RECOVERABLE ADHESION AND FRICTION

Michel Raous, Mathieu Schryve, Marius Cocou

REFERENCE de l'article

M. RAOUS, M. SCHRYVE, M. COCOU, Recoverable adhesion and friction, in *Nonsmooth/Nonconvex Mechanics with applications in Engineering, C.C. Banagiotopoulos (ed)*, Proceedings of II International Conference in memoriam of Prof. P.D. Panagiotopoulos in Thesssaloniki, Greece, 7-8 July, 2006, ISBN 960-243-623-9, pp. 165-172.

RECOVERABLE ADHESION AND FRICTION

M. RAOUS[1], **M. SCHRYVE** [1] **& M. COCOU**[1,2]
[1] *Laboratoire de Mécanique et d'Acoustique - C.N.R.S.*
31, chemin Joseph Aiguier, 13402 Marseille Cedex 20, France
E-mail: raous@lma.cnrs-mrs.fr
E-mail: schryve@lma.cnrs-mrs.fr
[2] *Université de Provence, U.F.R. M.I.M., Marseille, France*
E-mail: cocou@lma.cnrs-mrs.fr

1 SUMMARY

We gave in [11] and in [12] a model coupling adhesion to friction and unilateral contact, the RCCM model (Raous-Cangémi-Cocou-Monerie). In that model the loss of adhesion is non reversible. It has been used mainly for composite materials (ductile cracks, matrix/fiber interfaces, ...) and also for metal/concrete interfaces for reinforced concrete [8][13]. It is based on the notion of intensity of adhesion introduced by Frémond [4][5]. In the cases previously considered, adhesion can only decrease, which means that once the adhesion is partially or totally broken, it can not be regenerated.

In order to deal as well with surface forces (for example van der Waals forces for rubber or polyurethane contact with glass) as with recoverable adhesion (for example self-adhesive tape that can be used several times), a new class of models is presented where the intensity of adhesion may increase after having decreased. Convenient thermodynamic potentials are chosen and a new form of the differential equation controlling the evolution of the intensity of adhesion β is obtained. Because of the non differentiability of these potentials, the state and the complementary laws are written under the form of differential inclusions. These pseudo-potentials include non convex parts.

Variational inequalities are used to formulate either the quasi-static or the dynamic problems. The numerical methods previously developed by our group of Contact Mechanics at the LMA are here extended in order to compute the solutions of problems based on the present model.

The interface behavior corresponding to the new model is illustrated on numerical examples both for totally reversible adhesion and for partial recoverable adhesion i.e. for healing joining.

2 AN EXTENSION OF THE RCCM MODEL

The RCCM model developed at the LMA since several years couples adhesion to unilateral contact with friction. Details on the mechanical aspects, on the mathematical analysis, on the solvers and various applications can be found in [11][12][14][2]. It has been used in particular for composite materials [12] or in civil engineering [8][13]. In these cases, the intensity of adhesion was a strictly decreasing function that was convenient for the mechanical behaviors under consideration.

In this paper, we extend the model to recoverable adhesion in order to enlarge the field of applications considering notably: glued or self-adhesive supports elaborated for multiple use; van der Waals forces for polymers or elastomers; Schallamach waves; etc. Fig.1 illustrates one of our goals leading to the approximation of the van der Waals forces: the Signorini conditions will approximate the repulsive forces and the RCCM model with reversible adhesion will approximate the attractive ones.

3 THE MODEL AND THE THERMODYNAMIC FORMULATION

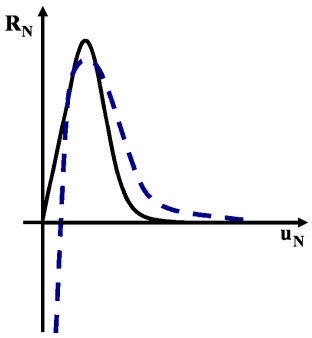

Figure 1: Approximation of the van der Waals interaction (dash line) by the recovering RCCM model (solid line).

3.1 Hypotheses and notations

We consider two linearly elastic solids which occupy the domains Ω^α of $\mathbb{R}^d$, $d = 2$ or 3, with Lipschitz boundaries $\Gamma^\alpha = \partial\Omega^\alpha$, $\alpha = 1, 2$. Deformations and displacements are assumed to be small. Let Γ^α_U, Γ^α_F and Γ^α_C be three sufficiently smooth open sets of Γ^α such that $\Gamma^\alpha = \overline{\Gamma}^\alpha_U \cup \overline{\Gamma}^\alpha_F \cup \overline{\Gamma}^\alpha_C$ and $\mathrm{mes}(\Gamma^\alpha_U) > 0$. We suppose that the solids are in unilateral contact with friction and adhesion along the initial contact surfaces Γ^1_C and Γ^2_C. We denote by $\boldsymbol{u}^\alpha = (u^\alpha_1, ..., u^\alpha_d)$ the displacement field, $\boldsymbol{\varepsilon}^\alpha = (\varepsilon_{ij}(\boldsymbol{u}^\alpha))$ the strain tensor and by $\boldsymbol{\sigma}^\alpha = \left(\sigma^\alpha_{ij}\right)$ the stress tensor corresponding to Ω^α, $\alpha = 1, 2$. Let $\boldsymbol{f} = (\boldsymbol{f}^1, \boldsymbol{f}^2)$ and $\boldsymbol{F} = (\boldsymbol{F}^1, \boldsymbol{F}^2)$ denote respectively the given body forces in $\Omega^1 \cup \Omega^2$ and the given tractions in $\Gamma^1_F \cup \Gamma^2_F$.

We adopt the following notations for the normal and tangential components of the displacement vectors, of the corresponding relative displacement ones and of the stress vectors (where $\boldsymbol{n}^\alpha$ is the outward unit normal vector to Γ^α):

$$v^\alpha_N = \boldsymbol{v}^\alpha \cdot \boldsymbol{n}^\alpha, \;\; \boldsymbol{v}^\alpha_T = \boldsymbol{v}^\alpha - v^\alpha_N \boldsymbol{n}^\alpha, \;\; [v_N] = -(v^1_N + v^2_N), \;\; [\boldsymbol{v}_T] = \boldsymbol{v}^2_T - \boldsymbol{v}^1_T,$$
$$\sigma^\alpha_N = (\boldsymbol{\sigma}^\alpha \boldsymbol{n}^\alpha) \cdot \boldsymbol{n}^\alpha, \;\; \boldsymbol{\sigma}^\alpha_T = \boldsymbol{\sigma}^\alpha \boldsymbol{n}^\alpha - \sigma^\alpha_N \boldsymbol{n}^\alpha, \;\; \alpha = 1, 2,$$

3.2 Thermodynamic formulation

Let $\boldsymbol{\mathcal{A}}^\alpha = (\mathcal{A}^\alpha_{ijkl})$ denotes the fourth-order tensor of elasticity satisfying the following classical symmetry and ellipticity conditions: $\mathcal{A}^\alpha_{ijkl} = \mathcal{A}^\alpha_{jikl} = \mathcal{A}^\alpha_{klij} \in L^\infty(\Omega^\alpha), \; \alpha = 1, 2, \; \forall\, i, j, k, l = 1, \ldots, d, \; \exists\; \alpha_A > 0, \quad \mathcal{A}^\alpha_{ijkl}\tau_{ij}\tau_{kl} \geq \alpha_A \; \tau_{ij}\tau_{ij}, \forall\, \boldsymbol{\tau} = (\tau_{ij})$ such that $\tau_{ij} = \tau_{ji}, \; \forall\, i, j = 1, \ldots, d.$

Considering the contact surfaces as material boundaries, we introduce a surface density of internal energy E and a density of entropy S on Γ_C (with the choice $\Gamma_C = \Gamma^1_C$). Then the Helmholtz free energies can be written $\Psi = E - ST$ on Γ_C and $\psi = e - sT$ in $\Omega^1 \cup \Omega^2$, where e is the specific internal energy, s the specific entropy and T the absolute temperature. According to M. Frémond [4][5], we introduce also the internal state variable β, which represents the intensity of adhesion ($\beta = 1$ means that the adhesion is total, $\beta = 0$ means that there is no adhesion and $0 < \beta < 1$ is the case of partial adhesion). In the following, we will consider only isothermic evolutions.

We choose the following state variables: the strain tensor $\boldsymbol{\varepsilon} = (\boldsymbol{\varepsilon}^1, \boldsymbol{\varepsilon}^2) = (\boldsymbol{\varepsilon}(\boldsymbol{u}^1), \boldsymbol{\varepsilon}(\boldsymbol{u}^2))$ in $\Omega^1 \cup \Omega^2$, the relative displacements $[\boldsymbol{u}_T]$, $[u_N]$ and the intensity of adhesion β on Γ_C. Let $(.)^r$ and $(.)^{ir}$ denote the reversible and the irreversible parts. Let $-\boldsymbol{\sigma}^r$ be the thermodynamic force associated with $\boldsymbol{\varepsilon}$. Let $-R^r_N$, $-\boldsymbol{R}^r_T$, where $\boldsymbol{R} = \boldsymbol{\sigma}^1 \boldsymbol{n}^1 = -\boldsymbol{\sigma}^2 \boldsymbol{n}^2$, denote the thermodynamic

forces associated with the relative displacements $[u_N]$, $[\boldsymbol{u}_T]$, and G_β the adhesive thermodynamic force associated with the internal variable β. Within the domain $\Omega^1 \cup \Omega^2$, the free energy ψ and the classical state law are as follows, where ρ^α are the mass densities of bodies α:

$$\psi^\alpha(\boldsymbol{\varepsilon}^\alpha) = \frac{1}{2\rho^\alpha}\,\boldsymbol{\mathcal{A}}^\alpha\,\boldsymbol{\varepsilon}^\alpha\cdot\boldsymbol{\varepsilon}^\alpha = \frac{1}{2\rho^\alpha}\,\mathcal{A}^\alpha_{ijkl}\,\varepsilon^\alpha_{ij}\,\varepsilon^\alpha_{kl}\ ,\ (\boldsymbol{\sigma}^\alpha)^r = \rho^\alpha\,\frac{\partial\psi^\alpha(\boldsymbol{\varepsilon}^\alpha)}{\partial\boldsymbol{\varepsilon}^\alpha},\ \alpha=1,2. \tag{1}$$

Now we focus on the contact boundary Γ_C, where the surface density of free energy Ψ is chosen under the form

$$\Psi([u_N],[\boldsymbol{u}_T],\beta) = \frac{C_N}{2}[u_N]^2\beta^2 + \frac{C_T}{2}||[\boldsymbol{u}_T]||^2\beta^2 - w\beta + I_{\mathbb{R}^+}([u_N]) + I_P(\beta), \tag{2}$$

where $P=\{\gamma\ ;\ 0\le\gamma\le 1\}$. The term w is the energy of decohesion.

The state laws on Γ_C can be written as

$$R^r_N \in \partial_{[u_N]}\Psi\ ,\ \boldsymbol{R}^r_T \in \partial_{[\boldsymbol{u}_T]}\Psi\ ,\ -G_\beta \in \partial_\beta\Psi\ , \tag{3}$$

where ∂_z denotes the subdifferential with respect to the variable z.

The two laws of thermodynamics give the following form of the Clausius-Duhem inequalities :

$$\rho^\alpha\,\dot\psi^\alpha \le \boldsymbol{\sigma}^\alpha\cdot\dot{\boldsymbol{\varepsilon}}^\alpha \ \text{ in }\ \Omega^\alpha\ ,\ \alpha=1,2,\quad \dot\Psi \le R_N\,[\dot u_N] + \boldsymbol{R}_T\cdot[\dot{\boldsymbol{u}}_T]\ \text{ on }\ \Gamma_C\,. \tag{4}$$

The irreversible parts of the contact forces are $R^{ir}_N = R_N - R^r_N$ and $\boldsymbol{R}^{ir}_T = \boldsymbol{R}_T - \boldsymbol{R}^r_T$. The only dissipative processes under consideration on Γ_C are related to the adhesion and to the friction. To formulate the corresponding complementary laws such that (4) are verified, we choose a pseudo-potential of dissipation $\Phi = \Phi([\dot{\boldsymbol{u}}_T],\dot\beta;\chi)$ with $\chi = (R^r_N,[u_N],[\boldsymbol{u}_T],\beta)$, positive, convex in $([\dot{\boldsymbol{u}}_T],\dot\beta)$ and zero in $(\boldsymbol{0},0)$, which constitutes an extension of standard generalized materials. We adopt the following form for the pseudo-potential of dissipation:

$$\Phi([\dot{\boldsymbol{u}}_T],\dot\beta;\chi) = \mu(1-\beta)|R_N - C_N\,[u_N]\beta^2|\,|||[\dot{\boldsymbol{u}}_T]||| + \frac{b}{2}\dot\beta^2 - C_T||[\boldsymbol{u}_T]||^2\beta\dot\beta, \tag{5}$$

so that the healing process was allowed and was governed only by the normal components. The parameter μ is the friction coefficient of the Coulomb law, the parameter b is the viscosity associated with the intensity of adhesion. The complementary laws are written as follows:

$$R^{ir}_N = 0\,,\ \boldsymbol{R}^{ir}_T \in \partial_{[\dot{\boldsymbol{u}}_T]}\Phi([\dot{\boldsymbol{u}}_T],\dot\beta;\chi)\,,\ G_\beta \in \partial_{\dot\beta}\Phi([\dot{\boldsymbol{u}}_T],\dot\beta;\chi). \tag{6}$$

On Γ_C the following expression for the dissipation D is obtained:

$$D = (\boldsymbol{R}_T - \boldsymbol{R}^r_T)\cdot[\dot{\boldsymbol{u}}_T] + G_\beta\,\dot\beta = \boldsymbol{R}^{ir}_T\cdot[\dot{\boldsymbol{u}}_T] + G_\beta\,\dot\beta \ge 0, \tag{7}$$

which ensures that the third Clausius-Duhem inequality of (4) holds.

3.3 The contact model

The contact constitutive laws are then obtained from the state laws (3) and the complementary laws (6) with the choice (2) for the free energy and (5) for the potential of dissipation.

$$[u_N]\ge 0,\quad -R_N + C_N\,[u_N]\beta^2 \ge 0,\quad (-R_N + C_N\,[u_N]\beta^2)\,[u_N] = 0\ \text{ on }\ \Gamma_C,$$

$$||\,\boldsymbol{R}_T - C_T[\boldsymbol{u}_T]\beta^2\,|| \le (1-\beta)\mu\,|\,R_N - C_N[u_N]\beta^2\,|\ \text{ on }\ \Gamma_C\ \text{ and}$$

$$||\,\boldsymbol{R}_T - C_T[\boldsymbol{u}_T]\beta^2\,|| < (1-\beta)\mu\,|\,R_N - C_N[u_N]\beta^2\,| \Rightarrow [\dot{\boldsymbol{u}}_T] = \boldsymbol{0},$$

$$||\,\boldsymbol{R}_T - C_T[\boldsymbol{u}_T]\beta^2\,|| = (1-\beta)\mu\,|\,R_N - C_N[u_N]\beta^2\,| \Rightarrow \exists\lambda\ge 0, [\dot{\boldsymbol{u}}_T] = \lambda(\boldsymbol{R}_T - C_T[\boldsymbol{u}_T]\beta^2),$$

$$b\,\dot\beta = w\ - C_N\,\beta\,[u_N]^2\ \text{ and }\ \beta\in[0,1]\ \text{ in }\ \Gamma_C,$$

for all $t \in]0,T[$, with the initial condition $\beta(0) = \beta_0 = 1$ on Γ_C and $R_N = R_N^1,\ \boldsymbol{R}_T = \boldsymbol{R}_T^1$.

4 THE VARIATIONAL FORMULATION

4.1 The dynamic problem

The formulation of the dynamic problem is given here (where $\rho^\alpha = 1$). The quasi-static problem can be easily deduced.

Problem $\mathbf{P_0}$: Find $\boldsymbol{u} = (\boldsymbol{u}^1, \boldsymbol{u}^2)$, β, $\boldsymbol{R}$ such that $\boldsymbol{u}(0) = \boldsymbol{u}_0 = (\boldsymbol{u}_0^1, \boldsymbol{u}_0^2)$, $\dot{\boldsymbol{u}}(0) = \boldsymbol{u_1} = (\boldsymbol{u}_1^1, \boldsymbol{u}_1^2)$ in $\Omega^1 \times \Omega^2$, $\beta(0) = \beta_0 = 1$ and $[u_N](0) = 0$ on Γ_C and

$$\ddot{\boldsymbol{u}}^\alpha - \operatorname{div} \boldsymbol{\sigma}^\alpha(\boldsymbol{u}^\alpha) = \boldsymbol{f}^\alpha,\ \boldsymbol{\sigma}^\alpha(\boldsymbol{u}^\alpha) = \boldsymbol{\mathcal{A}}^\alpha \boldsymbol{\varepsilon}(\boldsymbol{u}^\alpha) \text{ in }]0,T[\times \Omega^\alpha,$$
$$\boldsymbol{u}^\alpha = \boldsymbol{0} \text{ in }]0,T[\times \Gamma_U^\alpha,\ \boldsymbol{\sigma}^\alpha \boldsymbol{n}^\alpha = \boldsymbol{F}^\alpha \text{ in }]0,T[\times \Gamma_F^\alpha,\ \alpha = 1,2,$$
$$\boldsymbol{\sigma}^1 \boldsymbol{n}^1 + \boldsymbol{\sigma}^2 \boldsymbol{n}^2 = 0 \text{ in }]0,T[\times \Gamma_C,$$
$$\boldsymbol{\sigma}^1 \boldsymbol{n}^1 = \boldsymbol{R} \text{ in }]0,T[\times \Gamma_C^1,$$
$$\text{+ the contact law given in section 3.3}$$

4.2 The variational formulation

Let us first introduce two Hilbert spaces $(\boldsymbol{V},\ \|.\|)$, $(\boldsymbol{H},\ |.|)$, with the associated scalar product denoted respectively by $\langle .,. \rangle$, $(.,.)$, and the set $\boldsymbol{K}$ as follows:

$$\boldsymbol{V} = \boldsymbol{V}^1 \times \boldsymbol{V}^2, \text{ where } \boldsymbol{V}^\alpha = \{\boldsymbol{v}^\alpha \in [H^1(\Omega^\alpha)]^d;\ \boldsymbol{v}^\alpha = \boldsymbol{0} \text{ a.e. on } \Gamma_U^\alpha\},$$
$$\boldsymbol{H} = [L^2(\Omega^1)]^d \times [L^2(\Omega^2)]^d,\ \ \boldsymbol{K} = \{\boldsymbol{v} = (\boldsymbol{v}^1, \boldsymbol{v}^2) \in \boldsymbol{V};\ [v_N] \geq 0 \text{ a.e. in } \Gamma_C\}.$$

We assume that $\boldsymbol{F}^\alpha \in W^{1,\infty}(0,T; [L^2(\Gamma_F^\alpha)]^d)$, $\boldsymbol{f}^\alpha \in W^{1,\infty}(0,T; \boldsymbol{H}^\alpha)$, $\alpha = 1,2$, $\boldsymbol{u}_0 \in \boldsymbol{K}$, $\boldsymbol{u}_1 \in \boldsymbol{V}$, $\mu \in L^\infty(\Gamma_C)$, $\mu \geq 0$ a.e. on Γ_C. Let us define the bilinear, continuous and symmetric mapping $a(.,.)$ on $\boldsymbol{V} \times \boldsymbol{V} \to \mathbb{R}$ by

$$a(\boldsymbol{v}, \boldsymbol{w}) = a^1(\boldsymbol{v}^1, \boldsymbol{w}^1) + a^2(\boldsymbol{v}^2, \boldsymbol{w}^2), \quad \forall \boldsymbol{v}, \boldsymbol{w} \in \boldsymbol{V},$$

where $a^\alpha(\boldsymbol{v}^\alpha, \boldsymbol{w}^\alpha) = \displaystyle\int_{\Omega^\alpha} \boldsymbol{\mathcal{A}}^\alpha \boldsymbol{\varepsilon}(\boldsymbol{v}^\alpha) \cdot \boldsymbol{\varepsilon}(\boldsymbol{w}^\alpha)\, dx,$

Using the previous hypotheses, we consider $\boldsymbol{L}$ as an element of $W^{1,\infty}(0,T;\boldsymbol{V})$ such that

$$\langle \boldsymbol{L}, \boldsymbol{v} \rangle = \sum_{\alpha=1,2} \int_{\Omega^\alpha} \boldsymbol{f}^\alpha \cdot \boldsymbol{v}^\alpha\, dx + \sum_{\alpha=1,2} \int_{\Gamma_F^\alpha} \boldsymbol{F}^\alpha \cdot \boldsymbol{v}^\alpha\, ds \quad \forall\, \boldsymbol{v} = (\boldsymbol{v}^1, \boldsymbol{v}^2) \in \boldsymbol{V},\ \forall\, t \in [0,T].$$

We define the mappings c_N, c_T, c and J as follows:

$$c_N : L^\infty(\Gamma_C) \times \boldsymbol{V}^2 \to \mathbb{R},\ \ c_N(\beta, \boldsymbol{u}, \boldsymbol{v}) = \int_{\Gamma_C} C_N [u_N]\, [v_N]\, \beta^2\, ds,$$
$$c_T : L^\infty(\Gamma_C) \times \boldsymbol{V}^2 \to \mathbb{R},\ \ c_T(\beta, \boldsymbol{u}, \boldsymbol{v}) = \int_{\Gamma_C} C_T [\boldsymbol{u}_T] \cdot [\boldsymbol{v}_T]\, \beta^2\, ds,$$
$$c = c_T + c_N, \text{ and } J : L^\infty(\Gamma_C) \times \boldsymbol{V}^2 \to \mathbb{R},$$
$$J(\beta, \boldsymbol{u}, \boldsymbol{v}) = \int_{\Gamma_C} (1-\beta)\mu\, |\, R_N - C_N[u_N]\beta^2\, |\ \ ||\, [\boldsymbol{v}_T]\, ||\, ds \quad \forall\, \beta \in L^\infty(\Gamma_C),\ \forall\, \boldsymbol{u},\ \boldsymbol{v} \in \boldsymbol{V}.$$

We assume the following compatibility relation on initial conditions (no initial shocks):

$$\exists\, \boldsymbol{l} \in \boldsymbol{H},\ (\boldsymbol{l}, \boldsymbol{v}) + a(\boldsymbol{u}_0, \boldsymbol{v}) + c(\beta_0, \boldsymbol{u}_0, \boldsymbol{v}) + J(\beta_0, \boldsymbol{u}_0, \boldsymbol{v}) = \langle \boldsymbol{L}(0), \boldsymbol{v} \rangle\ \ \forall\, \boldsymbol{v} \in \boldsymbol{V}.$$

A variational formulation of the problem P_0 is then given.

Problem $\mathbf{P_1}$: Find $\boldsymbol{u} \in W^{1,2}(0,T;\boldsymbol{V}) \cap C^1([0,T];\boldsymbol{H}^{-1/2})$, $\boldsymbol{u}(t) \in \boldsymbol{K}$ for all $t \in]0,T[$, $\beta \in W^{1,\infty}(0,T;L^\infty(\Gamma_C))$ such that $\boldsymbol{u}(0) = \boldsymbol{u}_0$, $\dot{\boldsymbol{u}}(0) = \boldsymbol{u}_1$ in Ω, $\beta(0) = \beta_0$ on Γ_C and

$$\langle \dot{\boldsymbol{u}}(T), \boldsymbol{v}(T) - \boldsymbol{u}(T) \rangle_{-1/2,\,1/2} - (\boldsymbol{u}_1, \boldsymbol{v}(0) - \boldsymbol{u}_0)$$

$$+ \int_0^T \left\{ -(\dot{\boldsymbol{u}}, \dot{\boldsymbol{v}}) + ||\dot{\boldsymbol{u}}||^2 + a(\boldsymbol{u}, \boldsymbol{v} - \boldsymbol{u}) + c(\beta, \boldsymbol{u}, \boldsymbol{v} - \boldsymbol{u}) + J(\beta, \boldsymbol{u}, \boldsymbol{v} + \dot{\boldsymbol{u}} - \boldsymbol{u}) - J(\beta, \boldsymbol{u}, \dot{\boldsymbol{u}}) \right\} dt$$

$$\geq \int_0^T \langle \boldsymbol{L}, \boldsymbol{v} - \boldsymbol{u} \rangle \, dt \qquad \forall\, \boldsymbol{v} \in L^\infty(0,T;\boldsymbol{V}) \cap W^{1,2}(0,T;\boldsymbol{H}),\ \boldsymbol{v}(t) \in \boldsymbol{K} \text{ a.e. } t \in]0,T[,$$

$$b\,\dot{\beta} = w - C_N\,\beta\,[u_N]^2 \quad \text{and} \quad \beta \in [0,1] \quad \text{a.e. on } \Gamma_C, \text{ a.e. } t \in]0,T[.$$

where $\langle ., . \rangle_{-1/2,1/2}$ is the duality pairing between $\boldsymbol{H}^{-1/2}$ and $\boldsymbol{H}^{1/2}$ with

$$\boldsymbol{H}^{1/2} = [H^{1/2}(\Omega^1)]^d \times [H^{1/2}(\Omega^2)]^d.$$

The formal equivalence between the variational problem P_1 and the classical problem P_0 can be easily proved by using Green's formula and an integration by parts on time.

4.3 The solvers

The numerical methods given in [7] are extended here for solving problem P_1. Those are based on the Non Smooth Contact Dynamics method developed by J.J. Moreau and M. Jean [10] [6] and implemented in the LMGC code (Dubois [3]).

For quasi-static problem, the implementation has been done in the GYPTIS code that is a finite element code developed at the LMA by Latil and Raous [9]. Various solvers are used: Gauss-Seidel; conjugate gradient methods; mathematical programming methods.

5 AN EXTENSION TO PARTIAL RECOVERING ADHESION

In the model presented in section 3, the adhesion is totally reversible. It fits for example the approximation of surface forces as the van der Waals ones. We consider now phenomena where the adhesion can be only partially recovered. That could be the case of some self-adhesive tape which can be used several times but of which the efficiency decreases after every use. This new model can be also useful in the study of healing cracks. A new thermodynamic variable β_s is introduced and defined as follows, where $(\dot{\beta})^-$ denotes the negative part of $\dot{\beta}$:

$$\beta_s(t) = 1 - \alpha \int_0^t (\dot{\beta})^- d\tau \tag{8}$$

where α is a weakening factor ($0 \leq \alpha \leq 1$) and the condition $0 \leq \beta \leq \beta_s$ is imposed. Example 2 presented in the next section illustrates this behavior.

6 TWO EXAMPLES ILLUSTRATING THE RECOVERABLE ADHESION

6.1 Indentation and sliding on a polyurethane block (reversible adhesion)

This example is related to an ongoing research on the Schallamach waves. When a glass sphere is pressed and pushed horizontally on a polyurethane block, propagation of no-contact waves is observed. The global movement of the sphere is governed by the propagation of these waves. No sliding zones are observed [1][15]. We present here the finite element simulation with reversible adhesion of the alternate normal indentation of the block by the sphere. The computations have been conducted here in viscoelasticity (Kelvin-Voigt behavior) by using the LMGC

code and the MaTliB library. In that case the formulation of the unilateral contact has been generalized in order to take into account an initial gap g between the two solids and the initial condition on β is no more $\beta(0) = 1$ but a condition $\beta(0) \leq 1$ depending on the values of the gap g. That is the case for the point O (see in Fig.2) for which the evolution of β is given in Fig.4. We give in Fig.3 the evolution of the contact zone and of its totally adhesive part.

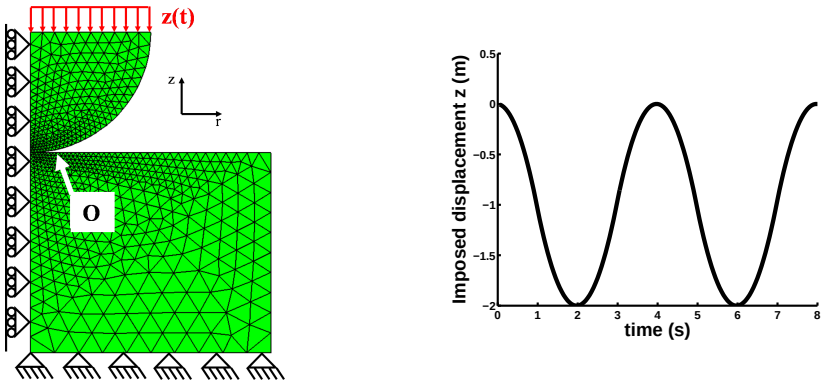

Figure 2: Initial configurations and prescribed displacement for indentation of a Polyuréthane block by a glass sphere.

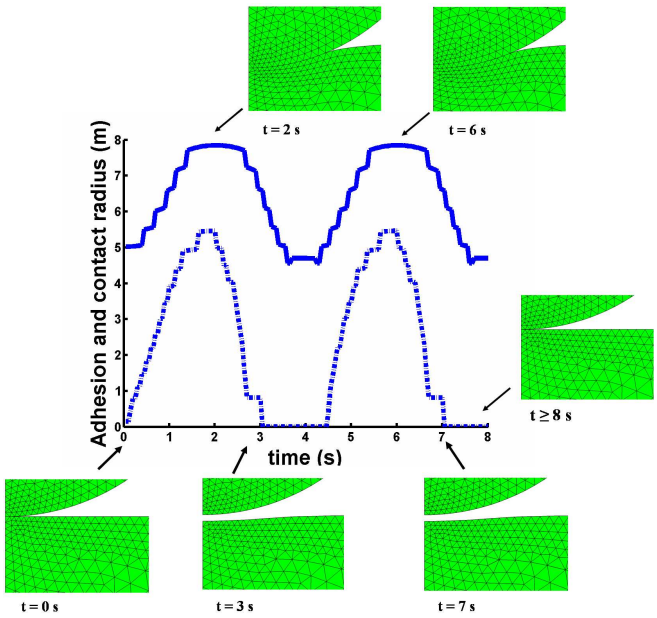

Figure 3: Adhesion radius (solid line) and contact radius (dot line).

6.2 Cycling loads with partial recovering adhesion

This is a simple one dimensional Matlab simulation (with a small value of b) of the classical RCCM model and of the new models first when the adhesion recovering is total (reversible adhesion) and secondly when it is partial (α = 0.5). The normal displacement is prescribed as presented in Fig.5 : loading, unloading and a reloading. The behavior is given in Fig.6. When an unloading is prescribed at the point A of the curve of Fig.6:

- the RCCM model gives a direct elastic return with a small stiffness (dot line),
- the partial recoverable model gives first some increase of β until the point B was reached and then an elastic return (solid line)
- the reversible model follows the same way back as for the loading (not discernible from the

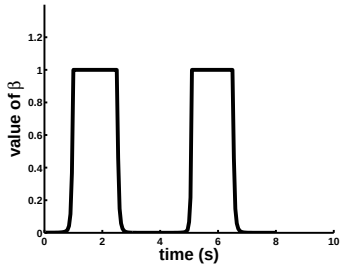

Figure 4: Intensity of adhesion at point O.

loading curve on the figure).
Fig.7 and Fig.8 show the evolution of β versus time and versus the normal displacement in the three cases. The dash zone corresponds to the energy dissipated by adhesion (part related to α).

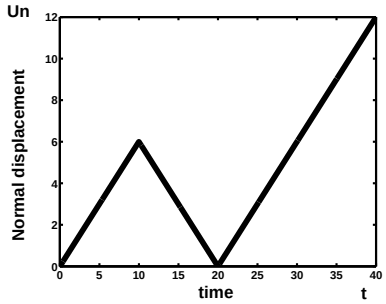

Figure 5: Prescribed displacement versus time.

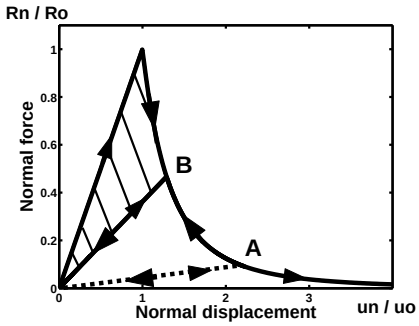

Figure 6: Normal behavior (for unloading: dot line=RCCM/ solid line=recoverable adhesion).

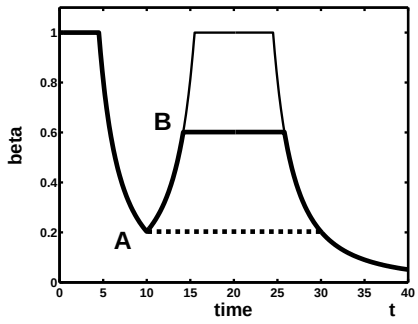

Figure 7: Evolution of β versus time (dot line =RCCM/ solid line=recoverable adhesion/ thin line=reversible adhesion).

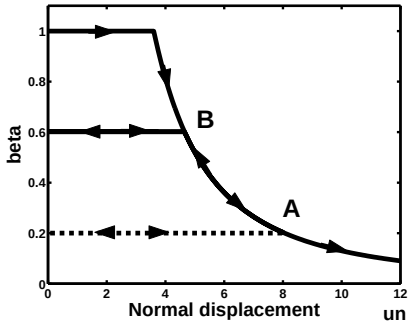

Figure 8: Evolution of β versus normal displacement (for unloading: dot line=RCCM/ solid line=recoverable adhesion).

7. REFERENCES

[1] M. Barquins, Sliding Friction of Rubber and Schallamach Waves - A Review. *Materials Science and Engineering*, **73**, 45-63, 1985.

[2] M. Cocou, M. Raous, Adhesive contact and implicit evolution variational inequalities. In *Nonsmoooth/nonconvex Mechanics with Applications in Engineering* (eds. C.C. Baniotopoulos), Ziti Publisher, Thessalonoki, 167-174, 2002.

[3] F. Dubois and M. Jean, LMGC90 une plateforme de développement dédiée à la modélisation de problèmes d'interaction. In: *Actes du sixième colloque national en calcul des structures* (eds. M. Potier-Ferry, M. Bonnet, and A. Bignonnet), Ecole Polytechnique, Paris, **1**, 111-118, 2003.

[4] M. Frémond, Adhérence des solides. *Journal de Mécanique Théorique et Appliquée*, **6**, 383-407, 1987.

[5] M. Frémond, Contact with adhesion. In *Nonsmooth Mechanics and Applications* (eds. J.J. Moreau and P.D. Panagiotopoulos), CISM - Courses and Lectures, Springer Verlag, **302**, 177-221, 1988.

[6] M. Jean, The Non Smooth Contact Dynamics method. *Computer Methods in Applied Mechanics and Engineering*, **177**, 235-257, 1999.

[7] M. Jean, V. Acary and Y. Monerie, Non-smooth contact dynamics approach of cohesive materials. *Phil. Trans. Royal Society of London*, A, **359**, 2497-2518, 2001.

[8] M.A. Karray, S. Barbarin and M. Raous, Traitement de la liaison béton-acier par un modèle d'interface couplant adhésion et frottement. *Annales Maghrébines de l'Ingénieur*, **18**, 2, 2004.

[9] J.-C. Latil and M. Raous, Module GYPTIS 1.0 - Contact unilatéral avec frottement en mécanique des structures. *Note interne LMA, CNRS*, **132**, 1991.

[10] J.-J. Moreau, Unilateral contact and dry friction in finite freedom dynamics. In: *Non Smooth Mechanics and Applications* (eds. J.-J. Moreau and P.D. Panagiotopoulos), CISM Courses and Lectures, Springer, **302**, 1-82, 1988.

[11] M. Raous, L. Cangémi and M. Cocou, A consistent model coupling adhesion, friction and unilateral contact. *Computer Methods in Applied Mechanics and Engineering*, **77**, 383-399, 1999.

[12] M. Raous and Y. Monerie, Unilateral contact, friction and adhesion: 3D cracks in composite materials. In: *Contact Mechanics* (eds. J.A.C. Martins and M.P.D. Monteiro Marques), Kluwer, Dordrecht, 333-346, 2002.

[13] M. Raous, M.A. Karray, Model coupling friction and adhesion for steel-concrete interfaces. In *Proceeding of Advances on Geomaterials and Structures*, Hammamet, 3-5 May, 2006. (to appear)

[14] R. Rocca and M. Cocou, Existence results for unilateral quasistatic contact problems with friction and adhesion. *Math. Modelling and Num. Analysis*, **34**, 981-1001, 2000.

[15] A. Schallamach, How does rubber slide ?. *Wear*, **17**, 301-312, 1971.