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Analytical and numerical prediction of acoustic radiation from a panel under turbulent boundary layer excitation

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Abstract

The vibroacoustic responses of a simply supported panel excited by turbulent flow are analytically and numerically investigated. In the analytical model, the radiated sound power is described in terms of the cross spectrum density of the wall pressure field and sensitivity functions for the acoustic pressure and fluid particle velocity. For the numerical model, a hybrid approach based on the finite element method is described in which the cross spectrum of the wall pressure field is represented by a set of uncorrelated wall plane waves. Realisations of the wall pressure field are used as deterministic input loads to the panel. The structural and acoustic responses of the panel subject to turbulent boundary layer excitation are then obtained from an ensemble average of the different realisations. Analytical and numerical results are compared with experimental data measured in an anechoic wind tunnel, showing good agreement. The effect of adding stiffeners on the vibroacoustic responses of the panel is also examined using the proposed

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numerical approach.

Keywords: turbulent boundary layer, acoustic radiation, wall pressure field, uncorrelated wall plane waves

1. INTRODUCTION

The vibroacoustic responses (i.e., the structural and acoustic responses) of elastic structures excited by the pressure field beneath a turbulent boundary layer (TBL) are a significant concern in naval and aircraft industries. An extensive number of analytical and numerical approaches as well as experiments have been conducted to predict the vibroacoustic responses of planar structures excited by turbulent flow, for example, see Refs. [1–8]. The structural responses of elastic panels excited by a turbulent flow or an acoustic diffuse field have been recently reviewed by the authors [9]; as such we herein concentrate on the acoustic responses of elastic panels excited by turbulent flow. The radiated sound power from a thin panel in air excited by a TBL was computed by Davies [10] using both deterministic and statistical methods. In the deterministic method, an estimation of the radiated sound power was obtained by summing over resonant modes from a modal analysis. Statistical Energy Analysis was also employed to predict the radiated power. Results from the two methods were shown to converge at frequencies above the hydrodynamic coincidence frequency. Han et al. [11] developed a hybrid computational fluid dynamics (CFD)-energy flow analysis (EFA) method to predict the vibroacoustic responses of a panel excited by turbulent flow. CFD was employed to obtain mean flow information. The travelling wave method was used to calculate the input power to the system. The structural and acoustic responses of the panel were then obtained using the EFA.

An analytical formulation in the wavenumber domain was proposed by Maury et al. [12; 13] to predict the vibroacoustic responses of a panel excited by either a diffuse acoustic field or a turbulent boundary layer. An increase in flow velocity was observed to more rapidly increase the radiated sound pressure than the turbulent pressure. Rocha [14] developed an analytical method to study the vibroacoustic responses of isotropic and composite plates under TBL excitation. It was shown that smaller composite panels generally produced lower levels of sound and vibration than longer and wider composite panels. Further, it was observed that apart from some distinct frequencies, the composite panels generated lower noise levels than that by isotropic pan-

33 els. Recently, Marchetto et al. [6; 15] studied the vibroacoustic responses of
34 panels under diffuse acoustic field and TBL excitation. They used a method
35 based on sensitivity functions and reciprocity principles. Two crucial quanti-
36 ties in the wavenumber domain were identified which had significant effect on
37 the panel vibroacoustic responses. They are the wall-pressure cross spectral
38 density function of the excitation and the sensitivity function at the receiver
39 location. Kou et al. [16] examined the modal averaged radiation efficiency
40 of a rectangular plate subject to TBL excitation. Below the hydrodynamic
41 coincidence frequency, the radiation efficiency was found to increase with an
42 increase in the convection velocity, attributed to the increase of the modal
43 radiation efficiency associated with lower order modes.

44 Cousin [17] experimentally investigated sound generation from a flat plate
45 excited on one side by turbulent flow. The sound levels were observed to in-
46 crease with decreasing frequency and increasing flow speed. Results showed
47 that the greatest radiated sound energy occurred at frequencies around and
48 below the aerodynamic coincidence frequency. Park et al. [18] carried out
49 an experiment to study vibration responses and the sound radiation of a
50 viscoelastically supported rectangular plate excited by turbulent flows. The
51 measured results were in good agreement with predictions from an analyt-
52 ical model described in previous publications. The vibroacoustic responses
53 of a fluid-loaded plate were numerically and experimentally investigated by
54 Ciappi et al. [19]. It was shown that among the TBL models considered
55 in their work, the Chase model provided good agreement between numer-
56 ical and experimental results. Liu et al. [4; 20] studied the vibroacoustic
57 responses of aircraft panels with and without ring frame attachments excited
58 by a TBL, using the modal expansion and receptance methods. They showed
59 that whilst the TBL pressure field produces the same vibration level for the
60 panel with and without ring frame attachments, the excited modes of the
61 stiffened plate are more efficient sound radiators. They also showed that
62 an increase in the skin loss factor dramatically reduced TBL-induced noise
63 radiation.

64 In this paper, analytical and numerical approaches to predict the radiated
65 acoustic power of an elastic panel subject to TBL excitation are presented.
66 In the analytical model, expressions for the cross-spectrum density of the
67 wall pressure field (WPF), and sensitivity functions for the acoustic pressure
68 and fluid particle velocity are derived. The acoustic power is then obtained
69 by integrating the cross spectrum between the acoustic pressure and fluid
70 particle velocity over the panel surface. The numerical model couples an

71 uncorrelated wall plane wave technique which describes the WPF with a vi-
72 broacoustic solver based on the finite element method (FEM), as described
73 previously in Refs [9; 21]. The turbulent boundary layer excitation is mod-
74 elled using a set of uncorrelated wall plane waves. Each realisation of the
75 wall pressure field is a deterministic input load to the vibroacoustic solver.
76 The structural and acoustic responses of the panel are then obtained from an
77 ensemble average of the different panel responses. Acoustic radiation from
78 a simply supported panel is initially examined, from which results obtained
79 analytically and numerically are compared with experimental data. The an-
80 alytical method is limited in its application to simple panels with simply
81 supported boundary conditions. In contrast, the numerical method can be
82 applied to complex structures with arbitrary boundary conditions. To show
83 one of the applications of the proposed numerical method, acoustic radi-
84 ation from a stiffened panel is then numerically predicted and the results are
85 compared with those of the unstiffened panel.

86 2. Analytical formulation

87 Figure 1 shows an elastic rectangular finite baffled panel excited by a
88 turbulent flow field. It is assumed that the TBL is homogeneous, stationary
89 and fully developed over the panel surface. Although in practice there are
90 inhomogeneities in the TBL, we are considering an idealised WPF associated
91 with the boundary layer developed on an extended flat surface in a subsonic
92 flow with zero pressure gradient. In such a flow condition, the boundary layer
93 thickness increases slowly in the flow direction which justifies representing the
94 TBL as a homogeneous spatial field and stationary in time. Further, it is
95 assumed that the WPF is not altered by the vibration of the panel. The
96 analytical model is herein developed in the wavenumber domain.

97 2.1. Panel velocity

The spatial average of the auto spectrum density (ASD) of the panel velocity is given by

$$\langle V^2 \rangle = \frac{1}{A} \int_A S_{vv}(\mathbf{x}, \omega) dA, \quad (1)$$

where S_{vv} is the ASD of the panel flexural velocity, \mathbf{x} is a point location on the panel, ω is the angular frequency, $A = L_x L_y$ is the panel surface area and

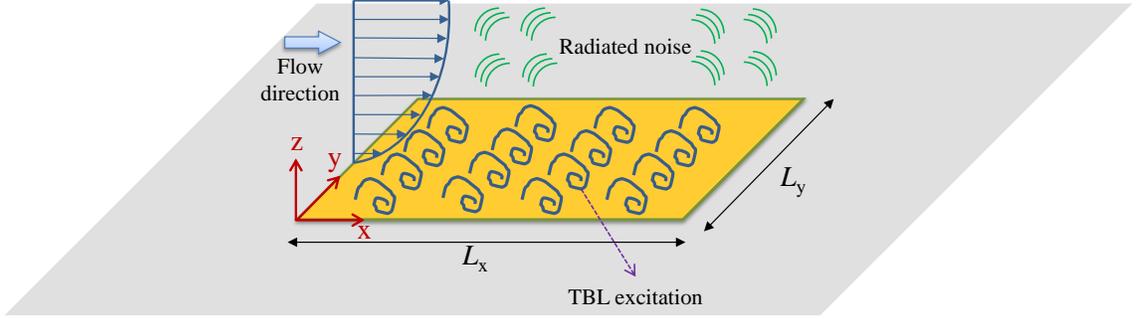


Figure 1: An elastic baffled panel under TBL excitation.

L_x, L_y are the panel length and width in the x and y directions, respectively. S_{vv} is obtained using [12; 22]

$$S_{vv}(\mathbf{x}, \omega) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} |H_{vs}(\mathbf{x}, \mathbf{k}, \omega)|^2 \phi_{pp}(\mathbf{k}, \omega) d\mathbf{k}, \quad (2)$$

98 where $H_{vs}(\mathbf{x}, \mathbf{k}, \omega)$ is the sensitivity function of the panel vibration velocity
 99 excited by a unit wall plane wave given in Appendix A. $\phi_{pp}(\mathbf{k}, \omega)$ is the cross
 100 spectrum density (CSD) of the pressure field in the wavenumber domain,
 101 and \mathbf{k} is the wavevector with components k_x and k_y in the streamwise and
 102 spanwise directions, respectively. Substituting equation (2) and the sensitiv-
 103 ity function given by equation (A.1) into equation (1), and using rectangular
 104 rule for numerical integration in the wavenumber domain, the spatial average
 105 of the auto spectrum of the panel velocity can be written as follows

$$\langle V^2 \rangle \approx \frac{1}{4\pi^2} \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \phi_{pp}(\mathbf{k}, \omega) \Lambda_{ij}(\mathbf{k}, \omega) \delta k_x \delta k_y, \quad (3)$$

$$\Lambda_{ij}(\mathbf{k}, \omega) = \sum_{m=1}^M \sum_{n=1}^N \sum_{h=1}^M \sum_{l=1}^N Z_{mn}^{ij} Z_{hl}^{ij*} \hat{\varphi}_{mnhl} \quad (4)$$

$$\{Z_{rs}^{ij} | (r, s) = (m, n) \vee (h, l)\} = \frac{i\omega \psi_{rs}(\mathbf{k})}{\Omega(\omega_{rs}^2 - \omega^2 + i\eta\omega\omega_{rs})} \quad (5)$$

$$\widehat{\varphi}_{mnhl} = \frac{1}{A} \int_A \sin\left(\frac{m\pi x}{L_x}\right) \sin\left(\frac{n\pi y}{L_y}\right) \sin\left(\frac{h\pi x}{L_x}\right) \sin\left(\frac{l\pi y}{L_y}\right) dA = \quad (6)$$

$$\left\{ \begin{array}{ll} \frac{1}{4}, & \text{if } m = h \wedge n = l \\ 0, & \text{otherwise} \end{array} \right\}.$$

106 where δk_x , δk_y are the wavenumber resolutions in the streamwise and span-
 107 wise directions, respectively, and N_x , N_y are the number of points considered
 108 for sampling the wavenumber space along the k_x and k_y directions. M and
 109 N are the cut-off modal orders in the x and y directions, respectively. ψ_{rs}
 110 and ω_{rs} are the modal forces and modal angular frequencies which are de-
 111 fined in Appendix A, Ω is the modal mass and η is the structural loss factor.
 112 According to equation (6), $\widehat{\varphi}_{mnhl}$ is non-zero only if $m = h \wedge n = l$. This can
 113 simplify equation (4) as follows

$$\Lambda_{ij}(\mathbf{k}, \omega) = \frac{1}{4} \sum_{m=1}^M \sum_{n=1}^N |Z_{mn}^{ij}|^2 \quad (7)$$

114 2.2. Acoustic power

The acoustic power of the panel can be obtained by integrating the nor-
 mal sound intensity corresponding to the cross spectrum between the sound
 pressure and the fluid particle velocity, over a virtual surface Γ surrounding
 the panel as follows [23; 24]

$$\Pi_{\text{rad}}(\omega) = \int_{\Gamma} \text{Re} \{ S_{pv_f}(\mathbf{x}, \omega) \} d\mathbf{x}. \quad (8)$$

The cross spectrum denoted by S_{pv_f} is given by the following analytical
 expression [12; 15]

$$S_{pv_f}(\mathbf{x}, \omega) = \frac{1}{4\pi^2} \int_{\infty} H_p(\mathbf{x}, \mathbf{k}, \omega) H_{v_f}^*(\mathbf{x}, \mathbf{k}, \omega) \phi_{pp}(\mathbf{k}, \omega) d\mathbf{k}, \quad (9)$$

115 where $H_p(\mathbf{x}, \mathbf{k}, \omega)$, $H_{v_f}(\mathbf{x}, \mathbf{k}, \omega)$ are sensitivity functions for the radiated pres-
 116 sure and the fluid particle velocity, respectively. The sensitivity functions in

117 the wavenumber domain, denoted by $\tilde{H}_p(\tilde{\mathbf{k}}, \mathbf{k}, \omega)$ and $\tilde{H}_{v_f}(\tilde{\mathbf{k}}, \mathbf{k}, \omega)$, are re-
 118 lated to the sensitivity functions in the spatial domain by inverse Fourier
 119 transform as follows

$$H_p(\mathbf{x}, \mathbf{k}, \omega) = \frac{1}{4\pi^2} \int_{\infty} \tilde{H}_p(\tilde{\mathbf{k}}, \mathbf{k}, \omega) e^{i\tilde{\mathbf{k}}\mathbf{x}} d\tilde{\mathbf{k}}, \quad (10)$$

$$H_{v_f}^*(\mathbf{x}, \mathbf{k}, \omega) = \frac{1}{4\pi^2} \int_{\infty} \tilde{H}_{v_f}^*(\tilde{\mathbf{k}}, \mathbf{k}, \omega) e^{-i\tilde{\mathbf{k}}\mathbf{x}} d\tilde{\mathbf{k}}, \quad (11)$$

120 where * denotes the complex conjugate. Using equations (8)-(11) and ex-
 121 tending virtual surface Γ to infinity in the equation (8) (note that except
 122 over the panel, the fluid particle velocity is zero everywhere on the virtual
 123 surface), the radiated acoustic power becomes

$$\Pi_{\text{rad}}(\omega) = \text{Re} \left[\left(\frac{1}{4\pi^2} \right)^2 \int_{\infty} \int_{\infty} \int_{\infty} \tilde{H}_p(\tilde{\mathbf{k}}, \mathbf{k}, \omega) \tilde{H}_{v_f}^*(\tilde{\mathbf{k}}, \mathbf{k}, \omega) \phi_{pp}(\mathbf{k}, \omega) \right. \\ \left. \left(\frac{1}{4\pi^2} \int_{\infty} e^{i(\tilde{\mathbf{k}}-\tilde{\mathbf{k}})\mathbf{x}} d\mathbf{x} \right) d\mathbf{k} d\tilde{\mathbf{k}} d\tilde{\mathbf{k}} \right]. \quad (12)$$

The integral in the parenthesis in equation (12) corresponds to the Dirac delta function which is given by

$$\frac{1}{4\pi^2} \int_{\infty} e^{i(\tilde{\mathbf{k}}-\tilde{\mathbf{k}})\mathbf{x}} d\mathbf{x} = \delta(\tilde{\mathbf{k}} - \tilde{\mathbf{k}}). \quad (13)$$

124 Using this definition, equation (12) can be simplified to

$$\Pi_{\text{rad}}(\omega) = \text{Re} \left[\left(\frac{1}{4\pi^2} \right)^2 \int_{\infty} \int_{\infty} \tilde{H}_p(\tilde{\mathbf{k}}, \mathbf{k}, \omega) \tilde{H}_{v_f}^*(\tilde{\mathbf{k}}, \mathbf{k}, \omega) \phi_{pp}(\mathbf{k}, \omega) d\mathbf{k} d\tilde{\mathbf{k}} \right]. \quad (14)$$

125 The sensitivity function of the sound pressure in the wavenumber domain
 126 is related to the sensitivity function of the the particle velocity at the panel
 127 surface as follows [23]

$$\tilde{H}_p(\tilde{\mathbf{k}}, \mathbf{k}, \omega) = \frac{\rho_f \omega}{\tilde{k}_z(\tilde{\mathbf{k}})} \tilde{H}_{v_f}(\tilde{\mathbf{k}}, \mathbf{k}, \omega) \quad (15)$$

128 where

$$\tilde{k}_z(\tilde{\mathbf{k}}) = \left\{ \begin{array}{ll} \sqrt{k_f^2 - \tilde{k}_x^2 - \tilde{k}_y^2}, & k_f^2 \geq \tilde{k}_x^2 + \tilde{k}_y^2 \\ i\sqrt{\tilde{k}_x^2 + \tilde{k}_y^2 - k_f^2}, & \text{otherwise} \end{array} \right\}, \quad (16)$$

129 and k_f is the acoustic wavenumber, ρ_f is the fluid density, and $\tilde{\mathbf{k}} = (\tilde{k}_x, \tilde{k}_y)$.
 130 Substituting equation (15) in equation (14), the radiated acoustic power
 131 becomes

$$\Pi_{\text{rad}}(\omega) = \text{Re} \left[\frac{\rho_f \omega}{16\pi^4} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{1}{\tilde{k}_z(\tilde{\mathbf{k}})} \left| H_{v_f}(\tilde{\mathbf{k}}, \mathbf{k}, \omega) \right|^2 \phi_{pp}(\mathbf{k}, \omega) d\tilde{\mathbf{k}} d\mathbf{k} \right]. \quad (17)$$

132 According to equation (16), when the wavenumbers are outside the acoustic
 133 circle defined by $\Omega_a = \left\{ \tilde{\mathbf{k}} \in \mathbb{R}^2, \left| \tilde{\mathbf{k}} \right| \leq k_f \right\}$, $\tilde{k}_z(\tilde{\mathbf{k}})$ becomes purely imaginary.
 134 As such, only wavenumbers inside the acoustic circle contribute to the radi-
 135 ated acoustic power. Hence equation (17) can be rewritten as

$$\Pi_{\text{rad}}(\omega) = \frac{\rho_f \omega}{16\pi^4} \int_{-\infty}^{\infty} \int_{\tilde{\mathbf{k}} \in \Omega_a} \frac{1}{\sqrt{k_f^2 - \tilde{k}_x^2 - \tilde{k}_y^2}} \left| H_{v_f}(\tilde{k}_x, \tilde{k}_y, \mathbf{k}, \omega) \right|^2 \phi_{pp}(\mathbf{k}, \omega) d\tilde{\mathbf{k}} d\mathbf{k}. \quad (18)$$

136 In equation (18), the term in the denominator becomes zero for wavenum-
 137 bers on the acoustic circle. This singularity is analytically removed in what
 138 follows. Using the following conversion formulas

$$\tilde{k}_x = \tilde{k}_r \sin\theta; \quad \tilde{k}_y = \tilde{k}_r \cos\theta, \quad (19)$$

139 equation (18) can be transformed to polar wavenumber coordinates as follows

$$\Pi_{\text{rad}}(\omega) = \frac{\rho_f \omega}{16\pi^4} \int_{-\infty}^{\infty} \left(\int_{\theta=0}^{\theta=2\pi} \int_{\tilde{k}_r=0}^{\tilde{k}_r=k_f} \frac{\tilde{k}_r}{\sqrt{k_f^2 - \tilde{k}_r^2}} \right. \\ \left. \left| H_{v_f}(\tilde{k}_r \sin\theta, \tilde{k}_r \cos\theta, \mathbf{k}, \omega) \right|^2 d\tilde{k}_r d\theta \right) \phi_{pp}(\mathbf{k}, \omega) d\mathbf{k}. \quad (20)$$

140 Finally, the change of variable, $\tilde{k}_r = k_f \sin\gamma$ analytically removes the singu-
 141 larity from the integral. As such, equation (20) can be expressed by

$$\Pi_{\text{rad}}(\omega) = \frac{\rho_f k_f \omega}{16\pi^4} \int_{-\infty}^{\infty} \left(\int_{\theta=0}^{\theta=2\pi} \int_{\gamma=0}^{\gamma=\frac{\pi}{2}} \sin\gamma \right. \\ \left. \left| H_{v_f}(k_f \sin\gamma \sin\theta, k_f \sin\gamma \cos\theta, \mathbf{k}, \omega) \right|^2 d\gamma d\theta \right) \phi_{pp}(\mathbf{k}, \omega) d\mathbf{k}. \quad (21)$$

142 Employing the rectangular method for the numerical integration in equa-
 143 tion (21), the radiated acoustic power becomes

$$\Pi_{\text{rad}}(\omega) = \frac{\rho_f k_f \omega}{16\pi^4} \sum_{-\mathbf{k}_{\text{cut-off}}}^{\mathbf{k}_{\text{cut-off}}} \left(\sum_{\theta \in [0, 2\pi]} \sum_{\gamma \in [0, \frac{\pi}{2}]} \sin\gamma \right. \quad (22)$$

$$\left. \left| H_{v_f}(k_f \sin\gamma \sin\theta, \tilde{k}_r \cos\theta, \mathbf{k}, \omega) \right|^2 \phi_{pp}(\mathbf{k}, \omega) \delta\gamma \delta\theta \right) \delta\mathbf{k},$$

144 where $\mathbf{k}_{\text{cut-off}}$ is cut-off wavenumber and $\delta\gamma$, $\delta\theta$, $\delta\mathbf{k}$ are the increments in the
 145 numerical integration. The sensitivity function H_{v_f} is given in Appendix A
 146 for a simply supported panel.

147 3. Numerical formulation

148 In the numerical formulation, the uncorrelated wall plane wave (UWPW)
 149 technique recently introduced by Maxit [21] is used to simulate the WPF
 150 generated by the TBL. The WPF generated by the TBL is represented by
 151 a set of deterministic pressure fields at each frequency of interest. Each re-
 152 alisation of the pressure field is applied as a deterministic input load to an
 153 element-based vibroacoustic solver. The structural and acoustic responses of
 154 the panel are then obtained for an ensemble average of the different panel
 155 responses at each frequency. This approach provides efficient coupling be-
 156 tween the statistical model to describe the wall pressure fluctuations and
 157 the deterministic model to describe the panel vibroacoustic responses. The
 158 pressure beneath the TBL for the l^{th} realisation can be represented by a set
 159 of UWPWs at the q^{th} node of an FEM mesh as follows [9; 21; 25]

$$p_{\text{inc}}^l(\mathbf{x}^q, \omega) = \sum_{i=1}^{N_x} \sum_{j=1}^{N_y} \sqrt{\frac{\phi_{pp}(k_x^i, k_y^j, \omega) \delta k_x \delta k_y}{4\pi^2}} e^{i(k_x^i x^q + k_y^j y^q + \varphi_{ij}^l)}, \quad (23)$$

160 where φ is a random phase uniformly distributed in $[0 \ 2\pi]$. The CSD can be
 161 expressed in terms of the ASD of the pressure field $S_{pp}(\omega)$ and the normalized
 162 CSD of the pressure field $\tilde{\phi}_{pp}(\mathbf{k}, \omega)$ as follows [21; 26]

$$\phi_{pp}(\mathbf{k}, \omega) = S_{pp}(\omega) \left(\frac{U_c}{\omega} \right)^2 \tilde{\phi}_{pp}(\mathbf{k}, \omega), \quad (24)$$

where U_c is the convective velocity. Using equation (23) as the deterministic load, the FEM is now implemented to simultaneously compute the l^{th} realisation of the structural displacement \mathbf{u}^l and the radiated pressure \mathbf{p}^l by solving the following fully coupled structural-acoustic equations [27]

$$\underbrace{\begin{bmatrix} -\omega^2 \mathbf{M}_s + \mathbf{K}_s & -\mathbf{H}_{fs} \\ -\omega^2 \rho_f c_f^2 \mathbf{H}_{fs}^T & -\omega^2 \mathbf{M}_f + \mathbf{K}_f \end{bmatrix}}_{\mathbf{A}} \begin{bmatrix} \mathbf{u}^l \\ \mathbf{p}^l \end{bmatrix} = \begin{bmatrix} \mathbf{f}_s^l \\ \mathbf{f}_f \end{bmatrix}, \quad (25)$$

where \mathbf{K} , \mathbf{H} and \mathbf{M} are respectively stiffness, coupling and mass matrices. Subscripts s and f respectively refer to the structure and fluid. \mathbf{f}_s^l is the structural force vector corresponding to the l^{th} realisation of the TBL pressure field given by equation (23). \mathbf{f}_f is the load from acoustic sources in the fluid domain, which is zero for the current case. After the inverse of the coefficient matrix \mathbf{A} is obtained, the panel displacement response and radiated pressure can be computed for each realisation as follows

$$\begin{bmatrix} \mathbf{u}^l \\ \mathbf{p}^l \end{bmatrix} = \mathbf{A}^{-1} \begin{bmatrix} \mathbf{f}_s^l \\ \mathbf{0} \end{bmatrix}. \quad (26)$$

The ASD of the panel vibration velocity S_{vv} and the cross spectrum between the sound pressure and the fluid particle velocity S_{pv_f} due to the TBL excitation is then calculated from the ensemble average of the different realisations by

$$S_{vv} = -\omega^2 \mathbf{E} \left[\mathbf{u}^l \mathbf{u}^{*l} \right]_l, \quad (27)$$

$$S_{pv_f} = \mathbf{E} \left[\mathbf{p}^l \mathbf{v}_f^{*l} \right]_l, \quad (28)$$

163 where $\mathbf{E} [\]$ represents the ensemble average of the realisations. This pro-
 164 cess is repeated for each frequency to obtain the spectra of the structural
 165 response. The method described here is termed the hybrid UWPW-FEM
 166 approach, whereby the UWPW technique was used to describe the WPF and
 167 a structural-acoustic solver based on the FEM method was employed to com-
 168 pute the vibroacoustic responses. It is worth noting that any element based
 169 approach can be used to obtain vibroacoustic responses of the panel. For
 170 example, instead of a coupled FEM-FEM model for the vibroacoustic solver
 171 as used in this work, a coupled finite element/boundary element method
 172 (FEM-BEM) can be employed to predict the vibroacoustic responses of the
 173 panel.

174 4. Results and discussion

175 A rectangular baffled aluminium panel with simply-supported boundary
176 conditions and excited by a TBL is examined. The aluminium panel of di-
177 mensions $0.6 \times 0.525 \text{ m}^2$ and 2.4 mm thickness was located in a $1.22 \times$
178 2.44 m^2 rigid panel made of medium density fibreboard of thickness 19 mm,
179 as shown in Figure 2(a). Fabrication of the simply-supported boundary con-
180 ditions for the experimental set-up of the aluminium test panel is described
181 in [28]. The panel was tested in an anechoic wind tunnel at the Université de
182 Sherbrooke [29]. The edge of the panel was placed 1.8 m from the convergent.
183 To help with the development of the TBL, a sandpaper strip was glued at
184 the intersection of the convergent and rigid baffle. This also removed any
185 discontinuity at the intersection of the baffle and the convergent. To reduce
186 the flanking transmission paths from the baffle, the test panel was decoupled
187 from the baffle and positioned on its own supporting stand. Sound absorbing
188 material was also placed on the stand to diminish possible sound reflections
189 (for more information on the characteristic of the absorbing material, see
190 [30]). The measurement space below the panel was acoustically treated us-
191 ing rigid tiles made of perlite-felt mix and backed by panels of compressed
192 0.0254 m thick glasswool.

193 The experiments were conducted at a free flow speed of 40 m/s. The
194 boundary layer was developed on an extended flat surface in a low Mach
195 number flow with zero pressure gradient. In such a flow condition, the bound-
196 ary layer thickness increased slowly in the flow direction as such the TBL
197 can be assumed as a homogeneous spatial field and stationary in time. The
198 panel mean quadratic velocity was computed from 20 discrete measurement
199 points using an array of accelerometers as described in [29]. To measure
200 the radiated sound, a shoebox-shaped microphone array was employed in
201 accordance with ISO3744 [31]. The array comprised 48 microphones placed
202 along a parallelepiped measurement surface and was located below the panel
203 in an acoustically treated box, as shown in Figure 2(b). The floor of the
204 measurement space was opened towards the anechoic room. The grid floor
205 was covered with compressed fibreglass panels. The stand of the test panel
206 was also covered by sound insulation material.

207 The wall pressure fluctuations of the turbulent flow generated over the
208 baffle were measured with a flush-mounted microphone array as described in
209 [6; 29]. The Mellen model was fitted to the measured WPF using the least
210 square method to estimate the decay rates, α_x and α_y , and the convective

211 velocity U_c . Using the experimentally fitted Mellen model and the measured
 212 ASD of the wall pressure presented in Appendix B, the CSD of the WPF
 213 was evaluated using equation (24) and used for all proceeding analytical and
 214 numerical results.

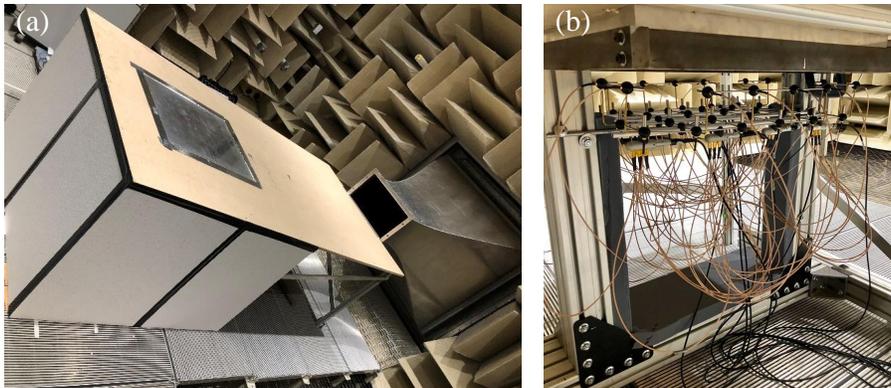


Figure 2: (a) Experimental set-up of the simply supported baffled rectangular plate in the anechoic wind tunnel at the Université de Sherbrooke [29], (b) parallelepiped microphone array used for the measuring radiated sound pressure.

215 Analytical results obtained using the sensitivity functions as well as nu-
 216 merical results obtained using the hybrid UWPW-FEM technique are com-
 217 pared with the experimental data obtained in an anechoic wind tunnel [29].
 218 The dimensions and material properties of the panel are listed in Table 1.
 219 The fluid density and kinematic viscosity were set to 1.225 kg/m^3 and 1.511
 220 $\times 10^{-5} \text{ m}^2/\text{s}$, respectively. The structural loss factor was experimentally es-
 221 timated using the -3dB bandwidth method for the first ten resonances of the
 222 plate. The mean value of the loss factors was used in the analytical and nu-
 223 merical models. The simulations were conducted using Matlab on a desktop
 224 personal computer with 32 GB of RAM and a total of four physical cores.
 225 For the UWPW-FEM technique, the wall pressure field was synthesized in
 226 Matlab and then imported as a load to an FEM model of the panel and
 227 acoustic domain in the commercial software COMSOL Multiphysics (v5.3a)
 228 using Matlab LiveLink. The acoustic domain in the FEM model was mod-
 229 elled using the Pressure Acoustics module and was discretised using tetra-
 230 hedral elements. The panel was modelled using the Shell Interface in the
 231 Structural Mechanics module and was meshed using structured quadrilateral

Table 1. Dimensions and material properties of the panel

Parameter	Value
Young's modulus, E (GPa)	68.9
Poisson's ratio, ν	0.3
Mass density, ρ_s (kg/m ³)	2740
Length, L_x (mm)	600
Width, L_y (mm)	525
Thickness, h_s (mm)	2.4
Damping loss factor, η	0.01

232 elements. The Acoustic-Structure Boundary coupling was applied to couple
 233 the acoustic domain to the panel. The MUMPS (MULTifrontal Massively Par-
 234 allel Sparse) direct solver was employed with the memory allocation factor
 235 of 1.2 [32].

236 Figure 3 illustrates the FEM model of the panel and acoustic domain.
 237 The anechoic chamber located below the panel in the experiment was sim-
 238 ulated using a hemisphere with a radius of 600 mm as shown in Figure 3.
 239 A perfectly matched layer (non-reflecting boundary condition) was applied
 240 on the boundary of the acoustic domain to allow the outgoing sound waves
 241 to leave the domain with minimal reflections. Similar to the experimental
 242 set-up, a rigid baffle was implemented. The 3D acoustic domain in the FE
 243 model was discretised using tetrahedral elements.

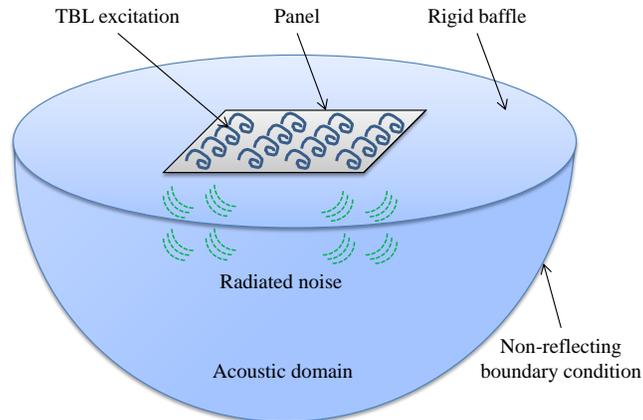


Figure 3: Schematic diagram of the FEM computational domain.

244 To obtain the panel response analytically and numerically, truncation
 245 of the wavenumber domain is required, whereby a truncated number of
 246 wavenumbers in the x and y directions need to be defined for equations (22)
 247 and (23). The criterion for defining the cut-off wavenumbers in the stream-
 248 wise and spanwise directions must be chosen such that the significant contri-
 249 butions of the integrands of these equations are correctly taken into account.
 250 For frequencies well above the aerodynamic coincidence frequency, it has
 251 been previously demonstrated that for prediction of the panel vibrational re-
 252 sponse, wavenumbers below or close to the flexural wavenumber of the plate
 253 are dominant [1; 6; 21]. A cut-off wavenumber of $\mathbf{k}_{\text{cut-off}} = \{2k_{p,\text{max}}, 2k_{p,\text{max}}\}$
 254 was used in both the streamwise and spanwise directions, where $k_{p,\text{max}} =$
 255 $(\omega_{\text{max}}\sqrt{\rho_s h/D})^{1/2}$ is the flexural wavenumber of the plate at the maximum
 256 frequency of interest denoted by ω_{max} . The wavenumber resolutions were set
 257 to $\delta k_x = \delta k_y = 0.25$ (1/m). The following criterion was used for the mesh
 258 size to take into account spatial variation of the WPF and to properly resolve
 259 structural modes

$$\Delta x = \Delta y = \min \left\{ \frac{\lambda_{p,\text{max}}}{10}, \frac{\pi}{2k_{p,\text{max}}} \right\} = \frac{\lambda_{p,\text{max}}}{10}, \quad (29)$$

260 where Δx , Δy are the element size in the x and y directions and $\lambda_{p,\text{max}}$ is
 261 the flexural wavelength of the plate at the maximum frequency of interest.
 262 In equation (22), $\delta\gamma = \pi/100$, $\delta\theta = \pi/50$ were used to numerically evaluate
 263 the acoustic power. A frequency resolution of 1 Hz was used in all numerical
 264 simulations.

265 4.1. Structural response of the panel

266 The UWPW technique described in Section 3 was used to synthesize the
 267 pressure field on the surface of the panel. Figure 4 shows the visualization
 268 of a single realisation of the surface pressure field at two discrete resonance
 269 frequencies corresponding to 224 Hz and 416 Hz, for a flow speed of 40 m/s.
 270 From Figure 4 it can be observed that at 416 Hz, a slightly higher spatial
 271 resolution is needed to properly describe and synthesize the WPF for plane
 272 waves compared to that needed at the lower frequency of 224 Hz. Figure 5
 273 presents the panel displacement at the two discrete frequencies using the
 274 single realisation of the WPF shown in Figure 4, and 30 realisations of the
 275 WPF. At the frequencies considered here, 30 realisations of the WPF are
 276 sufficient to obtain a converged solution. The converged panel displacement
 277 fields in Figure 5 reveals that the selected resonance frequencies correspond

278 to mode $(m, n)=(3, 2)$ at 224 Hz and $(m, n)=(5, 1)$ at 416 Hz. The effect of
 279 the number of realisations on the structural response of the panel is shown
 280 in Figure 6. Results for the velocity spectrum obtained analytically are also
 281 shown. An excellent match between the numerical and analytical results
 282 can be observed. Using 30 realisations, the maximum estimated error in
 283 the calculation of the panel response was less than 1 dB for the frequency
 284 range considered here. As such, 30 realisations were used for all subsequent
 285 calculations.

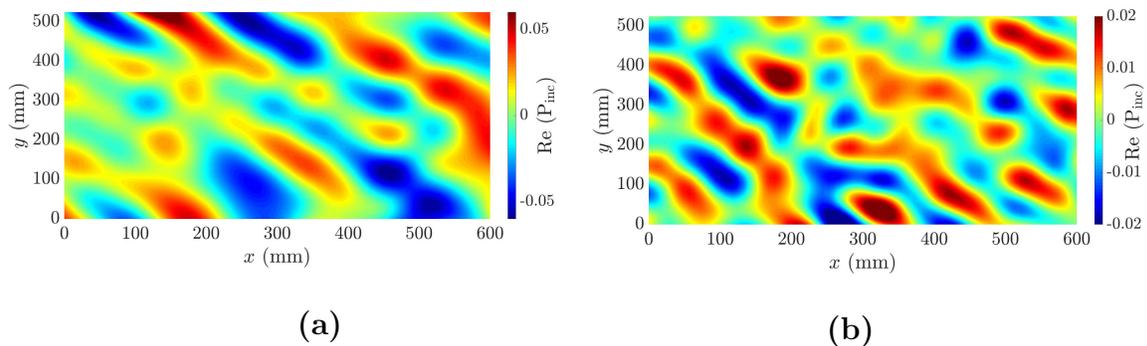


Figure 4: A single realisation of the wall pressure field for a flow speed of 40 m/s at (a) 224 Hz and (b) 416 Hz.

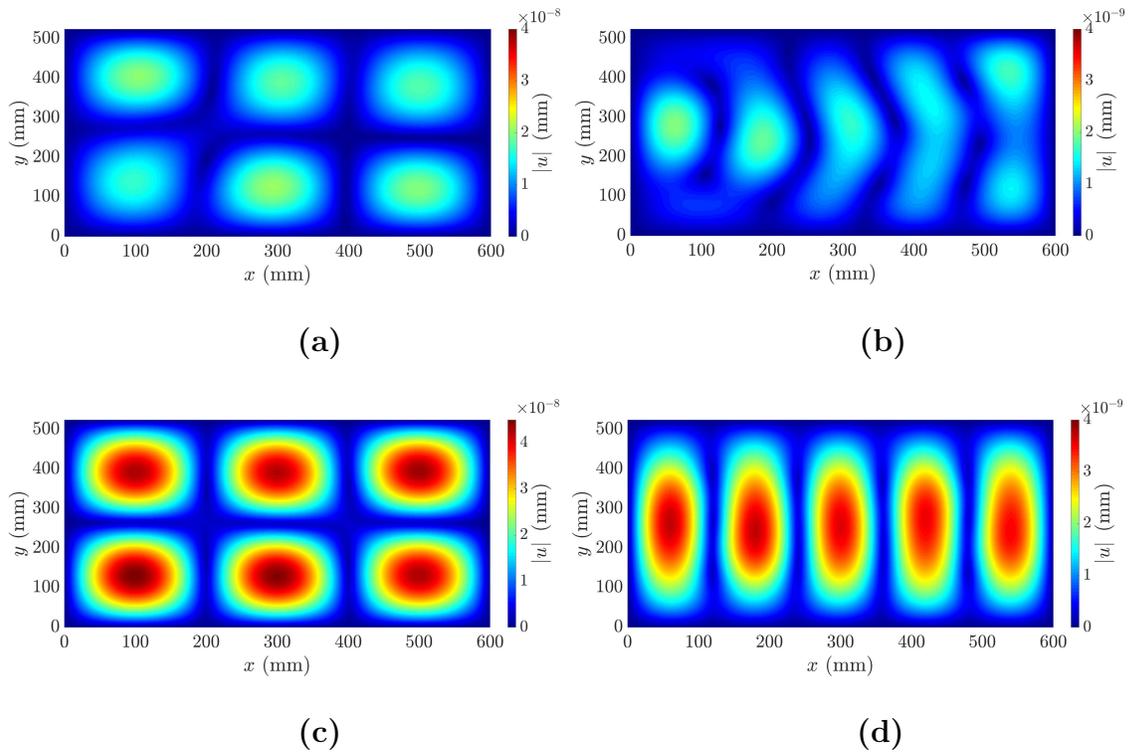


Figure 5: Panel displacement for a flow speed of 40 m/s at 224 Hz (a,c) and 416 Hz (b,d) using (a,b) a single realisation of the WPF corresponding to the WPF shown in Figure 4, and (c,d) the ensemble average of 30 realisations of the WPF.

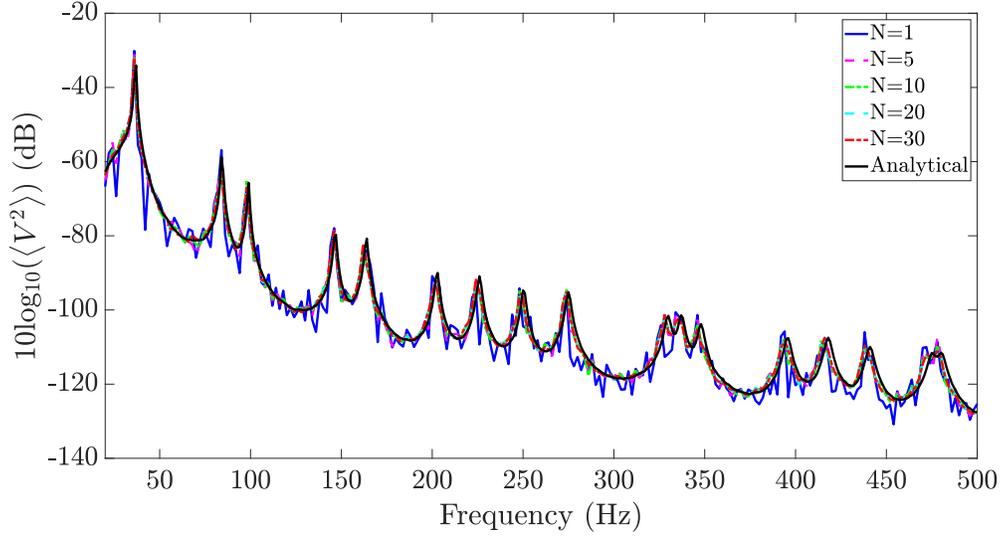


Figure 6: Predicted mean quadratic velocity spectra using different number of realisations at a flow velocity of $U_\infty = 40$ m/s (dB ref. $1 \text{ (m/s)}^2/\text{Hz}$).

286 Figure 7 compares the velocity spectra predicted analytically and nu-
 287 merically with experimental data. Very close agreement between predicted
 288 and measured results can be observed, with only a slight shift in resonance
 289 frequencies attributed to potential differences in the panel material proper-
 290 ties and implementation of the boundary conditions. The TBL excitation
 291 strongly excites the structure at the aerodynamic coincidence frequency, f_c ,
 292 which occurs when the flexural wavenumber given by $k_p = (\omega \sqrt{\rho_s h / D})^{1/2}$ is
 293 equal to the convective wavenumber $k_c = \omega / U_c$, that is, $f_c = U_c^2 \sqrt{\rho_s h / D} / (2\pi)$
 294 [6]. For the parameters chosen here and at a flow speed of 40 m/s, $f_c = 39$ Hz
 295 and is close to the first resonance of the plate.

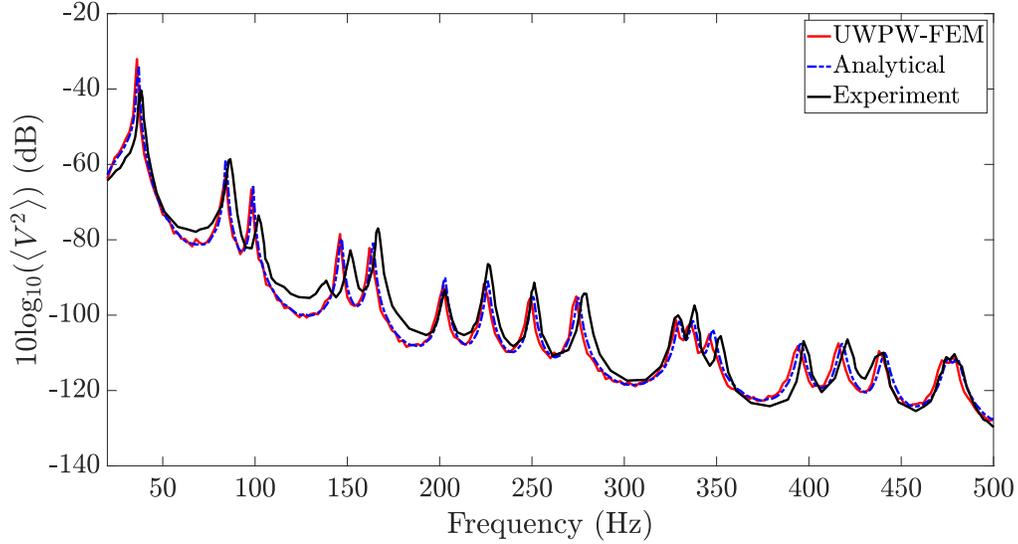


Figure 7: Predicted and measured mean quadratic velocity spectra at a flow velocity of $U_\infty = 40$ m/s (dB ref. 1 (m/s)²/Hz).

296 *4.2. Acoustic response of the panel*

297 It was already confirmed that for predicting structural response of a panel
 298 excited by TBL the cut-off wavenumber can be defined based on the flexural
 299 wavenumber and the effect of convected ridge can be neglected [9]. In this
 300 section, the validity of this criterion for evaluation the acoustic response of
 301 the panel is examined. To do this, the radiated acoustic power was obtained
 302 using a very large cut-off wavenumber (two times the convective wavenumber
 303 at the highest frequency of interest), as well as by using a cut-off wavenumber
 304 based on the flexural wavenumber. A maximum difference less than 0.5 dB
 305 was observed (results are not shown here), attributed to the filtering effect of
 306 the structure. This effect can be illustrated by plotting the forcing function
 307 and sensitivity function. To do this, equation (21) can be further written in
 308 a compact form as follows

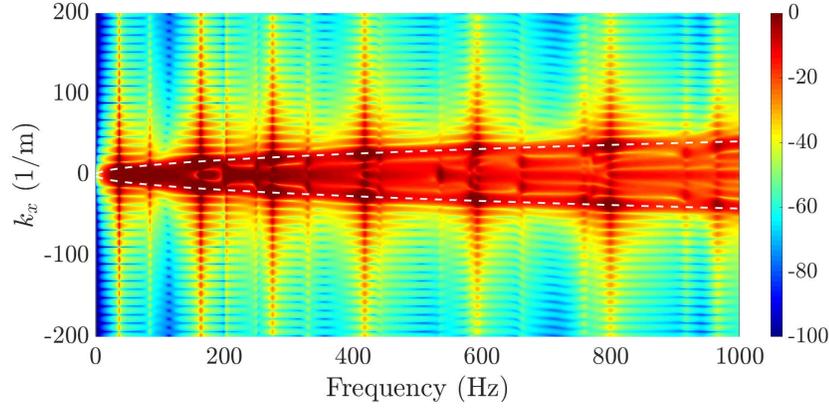
$$\Pi_{\text{rad}}(\omega) = \text{Re} \left[\left(\frac{1}{4\pi^2} \right) \int_{\infty} \left| \tilde{H}_{\Pi}(\mathbf{k}, \omega) \right|^2 \phi_{pp}(\mathbf{k}, \omega) d\mathbf{k} \right], \quad (30)$$

309 where $\left| \tilde{H}_{\Pi}(\mathbf{k}, \omega) \right|$ is the acoustic power sensitivity function given by

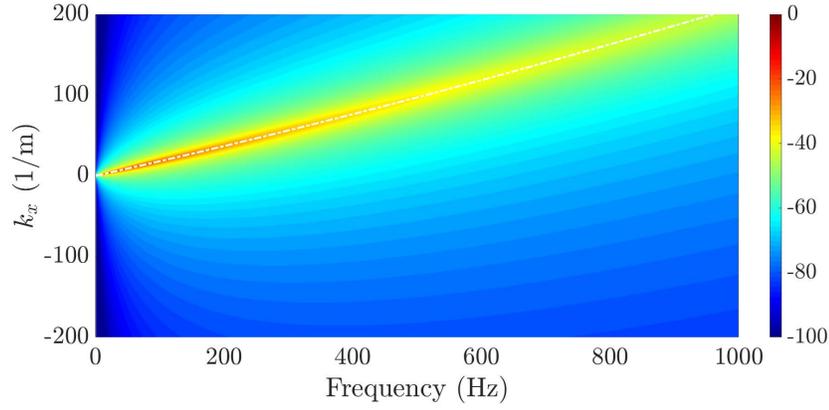
$$\left| \tilde{H}_{\Pi}(\mathbf{k}, \omega) \right|^2 = \frac{\rho_f k_f \omega}{4\pi^2} \left(\int_{\theta=0}^{\theta=2\pi} \int_{\gamma=0}^{\gamma=\frac{\pi}{2}} \sin\gamma \right. \quad (31)$$

$$\left. |H_{v_f}(k_f \sin\gamma \sin\theta, k_f \sin\gamma \cos\theta, \mathbf{k}, \omega)|^2 d\gamma d\theta \right).$$

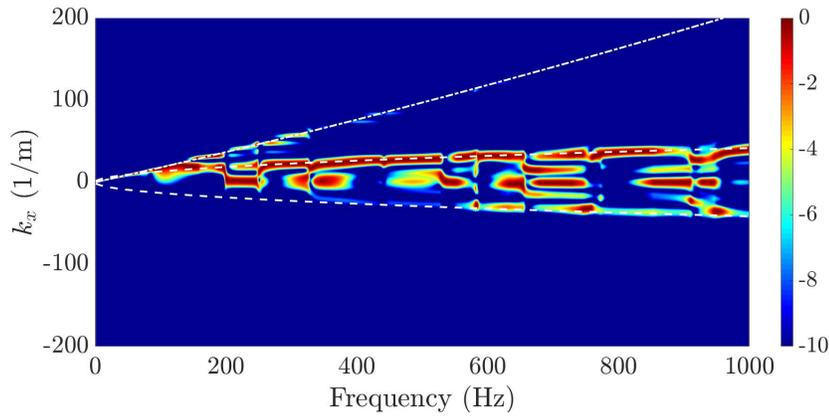
310 Figure 8(a) shows a map of the acoustic power sensitivity function. The
 311 white dashed lines correspond to the plate flexural wavenumbers. The maxi-
 312 mum values for the sensitivity function occur at wavenumbers smaller than or
 313 close to the flexural wavenumbers. However, the magnitude of the function is
 314 still significant for those wavenumbers larger than the flexural wavenumbers.
 315 Figure 8(b) presents the TBL forcing function, corresponding to the CSD of
 316 the WPF. The dash-dotted line corresponds to the convective wavenumbers
 317 where the TBL excitation is strong. The filtering effect of the structure is
 318 illustrated in Figure 8(c), which presents the product of the sensitivity func-
 319 tion and forcing function. It can be observed that the structure filters out
 320 most of the wavenumbers larger than flexural wavenumbers. Only wavenum-
 321 bers smaller than flexural wavenumbers contribute to the radiated acoustic
 322 power. However below 300 Hz, the effect of the convective ridge was not
 323 completely filtered by the panel. Whilst this contribution is not significant,
 324 the effect of the convective ridge was taken into account here as the cut-off
 325 wavenumber was defined as twice the flexural wavenumber at the highest
 326 frequency of interest.



(a)



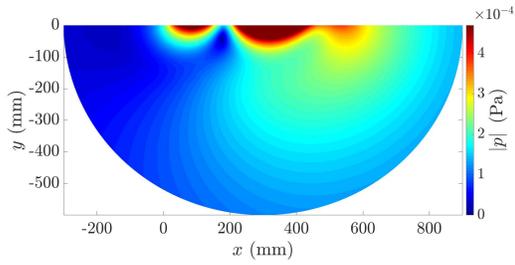
(b)



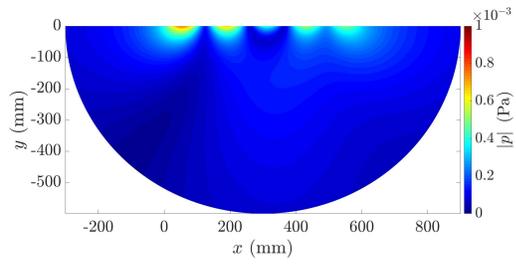
(c)

Figure 8: Maps of the (a) acoustic power sensitivity functions, $\left| \tilde{H}_{\Pi}(\mathbf{k}, \omega) \right|^2$ (dB, ref. $\text{Pa}^{-1}\text{m}^3\text{s}^{-2}\text{rad}^2$), (b) CSD function of the wall pressure spectrum using the Mellen and measured ASD function, $\phi_{pp}(\mathbf{k}, \omega)$ (dB, ref. $1 \text{ Pa}^2\text{m}^2\text{s rad}^{-2}$), and (c) result obtained by the product of (a) and (b) (dB, ref. 1 Wm^2). The dashed lines in (a) and (c) correspond to the plate flexural wavenumber; the dashed-dot line in (b) and (c) corresponds to the convective wavenumber.

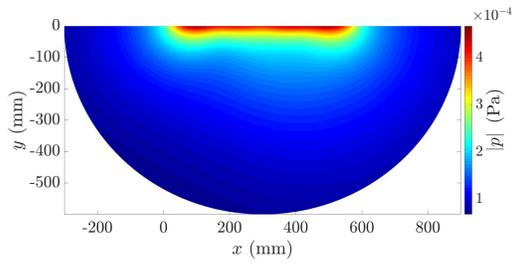
327 The absolute radiated pressure from the panel at the two discrete res-
328 onance frequencies of 224 Hz and 416 Hz is shown in Figure 9, using the
329 single realisation of the wall pressure (Figures 9(a) and (b)) as well as 30
330 realisations of the WPF (Figures 9(c) and (d)). A converged solution for
331 the pressure fields is obtained using 30 realisations of the WPF. This was
332 verified by comparing the numerical prediction of acoustic power with the
333 analytical results as well as measured acoustic power in Figure 10. It can
334 be seen that numerical results are in excellent agreement with analytical
335 results. The predicted acoustic power is also in good agreement with the
336 measured acoustic power. However, some discrepancies can be seen between
337 predicted and measured acoustic power. It can be observed that the exper-
338 imental data between resonance frequencies are relatively flat, this can be
339 attributed to the effect of background noise in the wind tunnel. In fact TBL
340 pressure fluctuations over the baffle generates noise which could be transmit-
341 ted into the anechoic chamber located below the panel. This noise was not
342 included in the simulation which may also explain the difference in magni-
343 tude between predicted and measured results. In addition, slight differences
344 in implementing panel boundary conditions and material properties used in
345 the numerical/analytical model and experiment can cause some discrepan-
346 cies between the measured and radiated acoustic power. Although, there is a
347 shift in frequency between numerical and experimental results in Figure 10,
348 the predicted resonance frequencies correspond to those obtained from mea-
349 surement. This was confirmed by comparing numerically calculated pressure
350 distribution in a plane 100 mm below the panel with those obtained from
351 the experiment using the microphone array as shown in Figure 11.



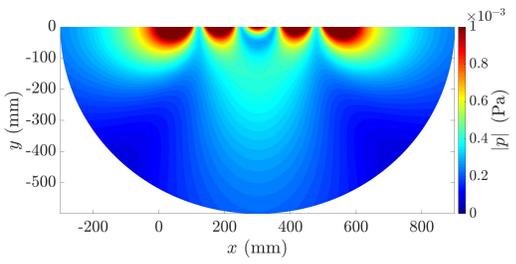
(a)



(b)



(c)



(d)

Figure 9: Pressure field in the xz -plane in the midspan of the panel at 224 Hz (a,c) and 416 Hz (b,d) using (a,b) a single realisation of the WPF corresponding to the WPF shown in Figure 4, and (c,d) the ensemble average of 30 realisations of the WPF.

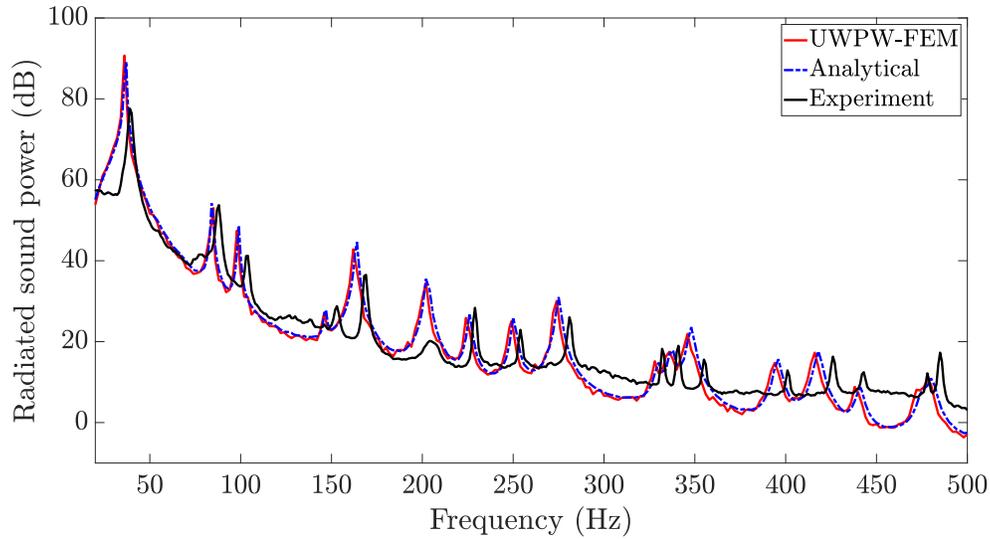


Figure 10: Predicted and measured acoustic power at flow velocity $U_\infty = 40$ m/s (dB ref. 1×10^{-12} (W)).

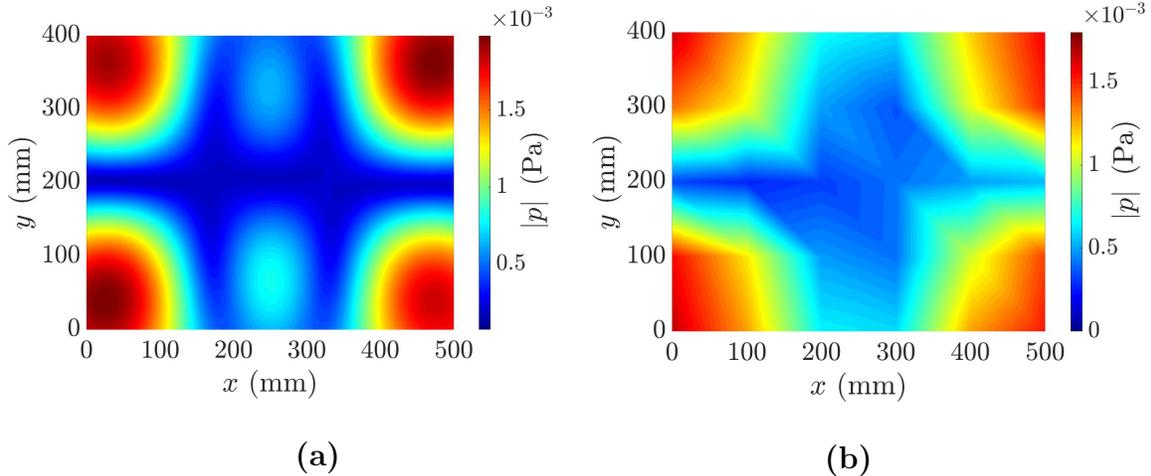


Figure 11: Maps of absolute radiated sound pressure in the xy -plane located 100 mm below the panel: (a) numerical results at 224 Hz; (b) experimental results at 229 Hz.

352 *4.3. Application to a stiffened panel*

353 To show one of the applications of the proposed numerical approach for
 354 complex panels, a simply supported stiffened panel is considered. Three
 355 stiffeners are added to the same panel considered in the previous section as
 356 shown in Figure 12. The stiffeners are located on the no-flow side of the
 357 panel. The cross-sectional dimensions of the stiffeners are 2.4 mm × 40 mm.
 358 The wavenumber characterizing the flexural motions of the stiffened panel is
 359 lower than that of the unstiffened panel as adding stiffeners increases panel
 360 stiffness. As such, the same cut-off wavenumber used for unstiffened panel
 361 could be used for the stiffened panel. Since the flow conditions and dimen-
 362 sions of stiffened panel are as same as those for unstiffened panel, there is
 363 no need to synthesize a new set of WPF, and the WPF generated using the
 364 UWPW technique in the previous section was used to predict the response of
 365 the stiffened panel. The stiffened panel is excited by TBL and its vibrational
 366 response and radiated acoustic power are compared with those obtained from
 367 the unstiffened panel as shown respectively in Figures 13 and 14. It was ob-
 368 served that stiffeners have significant effect on the vibroacoustic responses of
 369 the panel. The resonance frequencies of the stiffened panel are much higher
 370 than those of unstiffened panel, this is due to increase in panel stiffness by
 371 adding the stiffeners. At very low frequency (approximately at 28 Hz) a
 372 hump can be seen in both Figures 13 and 14. This can be attributed to the
 373 aerodynamic coincidences frequency for the stiffened panel. This frequency
 374 was estimated for the unstiffened panel to be 39 Hz. Adding stiffeners in-
 375 creases the flexural rigidity of the panel which decreases the aerodynamic
 376 coincidences frequency. The peak appeared at frequency close to 260 Hz
 377 in structural and acoustic responses of the stiffened panel corresponds to a
 378 group of modes with resonance frequencies close to each other. This was
 379 examined using an eigen-frequency analysis of the stiffened panel. Presence
 380 of a group of modes in a narrow frequency band explains why the peak ap-
 381 peared at frequency close to 260 Hz is not as sharp as those occurs at single
 382 resonance frequency.

To examine the effect of stiffeners on acoustic radiation from the panel, the radiation efficiency of the panel is computed for both unstiffened and stiffened panels as shown in Figure 15. The radiation efficiency of a panel is given by

$$\sigma_{\text{rad}} = \frac{\Pi_{\text{rad}}}{A\rho_f c_f \langle V^2 \rangle}, \quad (32)$$

383 Figure 15 shows that except at very low frequencies, adding stiffeners in-
384 creases radiation efficiency of the panel. This trend is well-known for panels
385 excited by a single point force and it can also be observed here for panels
386 excited by TBL excitation. Radiation efficiency is different for each mode.
387 For a simply supported panel, it is well-known that odd-odd modes (means
388 an odd mode in the x -direction and an odd mode in the y -direction) are
389 the most efficient modes and even-even modes are the least efficient modes
390 [33]. For example, at the resonance frequency around 150 Hz the unstiffened
391 panel has very low radiation efficiency, this sharp trough corresponds to the
392 mode number (2,2). In contrast, at frequency of approximately 200 Hz corre-
393 sponding to the mode number (1,3) the panel efficiently radiates sound to the
394 acoustic domain. As frequency increases both stiffened and unstiffened pan-
395 els show an upward trend in the radiation efficiency. The results presented in
396 this section highlights the capability of the proposed numerical approach to
397 predict the vibroacoustic responses of complex panels under TBL excitation
398 such as composite panels with stiffeners.

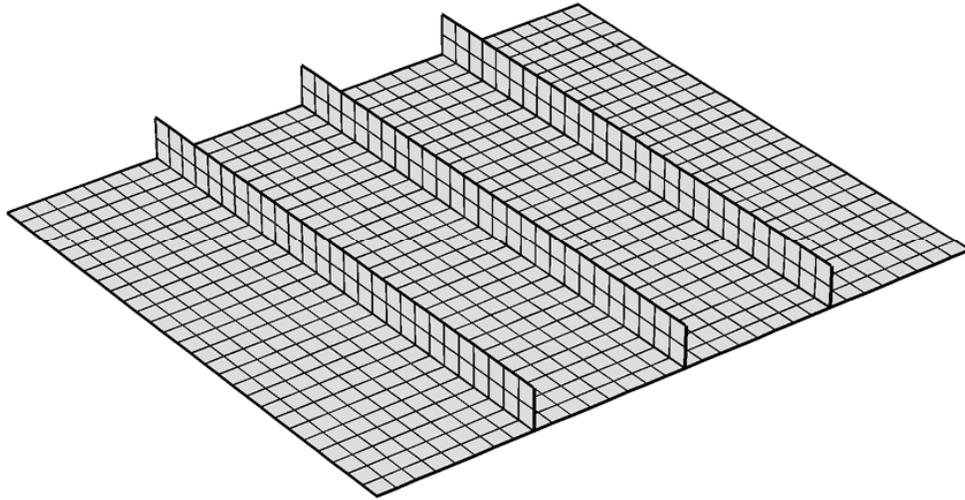


Figure 12: FEM model of the panel with three stiffeners oriented in spanwise direction.

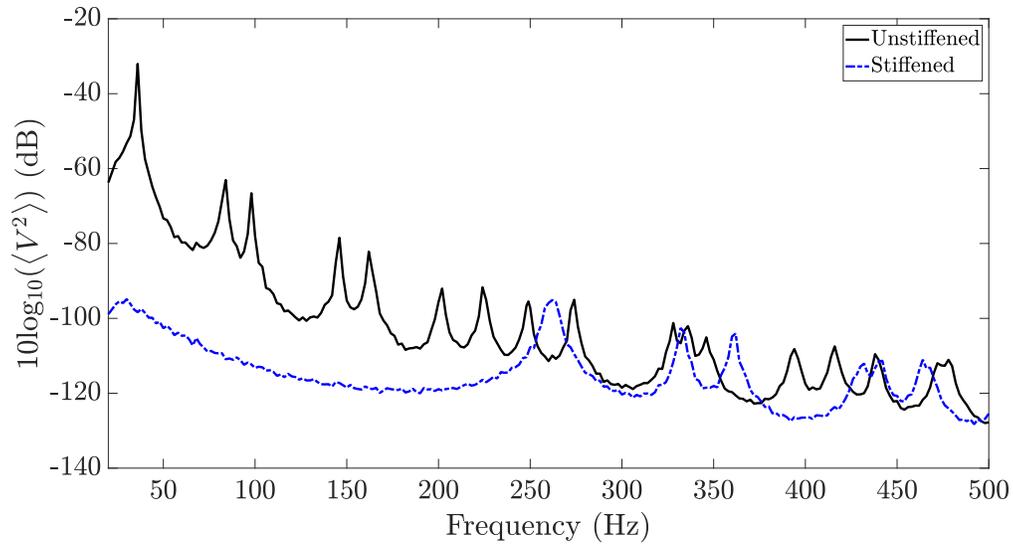


Figure 13: Predicted mean quadratic velocity spectra for unstiffened and stiffened panel, (dB ref. $1 \text{ (m/s)}^2/\text{Hz}$).

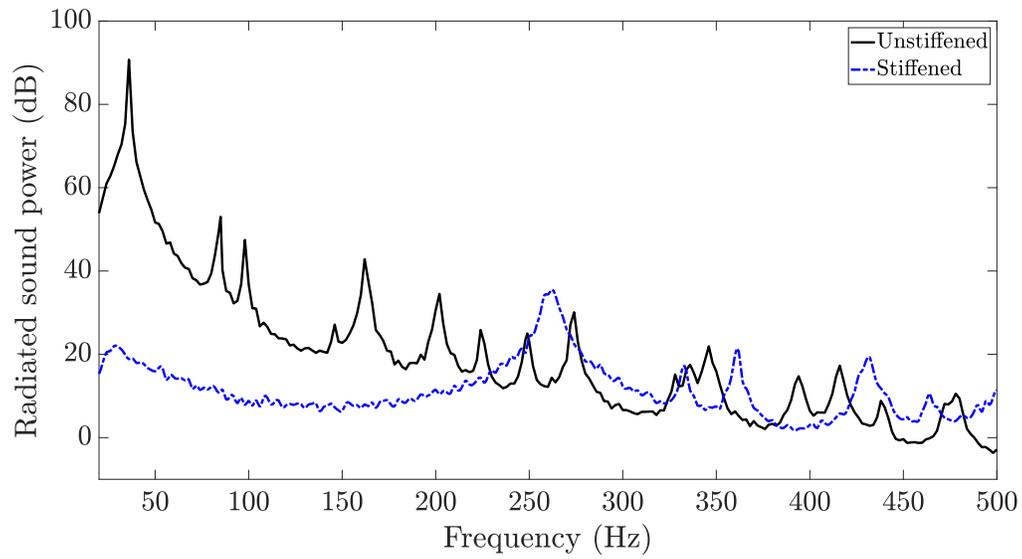


Figure 14: Predicted acoustic power for unstiffened and stiffened panel (dB ref. $1 \times 10^{-12} \text{ (W)}$).

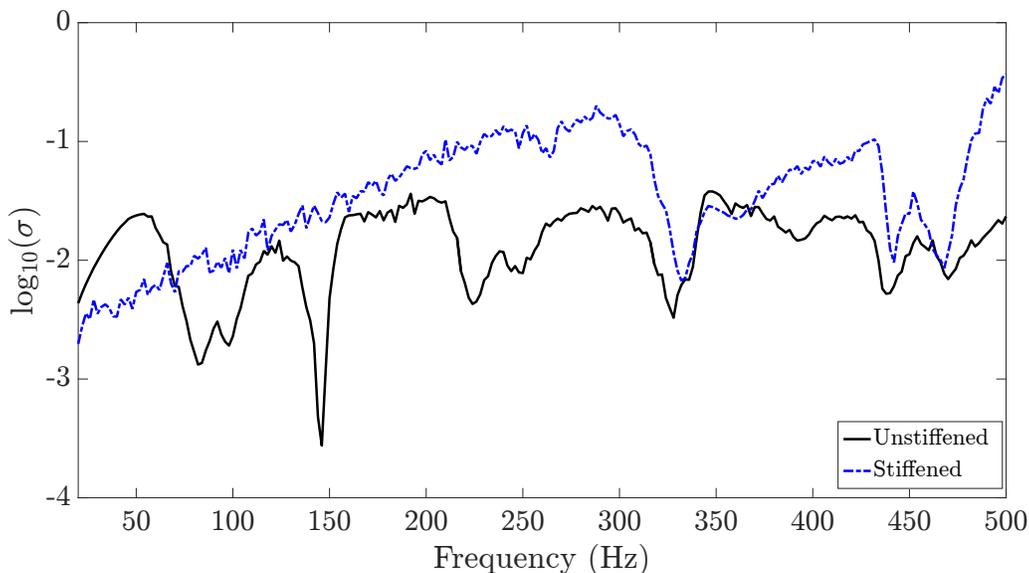


Figure 15: Radiation efficiency for stiffened and unstiffened panels.

399 5. Conclusions

400 The structural and acoustic responses of a simply supported panel under
 401 TBL excitation were numerically obtained using the hybrid UWPW-FEM
 402 approach. An analytical model based on sensitivity functions of the panel
 403 was used as a reference solution to verify the numerical method. Employing
 404 the acoustic power sensitivity functions, the effective wavenumber range for
 405 acoustic power calculation was identified and the filtering effect of the panel
 406 was illustrated. It was confirmed that both structural and acoustic responses
 407 of the panel can accurately be determined by choosing cut-off wavenumber
 408 based on the flexural wavenumber. The experimental data was also obtained
 409 from an anechoic wind tunnel to validate the numerical predictions. The
 410 numerical results were obtained by synthesizing the wall pressure field using
 411 the UWPW technique and applying different realisations of the WPF as deter-
 412 ministic loads to a fully coupled structural-acoustic solver based on the
 413 FEM. Since the WPF was obtained as deterministic loads, any element based
 414 methods such as the FEM and the boundary element method can be used for
 415 vibroacoustic calculation. Further, the proposed numerical approach can be
 416 applied to any complex planar structures with arbitrary boundary conditions.
 417 To illustrate one of the applications of the numerical approach, vibroacous-

418 tic responses of a stiffened panel subject to TBL excitation were studied. It
 419 was shown that adding stiffeners significantly increases the panel radiation
 420 efficiency. The main limitation of the proposed model corresponds to the
 421 accuracy in the simulation of the WPF. Errors in simulation of the WPF
 422 will inevitably result in errors in the prediction of the radiated acoustic pres-
 423 sure. Assumptions and limitations associated with the proposed model are (i)
 424 the TBL is homogeneous, stationary and fully developed over the structural
 425 surface in a subsonic flow, (ii) the influence of convection on the acoustic
 426 radiation from the structure is negligible, and (iii) the WPF is not altered
 427 by the vibration of the structure for which the fluid-structure interaction can
 428 be considered as weak coupling.

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 434 Lyon in France (ANR-10-LABX-0060/ANR-11-IDX-0007).

435 Appendix A

The sensitivity function $H_{v_s}(\mathbf{x}, \mathbf{k}, \omega)$ for a simply supported rectangular plate corresponding to the velocity at point \mathbf{x} when the panel is excited by a unit wall plane wave is given by

$$H_{v_s}(\mathbf{x}, \mathbf{k}, \omega) = i\omega \sum_{m=1}^M \sum_{n=1}^N \frac{\psi_{mn}(\mathbf{k})\varphi_{mn}(\mathbf{x})}{\Omega(\omega_{mn}^2 - \omega^2 + i\eta\omega\omega_{mn})}, \quad (\text{A.1})$$

$\Omega = \rho_s h L_x L_y / 4$ is the modal mass. The modal frequencies are given by

$$\omega_{mn} = \sqrt{\frac{D}{\rho_s h} \left(\left(\frac{m\pi}{L_x} \right)^2 + \left(\frac{n\pi}{L_y} \right)^2 \right)}, \quad (\text{A.2})$$

where $D = Eh^3/(12(1-\nu^2))$ is the flexural rigidity, E is the Young’s modulus and ν is Poisson’s ratio. The modal forces ψ_{mn} are calculated by integration over the panel surface as follows

$$\psi_{mn}(\mathbf{k}) = \int_A \varphi_{mn}(\mathbf{x}) e^{i(k_x x + k_y y)} dA = I_m^x(k_x) I_n^y(k_y), \quad (\text{A.3})$$

where $\varphi_{mn}(\mathbf{x})$ are the panel mode shapes given by

$$\varphi_{mn}(\mathbf{x}) = \sin\left(\frac{m\pi x}{L_x}\right)\sin\left(\frac{n\pi y}{L_y}\right), \quad (\text{A.4})$$

436 and

$$\{I_s^r(k_r)|(r, s) = (x, m) \vee (y, n)\} = \left\{ \begin{array}{ll} \left(\frac{s\pi}{L_r}\right) \frac{(-1)^s e^{i(k_r L_r)} - 1}{k_r^2 - \left(\frac{s\pi}{L_r}\right)^2}, & k_r \neq \frac{s\pi}{L_r} \\ \frac{1}{2}iL_r, & \text{otherwise} \end{array} \right\} (\text{A.5})$$

437 At the interface between the plate and the acoustic domain, the structural
 438 velocity v_s is equal to fluid particle velocity v_f in the normal direction, that
 439 is $H_{v_f}(\mathbf{x}, \mathbf{k}, \omega) = H_{v_s}(\mathbf{x}, \mathbf{k}, \omega)$. As such, the sensitivity function $H_{v_f}(\tilde{\mathbf{k}}, \mathbf{k}, \omega)$
 440 in equation (22) can be obtained analytically using a Fourier transform as
 441 follows

$$H_{v_f}(\tilde{\mathbf{k}}, \mathbf{k}, \omega) = \int_{\infty} H_{v_f}(\mathbf{x}, \mathbf{k}, \omega) e^{-i\tilde{\mathbf{k}}\mathbf{x}} d\mathbf{x} = \sum_{m=1}^M \sum_{n=1}^N a_{mn}(\tilde{\mathbf{k}}, \omega) \psi_{mn}(\mathbf{k}), \quad (\text{A.6})$$

442 where

$$a_{mn}(\tilde{\mathbf{k}}, \omega) = i\omega \frac{\psi_{mn}(-\tilde{\mathbf{k}})}{\Omega(\omega_{mn}^2 - \omega^2 + i\eta\omega\omega_{mn})}, \quad (\text{A.7})$$

443 and ψ_{mn} and I_s^r are given by equations (A.3)-(A.5).

444 Appendix B

445 The Mellen normalized wavenumber-frequency model is given by [34]

$$\tilde{\phi}_{pp}(k_x, k_y, \omega) = \frac{2\pi(\alpha_x \alpha_y)^2 k_c^3}{((\alpha_x \alpha_y k_c)^2 + (\alpha_x k_y)^2 + \alpha_y^2 (k_x - k_c)^2)^{3/2}}, \quad (\text{B.1})$$

446 where $k_c = \omega/U_c$. U_c , α_x and α_y were evaluated from experimental data
 447 as shown in Figure B1. The ASD of the WPF measured experimentally is
 448 shown in Figure B2.

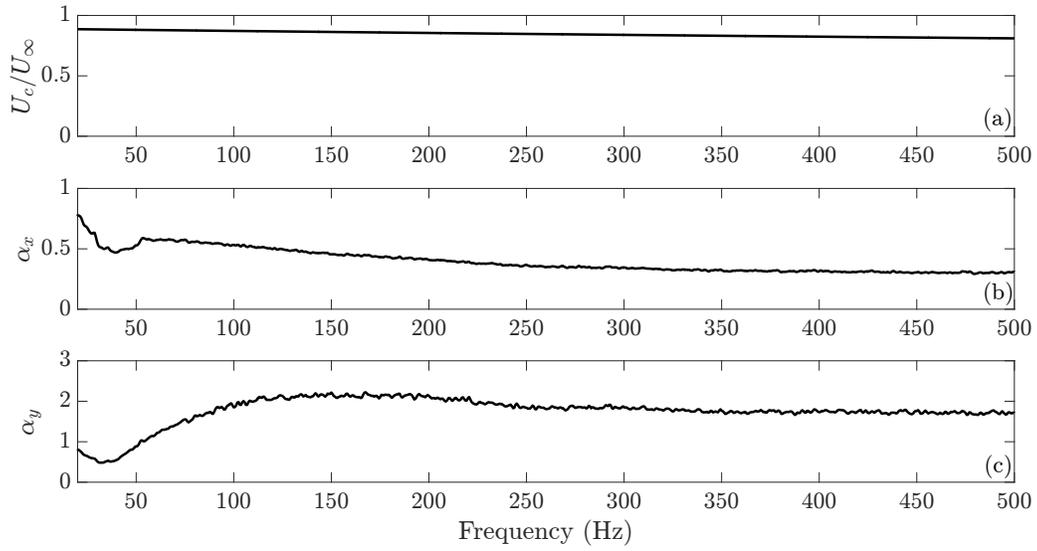


Figure B1: The TBL parameters extracted from measurements at $U_\infty = 40$ m/s: (a) convective speed normalized by the flow velocity; (b) streamwise exponential decay rate α_x ; (c) spanwise exponential decay rate α_y .

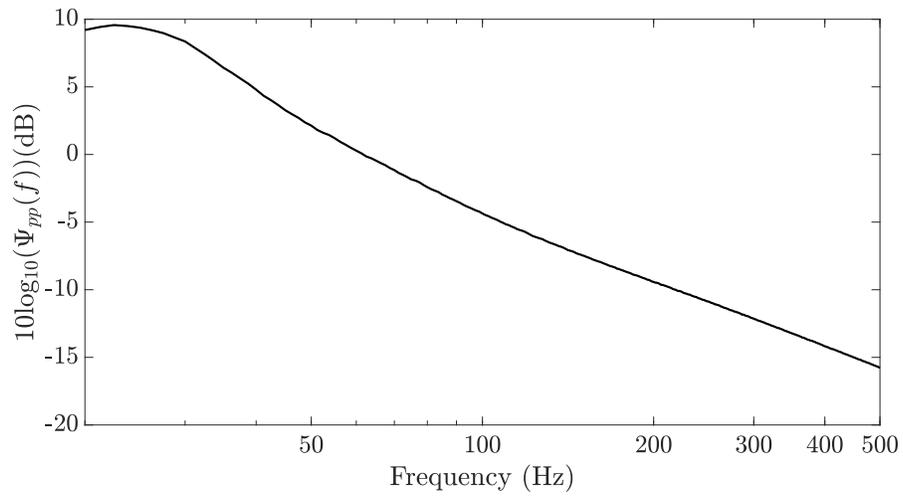


Figure B2: Measured ASD function of the wall pressure at $U_\infty = 40$ m/s (dB ref. $1 \text{ Pa}^2/\text{Hz}$).

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