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► **To cite this version:**

Jean-François Deü, Walid Larbi, Roger Ohayon. Piezoelectric structural acoustic problems: Symmetric variational formulations and finite element results. *Computer Methods in Applied Mechanics and Engineering*, 2008, 197 (19-20), pp.1715-1724. 10.1016/j.cma.2007.04.014 . hal-03177690

HAL Id: hal-03177690

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Submitted on 8 Nov 2023

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Piezoelectric structural acoustic problems: Symmetric variational formulations and finite element results

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This paper presents an original symmetric finite element formulation for piezoelectric structural acoustic coupled problems. First, an unsymmetric variational formulation in terms of structure displacement, electric potential and fluid pressure field is proposed. Then, through the introduction of an additional variable, namely fluid displacement potential, a symmetric finite element formulation is presented. Finally, after a rigorous condensation procedure and using the fluid added mass and piezoelectric added stiffness concepts, a reduced symmetric formulation is obtained. Some numerical examples concerning axisymmetric elastic/piezoelectric structures filled with internal compressible fluid are presented in order to validate the proposed formulation.

Keywords: Fluid–structure interaction; Piezoelectric structure; Finite element method; Symmetric formulation

1. Introduction

Over the past decade, a considerable amount of research has been devoted to the development, testing and modeling of noise reduction techniques using passive and/or active approaches. Classically, noise reduction by passive techniques is achieved by using sound absorbing materials such as porous media, which consists of an elastic matrix containing interconnected fluid-saturated pores. These materials are generally modeled by poroelastic Biot theory [1–4]. Alternatively, for structural–acoustic applications, a dissipative wall acoustic impedance can be used to take into account the absorbing properties of the materials at the fluid–structure interface [5–8]. It is well known that passive reduction methods are quite effective at high frequencies. In

the low frequency range, active techniques using piezoelectric materials are found to be an attractive alternative or complementary tool. In this case, sensor and actuator piezoelectric patches are surface-mounted or embedded in the structure. These patches are capable of self-sensing and self-actuation for active vibration and noise control. For the modeling of active-control structural–acoustic problems in the field of noise reduction applications (see Fig. 1 for a general sketch of the coupled problem), the finite element method is one of most powerful approaches. In this context, let us mention [9,10] where active controller designs are developed to reduce interior cabin noise levels and [11,12] where active/passive constrained layer damping treatments are proposed to control sound radiation from a vibrating thin structure into an acoustic cavity. Another methodology consists in combining the finite element approach for the smart structure and the boundary element formulation to calculate the acoustic response of the enclosed fluid. In such a case, a steady-state response of

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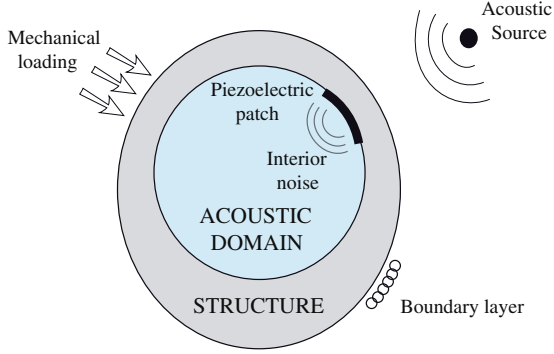


Fig. 1. The structural–acoustic interaction problem.

acoustic cavities bounded by piezoelectric composite shell structures is proposed in [13] and an active–passive control technique, based on an output feedback optimal controller design, is developed in [14].

In the previous finite element citations, different fields can be chosen to describe the acoustic fluid: pressure, fluid displacement, fluid displacement potential or fluid velocity potential [15]. The most natural and common formulation consists of choosing the scalar pressure field as main variable in the fluid domain. However, if pressure is taken as nodal variable, the coupled fluid–structure matrix system is not symmetric. In this context, the overall objective of the present study is to present a symmetric finite element formulation for electromechanical–acoustic problems using pressure variable to describe the acoustic medium.

Thus, we present in this paper an original variational formulation and the corresponding finite element matrix equations for fluid/piezoelectric-structure coupled problems. This formulation is derived from the test-function method. The coupled system consists of a piezoelectric structure (described by its displacement field u_i and electric potential ψ) containing an inviscid, compressible and barotropic fluid (described by its pressure field p), gravity effects being neglected. First, a non-symmetric variational formulation in terms of $(u_i, \psi; p)$ is proposed for transient and modal analysis of the coupled system. We next establish, through the introduction of an additional unknown field (namely displacement potential of the fluid φ), the symmetric variational formulation of the eigenvalue problem. Finally, it is shown that this problem can be reduced, by means of a rigorous elimination procedure in the discretized case, to a spectral problem in $(u_i; p)$ introducing the concepts of (i) generalized added mass operator for compressible fluid, and (ii) piezoelectric added stiffness operator.

2. Finite element formulation for the fluid/piezoelectric-structure coupled problem

Let us consider the linear vibrations of an elastic/piezoelectric-structure completely filled with a homogeneous, inviscid and compressible fluid, neglecting gravity effects.

We establish in this section the variational formulation of the coupled problem and the corresponding matrix equations. Note that standard indicial notations are adopted in this paper: subscripts denote the tensor components and repeated subscripts imply summation. In addition, a comma indicates a partial derivative.

2.1. Local equations of the coupled system

We consider a piezoelectric structure occupying the domain Ω_S at the equilibrium. The structure is subjected to a prescribed displacement u_i^d on a part Γ_u and to surface force density F_i^d on the complementary part Γ_σ of its external boundary. The interior fluid domain is denoted by Ω_F and the fluid–structure interface by Σ (see Fig. 2).

The electric boundary conditions are defined by a prescribed electric potential ψ^d on Γ_ψ and a surface density of electric charge Q^d on the remaining part Γ_D . Thus, the total structure boundary, denoted $\partial\Omega_S$, is such that $\partial\Omega_S = \Gamma_D \cup \Gamma_\psi = \Gamma_u \cup \Gamma_\sigma \cup \Sigma$ with $\Gamma_u \cap \Gamma_\sigma \cap \Sigma = \Gamma_\psi \cap \Gamma_D = \emptyset$.

The linearized deformation tensor is denoted by ε_{ij} and the corresponding stress tensor by σ_{ij} . Moreover, D_i denotes the electric displacement vector components and E_i the electric field vector components. ρ_S is the mass density of the structure and n_i^S is the unit normal external to Ω_S .

Since the compressible fluid is assumed to be inviscid, instead of describing its motion by a fluid displacement vector field, which requires an appropriate discretization of the fluid irrotationality constraint [16], we use the pressure scalar field p . Let us note c_F the constant speed of sound in the fluid, ρ_F the mass density of the fluid, and n_i the unit normal external to Ω_F .

The local equations of the fluid/piezoelectric-structure coupled problem described in Fig. 2 are

$$\sigma_{ij,j} = \rho_S \frac{\partial^2 u_i}{\partial t^2} \quad \text{in } \Omega_S, \quad (1a)$$

$$\sigma_{ij} n_j^S = F_i^d \quad \text{on } \Gamma_\sigma, \quad (1b)$$

$$u_i = u_i^d \quad \text{on } \Gamma_u, \quad (1c)$$

$$\sigma_{ij} n_j^S = p n_i \quad \text{on } \Sigma, \quad (1d)$$

$$D_{i,i} = 0 \quad \text{in } \Omega_S, \quad (2a)$$

$$D_i n_i^S = -Q^d \quad \text{on } \Gamma_D, \quad (2b)$$

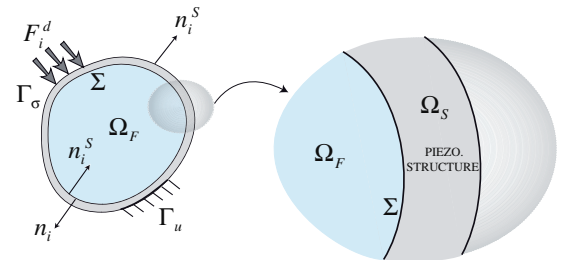


Fig. 2. Fluid/piezoelectric-structure coupled system.

$$\psi = \psi^d \quad \text{on } \Gamma_\psi, \quad (2c)$$

$$p_{,ii} = \frac{1}{c_F^2} \frac{\partial^2 p}{\partial t^2} \quad \text{in } \Omega_F, \quad (3a)$$

$$p_{,i} n_i = -\rho_F \frac{\partial^2 u_i}{\partial t^2} n_i \quad \text{on } \Sigma, \quad (3b)$$

with appropriate initial conditions.

Eq. (1a) corresponds to the elastodynamic equation in absence of body force; Eqs. (1b) and (1c) are the prescribed mechanical boundary conditions; Eq. (1d) results from the action of pressure forces exerted by the fluid on the structure; Eq. (2a) corresponds to the electric charge equation for a dielectric medium; Eqs. (2b) and (2c) are the previously described electric boundary conditions; Eq. (3a) is the Helmholtz equation; and Eq. (3b) is the kinematic interface fluid–structure condition on Σ .

The stress tensor σ_{ij} and electric displacement D_i are related to the linear strain tensor ε_{kl} and electric field E_k through the converse and direct linear piezoelectric constitutive equations:

$$\sigma_{ij} = c_{ijkl} \varepsilon_{kl} - e_{kij} E_k, \quad (4)$$

$$D_i = e_{ikl} \varepsilon_{kl} + \varepsilon_{ik} E_k, \quad (5)$$

where c_{ijkl} , e_{kij} and ε_{ik} denote elastic, piezoelectric and dielectric material constants.

Moreover, we have the following gradient relations between the linearized strain tensor ε_{kl} and the displacement u_k , and between the electric field E_k and the electric potential ψ :

$$\varepsilon_{kl} = \frac{1}{2} (u_{k,l} + u_{l,k}), \quad (6)$$

$$E_k = -\psi_{,k}. \quad (7)$$

We can note from the constitutive Eqs. (4) and (5) and from the gradient relations (6) and (7) that the stress tensor σ_{ij} and the electric displacement vector components D_i depend on the variables u_k and ψ :

$$\sigma_{ij} = \sigma_{ij}(u_k, \psi), \quad (8a)$$

$$D_i = D_i(u_k, \psi). \quad (8b)$$

For a detailed derivation of these classical equations, we refer the reader, for example, to [17] for piezoelectric aspects and to [15] for fluid–structure aspects.

2.2. Variational formulation

The local equations of Section 2.1 are expressed in terms of the chosen unknown fields of the fluid/piezoelectric-structure boundary value problem, i.e. the structural mechanical displacement u_i , the electric potential in the structure ψ , and the fluid pressure p .

In order to obtain the variational formulation associated with the local equations of the coupled fluid/piezoelectric-structure system given in Eqs. (1)–(3), the test-function method is applied. We proceed in three steps, successively considering the equations relating to the structure (subject

to fluid pressure actions), the electric charge equation for a dielectric medium, and the equations relating to the fluid (subject to a wall displacement).

First, we introduce the space C_u of sufficiently regular functions u_i defined in Ω_S and $C_u^* = \{u_i \in C_u \mid u_i = 0 \text{ on } \Gamma_u\}$. Multiplying Eq. (1a) by any time-independent test-function $\delta u_i \in C_u^*$, then applying Green's formula, and finally taking Eqs. (1b) and (1d) into account, leads to

$$\begin{aligned} & \int_{\Omega_S} \sigma_{ij}(u_k, \psi) \delta \varepsilon_{ij} dv + \rho_S \int_{\Omega_S} \frac{\partial^2 u_i}{\partial t^2} \delta u_i dv - \int_{\Sigma} p n_i \delta u_i ds \\ & = \int_{\Gamma_\sigma} F_i^d \delta u_i ds \quad \forall \delta u_i \in C_u^*, \end{aligned} \quad (9)$$

where $\delta \varepsilon_{ij} = \frac{1}{2} (\delta u_{i,j} + \delta u_{j,i})$.

Secondly, we consider the space C_ψ of sufficiently regular functions ψ in Ω_S and $C_\psi^* = \{\psi \in C_\psi \mid \psi = 0 \text{ on } \Gamma_\psi\}$. Multiplying Eq. (2a) by any time-independent test-function $\delta \psi \in C_\psi^*$ and integrating over Ω_S , we have

$$\int_{\Omega_S} D_{i,i}(u_k, \psi) \delta \psi dv = 0 \quad \forall \delta \psi \in C_\psi^*. \quad (10)$$

Finally, we consider the space C_p of sufficiently regular functions p defined in Ω_F . Multiplying Eq. (3a) by any time-independent test-function $\delta p \in C_p$, applying Green's formula, and taking Eq. (3b) into account, we obtain

$$\begin{aligned} & \frac{1}{\rho_F} \int_{\Omega_F} p_{,i} \delta p_{,i} dv + \frac{1}{\rho_F c_F^2} \int_{\Omega_F} \frac{\partial^2 p}{\partial t^2} \delta p dv \\ & + \int_{\Sigma} \frac{\partial^2 u_i}{\partial t^2} n_i \delta p ds = 0 \quad \forall \delta p \in C_p. \end{aligned} \quad (11)$$

Using the piezoelectric constitutive Eqs. (4) and (5), and taking the electric boundary condition (2b) into account, Eqs. (9) and (10) can be rewritten in the following forms

$$\begin{aligned} & \int_{\Omega_S} c_{ijkl} \varepsilon_{kl} \delta \varepsilon_{ij} dv - \int_{\Omega_S} e_{kij} E_k \delta \varepsilon_{ij} dv + \rho_S \int_{\Omega_S} \frac{\partial^2 u_i}{\partial t^2} \delta u_i dv \\ & - \int_{\Sigma} p n_i \delta u_i ds = \int_{\Gamma_\sigma} F_i^d \delta u_i ds \quad \forall \delta u_i \in C_u^*, \end{aligned} \quad (12)$$

and

$$\begin{aligned} & \int_{\Omega_S} e_{ikl} \varepsilon_{kl} \delta E_i dv + \int_{\Omega_S} \varepsilon_{ik} E_k \delta E_i dv \\ & = \int_{\Gamma_D} Q^d \delta \psi ds \quad \forall \delta \psi \in C_\psi^*, \end{aligned} \quad (13)$$

where $\delta E_i = -\delta \psi_{,i}$.

Thus, the variational formulation of the coupled fluid/piezoelectric-structure coupled problem consists in finding $u_i \in C_u$ such that $u_i = u_i^d$ on Γ_u , $\psi \in C_\psi$ such that $\psi = \psi^d$ on Γ_ψ and $p \in C_p$, satisfying Eqs. (11)–(13) with appropriate initial conditions.

2.3. Finite element formulation

Let us introduce \mathbf{U} , Ψ and \mathbf{P} corresponding to the vectors of nodal values of u_i , ψ and p respectively, and the

submatrices corresponding to the various terms involved in Eqs. (11)–(13) defined by

$$\int_{\Omega_S} c_{ijkl} \varepsilon_{kl} \delta \varepsilon_{ij} dv \Rightarrow \delta \mathbf{U}^T \mathbf{K}_u \mathbf{U}, \quad (14a)$$

$$\rho_S \int_{\Omega_S} u_i \delta u_i dv \Rightarrow \delta \mathbf{U}^T \mathbf{M}_u \mathbf{U}, \quad (14b)$$

$$\int_{\Omega_S} e_{kij} E_k \delta \varepsilon_{ij} dv \Rightarrow \delta \mathbf{U}^T \mathbf{C}_{u\psi} \boldsymbol{\Psi}, \quad (14c)$$

$$\int_{\Omega_S} e_{ikl} \varepsilon_{kl} \delta E_i dv \Rightarrow \delta \boldsymbol{\Psi}^T \mathbf{C}_{u\psi}^T \mathbf{U}, \quad (14d)$$

$$\int_{\Sigma} p n_i \delta u_i ds \Rightarrow \delta \mathbf{U}^T \mathbf{C}_{up} \mathbf{P}, \quad (14e)$$

$$\int_{\Sigma} u_i n_i \delta p ds \Rightarrow \delta \mathbf{P}^T \mathbf{C}_{up}^T \mathbf{U}, \quad (14f)$$

$$\frac{1}{\rho_F} \int_{\Omega_F} p_{,i} \delta p_{,i} dv \Rightarrow \delta \mathbf{P}^T \mathbf{K}_p \mathbf{P}, \quad (14g)$$

$$\frac{1}{\rho_F c_F^2} \int_{\Omega_F} p \delta p dv \Rightarrow \delta \mathbf{P}^T \mathbf{M}_p \mathbf{P}, \quad (14h)$$

$$\int_{\Omega_S} \varepsilon_{ik} E_k \delta E_i dv \Rightarrow \delta \boldsymbol{\Psi}^T \mathbf{K}_\psi \boldsymbol{\Psi}, \quad (14i)$$

$$\int_{\Gamma_\sigma} F_i^d \delta u_i ds \Rightarrow \delta \mathbf{U}^T \mathbf{F}, \quad (14j)$$

$$\int_{\Gamma_D} \mathcal{Q}^d \delta \psi ds \Rightarrow \delta \boldsymbol{\Psi}^T \mathbf{Q}, \quad (14k)$$

where \mathbf{M}_u and \mathbf{K}_u are the mass and stiffness matrices of the structure; $\mathbf{C}_{u\psi}$ is the electro-mechanical coupled stiffness matrix; \mathbf{K}_ψ is the electric stiffness matrix; \mathbf{M}_p and \mathbf{K}_p are the mass and stiffness matrices of the fluid; \mathbf{C}_{up} is the fluid–structure coupled matrix; \mathbf{F} and \mathbf{Q} are the applied mechanical force and charge vectors respectively.

Thus, the variational equations (11)–(13) for the fluid/piezoelectric-structure coupled problem can be written, in discretized form, as the following unsymmetric matrix system:

$$\begin{bmatrix} \mathbf{M}_u & 0 & 0 \\ 0 & 0 & 0 \\ \mathbf{C}_{up}^T & 0 & \mathbf{M}_p \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{U}} \\ \ddot{\boldsymbol{\Psi}} \\ \ddot{\mathbf{P}} \end{bmatrix} + \begin{bmatrix} \mathbf{K}_u & -\mathbf{C}_{u\psi} & -\mathbf{C}_{up} \\ \mathbf{C}_{u\psi}^T & \mathbf{K}_\psi & 0 \\ 0 & 0 & \mathbf{K}_p \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \boldsymbol{\Psi} \\ \mathbf{P} \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \mathbf{Q} \\ 0 \end{bmatrix} \quad (15)$$

with appropriate initial conditions.

3. Coupled fluid/piezoelectric-structure eigenvalue problem

In order to compute the natural vibration modes of the fluid/piezoelectric-structure coupled problem, we consider harmonic solutions for the structure displacement u_i , electric potential ψ , and fluid pressure p , i.e. we assume the following form for u_i , ψ and p :

$$u_i(x, t) = u_i(x) \exp(i\omega t) \quad \text{for } x \in \Omega_S, \quad t \geq 0,$$

$$\psi(x, t) = \psi(x) \exp(i\omega t) \quad \text{for } x \in \Omega_S, \quad t \geq 0,$$

$$p(x, t) = p(x) \exp(i\omega t) \quad \text{for } x \in \Omega_F, \quad t \geq 0,$$

where $\omega > 0$ is the circular frequency. Note that, for sake of simplicity, we keep the same notation for the variables in terms of (x, t) and those only in terms of (x) .

Substituting these expressions in Eqs. (11)–(13), the variational formulation of the fluid/piezoelectric-structure spectral problem consists in finding $\omega \in \mathbb{R}^+$ and $(u_i, \psi, p) \in (C_u^*, C_\psi^*, C_p)$, such that $\forall (\delta u_i, \delta \psi, \delta p) \in (C_u^*, C_\psi^*, C_p)$:

$$\int_{\Omega_S} c_{ijkl} \varepsilon_{kl} \delta \varepsilon_{ij} dv - \int_{\Omega_S} e_{kij} E_k \delta \varepsilon_{ij} dv - \int_{\Sigma} p n_i \delta u_i ds - \omega^2 \rho_S \int_{\Omega_S} u_i \delta u_i dv = 0, \quad (16)$$

$$\int_{\Omega_S} e_{ikl} \varepsilon_{kl} \delta E_i dv + \int_{\Omega_S} \varepsilon_{ik} E_k \delta E_i dv = 0, \quad (17)$$

$$\frac{1}{\rho_F} \int_{\Omega_F} p_{,i} \delta p_{,i} dv - \frac{\omega^2}{\rho_F c_F^2} \int_{\Omega_F} p \delta p dv - \omega^2 \int_{\Sigma} u_i n_i \delta p ds = 0. \quad (18)$$

In discretized form, the previous variational formulation can be written as

$$\left(\begin{bmatrix} \mathbf{K}_u & -\mathbf{C}_{u\psi} & -\mathbf{C}_{up} \\ \mathbf{C}_{u\psi}^T & \mathbf{K}_\psi & 0 \\ 0 & 0 & \mathbf{K}_p \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M}_u & 0 & 0 \\ 0 & 0 & \mathbf{0} \\ \mathbf{C}_{up}^T & 0 & \mathbf{M}_p \end{bmatrix} \right) \begin{bmatrix} \mathbf{U} \\ \boldsymbol{\Psi} \\ \mathbf{P} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}. \quad (19)$$

Using the second row of Eq. (19), the degrees-of-freedom associated with the electric potential can be expressed in terms of structure displacements as

$$\boldsymbol{\Psi} = -\mathbf{K}_\psi^{-1} \mathbf{C}_{u\psi}^T \mathbf{U}. \quad (20)$$

Thus, after substitution of $\boldsymbol{\Psi}$ into Eq. (19), we get the following spectral problem in terms of \mathbf{U} and \mathbf{P}

$$\begin{bmatrix} \mathbf{K}_u + \mathbf{K}^A & -\mathbf{C}_{up} \\ 0 & \mathbf{K}_p \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M}_u & 0 \\ \mathbf{C}_{up}^T & \mathbf{M}_p \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \mathbf{P} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad (21)$$

where the ‘‘added-stiffness matrix’’ \mathbf{K}^A , which is due to the electromechanical coupling [18], is given by

$$\mathbf{K}^A = \mathbf{C}_{u\psi} \mathbf{K}_\psi^{-1} \mathbf{C}_{u\psi}^T. \quad (22)$$

Remark. When the piezoelectric coupling constants are set to zero, i.e. $\mathbf{K}^A = 0$, we obtained the unsymmetric $(u_i; p)$ classical fluid–structure system:

$$\left(\begin{bmatrix} \mathbf{K}_u & -\mathbf{C}_{up} \\ 0 & \mathbf{K}_p \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M}_u & 0 \\ \mathbf{C}_{up}^T & \mathbf{M}_p \end{bmatrix} \right) \begin{bmatrix} \mathbf{U} \\ \mathbf{P} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}. \quad (23)$$

4. Symmetrized finite element formulation for the fluid/piezoelectric-structure eigenvalue coupled problem

In the previous section, a non-symmetric eigenvalue formulation has been obtained. The aim of this section is to establish a symmetric formulation of the fluid/piezoelec-

tric-structure coupled problem in view of a direct treatment by finite elements. This symmetrization is obtained through introduction of an intermediate unknown field [15], namely the fluid displacement potential field φ , defined up to an additive constant, such that $u_i^F = \varphi_{,i}$ (u_i^F is the fluid displacement). We shall see in the next section that this problem can be reduced, by means of an elimination procedure, to an eigenvalue problem in terms of $(u_i, \psi; p)$.

The equations of the coupled spectral problem in terms of $(u_i, \psi; p, \varphi)$ can be rewritten as follows:

$$\sigma_{ij,j} + \omega^2 \rho_S u_i = 0 \quad \text{in } \Omega_S, \quad (24a)$$

$$\sigma_{ij} n_j^S = 0 \quad \text{on } \Gamma_\sigma, \quad (24b)$$

$$u_i = 0 \quad \text{on } \Gamma_u, \quad (24c)$$

$$\sigma_{ij} n_j^S = \omega^2 \rho_F \varphi n_i \quad \text{on } \Sigma, \quad (24d)$$

$$D_{i,i} = 0 \quad \text{in } \Omega_S, \quad (25a)$$

$$D_i n_i^S = 0 \quad \text{on } \Gamma_D, \quad (25b)$$

$$\psi = 0 \quad \text{on } \Gamma_\psi, \quad (25c)$$

$$\rho_F \varphi_{,ii} + \frac{p}{c_F^2} = 0 \quad \text{in } \Omega_F, \quad (26a)$$

$$\varphi_{,i} n_i = u_i n_i \quad \text{on } \Sigma, \quad (26b)$$

$$\frac{p}{\rho_F c_F^2} = \frac{\omega^2}{c_F^2} \varphi \quad \text{in } \Omega_F. \quad (26c)$$

The corresponding variational formulation, involving structure displacement, electric potential, fluid displacement potential, and pressure can be easily obtained as described below:

- Multiplying Eq. (24a) by $\delta u_i \in C_u^*$, integrating by parts over Ω_S (using Green's formula), taking into account the boundary conditions (Eqs. (24b)–(24d)) and using the piezoelectric constitutive equation (Eq. (4)), leads to

$$\int_{\Omega_S} c_{ijkl} \varepsilon_{kl} \delta \varepsilon_{ij} dv - \int_{\Omega_S} e_{kij} E_k \delta \varepsilon_{ij} dv - \omega^2 \rho_S \int_{\Omega_S} u_i \delta u_i dv - \omega^2 \rho_F \int_{\Sigma} \varphi n_i \delta u_i ds = 0 \quad \forall \delta u_i \in C_u^*. \quad (27)$$

- Multiplying Eq. (25a) by $\delta \psi \in C_\psi^*$, integrating over Ω_S , taking into account the boundary conditions (Eqs. (25b) and (25c)) and using the piezoelectric constitutive equation (Eq. (5)), we have

$$\int_{\Omega_S} e_{ikl} \varepsilon_{kl} \delta E_i dv + \int_{\Omega_S} \epsilon_{ik} E_k \delta E_i dv = 0 \quad \forall \delta \psi \in C_\psi^*. \quad (28)$$

- Multiplying Eq. (26c) by a test-function $\delta p \in C_p$, integrating over Ω_F , we have

$$\frac{1}{\rho_F c_F^2} \int_{\Omega_F} p \delta p dv - \frac{\omega^2}{c_F^2} \int_{\Omega_F} \varphi \delta p dv = 0 \quad \forall \delta p \in C_p. \quad (29)$$

- Finally, we consider the space C_φ of sufficiently regular functions φ defined in Ω_F . Multiplying Eq. (26a) by

$\delta \varphi \in C_\varphi$, integrating over Ω_F , applying Green's formula, and taking Eq. (26b) into account, we obtain

$$-\rho_F \int_{\Omega_F} \varphi_{,i} \delta \varphi_{,i} dv + \rho_F \int_{\Sigma} u_i n_i \delta \varphi ds + \frac{1}{c_F^2} \int_{\Omega_F} p \delta \varphi dv = 0 \quad \forall \delta \varphi \in C_\varphi. \quad (30)$$

Then, the variational formulation consists in finding $\omega \in \mathbb{R}^+$ and $(u_i, \psi, p, \varphi) \in (C_u^*, C_\psi^*, C_p, C_\varphi)$ satisfying Eqs. (27)–(30).

The discretization of the preceding formulation leads to the following symmetric matrix equation:

$$\left(\begin{bmatrix} \mathbf{K}_u & -\mathbf{C}_{u\psi} & 0 & 0 \\ -\mathbf{C}_{u\psi}^T & -\mathbf{K}_\psi & 0 & 0 \\ 0 & 0 & \mathbf{M}_p & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M}_u & 0 & 0 & \mathbf{A} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathbf{B} \\ \mathbf{A}^T & 0 & \mathbf{B}^T & -\mathbf{F}_\varphi \end{bmatrix} \right) \times \begin{bmatrix} \mathbf{U} \\ \mathbf{\Psi} \\ \mathbf{P} \\ \mathbf{\Phi} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad (31)$$

where \mathbf{U} , $\mathbf{\Psi}$, \mathbf{P} and $\mathbf{\Phi}$ are the vectors of nodal values of u_i , ψ , p and φ respectively, and where the not yet defined submatrices of Eq. (31) are given by:

$$\rho_F \int_{\Sigma} \varphi n_i \delta u_i ds \Rightarrow \delta \mathbf{U}^T \mathbf{A} \mathbf{\Phi}, \quad (32a)$$

$$\rho_F \int_{\Sigma} u_i n_i \delta \varphi ds \Rightarrow \delta \mathbf{\Phi}^T \mathbf{A}^T \mathbf{U}, \quad (32b)$$

$$\frac{1}{c_F^2} \int_{\Omega_F} \varphi \delta p dv \Rightarrow \delta \mathbf{P}^T \mathbf{B} \mathbf{\Phi}, \quad (32c)$$

$$\frac{1}{c_F^2} \int_{\Omega_F} p \delta \varphi dv \Rightarrow \delta \mathbf{\Phi}^T \mathbf{B}^T \mathbf{P}, \quad (32d)$$

$$\rho_F \int_{\Omega_F} \varphi_{,i} \delta \varphi_{,i} dv \Rightarrow \delta \mathbf{\Phi}^T \mathbf{F}_\varphi \mathbf{\Phi}. \quad (32e)$$

After elimination of degrees-of-freedom associated with the electric potential $\mathbf{\Psi}$ using the second row of Eq. (31), we obtain the following symmetric system in terms of \mathbf{U} , \mathbf{P} and $\mathbf{\Phi}$:

$$\left(\begin{bmatrix} \mathbf{K}_u + \mathbf{K}^A & 0 & 0 \\ 0 & \mathbf{M}_p & 0 \\ 0 & 0 & 0 \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M}_u & 0 & \mathbf{A} \\ 0 & 0 & \mathbf{B} \\ \mathbf{A}^T & \mathbf{B}^T & -\mathbf{F}_\varphi \end{bmatrix} \right) \begin{bmatrix} \mathbf{U} \\ \mathbf{P} \\ \mathbf{\Phi} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad (33)$$

where \mathbf{K}^A is given by Eq. (22).

We recall that the matrix \mathbf{K}^A , which is due to the piezoelectric properties of the structure, is added to the structural elastic stiffness matrix. The main advantage of Eq. (33), when compared to Eq. (19), is the possibility of using symmetric eigenvalue solvers. The drawback is naturally the doubling of fluid degrees-of-freedom. In fact, this disadvantage can be avoided following the rigorous condensation procedure described in the next section.

5. Symmetric reduced model for fluid/piezoelectric-structure eigenvalue coupled problem

In this section, in order to obtain a symmetric reduced model for the fluid/piezoelectric-structure coupled problem, we propose to eliminate the fluid displacement potential Φ in the symmetric formulation given in the last section.

First, the third row of Eq. (33) is written:

$$\mathbf{A}^T \mathbf{U} + \mathbf{B}^T \mathbf{P} - \mathbf{F}_\varphi \Phi = 0. \quad (34)$$

In Eq. (34), the matrix \mathbf{F}_φ is singular and consequently Φ cannot be directly expressed in terms of \mathbf{U} and \mathbf{P} . In order to avoid this problem, we use the procedure described by Morand and Ohayon for elasto-acoustic problems [15] as detailed below.

Let us define a partitioning of Φ by letting $\Phi^T = [\Phi_1 \Phi_2^T]$ where Φ_1 denotes a particular component of Φ (here, the first), which induces the following partitioning of the matrices \mathbf{A} , \mathbf{B} and \mathbf{F}_φ in Eq. (33):

$$\left(\begin{array}{cccc} \mathbf{K}_u + \mathbf{K}^A & 0 & 0 & 0 \\ 0 & \mathbf{M}_p & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} - \omega^2 \begin{array}{cccc} \mathbf{M}_u & 0 & \mathbf{A}_1 & \mathbf{A}_2 \\ 0 & 0 & \mathbf{B}_1 & \mathbf{B}_2 \\ \mathbf{A}_1^T & \mathbf{B}_1^T & -F_{\varphi 11} & -F_{\varphi 12} \\ \mathbf{A}_2^T & \mathbf{B}_2^T & -F_{\varphi 21} & -F_{\varphi 22} \end{array} \right) \times \begin{array}{c} \mathbf{U} \\ \mathbf{P} \\ \Phi_1 \\ \Phi_2 \end{array} = \begin{array}{c} 0 \\ 0 \\ 0 \\ 0 \end{array}. \quad (35)$$

$\mathbf{F}_{\varphi 22}$ being nonsingular, Φ_2 can be expressed in terms of \mathbf{U} , \mathbf{P} and Φ_1 using the fourth row of Eq. (35):

$$\Phi_2 = -\mathbf{F}_{\varphi 22}^{-1} \mathbf{F}_{\varphi 21} \Phi_1 + \mathbf{F}_{\varphi 22}^{-1} \mathbf{A}_2^T \mathbf{U} + \mathbf{F}_{\varphi 22}^{-1} \mathbf{B}_2^T \mathbf{P}. \quad (36)$$

Considering this relation, the third row of Eq. (35) becomes:

$$\begin{aligned} & (\mathbf{A}_1^T - \mathbf{F}_{\varphi 12} \mathbf{F}_{\varphi 22}^{-1} \mathbf{A}_2^T) \mathbf{U} + (\mathbf{B}_1^T - \mathbf{F}_{\varphi 12} \mathbf{F}_{\varphi 22}^{-1} \mathbf{B}_2^T) \mathbf{P} \\ & = (F_{\varphi 11} - \mathbf{F}_{\varphi 12} \mathbf{F}_{\varphi 22}^{-1} \mathbf{F}_{\varphi 21}) \Phi_1. \end{aligned} \quad (37)$$

Moreover, it can be shown that:

$$F_{\varphi 11} - \mathbf{F}_{\varphi 12} \mathbf{F}_{\varphi 22}^{-1} \mathbf{F}_{\varphi 21} = 0 \quad (38)$$

because the kernel vector of \mathbf{F}_φ is constant (see Ref. [15, Chapter 8, p. 158]).

Thus, Eq. (37) can be written as:

$$\mathbf{a}^T \mathbf{U} + \mathbf{b}^T \mathbf{P} = 0, \quad (39)$$

where matrices \mathbf{a} and \mathbf{b} are defined by:

$$\mathbf{a}^T = \mathbf{A}_1^T - \mathbf{F}_{\varphi 12} \mathbf{F}_{\varphi 22}^{-1} \mathbf{A}_2^T, \quad (40a)$$

$$\mathbf{b}^T = \mathbf{B}_1^T - \mathbf{F}_{\varphi 12} \mathbf{F}_{\varphi 22}^{-1} \mathbf{B}_2^T. \quad (40b)$$

Finally, replacing Φ_2 by its expression Eq. (36) in the two first rows of Eq. (35) and using Eq. (39), the following matrix system is obtained:

$$\left(\begin{array}{ccc} \mathbf{K}_u + \mathbf{K}^A & 0 & 0 \\ 0 & \mathbf{M}_p & 0 \\ 0 & 0 & 0 \end{array} - \omega^2 \begin{array}{ccc} \mathbf{M}_u + \mathbf{A}_2 \mathbf{F}_{\varphi 22}^{-1} \mathbf{A}_2^T & \mathbf{A}_2 \mathbf{F}_{\varphi 22}^{-1} \mathbf{B}_2^T & \mathbf{a} \\ \mathbf{B}_2 \mathbf{F}_{\varphi 22}^{-1} \mathbf{A}_2^T & \mathbf{B}_2 \mathbf{F}_{\varphi 22}^{-1} \mathbf{B}_2^T & \mathbf{b} \\ \mathbf{a}^T & \mathbf{b}^T & 0 \end{array} \right) \times \begin{array}{c} \mathbf{U} \\ \mathbf{P} \\ \Phi_1 \end{array} = \begin{array}{c} 0 \\ 0 \\ 0 \end{array}, \quad (41)$$

which exhibits the matrix \mathbf{M}^S , called the ‘‘generalized added-mass operator’’ in the compressible case, which is expressed as

$$\mathbf{M}^S = \begin{array}{ccc} \mathbf{A}_2 \mathbf{F}_{\varphi 22}^{-1} \mathbf{A}_2^T & \mathbf{A}_2 \mathbf{F}_{\varphi 22}^{-1} \mathbf{B}_2^T & \mathbf{a} \\ \mathbf{B}_2 \mathbf{F}_{\varphi 22}^{-1} \mathbf{A}_2^T & \mathbf{B}_2 \mathbf{F}_{\varphi 22}^{-1} \mathbf{B}_2^T & \mathbf{b} \\ \mathbf{a}^T & \mathbf{b}^T & 0 \end{array}. \quad (42)$$

We note that the relation $\mathbf{a}^T \mathbf{U} + \mathbf{b}^T \mathbf{P} = 0$, corresponding to the last row of Eq. (41), is the discretization of the variational property (30) for $\delta\varphi$ constant:

$$\rho_F \int_{\Sigma} u_i n_i ds + \frac{1}{c_F^2} \int_{\Omega_F} p dv = 0. \quad (43)$$

This latter equation results from the global mass conservation in Ω_F .

6. Numerical results

We present in this section some finite element results, obtained with the previous formulation, for the free vibration analysis of elastic/piezoelectric structures coupled with compressible fluid. These results are computed via a specific Matlab software developed by the authors for axisymmetric geometries. The structure is discretized using an axisymmetric laminated piezoelectric conical shell element. This two-node element is based on the Kirchhoff–Love theory and combines an equivalent single layer approach for the mechanical behavior with a layerwise representation of the electric potential in the thickness direction. The fluid domain is discretized with quadrilateral axisymmetric elements. Moreover, a semi-analytical procedure combining the finite element method and Fourier series expansion in the circumferential direction, is used [19].

The first example concerns the free vibration of an axisymmetric elastic tank filled with fluid. In this example, we analyse the added mass effect of the fluid on the natural vibration of the coupled system. The second example, which concerns the free vibration of a simply supported piezoelectric cylindrical shell filled with compressible fluid, illustrates (i) the added stiffness effect due to the piezoelectric material and (ii) the added mass effect due to the fluid.

6.1. Free vibration of an axisymmetric tank filled with fluid

This example concerns the vibration analysis of an axisymmetric tank clamped along its central circumference edge and completely filled with a compressible fluid. The geometry of the tank is presented in Fig. 3 and the follow-

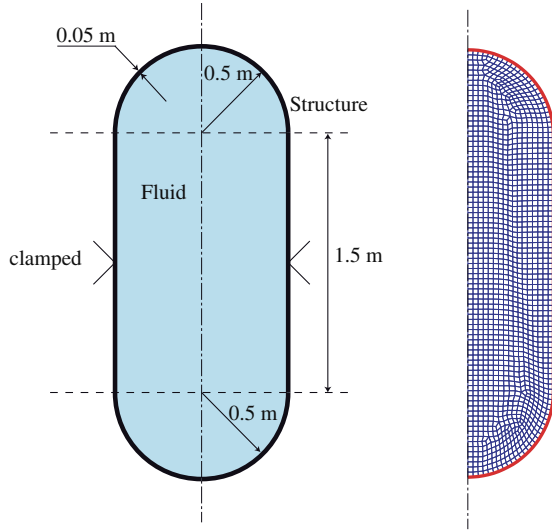


Fig. 3. Axisymmetric tank filled with fluid: geometrical data and meshes.

ing mechanical properties are used: Young's modulus $E = 210 \text{ GPa}$, Poisson's ratio $\nu = 0.3$ and mass density $\rho_S = 8000 \text{ kg/m}^3$. The considered fluid has a mass density $\rho_F = 565 \text{ kg/m}^3$ and a speed of sound $c_F = 730 \text{ m/s}$.

Table 1 and Figs. 4–7 present the natural frequencies and the mode shapes (fluid pressure and shell displacement) of (i) the fluid in a rigid cavity, (ii) the structure without fluid and (iii) the fluid–structure coupled system. All of these eigenmodes are computed for harmonic $n = 1$. The 3D mode shapes of the structure in uncoupled (Fig. 5)

Table 1

Frequencies (Hz) of (i) the fluid in a rigid cavity, (ii) the structure without fluid, and (iii) the fluid–structure coupled system, for harmonic $n = 1$

Fluid (F) in a rigid cavity	Structure (S) without fluid	Coupled (C) problem
$F_1 = 438.1$	$S_1 = 363.1$	$C_1 = 311.3$
$F_2 = 487.1$	$S_2 = 363.1$	$C_2 = 316.0$
$F_3 = 577.7$	$S_3 = 1052.2$	$C_3 = 470.5$
$F_4 = 693.7$	$S_4 = 1052.2$	$C_4 = 537.6$
$F_5 = 817.9$	$S_5 = 1372.8$	$C_5 = 595.9$
$F_6 = 940.6$	$S_6 = 1372.8$	$C_6 = 672.7$

and coupled (Fig. 7) cases have been constructed from the 1D element using the Fourier series expansion of the displacement field in the circumferential direction.

From the second column of Table 1 and from Fig. 5, it can be seen that there are two mode shapes at each natural frequency for the tank without fluid. This is due to the boundary condition at the centre of the tank.

Table 1 and Figs. 5 and 7 show that the eigenmodes C_1 and C_2 are structural modes corresponding to the first two first modes S_1 and S_2 of the empty shell. In fact, the mode shapes C_1 and C_2 are the sum of the mode shapes S_1 and S_2 in two different geometrical configurations. This is due to the presence of the fluid which couples, through the added mass effect, the mode shapes associated with the double frequencies 363.1 Hz. Moreover, the eigenmodes C_3, C_4, C_5 and C_6 correspond to the first four eigenmodes of the fluid in the rigid cavity (F_1, F_2, F_3, F_4) with a similar shape but different frequencies.

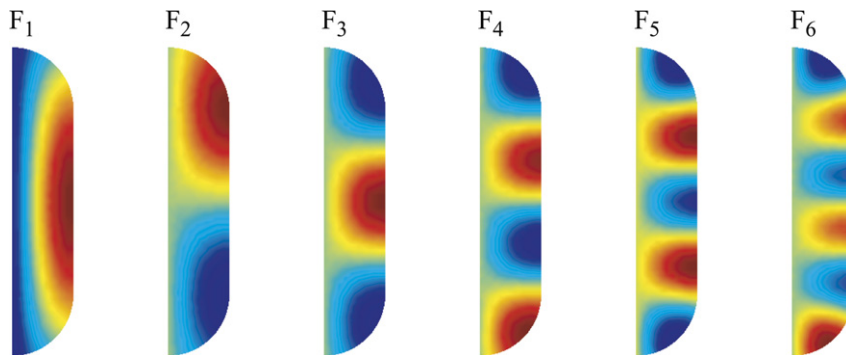


Fig. 4. First six mode shapes of the fluid in rigid cavity for harmonic $n = 1$.

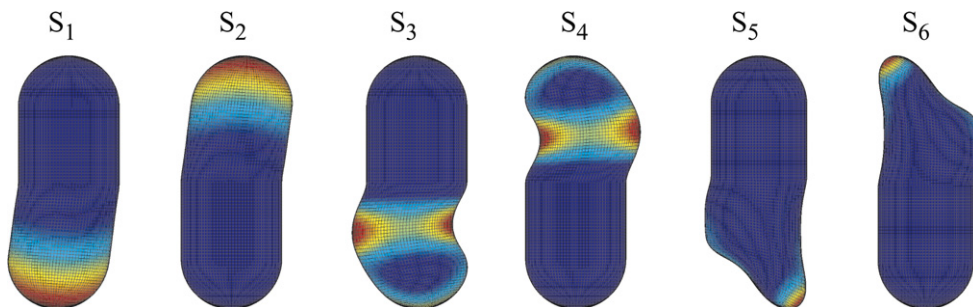


Fig. 5. First six mode shapes of the structure without fluid for harmonic $n = 1$.

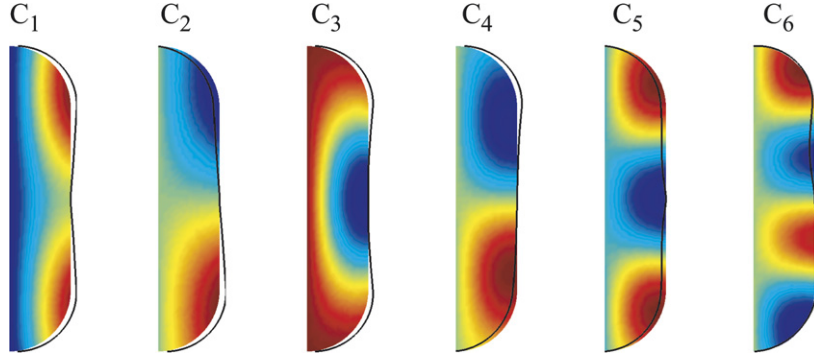


Fig. 6. Fluid–structure coupled modes: 2D fluid pressure and shell displacement.

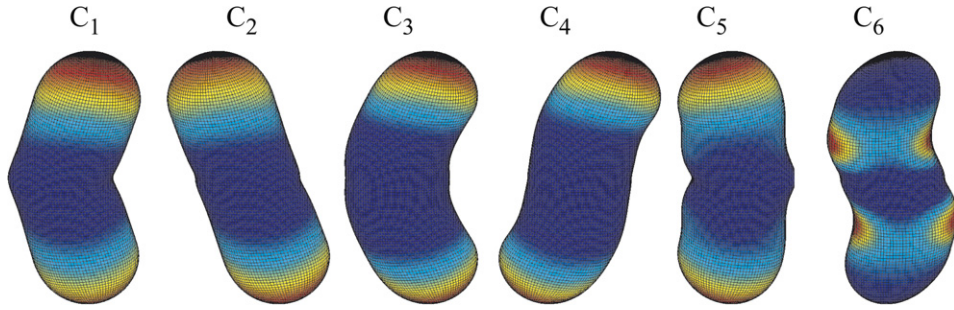


Fig. 7. Fluid–structure coupled modes: 3D shell displacement.

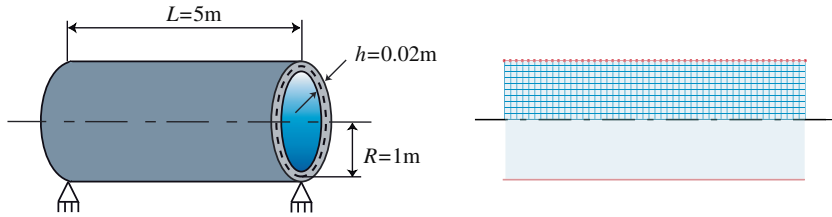


Fig. 8. Geometrical data and meshes of the piezoelectric cylindrical shell filled with compressible fluid.

6.2. Free vibration of a piezoelectric cylindrical shell filled with a compressible fluid

We present in this second example a comparative study of a simply supported piezoelectric cylindrical shell filled with compressible fluid. The geometrical properties are $L = 5$ m, $R = 1$ m and $h = 0.02$ m (Fig. 8). Moreover, the piezoelectric material is the PZT-5H [20] and the considered fluid has a mass density $\rho_F = 1000$ kg/m³ and a speed of sound $c_F = 1500$ m/s. Note that a null pressure is prescribed at both ends of the fluid cylinder.

Table 2 presents the frequencies of the empty piezoelectric cylinder calculated by our finite element method and those given by an exact three-dimensional solution proposed by the authors [20]. This exact solution is based on a mixed state-space approach previously developed for the free-vibration analysis of laminated piezoelectric plates actuated by transverse shear mechanisms [21].

Table 2

Frequencies (Hz) of a piezoelectric cylindrical shell: comparison with an exact state space solution [20]

n	Short circuited			Open circuited		
	Present	Exact	Error %	Present	Exact	Error %
1	87.81	88.23	0.48	93.37	93.79	0.45
	216.13	216.26	0.06	228.16	228.23	0.03
	278.71	275.96	-1.00	278.71	275.96	-1.00
	312.56	311.65	-0.29	331.70	330.65	-0.32
	368.99	366.97	-0.55	383.70	392.13	2.15
2	36.70	37.32	1.64	39.49	40.11	1.55
	113.54	114.21	0.59	121.98	122.68	0.57
	193.36	193.79	0.22	207.47	207.92	0.21
	259.39	259.35	-0.02	278.51	278.43	-0.03
	308.94	308.46	-0.15	332.12	331.55	-0.17

As it can be observed from this table, there is a good agreement between finite element and exact solutions for

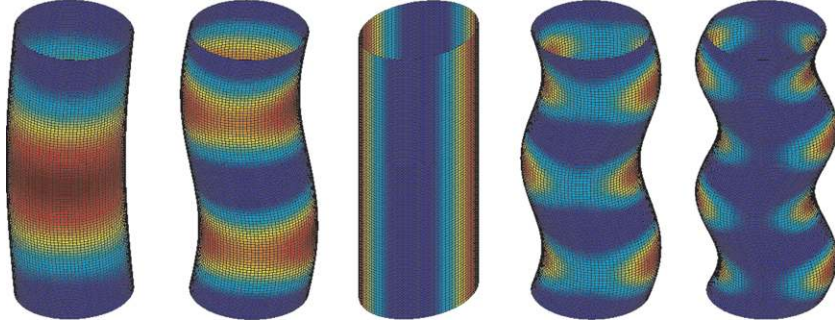


Fig. 9. First five mode shapes of the cylindrical shell for harmonic $n = 1$ in the short-circuited case.

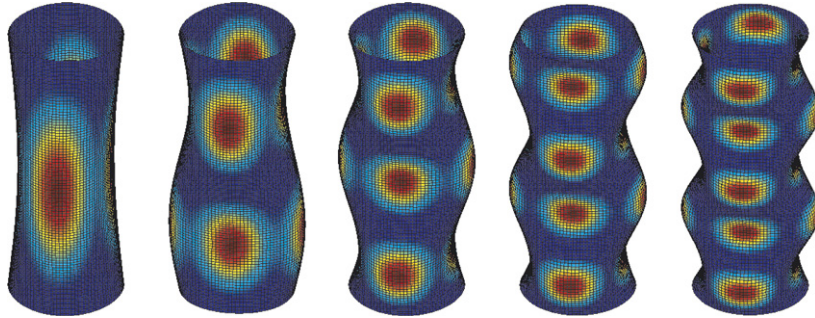


Fig. 10. First five mode shapes of the cylindrical shell for harmonic $n = 2$ in the short-circuited case.

different electric boundary conditions corresponding to short-circuited ($\psi^d = 0$ on $\partial\Omega_S$) or open-circuited ($Q^d = 0$ on $\partial\Omega_S$) configurations. These boundary conditions are prescribed on the inner and outer surface of the piezoelectric shell. Note that the error committed by the finite element approximation is lower than 3%. We can observe that the frequency of the third mode for $n = 1$ (278.71 Hz) is not influenced by the electric boundary conditions. This is due to the fact that this axial mode does not induce any electromechanical coupling due to the radial electric polarization. For the other modes, as expected, the natural frequencies are higher in the open-circuit case than in the closed-circuit one. This weak difference should not be neglected. It might be used to assess the piezoelectric effect through the so-called effective modal electromechanical coupling coefficient [21]. For illustration purpose, the mode shapes associated with the frequencies given in Table 2 are presented in Figs. 9 and 10.

Table 3 presents the eigenfrequencies of the fluid/piezoelectric-structure coupled system computed by our finite element approach and those obtained by the exact solution [20]. The results, given for the two first circumferential harmonics, show a good agreement between the two methods validating the finite element electromechanical-acoustic formulation. It is important to note that the difference between exact and finite element solutions can increase for higher modes, in particular due to the linearity assumption of the electric potential used in the finite element modeling. As expected, the frequencies are lower in the fluid-structure case due to the added mass effect of the fluid.

Table 3

Frequencies (Hz) of a piezoelectric cylindrical shell filled with a compressible fluid: comparison with an exact state space solution [20]

n	Short circuited			Open circuited		
	Present	Exact	Error %	Present	Exact	Error %
1	45.44	44.76	-1.53	49.49	48.27	-2.53
	108.92	107.39	-1.42	115.31	115.61	0.26
	154.56	154.13	-0.28	166.24	166.06	-0.10
	191.02	189.33	-0.89	209.63	204.21	-2.66
	220.02	217.19	-1.30	236.25	234.47	-0.76
2	19.64	19.92	1.40	21.00	21.46	2.16
	62.50	62.60	0.16	66.93	67.62	1.03
	108.11	108.57	0.43	116.42	117.25	0.71
	148.61	149.02	0.28	161.25	160.91	-0.21
	182.89	182.82	-0.04	197.33	197.40	0.04

7. Conclusions

In this paper, a symmetric finite element formulation for fluid/piezoelectric-structure coupled problem has been proposed. The local equations, in terms of structure displacement, electric potential and fluid pressure, and the corresponding unsymmetric variational formulation are first presented. Then, through the introduction of additional variable, namely fluid displacement potential, a symmetric formulation is established. It is shown that the electric potential and the fluid displacement potential can be eliminated from this symmetric formulation using a condensation procedure. This rigorous procedure is related to the fluid added mass and piezoelectric added stiffness

concepts. Some preliminary numerical examples concerning axisymmetric elastic/piezoelectric structures filled with internal compressible fluid are presented in order to validate the proposed formulation. Structural–acoustic applications using passive/active noise reduction techniques with damping materials and piezoelectric patches are in progress.

Acknowledgements

This research is partially supported by the European Community through the Marie Curie Research Training Network “A Computer Aided Engineering Approach to Smart Structures Design” under contract number MRTN-CT-2006-035559.

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