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Modeling of Plate Structures Equipped with Current Driven Electrostrictive Actuators For Active Vibration Control

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ABSTRACT: The study here reported is focused on active vibration control applications performed on plate host structures equipped with electrostrictive patches. In such applications the design of controllers requires to simulate the behavior of the coupled structure. These simulations can then be performed through a Finite Element Method which implies the elaboration of electrostrictive finite elements. The purpose of the present paper is to use the electrostrictive plate finite element elaborated in a previous paper such as to simulate active vibration control of a cantilever beam equipped with current driven or voltage driven electrostrictive actuators. From these numerical results it is shown that a linear controller is as efficient as a nonlinear one in terms of vibration absorption, whatever the driving input used. An experimental implementation of the current driving input is moreover presented. Based on experimental measurements, it is proved that current driving electrostrictive actuators induce a decrease of electric energy consumption compared to a classical voltage driving input.

Key Words: electrostrictive actuators, active vibration control, lightweighted structures, finite element method, numerical results

INTRODUCTION

With the view of improving micro-vibrations absorption, more powerful actuators have been searched for. In the last few years, electrostrictive ceramics characterized by important strains when subjected to an electric field have been elaborated and widely studied with the aim of supplying this need. These materials make it possible to design very attractive actuators for active vibrations control. Nevertheless they are characterized by a nonlinear electromechanical behavior and a thermal sensitivity which considerably complicate their use.

Studies focused on understanding the behavior of electrostrictive ceramics such as 0.9PMN-0.1PT under cycling electric field showed important sensitivity of this behavior to operating parameters. Moreover, recent papers (Pablo and Petitjean, 2000a; Pablo et al., 2001b) underlined a significant heating of electrostrictive patches under cycling electric field and showed that the sensitivity of electrostrictive behavior to operating parameters could in fact be reduced to a sensitivity to

the ceramic own temperature. From these observations some models of electrostrictive behavior giving good accuracy have been elaborated (Pablo and Petitjean, 2000a; Pablo et al., 2001b; Hom and Shankar, 1994; Fripp, 1995).

In active vibration control applications the design of controllers requires simulation models of the structure. These simulations can be performed through a Finite Element Method which implied the elaboration of some three-dimensional (Hom and Shankar, 1996; Ghandi and Hagood, 1997) and two-dimensional electrostrictive finite elements (Debus et al., 1998).

The study here reported is focused on active vibration control applications performed on plate host structures equipped with electrostrictive patches. The use of plate finite elements to simulate the behavior of such structures thus seems to be more suitable. As no plate electrostrictive finite element, with direct a priori plate assumptions, has been up to now presented (to our knowledge), we have presented in a previous paper (Pablo et al., 2001a) the elaboration of such a finite element for electrostrictive patches used as actuators.

The finite element formulations, briefly reminded here, are based on electromechanical constitutive equations derived in an earlier paper (Pablo and Petitjean, 2000a),

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mechanical and electrical considerations and direct a priori plate assumptions. It has been shown that the electromechanical problem can be reduced to an equivalent mechanical one. The electrical phenomena are then taken into account through prescribed stresses and non usual modifications of the elastic constitutive laws. This method considerably simplifies the resolution of the problem since classical finite elements for laminated plates can be used to model the electrostrictive plate.

From these finite element formulations and experimental measurements, we have moreover underlined that using current instead of voltage as the driving input of actuators may simplify active vibrations control with electrostrictive patches. Based on these considerations, two thin plate electrostrictive finite elements have been described depending on the driving input used (Pablo et al., 2001a).

In this paper, the electrostrictive finite elements are used such as to simulate active vibration control of a cantilever beam equipped with electrostrictive actuators and subjected to an external excitation. From these simulations, it will be shown that linear controllers are as efficient as nonlinear ones in terms of vibration reductions, whatever the driving input used. Finally, the advantages of a current driving input compared to a voltage one, previously noted will be here confirmed based on the numerical simulations and an experimental implementation of this first driving input.

PROBLEM STATEMENT

The present paper aims at presenting numerically simulating control loops using current driven, and voltage driven actuators (patches). In this paragraph the theory developed in a previous papers (Pablo et al., 2001a,b) is briefly reminded.

This section is focused on establishing the basic assumptions of a plate theory. As electrostrictive ceramics are characterized by an electromechanical behavior, the problem here to solve is a fully electromechanically coupled one. This coupling then induces the need for simultaneously solving mechanical and electrical relations taking into account the nonlinear electromechanical constitutive laws derived in (Pablo and Petitjean, 2000a,b).

We underline that in (Pablo and Petitjean, 2000a,b) we developed electromechanical constitutive equations for current driven actuators. Moreover, a similar development can be used to derive voltage driven constitutive equations.

Material Assumptions

This study is focused on a particular class of materials which will be called mechanically and electrically

orthotropic. These assumptions can then be translated as follows:

1. there is no coupling between transverse distortions and stresses on the one hand, and, plane strains and stresses on the other hand, through the constitutive law,
2. there is no coupling between transverse electric field and displacement on the one hand, and, plane electric fields and displacements on the other hand, through the constitutive law,
3. electrostriction couples transverse electric displacement and field, and plane stresses and strains on the one hand, and, in-plane electric displacements and field, and transverse stresses and strains on the other hand.

A Priori Plate Assumptions

We will suppose afterwards that the structures we are interested in are thin plates. We thus suppose that the ratio of the thickness h and the in-plane characteristic dimension L is small compared to 1. Given motion equations, transverse stresses T_{iz} ($i = x, y, z$) are negligible compared to in-plane stresses $T_{\alpha\beta}$ ($\alpha, \beta = x, y$).

From these assumptions, one can express displacements as linear functions of z (transverse variable):

$$U_x(x, y, z, t) = u(x, y, t) - zw_{,x}(x, y, t) \quad (1)$$

$$U_y(x, y, z, t) = v(x, y, t) - zw_{,y}(x, y, t) \quad (2)$$

$$U_z(x, y, z, t) = w(x, y, t), \quad (3)$$

where the subscript, i ($i = x, y, z$) denotes the derivative respectively to the variable i .

Given the strain–displacement relationship, strains associated with these displacements can be written as follows:

$$\mathbf{S}_{\alpha\beta} = \mathbf{S}^0 - z\mathbf{R}^1 = \begin{pmatrix} v_{x,x} \\ v_{y,y} \\ v_{x,y} + v_{y,x} \end{pmatrix} - z \begin{pmatrix} w_{,xx} \\ w_{,yy} \\ 2w_{,xy} \end{pmatrix}$$

$$\text{and } \mathbf{S}_{iz} \sim 0 \quad (i = x, y, z \text{ and } \alpha, \beta = x, y) \quad (4)$$

where \mathbf{S}^0 and \mathbf{R}^1 are respectively in-plane strains deriving from membrane and the curvature. This equations are the expression of the well-known Kirchhoff–Love assumptions.

PLATE THEORY

The plate theory here presented is based on a priori assumptions. We proved in an earlier paper (Pablo et al., 2001b) that for metallized patches for which in-plane electric field may be neglected, in-plane electric displacements are negligible compared to transverse electric displacement. The Gauss law can thus be reduced to: $\mathbf{D}_{z,z} = q^d$.

One is then able to integrate this relation, whatever the driving input used. The solution of the Gauss law will then make it possible to eliminate the unknown electrical variables in favor of the mechanical displacements and to obtain a purely mechanical problem with equivalent electric forces.

Current Driven Actuator

In this operating configuration a current is imposed through the electrostrictive patch. This current then induces charges on the upper and lower ceramic surfaces which can be associated with electric displacements. The transverse electric displacements (\mathbf{D}_z) is thus known for each time t on the upper and lower patch surfaces.

Given the electric boundary conditions (electric displacement D_z^d is imposed) the integration of the Gauss law leads to a constant transverse electric displacement through the thickness of the plate:

$$\mathbf{D}_z = \mathbf{D}_z^d(x, y) \quad (5)$$

Assuming plane stresses and introducing this solution in the stress constitutive law, the plane stresses–strains relationship can be read:

$$\mathbf{T}_{\alpha\beta} = \tilde{\mathbf{C}}_{\alpha\beta\gamma\delta}^D \mathbf{S}_{\gamma\delta} + \tilde{\mathbf{Q}}_{\alpha\beta z}^{TD} (D_z^d(x, y))^2, \quad (6)$$

where $\tilde{\mathbf{C}}_{\alpha\beta\gamma\delta}^D$ are the isolated elastic constants.

We thus have eliminated the electric displacement from the unknowns of the electromechanical problem. The problem now to solve is then a purely mechanical problem with electrically induced prescribed stresses. Nevertheless, the current density have to be measured at each point of the faces of the plate if we want to establish these stresses.

This approximation will be satisfied if the transverse electric displacement is almost constant when the faces of the patches are assumed to be voltage equipotentials. If not, this more critical case needs further investigations.

Voltage Driven Actuator

In this operating configuration a transverse voltage is imposed through the electrostrictive patch. As patches

are equipped with electrodes on the upper and lower surfaces, electric potentials (V) are supposed to be known and uniform on these faces.

When electrostrictive patches are voltage driven, the electromechanical problem is strongly nonlinear and difficult to solve. However, this problem can be simplified through a linearization around an operating point. Any variable v can then be read $v = v^P + \bar{v}$, where v^P and \bar{v} are respectively the known value at the operating point and an increment around this point for this variable. We then have to establish the value of increments.

The constitutive law, we have to use for voltage driven actuators, can also be read as a relation expressing stresses as a function of strains and electric displacement, where $\mathbf{D} = \mathbf{D}(\mathbf{E})$. Differentiating these constitutive laws with respect to strains \mathbf{S} and electric field \mathbf{E} , and, applying materials and plane stresses assumptions, one can read:

$$\bar{\mathbf{T}}_{\alpha\beta} = \tilde{\mathbf{C}}_{\alpha\beta\gamma\delta}^D \bar{\mathbf{S}}_{\gamma\delta} + \tilde{\mathbf{d}}_{\alpha\beta z}^{TD} \bar{\mathbf{D}}_z, \quad (7)$$

$$\bar{\mathbf{D}}_z = \tilde{\mathbf{d}}_{\alpha\beta z}^{DS} \bar{\mathbf{S}}_{\alpha\beta} + \tilde{\mathbf{h}}_{zz}^E \bar{\mathbf{E}}_z, \quad (8)$$

where submatrices $\tilde{\mathbf{C}}$, $\tilde{\mathbf{d}}$ and $\tilde{\mathbf{h}}$ are calculated at the operating point.

These “incremental” constitutive laws can be compared to linear piezoelectric constitutive laws. One can then use similar methods as for current driven actuators to eliminate the electric field from the problem unknowns. Considering Gauss law for the incremental problem, one can read:

$$\mathbf{D}_{z,z} = \mathbf{D}_{z,z}^P + \bar{\mathbf{D}}_{z,z} = q^d. \quad (9)$$

Integrating this relationship and taking into account the incremental constitutive law (8), one can establish the following value for the transverse electric displacement increment:

$$\bar{\mathbf{D}}_z = \Phi^{D0} \bar{\mathbf{S}}^0 - \Phi^{D1} \bar{\mathbf{R}}^1 + \bar{\mathbf{D}}_z^{\text{elec}}, \quad (10)$$

where Φ^{Di} ($i = 0, 1$) and $\bar{\mathbf{D}}_z^{\text{elec}}$ are submatrices depending on $\tilde{\mathbf{C}}$, $\tilde{\mathbf{d}}$, $\tilde{\mathbf{h}}$ and the electrical boundary conditions, and, are thus fully established.

Introducing this relationship in the incremental constitutive law (7) then implies:

$$\bar{\mathbf{T}}_{\alpha\beta} = \Phi_{\alpha\beta\gamma\delta}^{T0} \bar{\mathbf{S}}_{\gamma\delta}^0 - \Phi_{\alpha\beta\gamma\delta}^{T1} \bar{\mathbf{R}}_{\gamma\delta}^1 + \bar{\mathbf{T}}_{\alpha\beta}^{\text{elec}}, \quad (11)$$

where Φ^{Ti} ($i = 0, 1$) and $\bar{\mathbf{T}}^{\text{elec}}$ are submatrices depending on $\tilde{\mathbf{C}}$, $\tilde{\mathbf{d}}$, $\tilde{\mathbf{h}}$ and the electrical boundary conditions, and, are thus fully established.

In the particular case of homogeneous materials and null electrical volumic charges, incremental laws (10) and (11) :

$$\begin{cases} \bar{T}_{\alpha\beta} = \left(\tilde{\mathbf{C}}_{\alpha\beta\gamma\delta}^D + \tilde{\mathfrak{d}}_{\alpha\beta z}^{TD} \tilde{\mathfrak{d}}_{z\gamma\delta}^{DS} \right) \bar{\mathbf{S}}^0_{\gamma\delta} \\ \quad - z \tilde{\mathbf{C}}_{\alpha\beta\gamma\delta}^D \bar{\mathbf{R}}^1_{\gamma\delta} - \tilde{\mathfrak{d}}_{\alpha\beta z}^{TD} \tilde{\mathfrak{h}}_{zz}^E \frac{\bar{V}^+ - \bar{V}_-}{h} \\ \bar{D}_z = \tilde{\mathfrak{d}}_{z\alpha\beta}^{DS} \bar{\mathbf{S}}^0_{\alpha\beta} - \tilde{\mathfrak{h}}_{zz}^E \frac{\bar{V}^+ - \bar{V}_-}{h}, \end{cases} \quad (12)$$

where $\tilde{\mathbf{C}}_{\alpha\beta\gamma\delta}^D + \tilde{\mathfrak{d}}_{\alpha\beta z}^{TD} \tilde{\mathfrak{d}}_{z\gamma\delta}^{DS} = \tilde{\mathbf{C}}_{\alpha\beta\gamma\delta}^E$ and \bar{V}^+ and \bar{V}_- are electric potentials increments imposed on the electrodes. Thus, the elastic constants for membrane stresses are the short-circuited ones and elastic constants for bending stresses are the isolated ones.

We thus have eliminated the electric displacement from the unknowns of the electromechanical problem. The problem now to solve is then a purely mechanical problem with electrically induced prescribed stresses (known) and modification of the mechanical constitutive law.

Partial Conclusions

We thus proved that whatever the driving input used, the nonlinear electromechanical problem can be reduced to a purely mechanical problem where electrical phenomena are taken into account by additional prescribed stresses and consistent modification of the mechanical constitutive law. Moreover, the simplification of the problem when current is used instead of voltage as the driving input is highlighted by the previous developments.

CONTROL OF A CANTILEVER BEAM

Driving Input Considerations

We will here apply the theory proposed above on the active vibration control of a $40 \times 220 \times 1 \text{ mm}^3$ steel cantilever beam, for its first mode. We will thus study the time evolution of the mechanical displacements of a released beam. The beam is moreover equipped with a

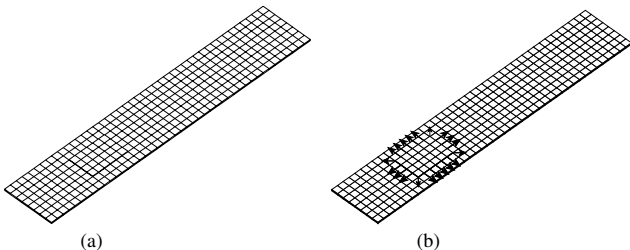


Figure 1. Modeled structure: (a) Cantilevered beam; (b) Electrical forces.

piezoelectric or electrostrictive actuator placed at 45 mm from the clamping. The whole structure can then be modeled as presented in Figure 1(a).

The first frequency of the beam respectively equipped with a short-circuited and isolated piezoelectric patch, obtained by a Finite Element Method, are about 17.95 and 18.02 Hz. The electric state of the patch thus does not significantly modify the first frequency.

Let us now suppose that a static voltage is applied to the actuator. The electrical forces applied to the beam through the patch are then plane forces and are distributed as presented in Figure 1(b).

If we now observe the transverse electric displacement D_z distribution over the patch, obtained through the constitutive laws, one can observe that mechanically induced electrical displacements, and thus total electric displacements are quasi uniform (cf. Figure 2). Indeed, from numerical results, the variation of mechanically induced electric displacement is less than 5% compared to its mean value.

The mean value of the transverse electric displacement can then be supposed to be uniform on the patch, and it is thus possible to use the current as the driving input given that $D_z = I/S$ (if I and S are respectively the imposed current and the patch area).

Active Vibration Control

In this section, results of active vibration controls, obtained by numerical simulation through a Finite Element Method, are presented. We will propose hereafter the design of various controllers so that to reduce the vibrations of a cantilever beam using a voltage or current driven electrostrictive patch. The beam is 220 mm long, 40 mm wide and 1 mm thick, an electrostrictive actuator which is 30 mm long, 20 mm wide and 0.4 mm thick is embedded at 45 mm from the clamping and a PVDF film sensor of similar in-plane dimensions and which is $40 \mu\text{m}$ thick is collocated with the actuator.

We will hereafter simulate the active vibration control of the first mode of the beam which frequency is about 18 Hz, using a controller proportional to the speed (efficiency similar to the optimal control one for this frequency).

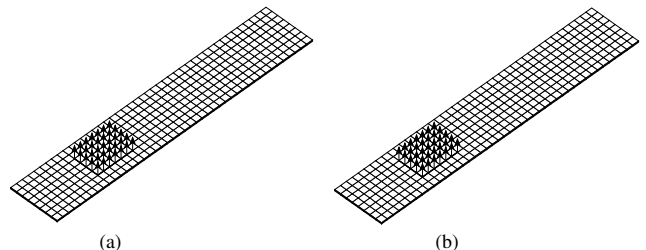


Figure 2. Transverse electric displacement distribution over the patch: (a) Mechanically induced D_z ; (b) Total D_z .

The force imposed by the control to the structure is then proportional to the speed (\dot{z}), and can be formulated as follows:

$$f^c = -C(\omega)\dot{z}, \quad (13)$$

where C and ω are respectively a gain and the pulsation to be controlled.

Moreover the force produced by the actuator is proportional to the driving input (electric field or electric displacement). As we want these two forces to be equal we can estimate the driving input needed for the reduction of the vibration, using the actuator electromechanical constitutive laws. From experimental measurements we have thus defined a characteristic electrostrictive behavior, not taking into account the hysteresis, which can be formulated through the strain-electric displacement and electric displacement–electric field laws:

$$S = \mathbb{Q}D^2, \quad (14)$$

$$D = \chi E^s \tanh(E/E^s), \quad (15)$$

where \mathbb{Q} , χ^* and E^s are respectively the electrostrictive coefficient, the pseudo-susceptibility and the saturation constant.

We have presented in Figure 3 the modeled behavior for nonheating and heating electrostrictive ceramics. The nonheating behavior (solid line) is characterized by the following parameters: $\mathbb{Q} = 0.96 \times 10^{-2} \text{ m}^4/\text{C}^2$, $\chi = 2.57 \times 10^{-7} \text{ F/m}$ and $E^s = 445 \text{ kV/m}$.

Finally, as the electrostrictive strains are only negative, we will hereafter polarize the ceramic so that to induce positive and negative variations of strain around the operating point. This operating point is then characterized by a 350 kV/m bias electric field, a 0.075 C/m² bias electric displacement and a 54 μdef strain.

The design of the control thus consists in establishing the magnitude of the driving input to impose to the

ceramic at each time t such as the control forces (13) meets:

$$f^c = -C(\omega)\dot{z} \quad \text{and} \quad \ddot{z} + \omega^2 z = f^c,$$

where C will be hereafter given and equal to 0.08.

Linear “Piezoelectric” Control

With the view of estimating the efficiency of the electrostrictive controls we need a reference. We thus define a “piezoelectric” control by assuming the behavior of the actuator to be linear. This linear behavior is then characterized by alternating strains and a “piezoelectric” coefficient obtained by the slope of the tangent to the electrostrictive behavior at the operating point. Measuring the slope of the dashed line plotted in Figure 3(a) and (c), one respectively obtain $g_{13} = -1.44 \times 10^{-3} \text{ V m/N}$ and $d_{13} = -211 \text{ pC/N}$ as piezoelectric coefficients for the current and voltage driven ceramics.

Using these “piezoelectric” actuators, the forces imposed to the structure are respectively proportional to the electric displacement D and the electric field E for the current and voltage driven control. The current and voltage driven problems to be solved are then respectively given by Equations (16) and (17). We underline that f_p and g_p are constants dependent on the mechanical characteristics of the actuator and which are obtained by projecting onto the first mode the piezoelectric nodal force vector deriving from the Finite Element Method.

$$f^a = f_p D = f^c = -C(\omega)\dot{z}, \quad (16)$$

$$f^a = g_p E = f^c = -C(\omega)\dot{z}. \quad (17)$$

Using such a control, the reference vibration reduction is given in Figure 4(a) and the reference driving input for current and voltage driven actuators are

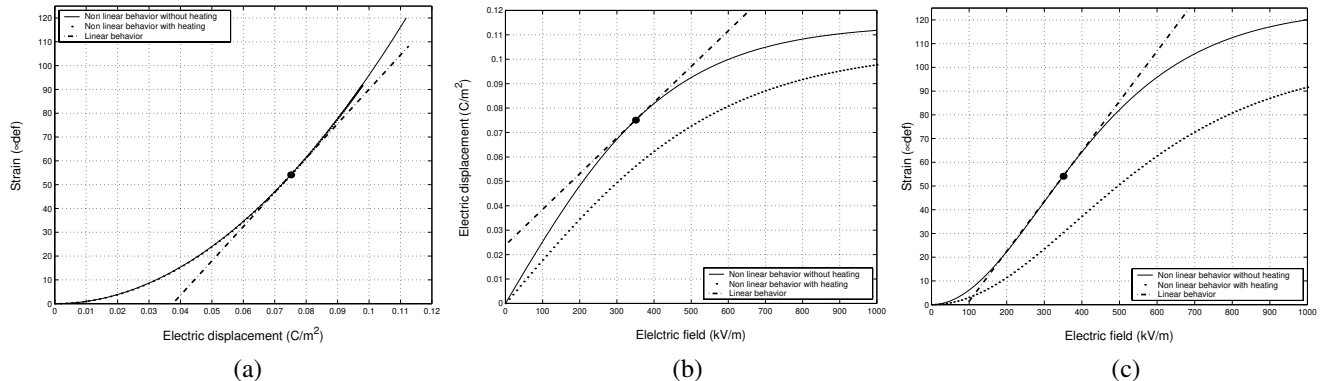


Figure 3. Modeled electrostrictive behavior: (-): without heating, (·): with heating): (a) Strain vs polarization; (b) Polarization vs electric field; (c) Strain vs electric field.

plotted in Figure 4(b) and (c). We underline that the reference vibration reduction here obtained (Figure 4(a)) is similar whatever the driving input used.

Nonlinear Electrostrictive Control

We are now interested in designing a controller, using a nonlinear electrostrictive actuator which is respectively characterized by its constitutive Equations (14) and (18) for current and voltage driving input.

$$S = \mathbb{Q} \left[\chi^* E^s \tanh\left(\frac{E}{E^s}\right) \right]^2 \quad (18)$$

The force supplied by the actuator (f^a) are then respectively proportional to the square of the electric displacement and the square of the hyperbolic tangent of the electric field imposed to the patch for current and voltage driven actuators. Moreover, it is to be underlined that the control forces (f^c) are still proportional to the speed whatever the driving input used.

The problems here to be solved for current and voltage driven patches, at each time t , can then been respectively expressed as follows :

$$f^a = f_p D^2 = f^c = -C(\omega)\dot{z}, \quad (19)$$

$$f^a = g_p \tanh^2\left(\frac{E^s}{E}\right) = f^c = -C(\omega)\dot{z}. \quad (20)$$

Consequently, the nonlinear electrostrictive control forces (f^c) will be similar to the piezoelectric force but the driving input will be modified. This phenomenon is clearly observable in Figure 5(b) and (c), and one can note that the electric variable modification depends on the driving input used.

Indeed, observing on a first hand the Figure 5(b), one can note that the nonlinear control implies a reduction of the electric displacement magnitude for the positive part of the signal and an increase of this magnitude for the negative part of the signal, compared to the ‘‘piezoelectric’’ control. On the other hand, one can note in Figure 5(c) that the nonlinear control implies an increase of the electric field magnitude for the positive part of the signal and a low reduction of this magnitude for the negative part of the signal, compared to the ‘‘piezoelectric’’ control.

These phenomena can easily be explained, based on the observation of Figure 3(a) and (c).

Indeed, using a current driven actuator one can note in Figure 3(a) that the tangent to the electrostrictive behavior at the operating point is placed below this curve. Consequently, a positive variation of electric

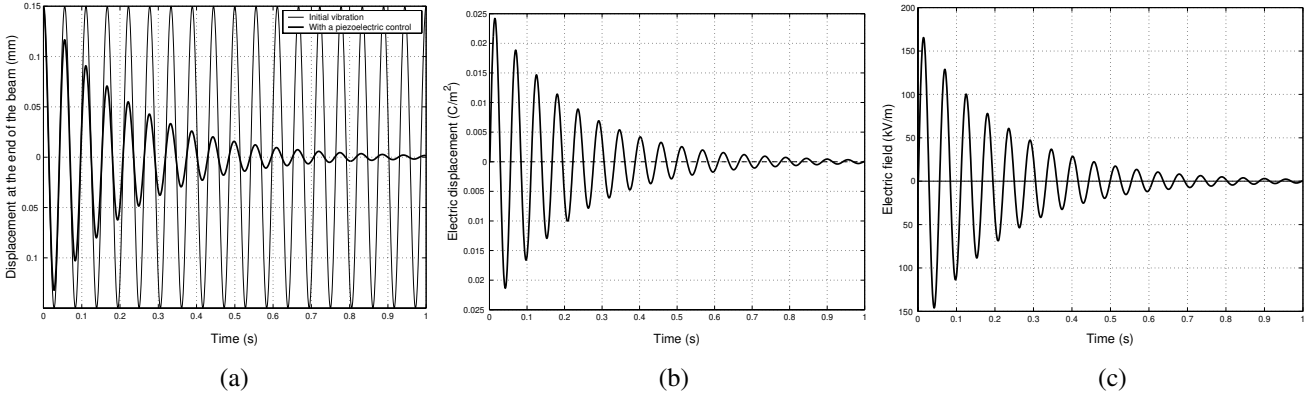


Figure 4. ‘‘Piezoelectric’’ active vibration control: (a) Vibration reduction; (b) Current driving input; (c) Voltage driving input.

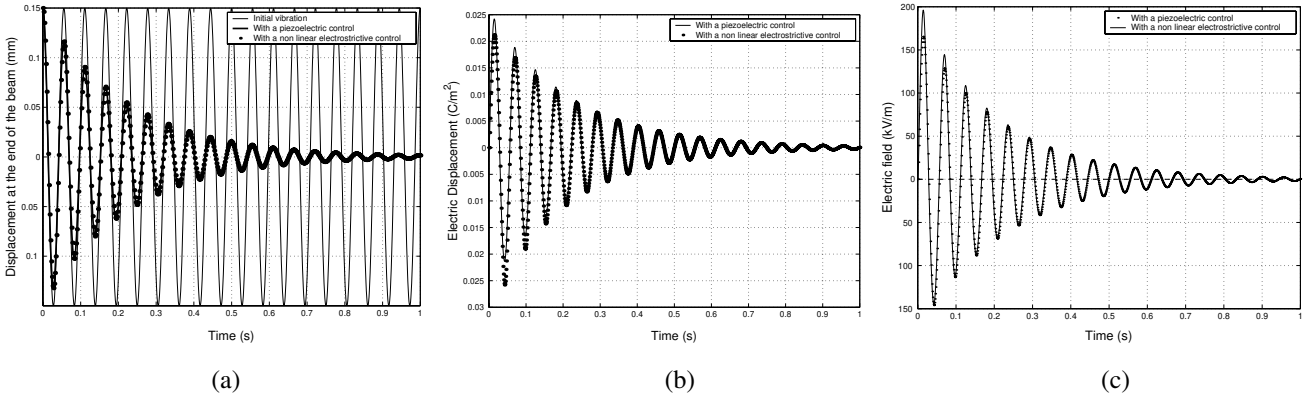


Figure 5. Nonlinear electrostrictive active vibration control: (a) Vibration reduction; (b) Current driving input; (c) Voltage driving input.

displacement induce a greater variation of distortion of the electrostrictive actuator than the piezoelectric one. For a similar driving input, the electrostrictive patch will thus impose a greater force to the structure than the “piezoelectric” patch. On the other hand, for a negative variation of electric displacement the previous observations can be reversed (greater force imposed by the “piezoelectric” actuator).

Conversely, using voltage driven patches, the operating point allowing the best efficiency of the control corresponds to the inflection point of the curve. The tangent to the electrostrictive behavior at this point is thus a secant of this curve (see Figure 3(c)). Consequently a variation of the electric field induces a greater variation of distortion for the “piezoelectric” actuator than for the electrostrictive one whatever the sign of this variation. A “piezoelectric” patch will thus impose a greater force to the structure than an electrostrictive patch, for a similar driving input. Moreover, as the electrostrictive behavior is not anti-symmetrical in relation to this operating point the variations of electric field magnitude are not similar for the two parts of the signal.

Finally, it is to be noticed that as the control forces (f^c) are similar for the nonlinear electrostrictive control and the “piezoelectric” one, so it is for the vibration reduction.

Linear Electrostrictive Control

Let us now apply the linear controllers previously designed for a “piezoelectric” actuator, given by the relationship (16) and (17), to the nonlinear electrostrictive patch.

As the driving input are linear (cf. Figure 6(b) and (c)), the force imposed to the structure by the electrostrictive actuator is nonlinear.

If one now observes the vibration reduction of these linear electrostrictive controls in Figure 6(a), one can note that their efficiency is similar to the

“piezoelectric” control, and consequently similar to the nonlinear electrostrictive control.

One can thus use a linearized behavior around the operating point to design the controller whatever the driving input used.

Actuator Heating

We have designed in paragraph a nonlinear electrostrictive controller, assuming that the ceramics are not subjected to a heating. Nevertheless, we proved in a previous paper (Pablo and Petitjean, 2000a) that voltage driven electrostrictive actuators undergo an important heating under cycling electric field and that their behavior is strongly sensitive to the ceramic own temperature. This dependence is then taken into account through the pseudo-susceptibility χ^* and the saturation constant E^S .

For example, if we suppose a heating of the ceramic from a 20°C initial temperature to a 40°C final temperature, experimental results revealed a 30% decrease of χ^* and a 30% increase of E^S . The behavior of the actuator is thus strongly modified by this heating, as one can note in Figure 3.

If one now uses the nonlinear electrostrictive controller designed in paragraph, using the relationship (20) and not taking into account the modification of the ceramic behavior, one obtains the vibration reduction presented in Figure 7(a).

As one can observe, the efficiency of the nonlinear controller (bold curve) is poorer than the one we could expect from the nonheating behavior (dotted curve) although the driving input imposed to the actuator has been increased (cf. Figure 7(b)).

The actuator heating thus need to be taken into account in the controller design for a voltage driven actuator such as to provide the greatest efficiency in vibration absorption terms.

In practice, the only way to take into account this heating in the controller consists in measuring the

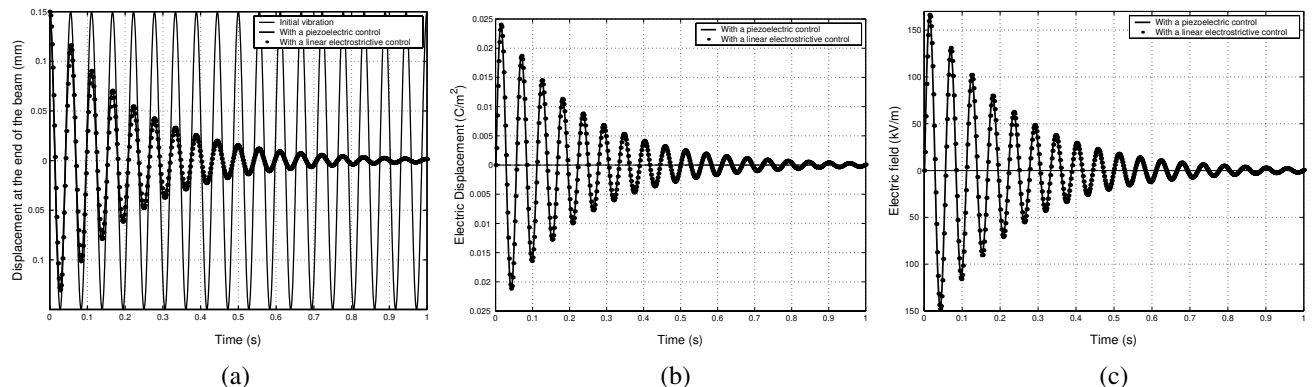


Figure 6. Linear electrostrictive active vibration control: (a) Vibration reduction; (b) Current driving input; (c) Voltage driving input.

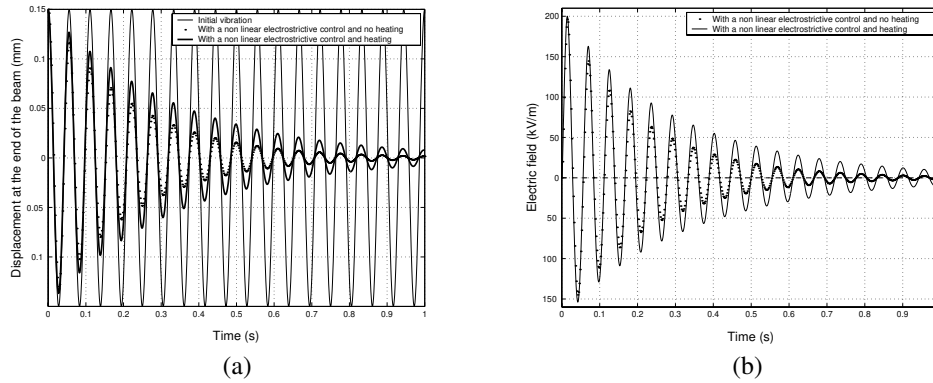


Figure 7. Nonlinear electrostrictive active vibration control with heating: (a) Vibration reduction; (b) Driving input.

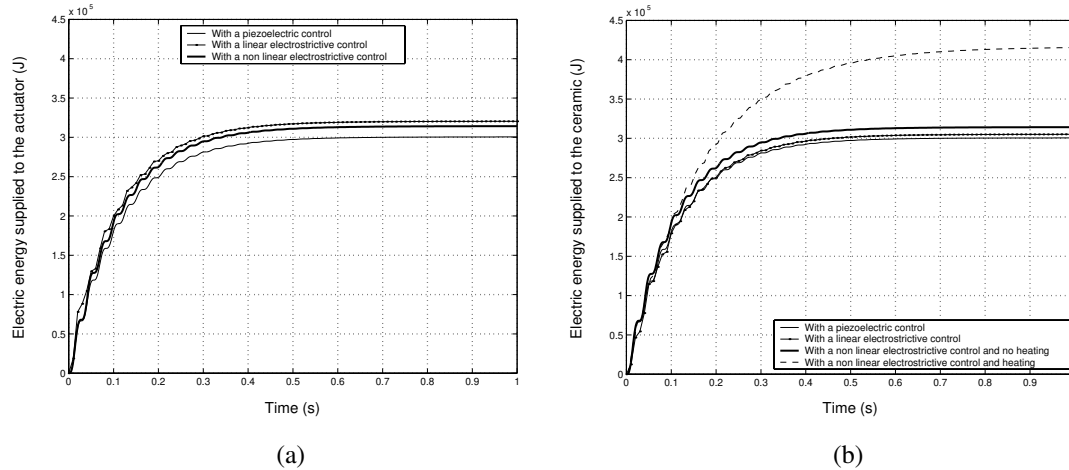


Figure 8. Electric energy supplied to the actuator: (a) Current driven actuators; (b) Voltage driven actuators.

actuator surface temperature using a thermoelectric couple. This measurement is then used to design a thermal controller which modifies at each time t the behavior of the ceramic used in the design of the active vibration controller. Nevertheless, this added controller induces an increase in calculation times and could lower the controller robustness.

Finally, we underline that the ceramic heating does not affect the current driven behavior. Using such a driving input thus induces an increase of the controller robustness.

Power Reflections

Using numerical results of active vibration control of a cantilever beam equipped with current driven or voltage driven actuators, we have shown that whatever the driving input used, linear controls are as efficient as nonlinear ones in terms of vibration reduction.

Moreover, the power needed for the vibration reduction is an important aspect in active vibration control. We thus have plotted in Figure 8 the electric energy supplied to the actuators for the various controllers previously designed. Figure 8(a) plots the electric energy supplied to current driven actuators and

Figure 8(b) plots the electric energy supplied to voltage driven ones.

Observing these two figures, it is first interesting to note that the “piezoelectric” control and the nonlinear electrostrictive control respectively use the same electric energy when they are current driven or voltage driven.

On the one hand, the use of the linear electrostrictive control have different effects on the electric energy consumption depending on the driving input chosen. Indeed, using a current driven actuator the electric energy consumption is increased compared to the one induced by the nonlinear electrostrictive control. On the other, using a voltage driven actuator the electric energy consumption is decreased. We nevertheless underline that the increase of electric energy consumption observed for the current driven actuator is low enough to be a real drawback.

From these energy considerations, the linear electrostrictive control thus seems to be a very interesting solution.

Finally, one can note the great increase in electric energy consumption of the heating actuator compared to the electric energy used by the nonheating one. This observation was predictable given the increase of the driving input plotted in Figure 7(b) and the vibration

reduction presented in Figure 7(a). As this heating is also available for voltage driven actuator, an active vibration control using current driven electrostrictive actuators thus seems more reliable than such a control using similar voltage driven patches. We moreover underline here that the modeled behavior used in these simulations do not take into account the hysteresis. As these hysteresis are more important in the strain-electric field behavior than in the strain-polarization one, the electric energy to be supplied to the ceramic is more important for a voltage driven electrostrictive patch than for a current driven one. This conclusion, based on numerical results have been experimentally confirmed as presented in the next section.

Current Driving Implementation

Two methods can be used to implement current driving of the electrostrictive patch. On a first hand, one can use a current amplifier which is able to impose great currents (several amperes) but low voltages to the ceramic.

On the other hand, one can use a classical power amplifier, which input (voltage) is piloted by a reference signal proportional to the current we want to impose to the actuator. We underline that such driving inputs have already been developed for piezoelectric stacks (Lindner and Chandrasekaran, 1999a,b) and electrostrictive stacks (Znovar et al., 1996; Znovar and Lindner, 1997, 1998).

We use, as for us, electrostrictive patches which are characterized by a nominal capacitance about 150 nF. Such a patch then need to use high voltages (about 500 V) and low currents (about 300 mA).

Elaborating such a current driving input, one can design the diagram presented in Figure 9, where V_e , u , R , i and V_s are respectively the reference signal, the ceramic voltage, a shunt resistance, the ceramic current and the resistance voltage which is similar to the ceramic current, given the linear voltage–current relationship.

Using such an assembly, one can then impose the current to the ceramic through the voltage V_e . The ceramic current is then known for each time t , but the ceramic voltage is not overcome.

Ceramic currents and voltages obtained for current driven and voltage driven electrostrictive patches are respectively plotted in Figure 10 (a) and (b). Bold curves have been obtained with current driven patches and solid curves have been obtained with voltage driven patches.

Let us first use a classical voltage driven actuator which is subjected to a sinusoidal electric field. Imposing a 250 kV/m static electric field, a dynamic electric field whose magnitude and frequency are respectively 250 kV/m and 900 Hz to this ceramic, one respectively obtains the ceramic voltage and the ceramic current plotted in solid curves in Figures 10(b) and (a). One can thus note that the ceramic voltage is a real sinusoidal curve but that the current is closer to a triangle signal than a sinusoidal one.

If we now use a current driven actuator subjected to a current whose magnitude is similar to the ceramic current magnitude obtained for the voltage driven ceramic, one can observe on Figure 10(a) that the ceramic current is nearly sinusoidal (bold curve). The use of the current driving circuit thus linearizes the ceramic current. Moreover, observing Figure 10(b)

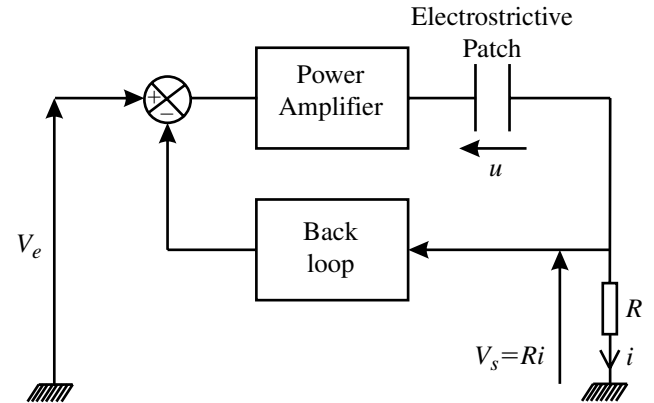


Figure 9. Current driving circuit.

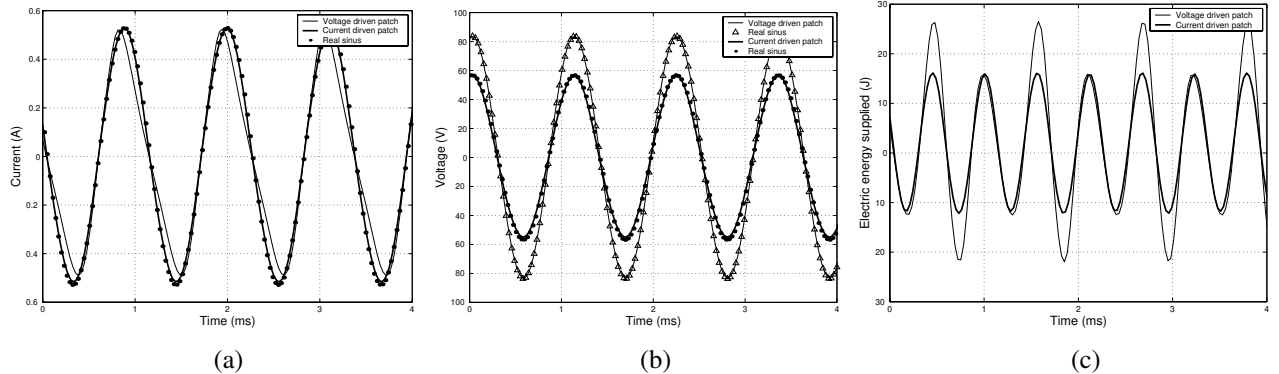


Figure 10. Electric characteristics of the current driving input. (a) Ceramic current; (b) Ceramic voltage; (c) Electric energy.

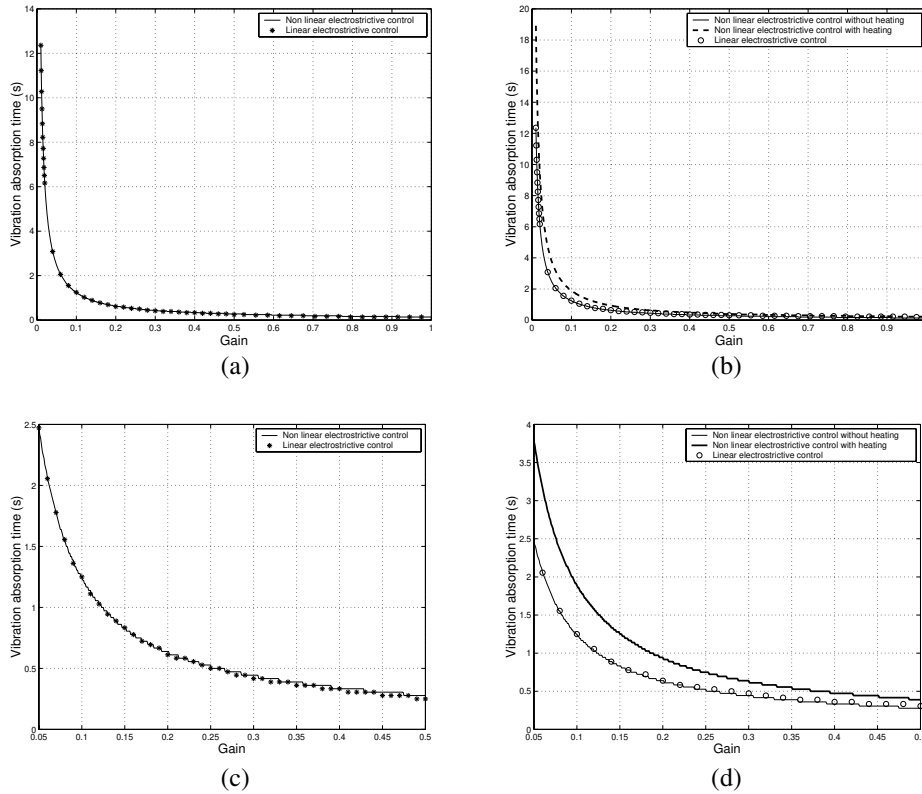


Figure 11. Vibration reduction time as a function of the gain: (a) Current driven actuators; (b) Voltage driven actuators; (c) Current driven actuators; (d) Voltage driven actuators.

one can note that the current driven ceramic voltage (bold curve) is slightly different from the real sinusoidal curve (dotted one). Nevertheless, the more important aspect of this curve does not consist in this distortion, but in the ceramic voltage magnitude. Indeed, one can note that using current instead of voltage as the driving input implies an important reduction of the ceramic voltage (about 35%).

From these observations it seems that using current driven actuators instead of voltage driven would imply a reduction in electric energy consumption. This fact is confirmed by the Figure 10(c) where the instantaneous electric energy supplied to the actuator is plotted for a voltage driven (solid line) and current driven (bold curve) ceramic.

The current driving circuit thus allows a linearization of the current imposed to the actuator, and an important reduction of electric energy to supply to this actuator such as to obtain similar strains (electrostrictive strains are only dependent on the polarization of the ceramic).

CONCLUSION

Numerical results of active vibration control of a cantilever beam using electrostrictive patches have been presented for current and voltage driving input, using

electrostrictive plate finite elements developed in previous papers.

From these results we have shown that using a linear electrostrictive control – a control designed using a linearized constitutive law of the electrostrictive material around the chosen operating point – the vibration reduction efficiency is similar to the one obtained with a non-linear electrostrictive control, whatever the driving input.

As the design of a linear control is easier than a nonlinear one, this result is very interesting for experimental applications. Indeed, the implementation of the linear controller induces a reduction of calculation times and thus implies a greater robustness than the nonlinear controller.

Nevertheless, results presented in this article have been obtained using a gain fixed to 0.08. So as to verify if the advantages of the linear control are still right for others value of the gain, we have calculated the vibration reduction time, here defined as the time needed for a 90% reduction of the initial vibration, for gains ranged between 0.01 and 1. Results obtained for this gain range are plotted in Figure 11 for current Figure 11(a) and (c) and voltage driven actuators (Figure 11(b) and (d)).

Observing this Figure one can note that whatever the driving input used, the vibration reduction time is quite similar for the linear electrostrictive control and the nonlinear one, on the studied gain range. Nevertheless,

a focus of these curves (Figure 11(c) and (d)) allows to note that the use of the linear controller induces a low modification of this vibration reduction time compared to the one obtained by the nonlinear controller. Indeed, a low decrease and a low increase of this time is respectively observable for the current driven actuator and the voltage driven one.

Moreover, it has been shown that using voltage driven electrostrictive actuator implies the need of taking into account the change in the ceramic behavior – due to the heating of the patch – through a thermal controller. Indeed, not taking into account the actuator heating induces a reduction of efficiency of the controller. This loss of efficiency is clearly observable in Figure 11(b) and (d).

As the current driven constitutive laws are not affected by the heating of the patch, a thermal controller is not needed. Using current as the driving input will thus increase the robustness of the controller.

Finally, we underline that results presented in this paper have been obtained neglecting the hysteresis. Nevertheless, we proved through experimental curves that the use of current as the driving input induces a reduction of electric energy consumed by the electrostrictive actuator.

Current thus seems to be the appropriate driving input to use with electrostrictive actuators in active vibration control applications.

NOMENCLATURE

T	= stress matrix
S	= strain matrix
S^0	= in-plane strain deriving from membrane
R^1	= in-plane strain deriving from curvature
U	= 3D mechanical displacement vector
u	= 2D displacement along the x -axis
v	= 2D displacement along the y -axis
w	= 2D displacement along the z -axis
C	= stiffness tensor
\tilde{C}^D	= modified isolated stiffness tensor
\tilde{C}^E	= modified short-circuited stiffness tensor
D	= electric displacement vector
D^d	= given electric displacement
E	= electric field vector
E^s	= saturation constant
χ^*	= pseudo-susceptibility
V	= electric potential
V^+	= electric potential on the upper electrode
V_-	= electric potential on the lower electrode
q^d	= imposed body charges
Q	= electrostrictive coefficients matrix
\tilde{Q}^{TD}	= modified electrostrictive coefficients matrix
$\tilde{d}^{TD}, \tilde{d}^{DS}$	= pseudo-piezoelectric coefficients matrix
\tilde{h}_{zz}^E	= pseudo-dielectric coefficients matrix

v^P	= variable v value at the operating point
\bar{v}	= variable v increment at the operating point
h	= thickness of the patch
L	= in-plane characteristic dimension
f^c	= force imposed by the control
f^a	= force induced by the actuator
\dot{z}	= speed
\ddot{z}	= acceleration

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