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► **To cite this version:**

Walid Larbi, Jean-François Deü, Roger Ohayon, Rubens Sampaio. Coupled FEM/BEM for control of noise radiation and sound transmission using piezoelectric shunt damping. *Applied Acoustics*, 2014, 86, pp.146-153. 10.1016/j.apacoust.2014.02.003 . hal-03177486

HAL Id: hal-03177486

<https://hal.science/hal-03177486v1>

Submitted on 27 Oct 2023

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Coupled FEM/BEM for control of noise radiation and sound transmission using piezoelectric shunt damping

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In this paper, we present a coupled finite element/boundary element method (FEM/BEM) for control of noise radiation and sound transmission of vibrating structure by passive piezoelectric techniques. The system consists of an elastic structure (with surface mounted piezoelectric patches) coupled to external/internal acoustic domains. The passive shunt damping strategy is employed for vibration attenuation in the low frequency range. The originality of the present paper lies in evaluating the classically used FEM/BEM methods for structural-acoustics problems when taking account smart systems at the fluid-structure interfaces.

1. Introduction

During the last two decades there has been an accelerating level of interest in the control of noise radiation and sound transmission from vibrating structures by passive piezoelectric techniques in the low frequency range. In this context, resonant shunt damping techniques have been recently used for interior structural-acoustic problems ([1,2] references included). The present work concerns the extension of this technique to internal/external vibroacoustic problems using a finite-element/boundary-element method (FEM/BEM) for the numerical resolution of the fully coupled electro-mechanical-acoustic system.

The problem is analyzed in the frequency domain. More specifically, the low-frequency range where the modal density is not high. The linear acoustic equations are used for the internal and external fluid (i.e. homogeneous, inviscid, compressible fluid, neglecting gravity effects). For such situation, the structure with the internal acoustic fluid is a resonant system while the external acoustic fluid, which is the cause of radiation effect, involves smooth operators as function of frequency. The radiation comes from the Sommerfeld condition at infinity. That is why, for the structure with the internal fluid, the FEM is used while the exterior problem is treated by a BEM (see for example [3,4]). More specifically, concerning the exterior problems, Fischer and Gaul present

in [4] a coupling algorithm based on Lagrange multipliers for the simulation of structure-acoustic field interaction. Finite plate elements are coupled to a Galerkin boundary element formulation of the acoustic domain. The interface pressure is interpolated as a Lagrange multiplier, thus, allowing the coupling of non-matching grids. A coupling scheme for fluid-structure interaction using a fast multipole boundary element method and a finite element method is presented in [5]. Special focus is set on partly immersed bodies, where a free fluid surface exists. A special half-space fundamental solution is applied, which allows to incorporate the Dirichlet boundary condition on the half-space plane (using for instance a mirror technique). In [6], the acoustic pressure distribution on the surface of a three dimensional structure and the total acoustic power radiated by the structure resulting from a known surface velocity distribution is obtained by FEM and BEM. The pressure distribution is computed by boundary integral equation. The sound radiation power of structure is then expressed as a positive definite quadratic form by impedance matrix.

For vibration and sound power reduction generated from vibrating structures, piezoelectric materials have been introduced since several years to overcome the trade-off in the width and height limits and the temperature dependence of viscoelastic materials. Most recently, piezoelectric materials have been used with passive energy dissipation devices by using an electrical impedance as a shunt. There are many kind of shunt damping circuits such as resistor, resistor-inductor, and negative capacitor. The passive shunt damping systems for reducing structural vibra-

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tion and noise from vibrating structure have been studied by many researchers. Passive vibration schemes using piezoelectric devices have been first presented by Hagood and Von Flotow [7]. A simple inductor–resistor network, when applied as an electrical shunt across the piezoelectric, produces a resonant device like a damped vibration absorber to suppress a selected mode. Recently, several researches have focused on modeling and implementing structures with piezoelectric shunted system. We can refer to [8–10] for the case of cantilever beam [11,12] for composite plate and [13,2] for composite plate coupled to internal acoustic fluid.

The outline of this paper is as follows:

- (i) Firstly, a finite element formulation of an elastic structure with surface-mounted piezoelectric patches and subjected to pressure load due to the presence of an internal/external fluid is derived from a variational principle involving structural displacement, electrical voltage of piezoelectric elements and acoustic pressure at the fluid–structure interface. This formulation, with only one couple of electric variables per patch, is well adapted to practical applications since realistic electrical boundary conditions, such that equipotentiality on the electrodes and prescribed global electric charges, naturally appear. The global charge/voltage variables are intrinsically adapted to include any external electrical circuit into the electromechanical problem and to simulate the effect of resistive or resonant shunt techniques. A reduced order model, based on a normal mode expansion using the eigenmodes of the structure in vacuo with short-circuited piezoelectric patches, is then presented. This methodology reduces significantly the system size and automatically shows the effective electromechanical modal coupling factor (EEMCF), characterizing the energy exchanges between the mechanical structure and the piezoelectric patches.
- (ii) Secondly, the direct boundary element method is used for modeling the scattering/radiation of sound by the structure coupled to an acoustic domain. The BEM is derived from boundary integral equation involving the surface pressure and normal acoustic velocity at the boundary of the acoustic domain. The coupled FEM-BEM model is obtained by using a compatible mesh at the fluid–structure interface.
- (iii) Finally, the efficiency of the proposed coupling methodology is examined on two numerical examples. First, the vibration reduction of an elastic plate backed by a closed acoustic cavity is considered. The second example is the simulation of the attenuation of the sound field emitted from a submerged simply-supported plate in a half-space acoustic domain by means of a piezoelectric shunt system.

2. Finite element formulation of elastic structure with piezoelectric shunt systems

2.1. System equations in the frequency domain

An elastic structure occupying the domain Ω_E is equipped with P piezoelectric patches and coupled to an inviscid linear acoustic fluid occupying the domain Ω_F (Fig. 1). Each piezoelectric patch has the shape of a plate with its upper and lower surfaces covered with very thin layer electrodes. The p th patch, $p \in \{1, \dots, P\}$, occupies a domain $\Omega^{(p)}$ such that $(\Omega_E, \Omega^{(1)}, \dots, \Omega^{(P)})$ is a partition of the whole solid domain Ω_S . In order to reduce the vibration amplitudes of the coupled problem, a resonant shunt circuit made up of a resistance $R^{(p)}$ and an inductance $L^{(p)}$ in series is connected to each patch [7,1,14].

We denote by Σ the fluid–structure interface and by \mathbf{n}^S and \mathbf{n}^F the unit normal external to Ω_S and Ω_F , respectively. Moreover, the

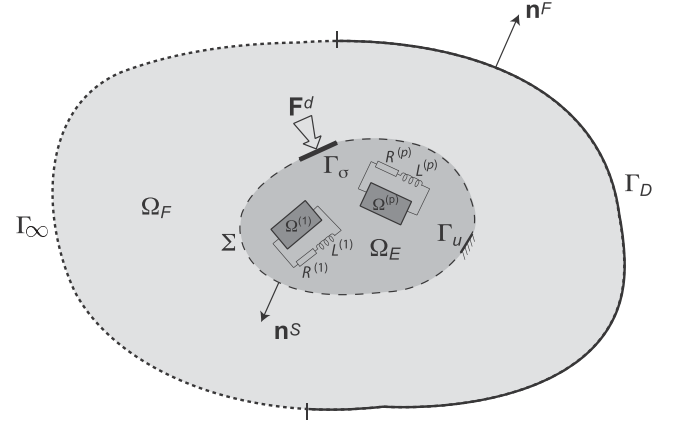


Fig. 1. Vibrating structure with piezoelectric shunt systems coupled to an acoustic domain.

structure is clamped on a part Γ_u and subjected to (i) a given surface force density \mathbf{F}^d on the complementary part Γ_σ of its external boundary and (ii) to a pressure field p due to the presence of the fluid on its boundary Σ . The electric boundary condition for the p th patch is defined by a prescribed surface density of electric charge Q^d on $\Gamma_D^{(p)}$.

The linearized deformation tensor is $\boldsymbol{\varepsilon} = \frac{1}{2}(\nabla \mathbf{u} + \nabla^T \mathbf{u})$ and the stress tensor is denoted by $\boldsymbol{\sigma}$. Moreover, \mathbf{D} denotes the electric displacement and \mathbf{E} the electric field such that $\mathbf{E} = -\nabla \psi$ where ψ is the electric potential. ρ_s is the mass density of the structure. The linear piezoelectric constitutive equations write:

$$\boldsymbol{\sigma} = \mathbf{c} \boldsymbol{\varepsilon} - \mathbf{e}^T \mathbf{E} \quad (1)$$

$$\mathbf{D} = \mathbf{e} \boldsymbol{\varepsilon} + \boldsymbol{\epsilon} \mathbf{E} \quad (2)$$

where \mathbf{c} denotes the elastic moduli at constant electric field, \mathbf{e} denotes the piezoelectric constants, and $\boldsymbol{\epsilon}$ denotes the dielectric permittivities at constant strain [15].

The local equations of elastic structure with piezoelectric patches and submitted to an acoustic pressure are [16]

$$\text{div } \boldsymbol{\sigma} + \omega^2 \rho_s \mathbf{u} = \mathbf{0} \quad \text{in } \Omega_S \quad (3a)$$

$$\boldsymbol{\sigma} \mathbf{n}^S = \mathbf{F}^d \quad \text{on } \Gamma_\sigma \quad (3b)$$

$$\mathbf{u} = \mathbf{0} \quad \text{on } \Gamma_u \quad (3c)$$

$$\boldsymbol{\sigma} \mathbf{n}^S = p \mathbf{n}^F \quad \text{on } \Sigma \quad (3d)$$

$$\text{div } \mathbf{D} = 0 \quad \text{in } \Omega^{(p)} \quad (4a)$$

$$\mathbf{D} \cdot \mathbf{n}^S = Q^d \quad \text{on } \Gamma_D^{(p)} \quad (4b)$$

where ω is the angular frequency.

For each piezoelectric patch, a set of hypotheses, which can be applied to a wide spectrum of practical applications, are formulated:

- the piezoelectric patches are thin, with a constant thickness, denoted $h^{(p)}$ for the p th patch;
- the thickness of the electrodes is much smaller than $h^{(p)}$ and is thus neglected;
- the piezoelectric patches are polarized in their transverse direction (i.e. the direction normal to the electrodes).

Under those assumptions, the electric field vector $\mathbf{E}^{(p)}$ can be considered normal to the electrodes and uniform in the piezoelectric patch [9], so that for all $p \in \{1, \dots, P\}$:

$$\mathbf{E}^{(p)} = -\frac{V^{(p)}}{h^{(p)}} \mathbf{n}^{(p)} \quad \text{in } \Omega^{(p)} \quad (5)$$

where $V^{(p)}$ is the potential difference between the upper and the lower electrode surfaces of the p th patch which is constant over $\Omega^{(p)}$ and where $\mathbf{n}^{(p)}$ is the normal unit vector to the surface of the electrodes.

2.2. Variational formulation

By considering successively each of the $P+1$ subdomains $(\Omega_E, \Omega^{(1)}, \dots, \Omega^{(P)})$, the variational formulation of the structure/piezoelectric-patches coupled system can be written in terms of the structural mechanical displacement \mathbf{u} , the electric potential difference $V^{(p)}$ constant in each piezoelectric patch, and the fluid pressure p at the fluid-structure interface:

$$\begin{aligned} & \int_{\Omega_S} [\mathbf{c}\boldsymbol{\varepsilon}(\mathbf{u})] : \boldsymbol{\varepsilon}(\delta\mathbf{u}) \, d\mathbf{v} + \sum_{p=1}^P \frac{V^{(p)}}{h^{(p)}} \int_{\Omega^{(p)}} \mathbf{e}^{(p)T} \mathbf{n}^{(p)} \\ & : \boldsymbol{\varepsilon}(\delta\mathbf{u}) \, d\mathbf{v} - \omega^2 \int_{\Omega_S} \rho_S \mathbf{u} \cdot \delta\mathbf{u} \, d\mathbf{v} - \int_{\Sigma} p \mathbf{n}^F \cdot \delta\mathbf{u} \, ds \\ & = \int_{\Gamma_\sigma} \mathbf{F}^d \cdot \delta\mathbf{u} \, ds \quad \forall \delta\mathbf{u} \in C_u^* \end{aligned} \quad (6)$$

where the admissible space C_u^* is defined by $C_u^* = \{\mathbf{u} \in C_u | \mathbf{u} = 0 \text{ on } \Gamma_u\}$. C_u being the admissible space of regular functions \mathbf{u} in Ω_S .

$$\begin{aligned} & \sum_{p=1}^P \delta V^{(p)} C^{(p)} V^{(p)} - \sum_{p=1}^P \frac{\delta V^{(p)}}{h^{(p)}} \int_{\Omega^{(p)}} [\mathbf{e}^{(p)} : \boldsymbol{\varepsilon}(\mathbf{u})] \cdot \mathbf{n}^{(p)} \, d\mathbf{v} \\ & = \sum_{p=1}^P \delta V^{(p)} Q^{(p)} \quad \forall \delta V^{(p)} \in \mathbb{R} \end{aligned} \quad (7)$$

where $C^{(p)} = \epsilon_{33}^{(p)} S^{(p)} / h^{(p)}$ defines the capacitance of the p th piezoelectric patch ($S^{(p)}$ being the area of the patch and $\epsilon_{33}^{(p)}$ the piezoelectric material permittivity in the direction normal to the electrodes) and $Q^{(p)}$ is the global charge in one of the electrodes (see [9]).

2.3. Finite element formulation

After discretization of the previous variational formulation by finite element method and using the following additional relation between electrical potential differences and electric charges due to the shunt circuits:

$$-\omega^2 \mathbf{L} \mathbf{Q} - i\omega \mathbf{R} \mathbf{Q} + \mathbf{V} = \mathbf{0} \quad (8)$$

we find the following matrix equation:

$$\begin{bmatrix} \mathbf{K}_u + \mathbf{C}_{uV} \mathbf{K}_V^{-1} \mathbf{C}_{uV}^T & \mathbf{C}_{uV} \mathbf{K}_V^{-1} & -\mathbf{C}_{up} \\ \mathbf{K}_V^{-1} \mathbf{C}_{uV}^T & \mathbf{K}_V^{-1} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \mathbf{Q} \\ \mathbf{P}_\Sigma \end{bmatrix} - i\omega \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \mathbf{Q} \\ \mathbf{P}_\Sigma \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{M}_u & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{U} \\ \mathbf{Q} \\ \mathbf{P}_\Sigma \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (9)$$

where $\mathbf{Q} = (Q^{(1)}, Q^{(2)}, \dots, Q^{(P)})^T$ and $\mathbf{V} = (V^{(1)}, V^{(2)}, \dots, V^{(P)})^T$ are the column vectors of electric charges and potential differences; $\mathbf{R} = \text{diag}(R^{(1)}, R^{(2)}, \dots, R^{(P)})$ and $\mathbf{L} = \text{diag}(L^{(1)}, L^{(2)}, \dots, L^{(P)})$ are the diagonal matrices of the resistances and inductances of the patches; \mathbf{U} and \mathbf{P}_Σ are the vectors of nodal values of \mathbf{u} and p ; \mathbf{M}_u and \mathbf{K}_u are the mass and stiffness matrices of the structure (elastic structure and piezoelectric patches); \mathbf{C}_{uV} is the electric mechanical coupled stiffness matrix; $\mathbf{K}_V = \text{diag}(C^{(1)}, C^{(2)}, \dots, C^{(P)})$ is a diagonal matrix filled with the P capacitances of the piezoelectric patches; \mathbf{C}_{up} is the fluid-structure coupled matrix; \mathbf{F} is the applied mechanical force vector.

2.4. Modal reduction

We present in this section a reduced-order formulation of the discretized problem obtained in the previous section. This formulation consists in expanding the structural displacement over the *in vacuo* structure modes in short circuited configuration.

We introduce the normalized modal base of the structure $\{\boldsymbol{\omega}_s, \boldsymbol{\Phi}_s\}$ truncated to N_s modes such that:

$$\boldsymbol{\omega}_s^2 = \text{diag}(\omega_{s1}^2, \omega_{s2}^2, \dots, \omega_{sN_s}^2), \quad \boldsymbol{\Phi}_s = [\boldsymbol{\Phi}_{s1}, \dots, \boldsymbol{\Phi}_{sN_s}] \quad (10)$$

and

$$\boldsymbol{\Phi}_s^T \mathbf{M}_u \boldsymbol{\Phi}_s = \mathbf{I}, \quad \boldsymbol{\Phi}_s^T \mathbf{K}_u \boldsymbol{\Phi}_s = \boldsymbol{\omega}_s^2 \quad \text{and} \quad \mathbf{U} = \boldsymbol{\Phi}_s \mathbf{q}_s(t) \quad (11)$$

where \mathbf{I} is the identity matrix of size N_s and the vector $\mathbf{q}_s = [q_{s1}, \dots, q_{sN_s}]^T$ is the modal amplitudes of the structure displacement.

By projecting the first row of the matrix Eq. (9) on the truncated modal basis of the structure, we obtain:

$$\begin{bmatrix} \boldsymbol{\omega}_s^2 + \boldsymbol{\Phi}_s^T \mathbf{C}_{uV} \mathbf{K}_V^{-1} \mathbf{C}_{uV}^T \boldsymbol{\Phi}_s & \boldsymbol{\Phi}_s^T \mathbf{C}_{uV} \mathbf{K}_V^{-1} & -\boldsymbol{\Phi}_s^T \mathbf{C}_{up} \\ \mathbf{K}_V^{-1} \mathbf{C}_{uV}^T \boldsymbol{\Phi}_s & \mathbf{K}_V^{-1} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{q}_s \\ \mathbf{Q} \\ \mathbf{P}_\Sigma \end{bmatrix} - i\omega \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{q}_s \\ \mathbf{Q} \\ \mathbf{P}_\Sigma \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{q}_s \\ \mathbf{Q} \\ \mathbf{P}_\Sigma \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_s^T \mathbf{F} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (12)$$

Using the effective electromechanical modal coupling factor (EEMCF), characterizing the energy exchanges between the mechanical structure and the piezoelectric patches and defined by:

$$k_{eff,i}^2 = \frac{\bar{\omega}_{si}^2 - \omega_{si}^2}{\omega_{si}^2} \quad (13)$$

where ω_{si} and $\bar{\omega}_{si}$ are, respectively, the short-circuit and open-circuit i th system natural frequencies, and the following relation demonstrated in [9]

$$k_{eff,i}^2 \simeq \frac{\boldsymbol{\Phi}_{si}^T \mathbf{C}_{uV} \mathbf{K}_V^{-1} \mathbf{C}_{uV}^T \boldsymbol{\Phi}_{si}}{\omega_{si}^2} \quad (14)$$

Eq. (12) becomes

$$\begin{bmatrix} \boldsymbol{\omega}_s^2 + \mathbf{k}_{eff}^2 \boldsymbol{\omega}_s^2 & \boldsymbol{\Phi}_s^T \mathbf{C}_{uV} \mathbf{K}_V^{-1} & -\boldsymbol{\Phi}_s^T \mathbf{C}_{up} \\ \mathbf{K}_V^{-1} \mathbf{C}_{uV}^T \boldsymbol{\Phi}_s & \mathbf{K}_V^{-1} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{q}_s \\ \mathbf{Q} \\ \mathbf{P}_\Sigma \end{bmatrix} - i\omega \begin{bmatrix} \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{R} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{q}_s \\ \mathbf{Q} \\ \mathbf{P}_\Sigma \end{bmatrix} - \omega^2 \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{q}_s \\ \mathbf{Q} \\ \mathbf{P}_\Sigma \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_s^T \mathbf{F} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (15)$$

where $\mathbf{k}_{eff}^2 = \text{diag}(k_{eff,1}^2, k_{eff,2}^2, \dots, k_{eff,N_s}^2)$.

This matrix equation represents the reduced order model of the structure with piezoelectric shunt damping and submitted to an acoustic pressure.

3. Boundary element formulation for external/internal acoustic fluid

3.1. Equations in the frequency domain

In this section, the direct boundary element method for exterior/interior acoustic domain is presented. The governing equations of the acoustic fluid are [17,3]

$$\Delta p + k^2 p = 0 \quad \text{in } \Omega_F \quad (16a)$$

$$\frac{\partial p}{\partial n} = 0 \quad \text{on } \Gamma_D \quad (16b)$$

$$\frac{\partial p}{\partial n} = \rho_F \omega^2 \mathbf{u} \cdot \mathbf{n}^F \quad \text{on } \Sigma \quad (16c)$$

$$\lim_{r \rightarrow \infty} \left(\frac{\partial p}{\partial r} + ikp \right) = \theta \left(\frac{1}{r} \right) \quad \text{on } \Gamma_\infty \quad (16d)$$

Eq. (16a) represents the Helmholtz equation where $k = \omega/c$ is the wave number, i.e. the ratio of the circular frequency ω and the sound velocity c ; Eq. (16b) is the rigid boundary condition on Γ_D ; Eq. (16c) is the kinematic interface fluid–structure condition on Σ ; Eq. (16d) represents the Sommerfeld condition at infinity.

3.2. Boundary element formulation

The boundary element formulation for acoustic problems can be used for the interior and exterior problems. The Helmholtz equation is valid for the pressure p at the arbitrary collocation point \mathbf{x} within the acoustic domain Ω_F . A weak form of this equation is obtained by weighting with the fundamental solution:

$$G(\mathbf{x}, \mathbf{y}) = \frac{e^{ik|\mathbf{x}-\mathbf{y}|}}{4\pi|\mathbf{x}-\mathbf{y}|} \quad (17)$$

where $|\mathbf{x}-\mathbf{y}|$ denotes the distance between an arbitrary point \mathbf{x} and the load source point \mathbf{y} .

Applying Green's second theorem, the Helmholtz equation can be transformed into a boundary integral equation, which can be expressed as follows

$$c(\mathbf{x})p(\mathbf{x}) = \int_{\partial\Omega_F} p(\mathbf{y}) \frac{\partial G(\mathbf{x}, \mathbf{y})}{\partial n_y} dS - \int_{\partial\Omega_F} \frac{\partial p(\mathbf{y})}{\partial n_y} G(\mathbf{x}, \mathbf{y}) dS \quad (18)$$

where

$$c(\mathbf{x}) = \begin{cases} 1 & \mathbf{x} \text{ in fluid domain} \\ \frac{1}{2} & \mathbf{x} \text{ on smooth boundary of fluid domain} \\ \frac{\Omega(\mathbf{x})}{4\pi} & \mathbf{x} \text{ on nonsmooth boundary of fluid domain} \\ 0 & \mathbf{x} \text{ outside fluid domain} \end{cases} \quad (19)$$

and

$$\Omega(\mathbf{x}) = 4\pi + \int_{\partial\Omega_F} \frac{\partial(|\mathbf{x}-\mathbf{y}|^{-1})}{\partial n_y} dS$$

is the solid angle seen from \mathbf{x} . Note that the value $c(\mathbf{x}) = \frac{1}{2}$ is valid if the surface $\partial\Omega_F$ is assumed to be closed and sufficiently smooth, i.e. there is a unique tangent to $\partial\Omega_F$ at every $\mathbf{x} \in \partial\Omega_F$. For the general case, where a non-unique tangent plane exists at $\mathbf{x} \in \partial\Omega_F$, we use $c(\mathbf{x}) = \frac{\Omega(\mathbf{x})}{4\pi}$ (for example, when \mathbf{x} is lying on a corner or an edge).

The fluid boundary is divided into N quadrilateral elements ($\partial\Omega_F = \sum_{j=1}^N S_j$) and Eq. (18) is discretized. After using the relation between the acoustic pressure and the fluid normal velocity $\frac{\partial p}{\partial n} = -i\rho_F \omega v$ (where $v = \mathbf{v}^F \cdot \mathbf{n}^F$), the discrete Helmholtz equation can be written for any point \mathbf{x} defined by the node i as

$$c_i p_i = \sum_{j=1}^N \int_{S_j} p_j(\mathbf{y}) \frac{\partial G(\mathbf{x}_i, \mathbf{y})}{\partial n_y} dS + i\rho_F \omega \sum_{j=1}^N \int_{S_j} v_j(\mathbf{y}) G(\mathbf{x}_i, \mathbf{y}) dS \quad (20)$$

For each quadrilateral element j , the pressure $p_j(\mathbf{y})$ and the normal velocity $v_j(\mathbf{y})$ can be expressed as a function of their nodal values

$$p_j(\mathbf{y}) = \sum_{k=1}^4 N_k p_j^k = \mathbf{N} \mathbf{p}_j, \quad v_j(\mathbf{y}) = \sum_{k=1}^4 N_k v_j^k = \mathbf{N} \mathbf{v}_j \quad (21)$$

and Eq. (20) becomes

$$c_i p_i = \sum_{j=1}^N \sum_{k=1}^4 \int_{S_j} N_k \frac{\partial G(\mathbf{x}_i, \mathbf{y}_j)}{\partial n_y} dS p_j^k + i\rho_F \omega \sum_{j=1}^N \sum_{k=1}^4 \int_{S_j} N_k G(\mathbf{x}_i, \mathbf{y}_j) dS v_j^k \quad (22)$$

or in the following form

$$c_i p_i = \sum_{j=1}^N \sum_{k=1}^4 H_{ij}^k p_j^k + i\rho_F \omega \sum_{j=1}^N \sum_{k=1}^4 G_{ij}^k v_j^k \quad (23)$$

where

$$H_{ij}^k = \int_{S_j} N_k \frac{\partial G(\mathbf{x}_i, \mathbf{y}_j)}{\partial n_y} dS, \quad G_{ij}^k = \int_{S_j} N_k G(\mathbf{x}_i, \mathbf{y}_j) dS \quad (24)$$

We place now the point \mathbf{x}_i at each of nodal points on the boundary \mathbf{y}_j successively, which is known as a ‘‘collocation point’’. We obtain

$$c_i \delta_{ij} p_j = \sum_{j=1}^N \sum_{k=1}^4 H_{ij}^k p_j^k + i\rho_F \omega \sum_{j=1}^N \sum_{k=1}^4 G_{ij}^k v_j^k \quad (25)$$

When the collocation scheme is repeated for all nodal points N_n of the boundary element mesh, a set of N_n expressions in the nodal field variables is obtained which can be assembled into the following matrix equation

$$\mathbf{H} \mathbf{P} = i\rho_F \omega \mathbf{G} \mathbf{V} \quad (26)$$

where \mathbf{P} and \mathbf{V} are the vectors with sound pressure and velocity in the normal direction to the boundary surface at the nodal position of the boundary element mesh.

Irregular frequencies: Let's remark that when the Boundary Element method is applied to exterior problems, the solution can be non-unique at some frequencies called irregular frequencies. These frequencies do not represent any kind of physical resonance but are due to the numerical method, which has no unique solution at some eigenfrequencies for a corresponding interior problem. In order to remove these frequencies, many different approaches were proposed. Among them, the two most popular categories are: (i) the Combined Helmholtz Integral Equation Formulation (CHIEF) method originally proposed by Schenck [18] and its variations, and (ii) the linear combination of the Kirchhoff-Helmholtz integral equation and its normal derivative originally proposed by Brakhage and Werner [19]. This approach was adapted to the Neumann problem by Burton and Miller [20]. We refer also to ([21,22] references included) for alternative procedures in order to obtain symmetric BEM without irregular frequencies.

4. FE/BE formulation for the fluid–structure with shunt systems coupled problem

The fluid boundary domain $\partial\Omega_F$ is divided into two parts including Γ_D (where the rigid boundary condition is applied) and the interface Σ (for the fluid–structure interface) such as $\partial\Omega_F = \Gamma_D \cup \Sigma$ and $\Gamma_D \cap \Sigma = \emptyset$. The boundary conditions given in Eqs. (16b) and (16c) can be expressed in discretized form

$$\mathbf{v} = \mathbf{0} \quad \text{on } \Gamma_D \quad (27a)$$

$$\mathbf{v} = i\omega \mathbf{T} \mathbf{U} \quad \text{on } \Sigma \quad (27b)$$

where \mathbf{T} is the global coupling matrix that transforms the nodal normal displacement of the structure to the normal velocity of the acoustic fluid at the interface. Using a partitioning of \mathbf{P} into $\mathbf{P} = [\mathbf{P}_\Sigma \mathbf{P}_D]^T$, applying this partitioning to the matrix equation (Eq. (26)) and introducing Eqs. (27) yields

$$\begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{P}_\Sigma \\ \mathbf{P}_D \end{bmatrix} = \begin{bmatrix} \mathbf{G}_{11} & \mathbf{G}_{12} \\ \mathbf{G}_{21} & \mathbf{G}_{22} \end{bmatrix} \begin{bmatrix} -\rho_F \omega^2 \mathbf{T} \mathbf{U} \\ \mathbf{0} \end{bmatrix} \quad (28)$$

By combining Eq. (15) with Eq. (28), we find the following coupled FE/BE matrix equation

$$\begin{bmatrix} \omega_s^2 + \mathbf{k}_{\text{eff}}^2 \omega_s^2 - \omega^2 \mathbf{I} & \Phi_s^T \mathbf{C}_{uv} \mathbf{K}_V^{-1} & -\Phi_s^T \mathbf{C}_{up} & \mathbf{0} \\ \mathbf{K}_V^{-1} \mathbf{C}_{uv}^T \Phi_s & \mathbf{K}_V^{-1} - i\omega \mathbf{R} - \omega^2 \mathbf{L} & \mathbf{0} & \mathbf{0} \\ \rho_F \omega^2 \mathbf{G}_{11} \mathbf{T} \Phi_s & \mathbf{0} & \mathbf{H}_{11} & \mathbf{H}_{12} \\ \rho_F \omega^2 \mathbf{G}_{21} \mathbf{T} \Phi_s & \mathbf{0} & \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{q}_s \\ \mathbf{Q} \\ \mathbf{P}_\Sigma \\ \mathbf{P}_D \end{bmatrix} = \begin{bmatrix} \Phi_s^T \mathbf{F} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix} \quad (29)$$

Remarks: acoustic indicators

- The sound intensity I at every point Q on the vibrating surface may be calculated from

$$I = \frac{1}{2} \text{Re}\{p(Q)v^*(Q)\} = \frac{1}{2} \text{Re}\{p(Q)(i\omega \mathbf{u}(Q) \cdot \mathbf{n}^s)^*\} \quad (30)$$

where Re denotes the real part of the expression in parentheses and $*$ denotes the complex conjugate.

- The intensity is the sound power radiated per unit area of the transmission; the total radiated sound power Π is found by integrating the intensity over the surface Σ :

$$\Pi = \int_{\Sigma} I ds = \frac{\omega}{2} \text{Im}\{\mathbf{U}^{*T} \mathbf{C}_{up} \mathbf{P}_\Sigma\} = \frac{\omega}{2} \text{Im}\{\mathbf{q}_s^{*T} \Phi_s^T \mathbf{C}_{up} \mathbf{P}_\Sigma\} \quad (31)$$

- The mean quadratic normal velocity $\langle V^2 \rangle$ is given by

$$\langle V^2 \rangle = \frac{1}{2\Sigma} \int_{\Sigma} v^2 ds = \frac{\omega^2}{2\Sigma} \mathbf{w}^* \bar{\mathbf{M}}_u \mathbf{w} \quad (32)$$

where \mathbf{w} is the vector of nodal normal displacements of the structure and $\bar{\mathbf{M}}_u$ is the structure mass matrix reduced to the dofs of interest.

- The radiation efficiency, σ , is the ratio of the sound power radiated by a vibrating structure to the sound power that would be radiated by an equivalent flat piston vibrating in an infinite baffle:

$$\sigma = \frac{\Pi}{\rho_F c \Sigma \langle V^2 \rangle} \quad (33)$$

5. Numerical examples

In the first example, the vibration reduction using an inductive shunt damping technique of an elastic plate backed by a closed acoustic cavity is analyzed. For this example, the FEM/FEM modal analysis of the electromechanical-acoustic problem is presented. Then, the FEM/BEM frequency response of the coupled system in short circuit and inductive shunt cases are compared in terms of vibration attenuation. The second example concerns the simulation of the attenuation of the sound field emitted from a submerged plate in a half-space acoustic domain by means of a piezoelectric shunt system.

5.1. Plate backed by a closed acoustic cavity

The first objective of this example is to compare the FEM [2] to the BEM for the acoustic domain in an internal fluid-structure coupled problem (the structure being represented by FEM reduced order model obtained by projection on the first ten "in vacuo" structure modes). The second objective is to evaluate the effect of the shunt system on the noise attenuation. Concerning the finite element description of the internal acoustic fluid, a reduced order model obtained by projection on the first ten acoustic modes in rigid fixed cavity will be used [23,24]. Moreover, the effects of the higher modes of each subsystem can be taken into account through an appropriate so-called "static correction". This aspect is not considered here because the number of retained normal modes was enough when comparing the results with those of the finite element non-reduced analysis.

We consider a 3D hexahedric acoustic cavity of size $A = 0.3048$ m, $B = 0.1524$ m and $C = 0.1524$ m along the directions x , y , and z , respectively. The cavity is completely filled with air (density $\rho = 1.2$ kg/m³ and speed of sound $c = 340$ m/s). The cavity walls are rigid except the top one, which is a simply-supported along its edges flexible aluminum plate of thickness 1.63 mm. The density of the plate is 2690 kg/m³, the Young modulus is 70 GPa and Poisson ratio 0.3. On the top surface of the plate, a PIC 151 patch is bonded, whose in plane dimensions are 0.0762×0.0508 m² along x and y and 0.5 mm thick (see Fig. 2). The mechanical characteristics of the piezoelectric material PIC 151 are given in [2].

Concerning the structural FEM/acoustic FEM discretization, we have used, for the structural part, 72 four nodes plate elements. The portion of the plate covered by the piezoelectric patch and the patch itself has been modeled according to laminated theory [2]. Moreover, only one electrical degree of freedom is used to represent the electrical charge Q in the patch. The acoustic cavity is discretized using $12 \times 6 \times 6$ hexahedric elements. The structural and acoustic meshes are compatible at the interface, and the fluid-structure coupling is realized through the \mathbf{C}_{up} matrix. Full details of this formulation are published by the authors in [2].

For the structural FEM/acoustic BEM formulation, besides the FE discretization of the plate, only the boundary Σ of the fluid domain is discretized with boundary elements. Notice that the BE nodes on this part must coincide with FE nodes.

Table 1 presents the first ten eigenfrequencies in three cases: (i) the 3D rigid acoustic cavity, (ii) the plate with the patch in short and open circuited cases and (iii) the plate/acoustic-cavity coupled system in the short circuited case. All results are computed with the finite element formulation presented in [2]. The fourth and ninth frequencies of the coupled system are associated with the first two acoustics modes in the rigid cavity lower than 1128.2 Hz, while the other frequencies correspond to the first eight vibration modes of the structure. This can be confirmed by comparing the mode shapes in case (iii) with those obtained in case

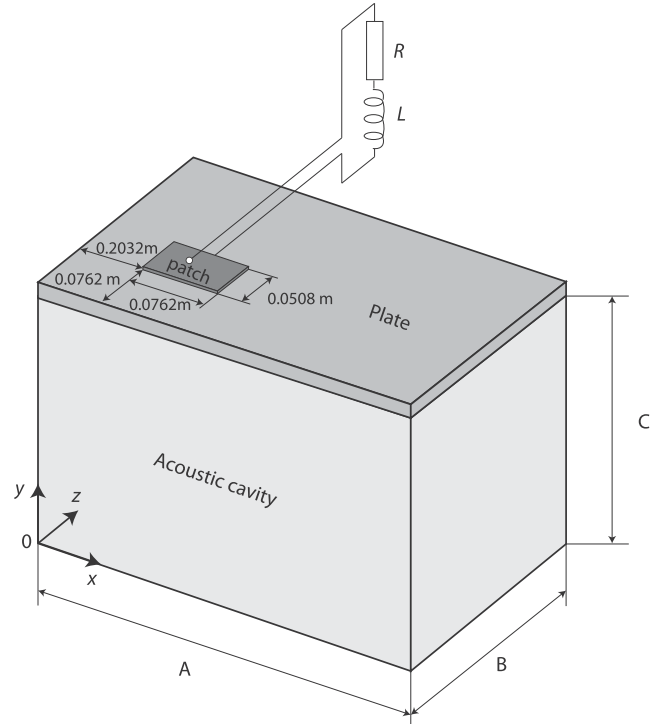


Fig. 2. Electromechanical-acoustic coupled system: geometrical data.

Table 1

Computed frequencies (Hz) of the structural–acoustic coupled system.

Fluid	Structure SC	Structure OC	Fluid–structure	Type ^a
559.3	210.6	210.7	215.1	S
1128.1	329.5	330.9	327.1	S
1128.3	540.6	544.4	538.9	S
1128.5	722.2	722.2	561.7	F
1259.1	828.4	830.6	719.4	S
1259.5	834.3	834.7	826.5	S
1595.5	1023.8	1024.7	832.1	S
1595.6	1204.7	1204.7	1021.0	S
1595.7	1296.1	1296.7	1128.1	F
1690.8	1567.0	1567.0	1129.9	S

^a S for structure predominant modal shape and F for fluid predominant modal shape.

(i) or case (ii), which are not shown here for the sake of brevity. Moreover, as expected, the natural frequencies of the coupled modes (structure dominated) are lower than those for the structure in vacuum (except for the first mode) due to the added-mass effect of the fluid.

The plate is now excited by a unit distributed time harmonic pressure load. In addition, mechanical damping was introduced through a modal damping coefficient $\zeta = 0.001$ for all eigenmodes in the selected reduced modal basis.

As can be seen in Fig. 3, the sound pressure level is calculated on the plate center. The results for the two methods are very similar at sound level peaks (resonance frequencies), which enable us to check the validity of the BEM formulation. For this particular example, the comparisons of computational times (using an implementation of the two methods in the Matlab software) showed that the FEM for the internal fluid is much more faster than BEM technique (the CPU time ratio is about 10).

In order to achieve maximum vibration dissipation of the third coupled mode (Fig. 4), the patch is tuned now to an RL shunt circuit. The resistance R and the inductance L can be adjusted and properly chosen to maximize the damping effect of a particular mode. The optimal resistance and inductance of the i th mode for a series resonant shunt are given by [7]:

$$R^{\text{opt}} = \frac{\sqrt{2k_{\text{eff},i}^2}}{C\omega_i(1 + k_{\text{eff},i}^2)} \quad (34a)$$

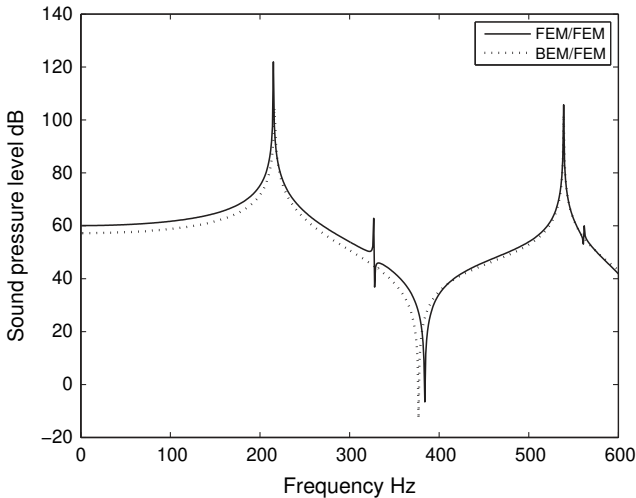


Fig. 3. Sound pressure level on plate center: comparison between FEM/FEM and FEM/BEM approaches.

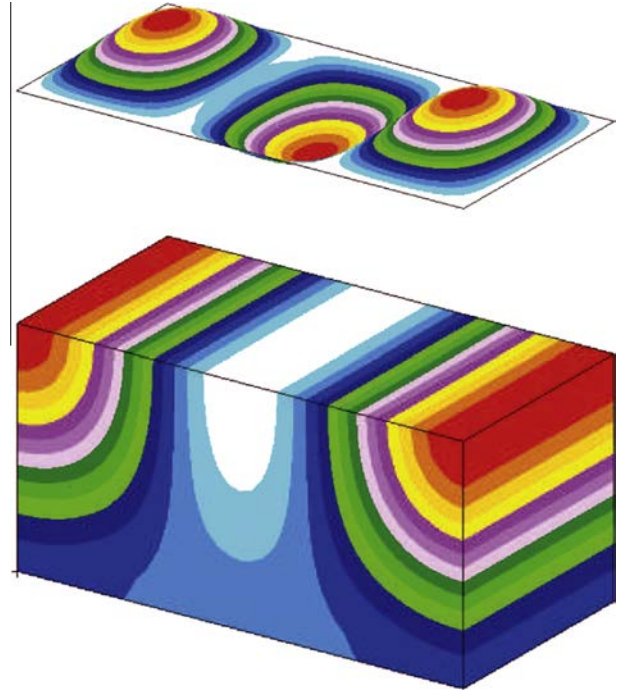


Fig. 4. Third fluid–structure coupled mode: fluid pressure level in the cavity and plate total displacement.

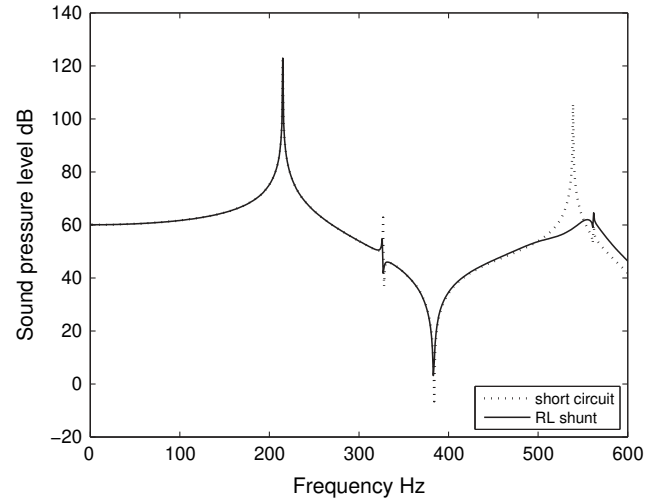


Fig. 5. Sound pressure level at the plate center with and without shunt system.

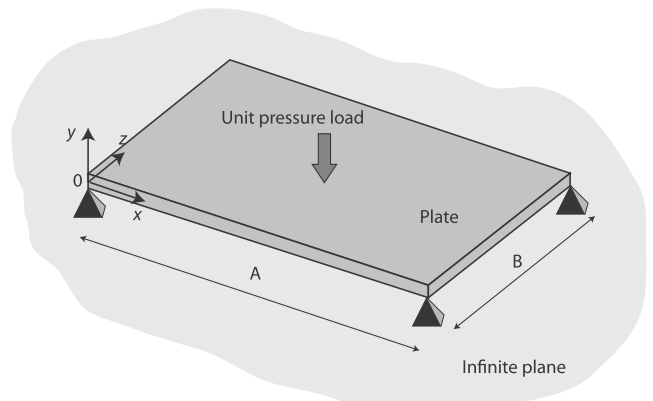


Fig. 6. Flexible plate in a half-space acoustic domain.

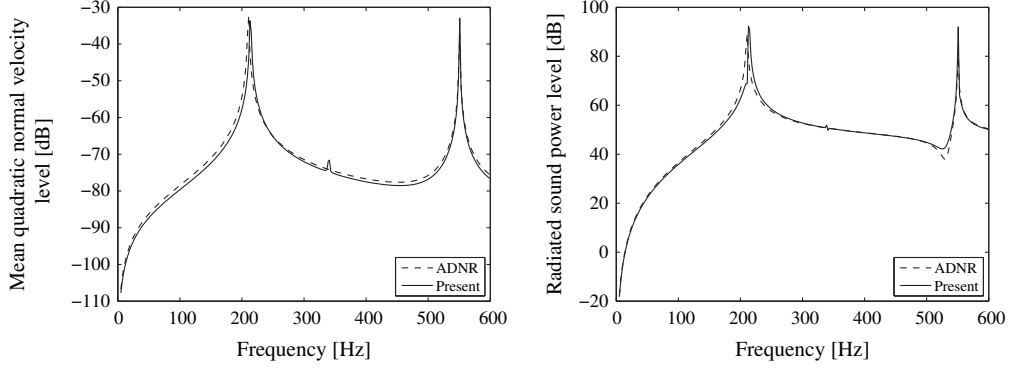


Fig. 7. The mean quadratic normal velocity and radiated sound power of the plate: comparison between the proposed FEM/BEM formulation and ADNR[®] results.

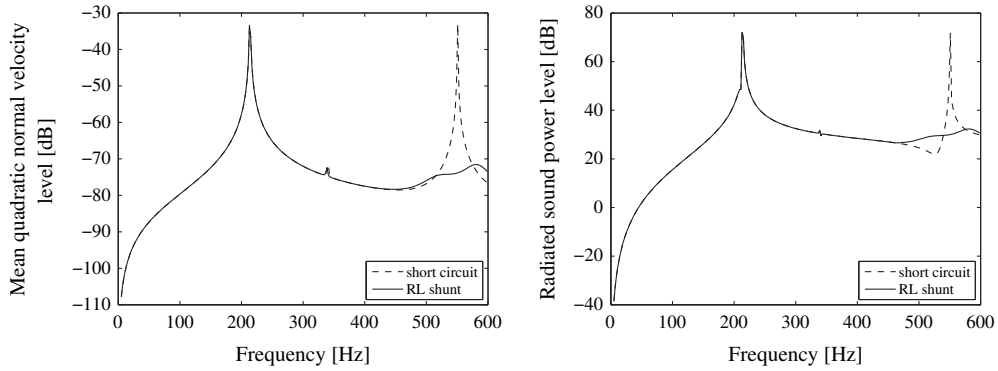


Fig. 8. The mean quadratic normal velocity and radiated sound power of the plate with and without shunt system.

$$L^{\text{opt}} = \frac{1}{C\omega_i^2(1 + k_{\text{eff},i}^2)} \quad (34b)$$

where ω_i is the short circuit natural frequency of the i th mode and $k_{\text{eff},i}$ is the effective electromechanical coupling coefficient given in Eq. (13). For the third coupled mode, these parameters are taken $R = 348 \Omega$ and $L = 0.61 \text{ H}$.

The system vibratory response is obtained with the BEM approach. Fig. 5 presents the sound pressure level at the plate center with and without shunt system. This figure shows that the resonant magnitude for the third mode has been significantly reduced due to the shunt effect. In fact, the strain energy present in the piezoelectric material is converted into electrical energy and hence dissipated into heat using the RL shunt device.

5.2. Vibrating plate in a half-space acoustic domain

In this example, we consider the radiation of the previous plate in a half-space acoustic domain (Fig. 6). As before, the plate is excited by a unit distributed time harmonic pressure load. The mean quadratic normal velocity of the plate and the radiated sound power are used as indicators for this example.

As in the first example, the structure is represented by FEM reduced order model obtained by projection on the first ten “in vacuo” structure modes (Eq. (29)) and mechanical damping was introduced through a modal damping coefficient $\xi = 0.001$ for all eigenmodes in the selected reduced modal basis. The external fluid is described by BEM.

The results are compared with those obtained with the available free software, ADNR[®]. This free software can calculate the

acoustic and vibration response of an elastic baffled and simply supported rectangular plate mechanically or acoustically excited. It uses a semi-analytical method based on the variational approach and on the Rayleigh–Ritz method to calculate the plate movement equation.

The results (mean quadratic normal velocity of the plate and the radiated sound power) presented in Fig. 7 show the accuracy of the BEM procedure described in the paper for external problems compared to the semi-analytical method results given by ADNR[®].

The series single-mode shunt damping circuit is performed to damp the third bending mode of the plate, as presented in the previous section. The damping performances of the shunted piezoelectric can be observed in Fig. 8 by comparing the mean quadratic normal velocity and the radiated sound power of the plate with and without shunt. The peak amplitude of the third mode was reduced by 38 dB.

6. Conclusions

In this work, a coupled finite element/boundary element method (FEM/BEM) for control of noise radiation and sound transmission of vibrating structure by active piezoelectric techniques is analysed. The passive shunt damping strategy is employed for vibration attenuation in the low frequency range. A reduced order model for the structure/piezoelectric shunt system with introduction of the effective electromechanical modal coupling factor is developed.

Numerical examples show that for interior acoustic problems, the use of reduced order model for the fluid is recommended, but for the external domain the BEM is more efficient even if costly

(the use of recent fast multipole method should improve the convergence [5]).

The more complex system constituted of structure equipped with shunted piezoelectric patches and coupled with an interior and exterior acoustic fluid is the subject of present and future investigations from computational and experimental aspects.

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