



**HAL**  
open science

## Optimal actuator torques distribution of articulated rovers on rough terrain

Faïz Ben Amar, Pierre Jarrault, Philippe Bidaud, Christophe Grand

► **To cite this version:**

Faïz Ben Amar, Pierre Jarrault, Philippe Bidaud, Christophe Grand. Optimal actuator torques distribution of articulated rovers on rough terrain. Twelfth International Conference on Climbing and Walking Robots and the Support Technologies for Mobile Machines „, 2009, Istanbul, Turkey. pp.791-799, 10.1142/9789814291279\_0097 . hal-03177938

**HAL Id: hal-03177938**

**<https://hal.science/hal-03177938>**

Submitted on 24 Mar 2021

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

## Optimal actuator torques distribution of articulated rovers on rough terrain

F. Ben Amar, P. Jarrault, Ph. Bidaud, Ch. Grand

*ISIR - Institut des Systèmes Intelligents et Robotiques  
Université Pierre et Marie Curie Paris 6, CNRS UMR 7220  
4, place Jussieu 75252 Paris Cedex 05 France  
{amar}@isir.fr*

The paper develops a method for analyzing and improving by control obstacle clearance capacities of articulated multi-wheeled rovers. On uneven ground surface, load and traction force distributions through the wheel/ground contact system are highly coupled. They are both conditioned by the global equilibrium of the mechanical system and the contact stability constraints. The optimal traction force distribution problem is formulated here as a convex optimization problem using Linear Matrix Inequalities (LMIs). Velocity and force transmissions in articulated multi-wheeled mobile robots are introduced under a generic form decomposed in task, joint and contact levels. Simulation results show that the traction distribution forces which is so determined lead to a significant increase in obstacle clearance capacities compared to a simple velocity control technique.

*Keywords:* Rovers, obstacle clearance, mobility, kinematics.

### 1. Introduction

Locomotion systems can be seen as multi-body articulated systems interacting with the environment by a set of unilateral contacts with adhesion or slippage. From a topological point of view, locomotion systems can be compared to articulated hands and the analysis of the mechanical properties of locomotion systems can be inspired by grasp analysis. Force and velocity transmissions in these systems can be analyzed by the use of similar mathematical tools, as for the optimization of contact force distribution<sup>1</sup> or for evaluating quantitatively the obstacle clearance capabilities.<sup>2</sup>

The work developed in this paper tries to bring an answer to the evaluation of traction capabilities and the optimal traction distribution for obstacle clearance of wheeled-based mobile robots evolving on uneven surfaces.

Off-road mobile robots have generally complex structure (several joints for suspension or for auxiliary locomotion modes). Most of articulated wheeled robots have 6 wheels, which are either multi-platforms or mono-platform. The formers are generally composed by 3 articulated axles called modules,<sup>3</sup> and the seconds one have a main body and more complex mechanisms such as rocker-bogie structure<sup>4,5</sup> or with multi-parallelogram systems.<sup>6</sup> The problem of obstacle clearance of off-road robots has been addressed slightly, in particular either from experimental point of view or by using dynamic simulation<sup>7,5</sup>.

The paper proposes a general framework for analysis and optimization of the obstacle clearance process. The framework can be applied to any articulated wheeled system with active or passive mobilities. The method is based on a kineto-static model which takes into account the slippage and friction condition in wheel-ground contacts. We start with the description of the robot kinematic structure. Section (3) draws up the problem of contact force distribution in multi-wheeled articulated robots and proposes a formulation based on a convex optimization that involves linear matrix inequalities LMIs. The method defines the stability contact by using the maximal friction condition. Simulation results developed in the last section show the efficiency of the method and its robustness in relation to a realistic tyre model that considers wheel slippage. Results are also compared to a simple control method based on an equi-distribution of wheel's rate and demonstrate the relevance of the optimization of force distribution.

## 2. Robot kinematics



Fig. 1. RobuRoc6 negotiating an obstacle.

The vehicle RobuROC6 considered in this paper can be considered as a series of 3 monocycles modules linked together by two orthogonal revolute joints allowing roll and pitch motions of each module (figure 1). Each monocycle module is steered and driven by two actuated conventional wheels on which a lateral slippage may occur. The rear and the front modules are symmetrically arranged about the central one. The two revolute joints along the pitch axis are coupled by means of 4 hydraulic actuators with interconnected cylinders. This interconnection ensures that the front and rear pitch joints rotate symmetrically with respect to the middle axle. This kinematics permits to transform RobuRoc6 into a 4-wheel configuration as shown in figure (2) mainly to increase its manoeuvrability when needed. However the robot could operate without actuating the hydraulic pump and then the pitch suspension works as a differential mechanical system.

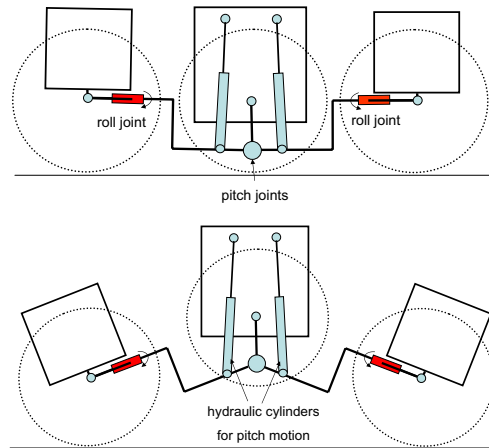


Fig. 2. Kinematics scheme of RobuROC6 : 2 configurations illustrating the central module manipulation.

We restrain the analysis to the sagittal plane and we will not consider the parallel mechanisms composed by the hydraulic cylinders controlling the pitch rotoide joints. We consider that the pitch suspension is not actuated, implying that the robot has all its wheels in contact with the ground.

### 3. Optimal traction distribution

Traction and load distributions are of great importance when contact geometrical characteristics are uneven i.e. contact normals are not parallel

and contact points are not coplanar. In this case, traction and load distribution problems can not be decoupled. We can then use the well known frameworks developed for the analysis of grasping systems. Wheeled mobile systems can be considered as a system where multiple interconnected wheels “grasp” the ground.

If we consider quasi-static equations associated to the generalized parameters  $\mathbf{x} = (x, z, o)^t$  defining the platform and  $\theta_s = (\psi)$  defining the joint suspension parameter, we can express equilibrium between contact forces  $\mathbf{f}$  and external gravitational forces  $\mathbf{g}$  by the following matrix form :

$$\begin{bmatrix} \mathbf{G}(\mathbf{x}, \theta, \alpha) \\ -\mathbf{J}_\psi^t(\mathbf{x}, \theta, \alpha) \end{bmatrix} \mathbf{f} = \begin{bmatrix} \mathbf{g}_x \\ \mathbf{g}_\psi \end{bmatrix} \quad (1)$$

or in a more compact form

$$\bar{\mathbf{G}}(\mathbf{x}, \theta, \alpha) \mathbf{f} = \bar{\mathbf{g}} \quad (2)$$

$\mathbf{G}$  and  $\mathbf{J}$  are called the grasp and the Jacobian matrices and are depending on the robot configuration and on the ground configuration characterized by the angles  $\alpha = (\alpha_i)$  of the contact tangent plane with respect to the horizontal (fig.3).

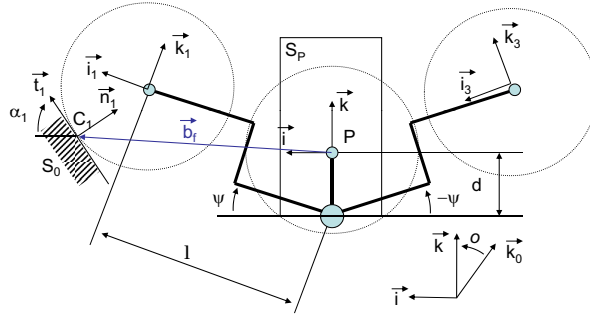


Fig. 3. Planar scheme of the suspension kinematics and geometrical parameters definition.

Solving this model consists in computing  $(\mathbf{f})$  for a given configuration  $\mathbf{q} = (\mathbf{x}, \theta)$  and a given external gravitational generalized force  $\mathbf{g}$ . Most of

models of articulated rovers have a high number of static indeterminacy. This force indeterminacy has two sources : (1) internal, because of the use of a redundant actuation (all the wheels are actuated) and (2) external, because of the multiple frictional wheel/ground contacts. For the considered system, the indeterminacy is equal to 2 when the pitch joint is passive, and is equal to 3 when this joint is actuated.

The main issue of the contact stability problem is to determine a contact force distribution which satisfies :

- the unilateral contact condition :  $f_{i,N} > 0$ ,
- no (or small) slippage condition :  $(f_{i,T})^2 < (\mu f_{i,N})^2$ .

$f_{i,N}$ ,  $f_{i,T}$  denote respectively the normal and the tangential components of the  $i$ th contact force.

These last conditions can be transformed, as proven by<sup>8</sup> and extended by,<sup>9</sup> into positive definiteness of certain symmetric matrices which is for a punctual contact with friction (PCWF), restricted here to a planar problem,

$$\mathbf{P}(\mathbf{f}) = \sum_{i=1}^3 \mathbf{f}_{i,T} \mathbf{S}_{i,T} + \mathbf{f}_{i,N} \mathbf{S}_{i,N} > \mathbf{0} \quad (3)$$

with  $\mathbf{S}_{i,T}$ ,  $\mathbf{S}_{i,N}$  are constant block diagonal symmetric matrix. For  $i = 1$

$$\mathbf{S}_{1,T} = \text{blockdiag}(\mathbf{E}_{12}^2 + \mathbf{E}_{21}^2, \mathbf{0}_{2 \times 2}, \mathbf{0}_{2 \times 2})$$

$$\mathbf{S}_{1,N} = \text{blockdiag}(\mu(\mathbf{E}_{11}^2 + \mathbf{E}_{22}^2), \mathbf{0}_{2 \times 2}, \mathbf{0}_{2 \times 2})$$

and et cetera for  $i = 2, 3$ . In these relations,  $\mathbf{E}_{bc}^a$  is a square matrix of dimension  $a$  with element  $(b, c)$  to be 1 and all others to be zero.

The problem can be formulated as a set of convex optimization problems involving Linear Matrix Inequalities (LMIs) which can be handled by general-purpose LMI solvers in computationally viable conditions.

We define a measure of optimality for traction forces by

$$\Psi(\mathbf{f}) = w^2 \mathbf{f}_T^t \mathbf{f}_T + \log \det \mathbf{P}^{-1}(\mathbf{f})$$

where  $\mathbf{f}_T = (f_{1,T}, f_{2,T}, f_{3,T})^t$  depicts the vector of traction forces,  $w$  is a weighting factor. The first term of the measure will grow with the contact

tangential forces, and the second term grows to infinity as any contact force approaches the boundary of its friction cone.

The traction force optimization problem can therefore be stated as follows

$$\begin{aligned} & \text{minimize} && \Psi(\mathbf{f}) = \mathbf{f}^t \mathbf{W}^t \mathbf{W} \mathbf{f} + \log \det \mathbf{P}^{-1}(\mathbf{f}) \\ & \text{subject to} && \bar{\mathbf{G}} \mathbf{f} = \bar{\mathbf{g}} \end{aligned} \quad (4)$$

with  $\mathbf{W}$  corresponds to a weighting matrix of dimensions (6,6) where elements are 0, except  $W_{11} = W_{33} = W_{55} = w$ .

Finally, wheel torques can be computed by using

$$\boldsymbol{\tau} = \mathbf{J}^t \mathbf{f} \quad (5)$$

which gives more simply

$$\tau_i = r f_{i,T} \quad (6)$$

$r$  is the wheel radius.

#### 4. Simulation results

This section gives a preliminary simulation result of a step-like obstacle crossing, using the vehicle and ground parameters given in table (1). The robot crosses a step of height equal to the wheel radius. Motions equations integrate the brush tyre model<sup>10</sup> which considers longitudinal wheel slippage. Because of velocity displacement is very small while obstacle crossing, centrifugal and coriolis forces  $\mathbf{C}(\mathbf{q}, \dot{\mathbf{q}})$  can be neglected in the dynamic model. However, second order terms of inertial forces  $\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}}$  are well and truly considered in our simulation model.

Simulations are carried out by Matlab software<sup>TM</sup> and *cvx* toolbox<sup>11</sup> dealing with convex programming. Figure (4) shows the traction coefficients ( $f_{i,T}/f_{i,N}$ ) as function of time, with two control strategies. The first one is based on the LMI-based torque optimization described in the previous section. The second one is based on usual velocity control with equal wheel's rate distribution  $\omega_i = \omega$ . These curves have three parts, each corresponds to the phase when a wheel is climbing the step. We verify that traction

module mass $m_{1,2,3}$	50Kg
arm length $l$	0.60m
pitch joint position $d$	0.10m
wheel radius $r$	0.25m
Friction coefficient $\mu$	0.8
Step height	0.25m
critical slippage ratio $s_c$	0.2

coefficient obtained by optimization is less than the friction coefficient and then each contact force is inside its friction cone. With the velocity control, the robot can not cross the step, as at each frontal contact two wheels are highly spinning while the other is braking; this latter has a rate smaller than the theoretical ideal rolling rate. We can notice also that high internal forces are created by applying simultaneously tractive and braking torques.

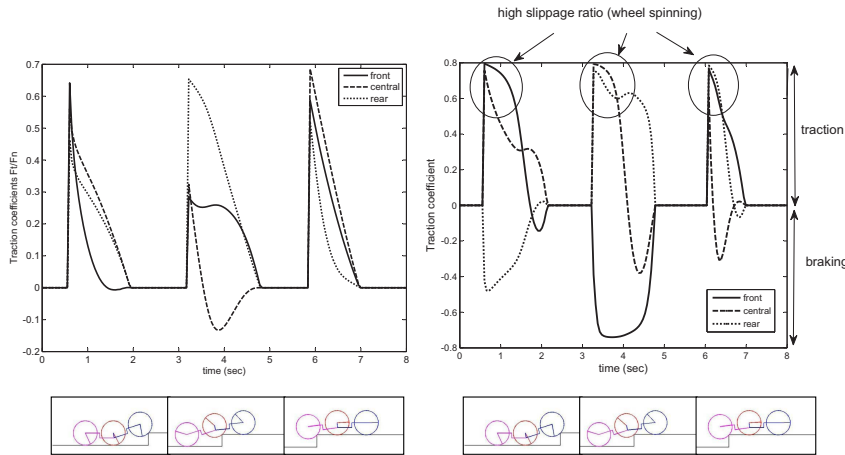


Fig. 4. Traction coefficients  $f_t/f_n$ : at left with optimization (weight factor  $w = 0.01$ ) and at right without optimization (with equal rolling velocity).

### 5. Conclusion

In this paper, a new method is proposed for determining an optimal traction force distribution in multi-wheeled articulated robot. The method, inspired from multifingered grasping tasks, is based on a Linear Matrix Inequalities



formulation which leads directly to a simple convex optimization problem that can be solved efficiently in polynomial time. The magnitude of traction forces is used as a measure of optimality of the clearance task. The approach considers cone friction constraint and turns out to be robust to slippage phenomena. This approach has to be extended to 3D motion in order to study for example the effect, of the robot configuration angle along the yaw direction, on the clearance capacity. Experimental validation of such optimal torque distribution requires the estimation of contact parameters (position and normal). However for structured obstacles (stair, step, etc...), this problem can be solved easily by using a ground elevation map and an on-line obstacle sensing.

## References

1. Y. P. Li, T. Zielinska, M. H. A. Jr. and W. Lin, Vehicle dynamics of redundant mobile robots with powered caster wheels, in *Proc. of the 16th CISM IFToMM Symposium, Romansy*, (Warsaw, 2006).
2. F. Le Menn, P. Bidaud and F. Ben Amar, Generic differential kinematic modeling of articulated multi-monocycle mobile robots, in *IEEE/ICRA'06 Proceedings of the Int. Conference on Robotics and Automation*, (Orlando, USA, 2006).
3. S. Sreenivasan and K. Waldron, *Displacement analysis of an actively articulated wheeled vehicle configuration with extensions to motion planning on uneven terrain Transactions of the ASME* **118**, 312 (1996).
4. R. Volpe, *Rocky 7: A Next Generation Mars Rover Prototype Journal of Advanced Robotics* **11**, 341 (1997).
5. T. Thueer, A. Ambroise Krebs, R. Siegwart and P. Lamon, *Performance comparison of rough-terrain robots simulation and hardware J. of Field Robotics* **24**, 251 (2007).
6. R. Siegwart, P. Lamon, T. Estier, M. Lauria and R. Pigué, *Innovative design for wheeled locomotion in rough terrain Robotics and Autonomous Systems* **40**, 151 (2002).
7. B. C. Bouzgarrou, F. Chapelle and J. C. Fauroux, A new principle for climbing wheeled robots: Serpentine climbing with the open WHEEL platform, in *2006 IEEE/RSJ International Conference on Intelligent Robots and Systems*, (Beijing, 2006).
8. M. Buss, H. Hashimoto and J. Moore, *Dextrous hand grasping force optimization IEEE Transactions on Robotics and Automation* **12**, 406 (1996).
9. L. Han, J. C. Trinkle and Z. Li, *Grasp Analysis as Linear Matrix Inequality Problems IEEE Transactions on Robotics and Automation* **16**, 1261 (2000).
10. J. Wong, *Terramechanics and off-road Vehicles* (Elsevier, 1989).
11. M. Grant and S. Boyd, Cvx: Matlab software for disciplined convex programming, in *www.stanford.edu/boyd/cvx/*, (1997).