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Global Sensitivity Analysis of Piezoelectric Energy Harvesters

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Abstract

Vibration energy harvesting using the direct effect of piezoelectricity has attracted increasing attention during the last two decades. Different modeling techniques have been applied to describe the electromechanical coupling effect of a piezoelectric harvester and to predict its electrical output. This study aims to identify the most important properties of both harvester substrate material and piezoelectric material that cause uncertainty in the predicted performances of the harvester. Global sensitivity analysis, applied in this paper, is a promising method used to identify systems parameters which have significant impact on the system output. In this paper, the Elementary Effects method (EEs), a particular implementation of the global sensitivity method, is used to identify the impact of substrate and piezoelectric material properties on the voltage frequency response function of a typical bimorph piezoelectric energy harvester with fixed geometry. With a small number of model evaluations at selected ranges of material properties,

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it has been found that the elastic modulus and density of the piezoelectric layer are the parameters which lead to the largest output variability. Furthermore, it has been found that the order of importance of the parameters can change from short-circuit to open-circuit conditions.

Keywords: Global Sensitivity Analysis, Elementary Effects, Piezoelectric Energy Harvesting, Finite Element Method

1. Introduction

Energy harvesting using piezoelectric material has attracted considerable interest in the last decade. Piezoelectric material displays converting capabilities due to its direct and indirect electromechanical effect. Several approaches have been developed to model Piezoelectric Energy Harvesters (PEHs). In particular, two methods have been accepted and are the most widely used in the scientific community: (i) the analytical distributed parameter model introduced by Erturk and Inman [1, 2] is applied for unimorph and bimorph harvesters to obtain a modal solution of second-order ordinary differential equations, (ii) the finite element method, adapted by De Marqui Junior et al. [3, 4], uses standard discretization to provide discrete models with less restrictive assumptions on the global electrical variables. The two modeling approaches are particularly useful as they have been amply tested showing close correlation with experimental data.

In the frame of numerical approaches, sensitivity analysis is a very useful tool for modeling and design analysis of mechatronic systems. It is applied in various engineering problems such as structural analysis, model updating, design optimization of structures, system control and uncertainty propagation [5]. Sensitivity analysis enables the evaluation of the degree of influence

20 of input parameters on the output responses of the modeled system. Fur-
21 thermore, this method process is widely considered as the most important
22 step in design optimization since significant number of optimization algo-
23 rithms require the prediction of the evolution of the output responses of the
24 system for different values of design variables [6].

25 Sensitivity analysis techniques can be classified into local and global
26 methods. Local sensitivity analysis, often referred to as a one factor at time
27 analysis, is based on the approximation of partial derivatives to assess how
28 uncertainty in one factor affects the model response keeping other factors
29 fixed at their nominal values. Global Sensitivity Analysis (GSA), on the
30 other hand, offers a comprehensive approach to the studied model, since it
31 evaluates the effect of factors that are varying within the considered multi-
32 dimensional input space [7].

33 In recent published research, Sharp and Brooks [8] introduced expres-
34 sions for the sensitivities of the Frequency Response Functions (FRFs) of
35 linear constant coefficient systems with distinct eigenvalues to perturba-
36 tions of the design parameters suggesting several engineering applications of
37 the developed theory. Lasecka-Plura et al [5] studied the sensitivity of the
38 FRF and steady-state vibration response of planar frames with viscoelastic
39 dampers. A comparison was made between the first-order, the second-order
40 and the exact solution sensitivities. Analysis show that the second order
41 sensitivity gives results which are very close to the exact solution, when the
42 perturbation of the parameter is smaller than 20%. Lima et al [6, 9] inves-
43 tigated the first order sensitivity of complex frequency response functions
44 for composite sandwich plates composed of a combination of fiber-reinforced
45 and elastomeric viscoelastic layers, frequently used for noise and vibration
46 attenuation. The study aims to find the sensitivity of viscoelastic struc-

47 tures to variation of temperature and structural parameters. Ng and Liao
48 [10] used the first order sensitivity to evaluate the performances of sensing
49 and power generation of the piezoelectric energy harvesters to variation of
50 elastic substrate properties for three type of sensors. The reported research
51 results show that for the series bimorph piezoelectric sensor, the voltage
52 sensitivity is the highest and the charge sensitivity is the lowest. For the
53 parallel bimorph piezoelectric sensor, the voltage sensitivity is the lowest
54 and charge sensitivity is highest. A unimorph sensor, however, has rela-
55 tively medium sensitivities of charge and voltage responses. Ruiz et al [11]
56 analyzed the uncertainty propagation and global sensitivity of the FRFs of
57 the output voltage of piezoelectric energy harvesters. Their approach is
58 based on the identification of Sobol' indices to assess the robustness of the
59 stochastic prediction. Peralta et al [12] introduced a procedure to update the
60 electromechanical properties of the PEHs based on Bayesian updating tech-
61 niques and global sensitivity analysis. The introduced approach constitutes
62 a powerful method for the robust design and prediction of the performances
63 of piezoelectric energy harvesters.

64 In the literature, several GSA methods are applied [13, 14, 15] ranging
65 from qualitative screening techniques [16, 17] to quantitative methods based
66 on variance decomposition [18, 19]. The Morris method [16] is a global
67 sensitivity screening technique used to derive sensitivity measures from a set
68 of local derivatives sampled on a grid throughout the parameter space [20].
69 This technique investigates the model response to a change in the inputs by
70 varying One-At-a-Time (OAT), while keeping all other variables fixed. The
71 local sensitivity measure associated to each factor is called elementary effect
72 and is defined as the ratio between the variation in the model response
73 output and the variation in the input factor. In order to obtain a global

74 sensitivity measure, different elementary effects (local derivatives taken at
75 points sampled throughout the parameter space) for each parameter are
76 estimated and averaged. Two sensitivity measures are carried out, the first
77 one is the *mean* of the EEs which is refined by Campolongo et al [7, 21]
78 consisting in averaging the absolute values of the elementary effects. The
79 second measure is the *standard deviation* of the EEs. By plotting both
80 sensitivity measures, the Morris method identifies the hierarchical order of
81 influence of the input parameters on the model output.

82 Most of the previous researches were limited to local sensitivities for
83 variation of system parameters in the vicinity of the considered nominal
84 configuration. Generally, an interval variation of system parameter is more
85 acceptable than a nominal parameter, hence the importance of applying
86 global sensitivity. Morris method computes global sensitivity in an efficient
87 way. Previously published research work does not seem to have applied
88 this method in sensitivity analysis of piezoelectric harvesters. In this paper,
89 we propose combining the implementation of the PEH model with Morris
90 elementary effect estimation method to determine the effect of uncertain
91 electromechanical parameters of a bimorph piezoelectric energy harvester
92 on the output voltage for short-circuit and open-circuit conditions. Further-
93 more, the effective electromechanical coupling factor is monitored for the
94 model evaluations used to compute elementary effects. In order to predict
95 the output voltage of the studied harvester, a finite element formulation of a
96 laminated piezoelectric harvester is applied. The mathematical development
97 of the model is given in Section 2. This model is used in Section 3 to derive
98 the complex FRFs of a bimorph piezoelectric energy harvester considered
99 as a case study in this paper. In Section 4, the Morris method is applied
100 using the modulus of the voltage FRF of the harvester as an output and

101 a set of uncertain physical parameters. The considered numerical example
 102 presented in Section 5 shows the robustness of the EEs method compared
 103 to others GSA methods. Furthermore, the global sensitivities of the volt-
 104 age FRF of a bimorph piezoelectric harvester based on elementary effects
 105 have been compared with those computed using the variance decomposition
 106 based sensitivity analysis method described in reference [11] for different
 107 configurations of the PEHs.

108 2. Finite Element Modeling of a Piezoelectric Composite Beam

109 Considering a laminated piezoelectric beam composed of K layers includ-
 110 ing P piezoelectric layers excited under base motion at the clamped end
 111 as shown in Figure 1. In the absence of mechanical dissipative effects, the
 112 extended Hamilton's principle for an electromechanical body, between the
 113 two known times t_1 and t_2 , is [22]:

$$\int_{t_1}^{t_2} (\delta T - \delta U + \delta W_{nc}) dt = 0 \quad (1)$$

114 where δT is the first variation of the kinetic energy, δU is the first variation of
 115 the internal energy including the piezoelectric effect contribution and δW_{nc}
 116 is the virtual work done by the non-conservative electric charge components.

117

118 The thin piezoelectric layers of the cantilever beam are poled in the
 119 thickness direction with an electrical field applied parallel to this polariza-
 120 tion direction. Such a configuration is characterized in particular by the
 121 electromechanical coupling between the axial strain and the transverse elec-
 122 trical field [23]. The reduced constitutive equations corresponding to the p^{th}
 123 piezoelectric layer is written as follows, $p \in \{1, \dots, P\}$:

$$\sigma_1^{(p)} = \bar{c}_{11}^{(p)} \varepsilon_1^{(p)} - \bar{e}_{31}^{(p)} E_3^{(p)} \quad (2)$$

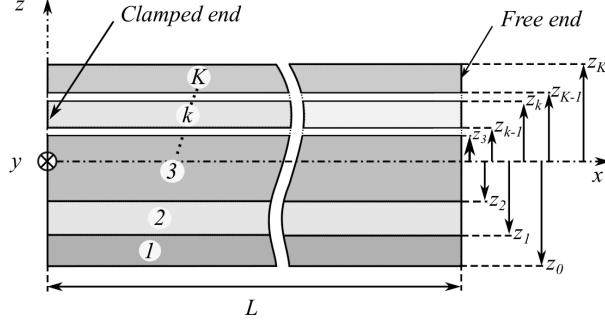


Figure 1: A multilayer piezoelectric cantilever beam: $k \in \{1, \dots, K\}$

$$D_3^{(p)} = \bar{e}_{31}^{(p)} \varepsilon_1^{(p)} + \bar{e}_{33}^{(p)} E_3^{(p)} \quad (3)$$

124 where $\sigma_1^{(p)}$, $\varepsilon_1^{(p)}$, $E_3^{(p)}$ and $D_3^{(p)}$ are respectively the normal stress, the normal
 125 strain, the electric field and the electric displacement; $\bar{c}_{11}^{(p)}$ is the elastic mod-
 126 ulus, $\bar{e}_{31}^{(p)}$ is the piezoelectric coupling coefficient and $\bar{\varepsilon}_{33}^{(p)}$ is the permittivity
 127 at constant strain. The electric field $E_3^{(p)}$ is normal to the electrodes of the
 128 piezoelectric layer and its intensity is assumed to be uniform in the piezo-
 129 electric thickness direction. The superscript (p) refers to the piezoelectric
 130 material. Moreover, the thickness of the electrodes is much smaller than
 131 the thickness of piezoelectric layer so that the equipotentiality assumption
 132 in the piezoelectric electrodes is well satisfied. Thus the electric field can be
 133 expressed as:

$$E_3^{(p)} = -\frac{V^{(p)}}{h^{(p)}} \quad (4)$$

134 where $V^{(p)}$ is the potential difference between the upper and the lower elec-
 135 trode surfaces and $h^{(p)} = z_p - z_{p-1}$ is the thickness of the p^{th} piezoelectric
 136 layer.

137 The equations of the coupled electromechanical variational formulation

138 can be rewritten in the following form [23, 24]:

$$\sum_{k=1}^K \int_{\Omega^k} \rho^k (\ddot{u}_x \delta u_x + \ddot{u}_z \delta u_z) d\Omega + \sum_{k=1}^K \int_{\Omega^k} \bar{c}_{11}^k \varepsilon_1 \delta \varepsilon_1 d\Omega + \sum_{p=1}^P \frac{V^{(p)}}{h^{(p)}} \int_{\Omega^{(p)}} \bar{e}_{31}^{(p)} \delta \varepsilon_1 d\Omega = 0$$

139 (5)

$$- \sum_{p=1}^P \frac{\delta V^{(p)}}{h^{(p)}} \int_{\Omega^{(p)}} \bar{e}_{31}^{(p)} \varepsilon_1 d\Omega + \sum_{p=1}^P \frac{\delta V^{(p)}}{h^{(p)}} \int_{\Omega^{(p)}} \bar{e}_{33}^{(p)} \frac{V^{(p)}}{h^{(p)}} d\Omega + \sum_{p=1}^P \delta V^{(p)} Q^{(p)} = 0$$

140 (6)

140 where ρ^k and Ω^k are respectively the mass density and the domain occupied
 141 by the k^{th} layer, and $Q^{(p)}$ is the electrical charge of the p^{th} piezoelectric
 142 layer. Furthermore, the mechanical displacements fields u_x and u_z of the
 143 laminated harvester under base excitation, based on Euler-Bernoulli beam
 144 assumptions, are defined as follow:

$$u_x(x, z, t) = u(x, t) - z\theta(x, t)$$

145 (7)

$$u_z(x, z, t) = w(x, t) = w_b(t) + w_{rel}(x, t)$$

(8)

146 where u is the axial displacement of the center line of the beam, $\theta =$
 147 $\partial w_{rel}/\partial x$ is the beam section rotation, w is the transverse displacement,
 148 $w_b(t) = w_b e^{j\omega t}$ is the base displacement and w_{rel} is the relative transverse
 149 displacement of clamped-free beam.

150 Applying the finite element method, the previous formulations uses a
 151 standard discretization of N mechanical Degrees Of Freedom (DOFs) and
 152 provides less restrictive assumptions on the electrical DOFs: only one elec-
 153 trical DOF for each piezoelectric layer is needed due to the equipotentiality
 154 in the electrodes. In this paper, three mechanical degrees of freedom per
 155 node are assumed (u , w , θ). Details of the stratified beam finite element
 156 discretization are provided in [23] for the master element shape functions.
 157 The various terms appearing in the variational formulation of Equations (5)

158 and (6) in terms of Hamilton's principle are now discussed. The discrete
 159 form for each term is provided [22]:

160 (i) The kinetic energy variation is:

$$\delta T = - \sum_{k=1}^K \int_{\Omega^k} \rho^k (\ddot{u}_x \delta u_x + \ddot{u}_z \delta u_z) d\Omega \Rightarrow -\delta \mathbf{U}^T \mathbf{M}_m \ddot{\mathbf{U}}(t) + \delta \mathbf{U}^T \mathbf{F}(t) \quad (9)$$

161 where \mathbf{M}_m is the global ($N \times N$) mass matrix, $\mathbf{U}(t) = \mathbf{U} e^{j\omega t}$ is the global
 162 ($N \times 1$) vector of mechanical coordinates displacements, $\mathbf{F}(t) = \mathbf{F} e^{j\omega t}$ is the
 163 global ($N \times 1$) forcing vector due to base excitation, which can be expressed
 164 as an ($N \times 1$) effective mass vector \mathbf{m}^* multiplied by the base acceleration
 165 [3, 22] as follows:

$$- \sum_{k=1}^K \int_{\Omega^k} \rho^k \delta w_{rel} \ddot{w}_b d\Omega \Rightarrow \delta \mathbf{U}^T \mathbf{m}^* \ddot{w}_b(t) = \delta \mathbf{U}^T \mathbf{F}(t) \quad (10)$$

166 (ii) The internal energy variation is:

$$\begin{aligned} \delta U = & \sum_{k=1}^K \int_{\Omega^k} \bar{c}_{11}^k \varepsilon_1 \delta \varepsilon_1 d\Omega + \sum_{p=1}^P \frac{V^{(p)}}{h^{(p)}} \int_{\Omega^{(p)}} \bar{e}_{31}^{(p)} \delta \varepsilon_1 d\Omega \\ & + \sum_{k=1}^P \frac{\delta V^{(p)}}{h^{(p)}} \int_{\Omega^{(p)}} \bar{e}_{31}^{(p)} \varepsilon_1 d\Omega - \sum_{k=1}^K \frac{\delta V^{(p)}}{h^{(p)}} \int_{\Omega^{(p)}} \bar{\epsilon}_{33}^{(p)} \frac{V^{(p)}}{h^{(p)}} d\Omega \end{aligned} \quad (11)$$

167 • The first term of Equation (11) is the mechanical contribution to the
 168 internal energy variation:

$$\sum_{k=1}^K \int_{\Omega^k} \bar{c}_{11}^k \varepsilon_1 \delta \varepsilon_1 d\Omega \Rightarrow \delta \mathbf{U}^T \mathbf{K}_m \mathbf{U}(t) \quad (12)$$

169 where \mathbf{K}_m is the global ($N \times N$) stiffness matrix.

170 • The second and third terms in Equation (11) are the piezoelectric
 171 contributions to the internal energy variation related to the direct and
 172 inverse effects:

$$\sum_{p=1}^P \frac{V^{(p)}}{h^{(p)}} \int_{\Omega^{(p)}} \bar{e}_{31}^{(p)} \delta \varepsilon_1 d\Omega \Rightarrow \delta \mathbf{U}^T \mathbf{K}_c \mathbf{V}(t) \quad (13)$$

173

$$\sum_{k=1}^P \frac{\delta V^{(p)}}{h^{(p)}} \int_{\Omega^{(p)}} \bar{\epsilon}_{31}^{(p)} \varepsilon_1 d\Omega \Rightarrow \delta \mathbf{V}^T \mathbf{K}_c^T \mathbf{U}(t) \quad (14)$$

174

where $\mathbf{K}_c = (\mathbf{K}_c^{(1)} \cdots \mathbf{K}_c^{(P)})$ is the global $(N \times P)$ electromechanical

175

coupling matrix, $\mathbf{V}(t) = \mathbf{V}e^{j\omega t}$ is the global $(P \times 1)$ vector of voltage

176

outputs.

177

- The last term in Equation (11) is the electrostatic contribution to the

178

internal energy variation is:

$$\sum_{k=1}^K \frac{\delta V^{(p)}}{h^{(p)}} \int_{\Omega^{(p)}} \epsilon_{33}^{(p)} \frac{V^{(p)}}{h^{(p)}} d\Omega \Rightarrow \delta \mathbf{V}^T \mathbf{K}_e \mathbf{V}(t) \quad (15)$$

179

where $\mathbf{K}_e = \text{diag}(C^{(1)} \cdots C^{(P)})$ is the diagonal global $(P \times P)$ ca-

180

pacitance matrix, and $C^{(p)} = \frac{b \times L}{h^{(p)}} \bar{\epsilon}_{33}^{(p)}$ is the capacitance of the p^{th}

181

piezoelectric layer.

182

(iii) The variation of work due to the charges is:

$$\delta W_{nc} = \sum_{p=1}^P \delta V^{(p)} Q^{(p)} \Rightarrow \delta \mathbf{V}^T \mathbf{Q}(t) \quad (16)$$

183

where $\mathbf{Q}(t) = \mathbf{Q}e^{j\omega t}$ is the global $(P \times 1)$ vector of electric charge outputs.

184

The harvested energy is dissipated through a resistive electrical load R .

185

Using Ohm's law, the following additional equation relates the voltage vector

186

\mathbf{V} to the time derivative of the charge vector \mathbf{Q} :

$$\mathbf{V}(t) = R\dot{\mathbf{Q}}(t) \quad (17)$$

187

To account for system damping, a proportional viscous damping is assumed

188

and included in the model. The global $(N \times N)$ damping matrix \mathbf{C}_m is

189

assumed to be a linear combination of the mass and stiffness matrices (the

190

Rayleigh damping), so that:

$$\mathbf{C}_m = \alpha \mathbf{M}_m + \beta \mathbf{K}_m \quad (18)$$

191 where α and β are real proportional coefficients which are typically deter-
 192 mined from the measurement of modal damping.

193 The general damped electromechanical equations obtained from the pre-
 194 vious finite element formulation can be expressed as [25, 26, 27]:

$$\mathbf{M}_m \ddot{\mathbf{U}}(t) + \mathbf{C}_m \dot{\mathbf{U}}(t) + \mathbf{K}_m \mathbf{U}(t) + \mathbf{K}_c \mathbf{V}(t) = \mathbf{F}(t) \quad (19)$$

195

$$\mathbf{K}_e \mathbf{V}(t) - \mathbf{K}_c^T \mathbf{U}(t) + \mathbf{Q}(t) = 0 \quad (20)$$

196 where Equation (19) corresponding to the mechanical equation of motion
 197 includes the inverse effect electromechanical coupling term, whereas Equa-
 198 tion (20) corresponding to the electrical equation includes the direct effect
 199 electromechanical coupling term introduced by the piezoelectric behavior.

200 *2.1. Natural frequencies of the harvester for the short-circuit and the open-*
 201 *circuit conditions*

202 The natural frequencies of the harvester are calculated by solving the
 203 free undamped system corresponding to Equation (19). For piezoelectric
 204 energy harvesters, two natural frequencies are defined as a characteristic
 205 limit bounds for each vibration mode: (i) the short-circuit natural frequen-
 206 cies are calculated by assuming that no potential difference exists across the
 207 piezoelectric layers in free vibration. As a result, the electromechanical cou-
 208 pling in Equation (19) is omitted, (ii) the open circuit natural frequencies,
 209 however, are calculated by assuming that no charge flows in the electrical
 210 circuit [28]. In practice, these natural frequencies are determined in terms of
 211 the load resistance value. The short-circuit condition corresponds to a low
 212 resistive load connected to the electrodes of the piezoelectric layers ($R \rightarrow 0$),
 213 whereas the open-circuit condition is determined for a high load resistance
 214 ($R \rightarrow \infty$). The main motivation of computing these particular limits is

215 to determine the electromechanical coupling factor, using a classical elastic
 216 mechanical formulation, which corresponds to the electromechanical energy
 217 ratio.

218 The normal modes of the short-circuit system are obtained assuming
 219 harmonic solutions of the undamped equations of motion (19) for $\mathbf{V} = \mathbf{0}$
 220 and $\mathbf{F} = \mathbf{0}$.

$$\left[\mathbf{K}_m - \omega_i^2 \mathbf{M}_m \right] \Phi_i = \mathbf{0} \quad (21)$$

221 where ω_i is the natural frequency and Φ_i is eigenvector for the i^{th} mode
 222 for short-circuit condition. These modes verify the following orthogonality
 223 properties:

$$\Phi_i^T \mathbf{M}_m \Phi_j = \delta_{ij} \quad \text{and} \quad \Phi_i^T \mathbf{K}_m \Phi_j = \omega_i^2 \delta_{ij} \quad (22)$$

224 where δ_{ij} is the Kronecker symbol and the mode Φ_i is normalized with
 225 respect to the structural mass matrix.

226 The eigenvalue problem for the open-circuit condition is derived by cou-
 227 pling Equations (19) and (20) substituting $\mathbf{Q} = \mathbf{0}$ and $\mathbf{F} = \mathbf{0}$:

$$\left[\hat{\mathbf{K}}_m - \hat{\omega}_i^2 \mathbf{M}_m \right] \hat{\Phi}_i = \mathbf{0} \quad (23)$$

228 where $\hat{\mathbf{K}}_m = \mathbf{K}_m + \mathbf{K}_c \mathbf{K}_e^{-1} \mathbf{K}_c^T$ is the stiffness matrix for the open-circuit
 229 condition which depends on the piezoelectric system properties, $\hat{\omega}_i$ is the
 230 natural frequency and $\hat{\Phi}_i$ is eigenvector for the i^{th} vibration mode. These
 231 modes verify the following orthogonality properties:

$$\hat{\Phi}_i^T \mathbf{M}_m \hat{\Phi}_j = \delta_{ij} \quad \text{and} \quad \hat{\Phi}_i^T \hat{\mathbf{K}}_m \hat{\Phi}_j = \hat{\omega}_i^2 \delta_{ij} \quad (24)$$

232 2.2. Effective Electromechanical Coupling Factor

233 The dimensionless Effective Electromechanical Coupling Factor (EEMCF)
 234 of the piezoelectric laminated beam for the i^{th} mode can be expressed in

235 terms of the natural frequencies of the short and open-circuit conditions.
 236 This factor defines the exchange of energies of the vibrating structure at the
 237 i^{th} mode. It is usually defined, as follows [29, 30, 31]:

$$k_{eff,i}^2 = \frac{\widehat{\omega}_i^2 - \omega_i^2}{\omega_i^2} \quad (25)$$

238 3. The Bimorph Piezoelectric Energy Harvester

239 The finite element formulation, presented in Section 2, can be easily
 240 adapted to the case of a bimorph piezoelectric energy harvester considered
 241 as a case study in this paper. The harvester is composed of two identical
 242 piezoelectric layers, so that $P = 2$ and $C^{(1)} = C^{(2)} = C$, bonded to an
 243 elastic substrate beam as shown in Figure 2. Thus, the harvester has a total
 244 of three layers ($K = 3$). The harvester is assumed to be a linear input-
 245 output system which is characterized by its Frequency Response Functions.

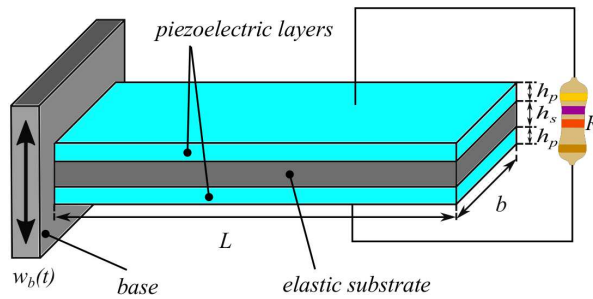


Figure 2: A Clamped free bimorph piezoelectric energy harvester under base excitation with a load resistance mounted in series

246

247 The FRFs are defined here as the output responses of the harvester (dis-
 248 placement or voltage) per base acceleration versus the excitation frequency.

249 Transforming Equations (19) and (20) to the frequency domain, the equiva-
 250 lent expression for nodal displacements FRFs relative to the electrical config-
 251 urations (load resistance mounted in series or parallel connection) is given in
 252 the following equation in which the electrical DOFs are eliminated [32, 33].

$$\frac{\mathbf{U}}{-\omega^2 w_b} = \left[-\omega^2 \mathbf{M}_m + j\omega \mathbf{C}_m + \mathbf{K}_m + \frac{j\omega \mathbf{K}_c^{eq} \mathbf{K}_c^{eqT}}{\left(\frac{1}{R} + j\omega C^{eq}\right)} \right]^{-1} \mathbf{m}^* \quad (26)$$

253 where \mathbf{K}_c^{eq} and C^{eq} are respectively the equivalent electromechanical cou-
 254 pling vector and the equivalent capacitance for a series and parallel connec-
 255 tions of the load resistance with piezoelectric layers [33], which are given in
 256 Table 1. For the mechanical vibration response, only the transverse tip dis-
 257 placement FRF $\left(\frac{w_n}{-\omega^2 w_b}\right)$ is considered in this study. The index $n = N/3$ is
 258 the total number of nodes used to discretize the beam harvester with linear
 259 elements.

Table 1: Equivalent terms of a bimorph energy harvester for the series and the parallel connections of the piezoceramic layers

Terms	Series connection	Parallel connection
\mathbf{K}_c^{eq}	$\frac{\mathbf{K}_c^{(1)} + \mathbf{K}_c^{(2)}}{2}$	$\mathbf{K}_c^{(1)} + \mathbf{K}_c^{(2)}$
C^{eq}	$\frac{C}{2}$	$2C$

260 Similarly, for the electrical outputs, the resultant voltage FRF of the har-
 261 vester is obtained as a function of the nodal displacements FRFs as follows
 262 [32]:

$$\frac{V}{-\omega^2 w_b} = \frac{j\omega \mathbf{K}_c^{eqT}}{\left(\frac{1}{R} + j\omega C_p^{eq}\right)} \left(\frac{\mathbf{U}}{-\omega^2 w_b}\right) \quad (27)$$

263 The current I and the electrical power P FRFs are derived from the

264 voltage FRF:

$$\frac{I}{-\omega^2 w_b} = \frac{1}{R} \left(\frac{V}{-\omega^2 w_b} \right) \quad (28)$$

265

$$\frac{P}{(-\omega^2 w_b)^2} = \frac{1}{R} \left(\frac{V}{-\omega^2 w_b} \right)^2 \quad (29)$$

266 The global finite element matrices appearing in the frequency response
267 functions Equations (26) and (27) establish the dependence of the response
268 of the system on a set of parameters representing the physical characteristics
269 of the harvester structure. In general, we assume that any complex frequency
270 response function H of the BPEH can be expressed in the following form:

$$H = H(\omega, \boldsymbol{\theta}) \quad (30)$$

271 where $\boldsymbol{\theta}$ is a vector of physical parameters of the harvester.

272 4. Global Sensitivity Analysis of the Frequency Response Func- 273 tions

274 4.1. The Elementary Effects Method

275 The Elementary Effects Method constitutes a simple way for screening
276 the effect on the output responses of a few important input factors among the
277 many factors that can be contained in the studied model. The fundamental
278 idea behind this method is given by Morris [16, 34], who introduced the
279 concept of elementary effects, proposing the construction of two sensitivity
280 measures: the *mean* and the *standard deviation* of the EEs for each input
281 parameter. In practice, this method is conducted by: (a) defining the model
282 and its input parameters, (b) assigning Probability Density Function (PDF)
283 to each input parameter, (c) generating an input matrix using an appropriate
284 random sampling method, (d) calculating the corresponding output response

285 vector, and (e) computing the EEs and then the sensitivity measures of each
 286 input/output relationship [35]. The average and standard deviation are then
 287 computed for a set of elementary effects for each factor. The main goal is
 288 to determine which input factors could be considered to have effects which
 289 are [16, 17]:

- 290 1. Negligible (low mean, low standard deviation).
- 291 2. Linear and additive (high mean, low standard deviation).
- 292 3. Nonlinear or involved in interactions with other factors (high standard
 293 deviation).

294 This method computes the sensitivities of the outputs considering one factor
 295 at time among the input factors through the concept of EEs, which are
 296 approximations of the first order partial derivatives of the response. These
 297 elementary effects are estimated at a set of sampled points in the input
 298 space.

299 The finite element model of the bimorph piezoelectric energy harvester
 300 defined in Equations (26) and (27), is the model-parameters considered. The
 301 frequency response function H in Equation (30) is the model-output and the
 302 k independent parameters in vector θ are the model-inputs. Each parameter
 303 θ_i is assumed to be scaled to take on values in the interval $[0, 1]$ as follows:

$$\bar{\theta}_i = \frac{\theta_i - \theta_{i,min}}{\theta_{i,max} - \theta_{i,min}} \quad (31)$$

304 where $\theta_{i,max}$ and $\theta_{i,min}$ are respectively the maximum and minimum bounds
 305 of the i^{th} parameter. The bar is omitted below to simplifying the nota-
 306 tion. Thus, these non-dimensional parameters are randomly selected in a
 307 k -dimensional unit hypercube.

308 A discretized approach of the input space is proposed. In fact, the pos-
 309 sible input values will be restricted to an experimental design, which is a

310 regular k -dimensional p -levels grid, where each θ_i may take a value from the
 311 set $\{0, 1/(p-1), 2/(p-1), \dots, 1\}$. For a given vector $\boldsymbol{\theta}$, the elementary
 312 effect of the i^{th} input parameter is defined as follows [36, 16]:

$$EE_i(\boldsymbol{\theta}) = \frac{|H(\theta_i + \Delta, \boldsymbol{\theta}_{\sim i})| - |H(\boldsymbol{\theta})|}{\Delta} \quad (32)$$

313 where $|\cdot|$ is the modulus, θ_i varies between 0 and $(1 - \Delta)$; Δ is the variation
 314 size which is a predetermined multiple of $1/(p-1)$, and $\boldsymbol{\theta}_{\sim i}$ is the set of
 315 all parameters except the i^{th} parameter. The originality of the elementary
 316 effects method is based on selecting a set of r trajectories where parameters
 317 are changed one at a time on the design grid [7]. A trajectory enables
 318 the calculation of an elementary effect for each input parameter i between
 319 two points of the trajectory ($\boldsymbol{\theta}^{(j)}$ and $\boldsymbol{\theta}^{(j+1)}$, where $j \in \{1, \dots, k\}$) [17, 37].
 320 Therefore, each trajectory requires $(k+1)$ model evaluations (simulations)
 321 to calculate k elementary effects (one EE per parameter).

322 Figure 3 presents a trajectory in a 3-dimensional space which is com-
 323 posed of a four sampling points $\{\boldsymbol{\theta}^{(1)}, \dots, \boldsymbol{\theta}^{(4)}\}$ and $\Delta = 0.25$. Along a
 324 trajectory each input parameter is increased or decreased by the same step
 325 Δ . Considering, for example, the third component θ_3 . This components
 326 differs only in the two consecutive sample points $\boldsymbol{\theta}^{(3)}$ and $\boldsymbol{\theta}^{(4)}$, thus:

$$EE_3 = \frac{|H(\boldsymbol{\theta}_4)| - |H(\boldsymbol{\theta}_3)|}{0.25} \quad (33)$$

327

328 A set of r different trajectories (with index t , where $t = 1, \dots, r$) provides
 329 r estimates of elementary effects related to each input parameter i , at the
 330 cost of $r \times (k+1)$ simulations.

331 The first measure of sensitivity is the average of the elementary effects

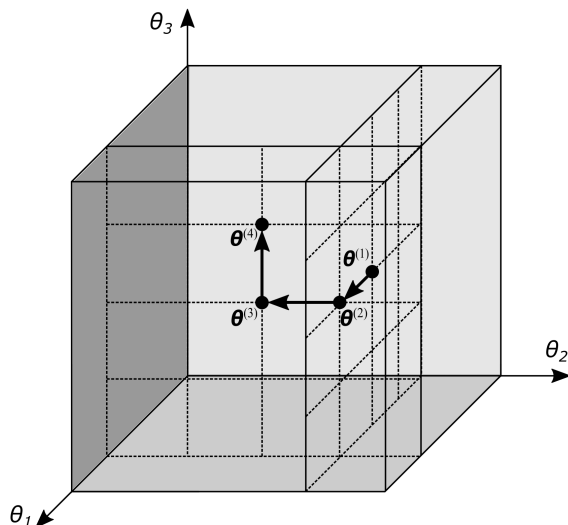


Figure 3: An example of a trajectory in the input domain for $k = 3$ and $p = 5$

332 which is computed for each input parameter i :

$$\mu_i = \frac{1}{r} \sum_{t=1}^r EE_i^{*(t)} \quad (34)$$

333 where $EE_i^{*(t)}$ is the normalized elementary effects of $EE_i^{(t)}$. This measure
 334 has been rectified by Campolongo et al. [21], who has recommended the
 335 use of the mean of absolute elementary effects, defined in Equation (32),
 336 rather than the usual average, since some EEs can eliminate each other in
 337 non-monotonic models. Thus, the mean of the absolute values of EEs is:

$$\mu_i^* = \frac{1}{r} \sum_{t=1}^r |EE_i^{*(t)}| \quad (35)$$

338 For a given parameter i , if the r estimates of EEs have the same sign, that
 339 means that this parameter has a monotonic effect on the output response,
 340 increasing or decreasing, depending on the sign of the EEs. In this case, μ_i^*

341 is equal to the absolute value of μ_i . The reverse is also true; if $\mu_i^* = \text{abs}(\mu_i)$,
 342 the effects of the i^{th} parameter are monotonic.

343 The second proposed measure is the standard deviation of the EEs σ_i ,
 344 which assesses the extent of interactions and non-linear effects of each input
 345 parameter i . This measure is defined as follows:

$$\sigma_i = \sqrt{\frac{1}{r-1} \sum_{t=1}^r (|EE_i^{*(t)}| - \mu_i^*)^2} \quad (36)$$

346 A graphical representation of the (μ_i^*, σ_i) points in the (μ^*, σ) plane
 347 allows a comparison of the sensitivities of different parameters taking into
 348 account at the same time the two sensitivity measures.

349 4.2. Latin Hypercube Sampling for Elementary Effects Method

350 Latin Hypercube Sampling (LHS) has become a popular technique for
 351 generating designs to computational experiments [38, 39]. To do that for the
 352 elementary effects method, three steps are essential: (1) The LHS generates
 353 $p = 2 \times r$ random samples of k uncorrelated parameters in the k -dimensional
 354 unit hypercube; where each component of θ has an uniform distribution in
 355 the interval $[0, 1]$. (2) Returning to the specified distribution of the uncertain
 356 parameters by inverting the Cumulative Distribution Function (CDF). (3)
 357 Applying the One factor at a time strategy, which consists in adding the
 358 $r \times (k-1)$ points in order to establish r trajectories, each trajectory contains
 359 $k+1$ points. Therefore, the computational experiments matrix is composed
 360 of $r \times (k+1)$ random points of k uncorrelated parameters. Each row of the
 361 matrix is a point in the input space. The rows of the matrix are sorted in
 362 r blocks, each block includes $k+1$ rows (each block presents a trajectory).
 363 Inside each block, points differ only in one component at a time. Thus, each
 364 block allows to compute one Elementary Effect per input parameter.

365 Figure 4 shows an example of two parameters uniformly distributed in
 366 $[0\ 1]^2$ space. In order to determine 4 elementary effects per parameter, the
 367 LHS method generates 8 random points given in Figure 4.a. Adding 4 points
 368 whose one parameter changes at the time, provides 4 trajectories as shown
 369 in Figure 4.b. The LHS with the one factor at the time strategy ensure two
 370 levels per input parameter for one trajectory of the EEs method.

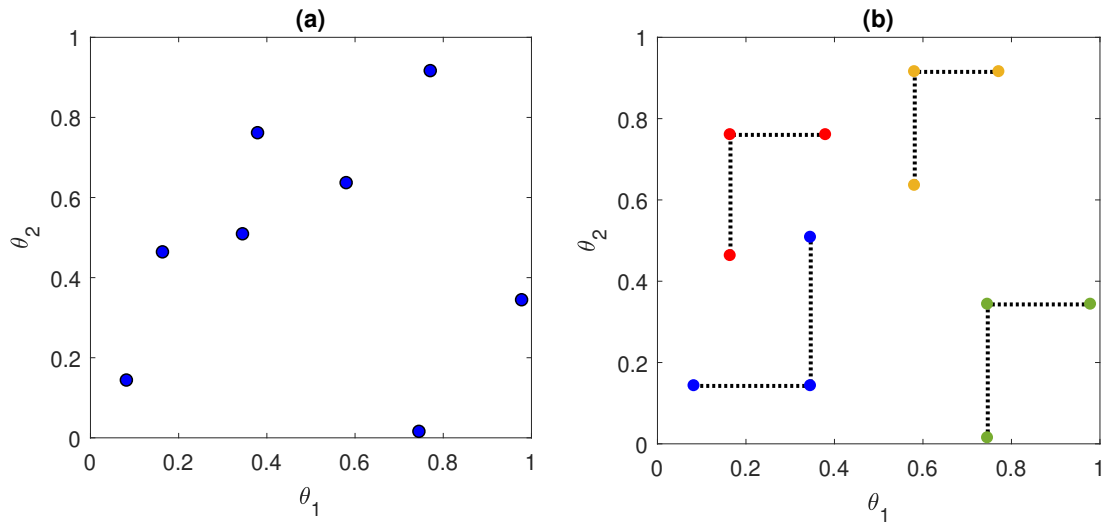


Figure 4: (a) Random points generated by the LHS method in two unit dimensions, (b) 4 trajectories in $[0\ 1]^2$ space for the EEs method

371 5. Numerical Example

372 This section presents a numerical example of the harvester introduced in
 373 Section 4, which is composed of two identical piezoceramic layers of PZT-
 374 5A bonded on the top and the bottom surfaces of an aluminum substrate.
 375 The geometrical dimensions and material properties of the harvester used
 376 in this example are given in Table 2 (adapted from Erturk and Inman [1]).

377 The ratio of the overhang length to the total thickness of the harvester
378 is about 85.7, which makes it reasonable to neglect the shear deformation
379 and the rotary inertia effects for the first few vibration modes. For the
380 numerical simulation, we used modal mechanical damping ratios $\zeta_1 = 0.01$
381 and $\zeta_2 = 0.012$ for the first two modes. The proportional coefficients α and
382 β given in Equation (18) are computed using these two damping ratios. The
383 load resistance R is mounted in series with the piezoceramic layers as shown
in Figure 2.

Table 2: Physical parameters values of the harvester structure of Fig.2

Parameters	Descriptions	Units ¹	PZT-5A	Aluminum
L	Beam length	mm	30	30
b	Beam width	mm	5	5
h_p, h_s	Layers thickness	mm	0.15	0.05
E_p, E_s	Young's modulus	GPa	61	70
ρ_p, ρ_s	Mass density	kg/m^3	7750	2700
e_{31}	Piezoelectric constant	C/m^2	-10.4	–
ϵ_{33}	Permittivity constant	nF/m	13.3	–

384

385 For the finite element discretization, the harvester is modeled in 1D using
386 30 linear elements with three degrees of freedom per node.

387 The model is validated for the frequency range from 0 to 5000 Hz by
388 comparing the first three resonance frequencies computed by the finite ele-
389 ment model to those given by the analytical solution computed by Erturk
390 et Inman [1] for the same harvester for the short-circuit and open circuit

¹Unit of parameter (PU), it will be used to define sensitivities units

391 conditions. The first three natural frequencies are listed in Table 3 with
 392 the effective electromechanical coupling factor k_{eff} defined in Equation (25)
 393 which is computed for the FE-model. We can notice that the natural fre-
 394 quencies for the open-circuit condition are higher than those obtained in
 395 the short-circuited case due to the electromagnetical coupling effects and
 396 the added stiffness in the open-circuit condition. Therefore, the natural fre-
 397 quencies for the two approaches are in excellent agreement for all modes.
 398 An infinitesimal error for the third mode can be due to the numerical com-
 399 putational errors caused by the approximation solution of the finite element
 400 method, which is so sensitive to the assigned discretization. These results
 401 confirm the validation of the finite element harvester model and allow for
 pursuing the study.

Table 3: The first three natural frequencies for the short circuit and open circuit conditions computed from the Finite Element Method (FE) and the Analytical solution (Anal.) [1]

Mode (r)	f_r^{cc} (FE)	f_r^{cc} (Anal. [1])	f_r^{co} (FE)	f_r^{co} (Anal. [1])	$k_{eff,r}$ (FE)
1	185.1	185.1	191.1	191.1	0.2561
2	1159.8	1159.7	1171.7	1171.6	0.1435
3	3246.7	3245.3	3258.2	3254.1	0.0845

402

403 The selected uncertain parameters are the Young modulus and the ma-
 404 terial densities for both the substrate and the piezoelectric layers, also the
 405 permittivity and piezoelectric constants for the piezoceramic layers. Table 4
 406 lists the tolerances provided by the manufacturers as percent variation from
 407 the nominal values [11]. A tolerance of $\pm 20\%$ is reported for the piezoceramic
 408 PZT-5A electromechanical properties and $\pm 10\%$ for the Young's modulus

409 and density of the aluminum substrate material. An uniform distribution is
 410 assumed for all materials properties of the harvester.

Table 4: Bounds for the uniform PDF distribution associated to the electromechanical parameters; bounds are expressed in terms of the nominal values [11]

Parameters	Bounds
E_p	$\pm 20\%$
E_s	$\pm 10\%$
ρ_p	$\pm 20\%$
ρ_s	$\pm 10\%$
e_{31}	$\pm 20\%$
ϵ_{33}	$\pm 20\%$

411 The application of the Morris method for the voltage FRF of the bimorph
 412 piezoelectric energy harvester consists in computing $r = 10$ elementary ef-
 413 fects for each parameter listed in table 4. Thus, 70 simulations are required
 414 for a fixed excitation frequency and load resistance. The aim of the study is
 415 to identify the effect of the excitation frequency on the importance order of
 416 the parameters particularly in the vicinity of the first mode. Furthermore,
 417 the system is be studied for both a short-circuit condition simulated using
 418 a load resistance $R = 100 \Omega$ and an open-circuit condition simulated using
 419 for a load resistance $R = 10^7 \Omega$.

420 Figure 5 shows the voltage and tip displacement FRFs responses of the
 421 harvester versus the excitation frequencies in the vicinity of the first mode for
 422 70 samples. One can be observe the variability in the first natural frequency
 423 and in amplitude peaks. The voltage FRF peak varies between 0.02032 and
 424 0.01152 V/g whereas the tip displacement FRF peak varies between 442.46

and $756.55 \mu\text{m}/g$.

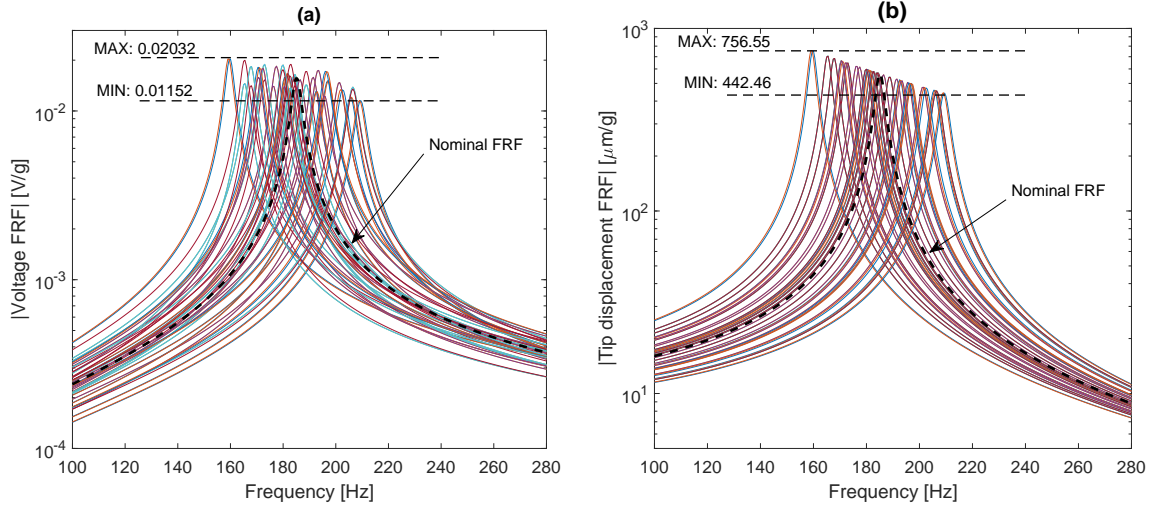


Figure 5: Modulus of FRFs versus excitation frequencies of the first mode for the 70 simulations of: (a) voltage and (2) Tip displacement for the short circuit condition $R = 100\Omega$

425

426 Figure 6 shows the average μ^* and the standard deviation σ of the EEs
 427 to the electromechanical characteristics of the bimorph piezoelectric energy
 428 harvester for the output voltage FRF at the selected excitation frequencies
 429 in the vicinity of the first natural frequency. The harvester has a 100Ω load
 430 resistance connected in series between the two piezoelectric layers. One can
 431 notice that the hierarchical influence of the input parameters is unchange-
 432 able and independent of the excitation frequency. Moreover, the density
 433 and the elastic modulus of the piezoceramic layers have the highest val-
 434 ues of the EEs indicators whereas the permittivity constant has the lowest
 435 values. These results provide a useful information about how significant un-
 436 certain parameters affect the output voltage of the harvester and how much

interaction exists between the parameters.

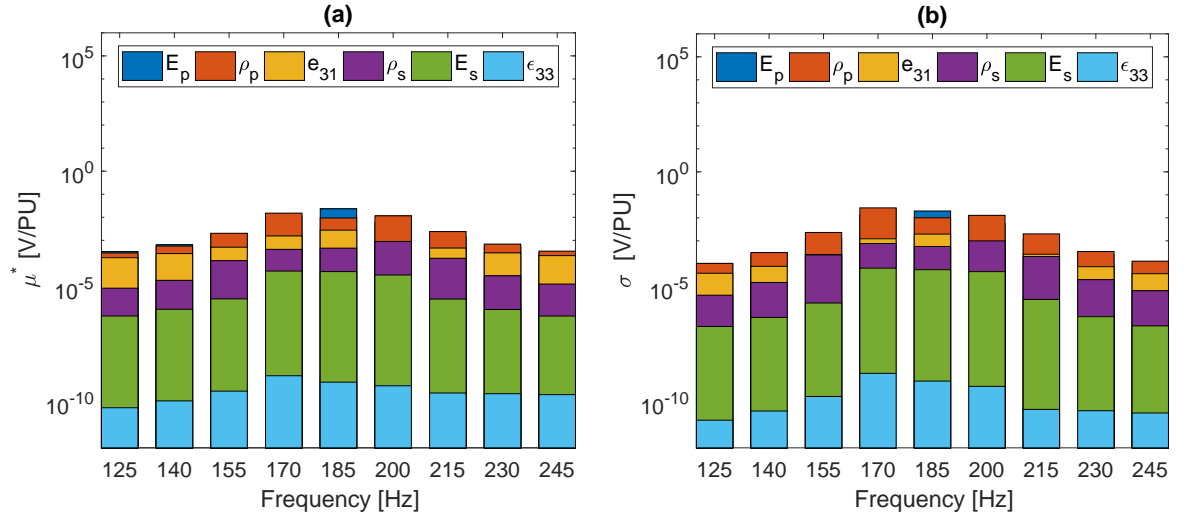


Figure 6: EEs measures for the voltage FRF of the bimorph piezoelectric energy harvester for a load resistance $R = 100 \Omega$ at excitation frequencies in the vicinity of the first natural frequency (a) the mean and (b) the standard deviation

437

438 Figure 7 shows the (μ^*, σ) scatter graph of the FRF voltage for the six
 439 parameters to the variation of the electromechanical characteristics for a
 440 load resistance $R = 100 \Omega$ and and excitation frequency equal $185 Hz$. The
 441 limits of the boxes correspond to a confidence interval of 95%. Clearly,
 442 we can see that the sensitivities of the absolute mean and standard devia-
 443 tion of the piezoelectric density ρ_p and the elastic modulus E_p are relatively
 444 significant compared to those of the piezoelectric constant, permittivity con-
 445 stant and substrate properties. Therefore, the density and elastic modulus
 446 of piezoelectric layers are the most important parameters that affect the
 447 output voltage FRF. Furthermore, the average and the standard deviation
 448 are of the same order of values. This means that the elastic modulus and

449 density of the piezoelectric layers are strongly influenced by the nonlinear
 effects and interactions with others parameters.

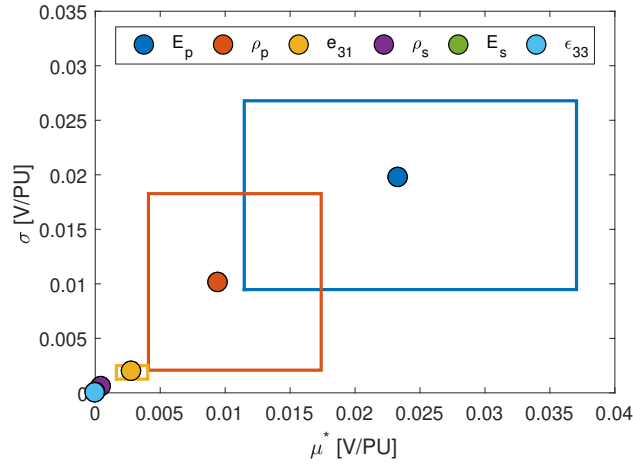


Figure 7: (μ^*, σ) Scatter graph of the voltage FRF for bimorph piezoelectric energy harvester for a load resistance $R = 100 \Omega$ and an excitation frequency 185 Hz with 95% confidence interval

450

451 Considering next the sensitivity analysis for a roughly open circuit con-
 452 dition depicted by a load resistance $R = 10^7 \Omega$. Figure 8 shows the average
 453 μ^* and the standard deviation σ of the EEs for the output voltage FRF
 454 at a set of excitation frequencies in the vicinity of first natural frequency.
 455 One can notice the EEs sensitivity measures are in the same order for all
 456 considered frequencies. This means that the parameters importance is the
 457 same for any of the excitation frequency. However, the order influence of
 458 the parameters change from the short to the open circuit condition. The
 459 influence of the permittivity constant becomes higher than the Aluminum
 460 substrate material properties. The density and the elastic modulus of the
 461 piezoceramic layers remain the highest EEs indicators similarly to the case

462 of a roughly closed circuit whereas the EEs indicators for the aluminum
 463 properties become the lowest.

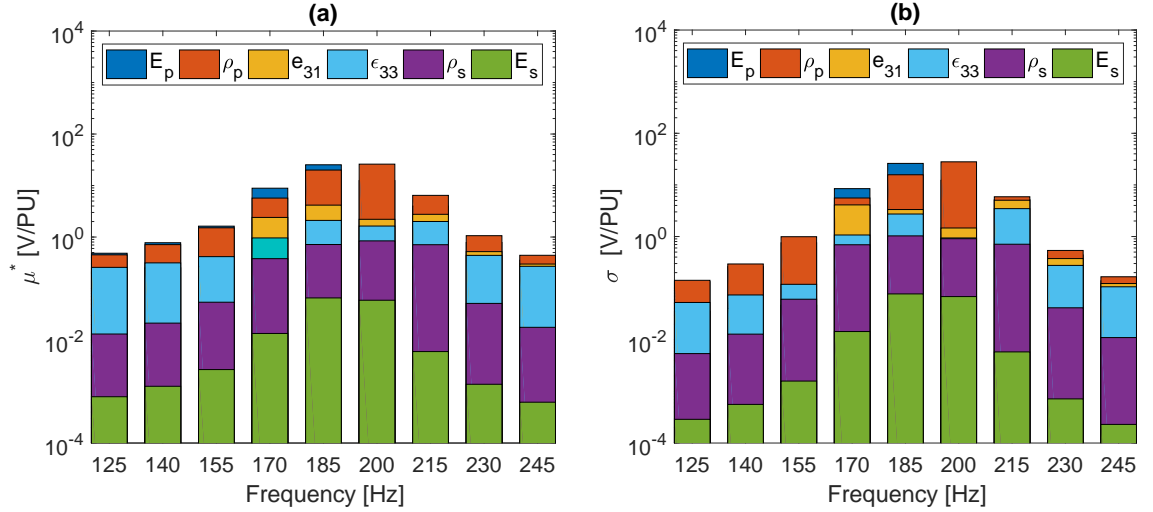


Figure 8: EEs measures for the voltage FRF of the bimorph piezoelectric energy harvester for a load resistance $R = 10^7 \Omega$ at excitation frequencies in the vicinity of the first natural frequency (a) the mean and (b) the standard deviation

463

464 Figure 9 presents the standard deviation versus the absolute mean of the
 465 EEs and the confidence interval of 95% for the k electromechanical charac-
 466 teristics of the harvester for the case of a roughly open circuit condition
 467 depicted by a load resistance $R = 10^7 \Omega$ and for an excitation frequency of
 468 185 Hz. One can clearly notice that the piezoelectric density and elastic
 469 modulus remain the major contributors for the uncertainty of the voltage
 470 FRF. The permittivity constant becomes more significant compared to the
 471 Aluminum substrate characteristics which have negligible effect on the har-
 472 vester voltage response in both short and open circuit.

473 In order to evaluate the harvester performance, The EEMCF defined in

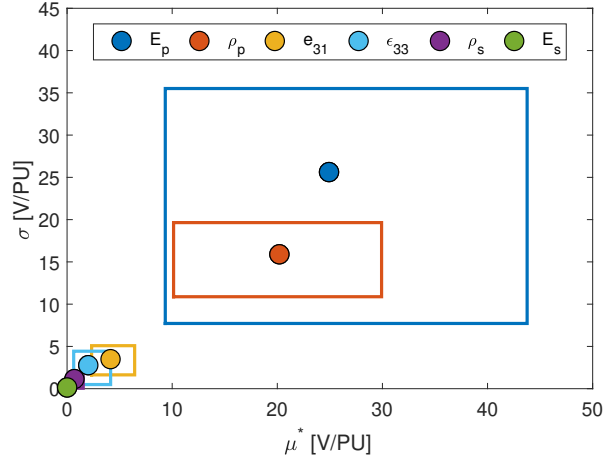


Figure 9: (μ^*, σ) Scatter graph of the voltage FRF for the bimorph piezoelectric energy harvester harvester for a load resistance $R = 10^7 \Omega$ and excitation frequency $185 Hz$ with 95% confidence interval

474 (30) as the ratio of the exchanged mechanical energy and electrical energies
 475 is shown in Figure 10 for each of the 10 electromechanical characteristic
 476 sample vectors used in this GSA study. One can observe that the EEMCF
 477 can reach a maximum value of 0.34 for the 42th sample vector. This results
 478 can be very useful for the research of optimal harvester parameters that
 479 maximize this factor.

480 6. Conclusion

481 In this paper, a finite element formulation of the electromechanical cou-
 482 pling problem for a laminated piezoelectric cantilever beam introduced. The
 483 finite element formulation is applied in the case of a symmetric bimorph
 484 piezoelectric energy harvester. The FRFs of the harvester are used as the
 485 performance predictors. The Morris global sensitivity method using the el-

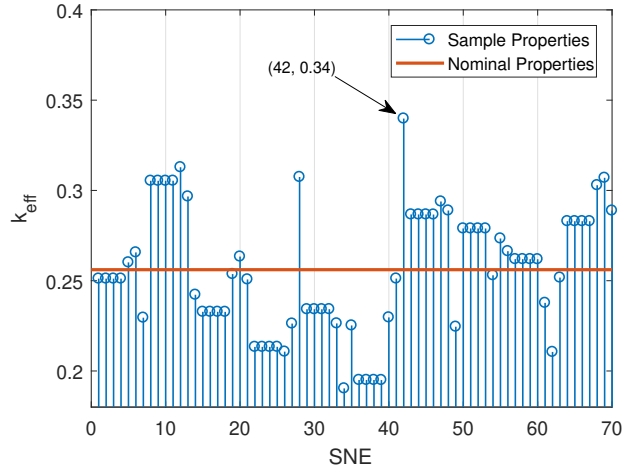


Figure 10: The EEMCF k_{eff} for the 70 simulations used in the GSA

486 elementary effects is used to study the effect of the variability of the material
 487 properties of the substrate and piezoelectric layers on the voltage FRF out-
 488 put of the harvester. The EEs method can estimate the effect of parameters
 489 and their interactions by considering both the mean and the variance of
 490 the elementary effects. Using only 70 simulations, the sensitivities measures
 491 show that the piezoelectric elastic modulus and its density are the most
 492 influential parameters on the voltage FRF. These results are in agreement
 493 with the published research of Ruiz et al [11] which used the Sobol' indices
 494 as a variance based sensitivity analysis method. The method of Morris cor-
 495 rectly screens the most and least sensitive parameters among few selected
 496 parameters for a spatially distributed bimorph piezoelectric energy harvester
 497 FE-model with fewer model evaluations than the Sobol's method.

498 Future research may consider second order sensitivity analysis using the
 499 elementary effects method to minutely study the interactions between pa-

500 rameters properties of the harvester.

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