  $(N_s + P)$  which is much smaller than the full system  $(N + P)$ .

The major interest of choosing the short-circuit eigenmodes as the expansion basis is that its can be computed with a classical elastic mechanical problem. This operation can thus be done by any standard finite elements code.

### 3 APPLICATION TO A BIMORPH PIEZOELECTRIC ENERGY HARVESTER

In this section, the previous formulation is used for a bimorph piezoelectric vibration energy harvester excited by a harmonic base motion. The harvester consists in an Euler Bernoulli beam composed of two layers of piezoceramic and an elastic substrate which are assumed to be perfectly bonded to each other as shown in Figure 1. Thus, the total number of layers is equal to 3 and the number of piezoelectric elements is equal to 2 ( $P = 2$ ). The piezoceramic layers (which are poled in the thickness direction) are covered by continuous electrodes (which are assumed to be perfectly conductive) of negligible thickness. A resistive electrical load is mounted in series with the two piezoceramic layers. The electrical voltage and the mechanical vibration of the harvester are presented by its Frequency Response Functions (FRFs) which are defined here as the response outputs of the harvester per base acceleration  $a_b(t)$  (in terms of the gravitational acceleration  $g = 9.81m.s^{-2}$ ) given in [5, 13].

The geometrical and physical parameters (adapted by Erturk and Inman [2]) used in this study are given in Table 1. The ratio of the overhang length to the total thickness is about 85.7, which makes it reasonable to neglect the shear deformation and the rotary inertia effects of the harvester for the first few vibration modes. For the purpose of simulation, one takes  $\xi_1 = 0.010$  and  $\xi_2 = 0.012$  as the mechanical damping ratios of the first two modes. The discretization provides  $N = 90$  mechanical degrees of freedom using linear elements.



**Figure 1:** A Clamped free bimorph piezoelectric energy harvester under base excitation with a load resistance mounted in series

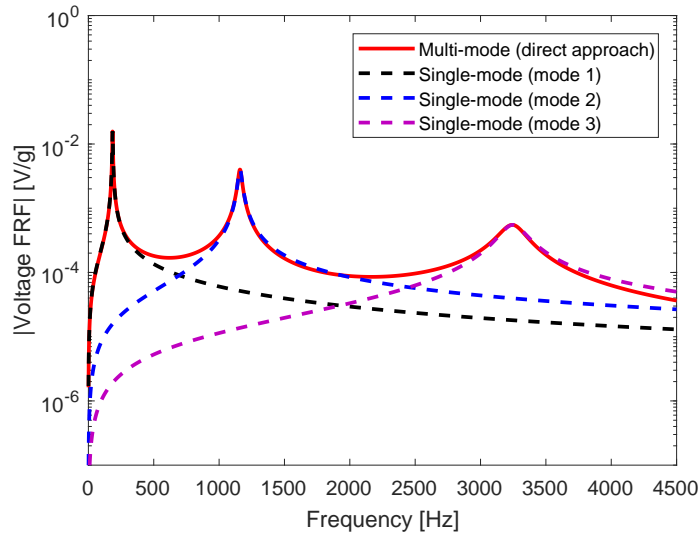
The analysis given here considers the frequency range from  $f_s = 1 Hz$  to  $f_f = 4500 Hz$ . In this range three natural frequencies are appeared. The multi-mode and the single-mode of tip displacement and voltage FRFs are normalized with respect to the base acceleration  $g$ , and are computed from Equations (12) and (13). They are plotted in modulus (appointed  $H_D$  and  $H_V$  respectively for the modulus of the tip displacement FRF and the modulus of the voltage FRF)

**Table 1:** Material and electromechanical properties of the harvester shown in Fig.2

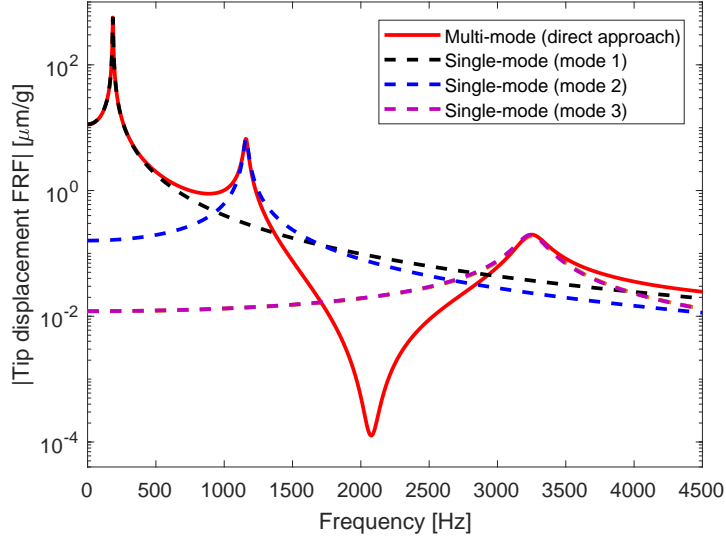
Parameters	Descriptions	Units	PZT-5A	Aluminum
$L$	Beam length	$mm$	30	30
$b$	Beam width	$mm$	5	5
$h_p, h_s$	Layers thickness	$mm$	0.15	0.05
$E_p, E_s$	Young's modulus	$GPa$	61	70
$\rho_p, \rho_s$	Mass density	$kg/m^3$	7750	2700
$e_{31}$	Piezoelectric constant	$C/m^2$	-10.4	-
$\epsilon_{33}$	Permittivity constant	$nF/m$	13.3	-

for a set of resistance load.

In the following, the simulations of these single-mode expressions are compared to those of the multi-mode (obtained by direct method) for the first three vibration modes with a load resistance mounted in series with piezoelectric layers. Figure 2 shows the single-mode of voltage FRFs for  $i = 1$ ,  $i = 2$ , and  $i = 3$  along with the multi-mode for a load resistance equal to  $R = 100 \Omega$ . As can be seen in this figure, the single-mode solutions agree with the multi-mode solution only at the vicinity of the resonance frequency of the respective mode of interest.


**Figure 2:** Comparison of the multi-mode (direct approach) and the single-mode (modal projection) of voltage FRFs with  $R = 100 \Omega$ 

The frequency response predictions of the single-mode tip displacement FRFs for  $i = 1$ ,  $i = 2$ , and  $i = 3$  are shown in Figure 3 along with the multi-mode tip displacement FRFs. Again, the single-mode FRFs exhibit agreement with the multi-mode FRFs around the modes of interest. The slight overestimation of the resonance frequencies due to ignoring the neighboring vibration modes is the case here too, and the error in the single-mode resonance frequencies is less than 0.1% for these vibration modes (compared to the resonance frequencies of the multi-mode) for excitations at the fundamental short- and open-circuit resonance frequencies. Note that the



**Figure 3:** Comparison of the multi-mode (direct approach) and the single-mode (modal projection) of Tip displacement FRF  $R = 100 \Omega$

slightly overestimated open-circuit resonance frequency is used in the single-mode simulations. The predictions of the single-mode FRFs for this most important vibration mode are very accurate. Therefore, it can comfortably be used as a first approximation in the modeling of a piezoelectric energy harvester beam for modal excitations.

To study the reduced order harvester model efficiency, we choose truncated basis contain a number of eignmodes which is a multiple of the number of the natural frequencies in the studied frequency range ( $N_s = \{3, 6, 9, 69\}$ ) comparing with the full system projection ( $N_s = N$ ).

Figures 4 and 5 show that the FRFs of the harvester are agree at the vicinity of the resonance frequencies for all basis of projection with the finite element direct method resolution but is not the case for the other excitation frequencies.

Two criteria are opted in this study: (1) the sum of the least square between the projected responses on the truncated basis and the direct method given in the following Equations (14) and (15) respectively for the tip displacement FRF and voltage FRF, which can give information about the accuracy of the reduced model simulation.

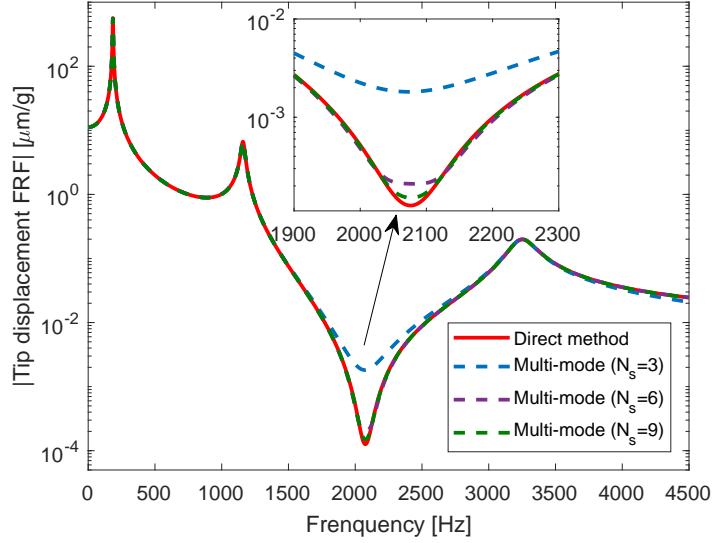
$$Obj^D = \sum_{f=f_s}^{f_f} \left( H_D^D(f) - H_D^{N_s}(f) \right)^2 \quad (14)$$

$$Obj^V = \sum_{f=f_s}^{f_f} \left( H_V^D(f) - H_V^{N_s}(f) \right)^2 \quad (15)$$

where  $H_D^D$  and  $H_V^D$  are respectively the modulus of tip displacement and voltage FRFs computed by the direct method and  $H_D^{N_s}$  and  $H_V^{N_s}$  are respectively the modulus of the tip displacement and voltage FRFs calculated by modal projection on a short-circuit truncated basis. (2) the second criteria is the time required for only one simulation.

Table 2 lists the two criteria for the harvester responses. Figure 6 shows the two criteria of





**Figure 4:** Comparison of the multi-mode Tip displacement FRF for different basis size with direct method,  $R = 10^2\Omega$

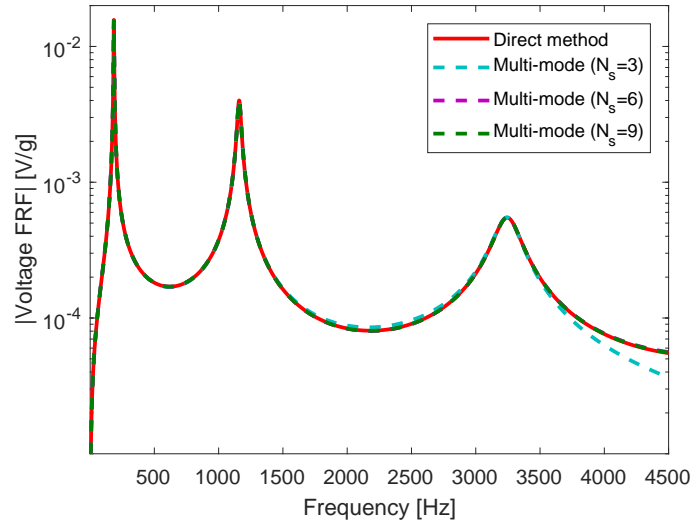
**Table 2:** Criteria of comparison of tip displacement and voltage FRFs for the short-circuit basis size

$N_s$	Tip displacement		Voltage	
	$Obj^D (\mu m/g)^2$	Time (s)	$Obj^V (\mu m/g)^2$	Time (s)
3	2.00 E-02	38.248	2.51 E-07	38.949
6	2.77 E-05	39.638	1.20 E-09	39.944
9	2.48 E-06	41.031	2.35 E-10	41.199
69	9.10 E-11	81.409	7.73 E-13	81.685
N	5.32 E-11	94.852	5.15 E-13	95.302

comparison for the harvester responses ((a) tip displacement FRF and (b) voltage FRF) versus the short-circuit basis size. Results show that for a small basis size, reduced order model simulation give responses with low precision (high  $Obj$  value) in a short computation time. Furthermore, the accuracy of the responses increase monotonically ( $Obj$  decreases) with the increase of the projection basis size, whereas the time of computation increase also. Indeed, high accuracy of simulation causes high numerical computation cost. For our case study,  $N_s = 6$  is recommended for this frequency range.

## 4 CONCLUSIONS

In this research, the general finite element governing equations of the general electromechanical problem is firstly presented. Model reduction technique has been developed which consists in a modal projection of the system equations on a truncated basis of eigenvectors in order to reduce its matrices size. This basis is obtained with short circuit boundary conditions. The main goal is to predict the output responses of the harvester with high accuracy and low computational cost. Although an example of a bimorph piezoelectric energy harvesting, results show that the computational cost and the responses accuracy of the full-scale model have been reduced

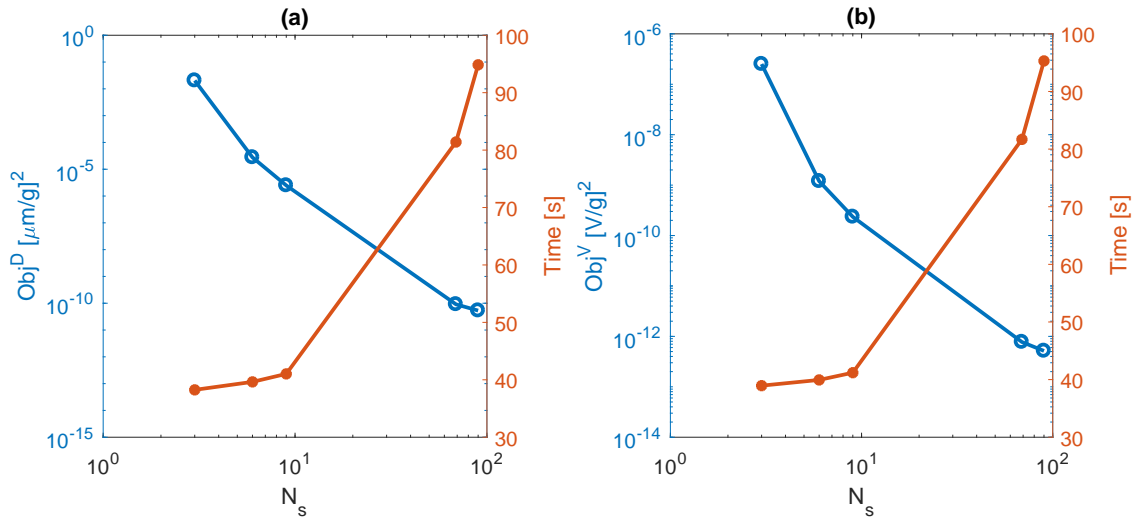


**Figure 5:** Comparison of the multi-mode voltage FRF for different basis size with direct method,  $R = 10^2\Omega$

monotonically with the reduction of the model scale. Moreover, the responses accuracy of the reduced order model is affected by errors introduced by the process of mode truncation that can be controlled or minimized by a modal truncation augmentation method [14]. In this method, the effects of the truncated modes are considered by their static effect only.

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**Figure 6:** (a) Co-plot of criteria versus the basis size for the tip displacement FRF (b) Co-plot of criteria versus the basis size for the voltage FRF

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