



HAL
open science

Simulation of airfoil surface pressure due to incident turbulence using realizations of uncorrelated wall plane waves

M. Karimi, P. Croaker, A. Skvortsov, Laurent Maxit, R. Kirby

► To cite this version:

M. Karimi, P. Croaker, A. Skvortsov, Laurent Maxit, R. Kirby. Simulation of airfoil surface pressure due to incident turbulence using realizations of uncorrelated wall plane waves. *Journal of the Acoustical Society of America*, 2021, 149 (2), pp.1085-1096. 10.1121/10.0003498 . hal-03176229

HAL Id: hal-03176229

<https://hal.science/hal-03176229v1>

Submitted on 17 May 2024

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Simulation of Airfoil Surface Pressure due to Incident Turbulence using Realizations of Uncorrelated Wall Plane Waves

M. Karimi,^{1, a)} P. Croaker,² A. Skvortsov,² L. Maxit,³ and R. Kirby¹

¹⁾*Centre for Audio, Acoustics and Vibration, University of Technology Sydney, Sydney, Australia^{b)}*

²⁾*Maritime Division, Defence Science and Technology, Melbourne, Australia*

³⁾*Univ Lyon, INSA–Lyon, Laboratoire Vibrations-Acoustique (LVA), 25 bis, av. Jean Capelle, F-69621, Villeurbanne Cedex, France*

1 A numerical technique is proposed for synthesizing realizations of airfoil surface pressure
2 induced by incoming turbulence. In this approach, realization of the surface pressure field
3 is expressed as a set of uncorrelated wall plane waves. The amplitude of these plane waves
4 are determined from the power spectrum density function of the incoming upwash velocity
5 fluctuation and the airfoil aeroacoustic transfer function. The auto spectrum of the surface
6 pressure is obtained from an ensemble average of different realizations. The numerical
7 technique is computationally efficient as it rapidly converges using relatively small number
8 of realizations. The surface pressures for different airfoils excited by incoming turbulence
9 are numerically predicted and the results are compared with experimental data in literature.
10 Further, the unsteady force exerted on an airfoil due to the airfoil-turbulence interaction is
11 also computed and it is shown to be in very good agreement with analytical results.

^{a)}Mahmoud.karimi@uts.edu.au

^{b)}Also at: School of Mechanical and Manufacturing Engineering, UNSW Sydney, Australia

12 I. INTRODUCTION

13 The noise generated from interaction of incoming turbulence with an airfoil occur in many
14 industrial applications, and has been studied by many researchers in the literature. The interaction
15 between an airfoil and incoming turbulence induces lift fluctuations which radiates noise to the
16 far field as acoustic dipoles. A turbulent flow field can be formed upstream of an airfoil due to
17 the presence of inflow distortions and/or other aerodynamic elements. When there is also a steady
18 inflow, a turbulent flow can be produced by the development of a turbulent boundary layer over
19 the airfoil surface. These noise generation mechanism are respectively called as leading edge
20 noise (incoming turbulence noise) mechanism ([Amiet, 1975, 1976b](#); [Ayton and Chaitanya, 2017](#);
21 [Ayton and Peake, 2016](#); [Paterson and Amiet, 1976](#)) and trailing edge noise (self noise) mechanism
22 ([Amiet, 1976a, 1978](#); [Brooks, 1981](#); [Moreau *et al.*, 2008](#)). These two mechanisms can be present
23 at the same time. Nevertheless, if the turbulence intensity of incoming flow field is large enough,
24 the pressure fluctuations associated with boundary layer eddies will be small compared with the
25 unsteady pressure due to the incoming turbulence. In this case the turbulence ingestion noise
26 becomes the predominant noise ([Moreau and Roger, 2007](#)).

27 The leading edge noise has been extensively studied both numerically and experimentally
28 ([Amiet, 1975](#); [de Santana *et al.*, 2015](#); [Devenport *et al.*, 2010](#); [Glegg and Devenport, 2010](#); [Hutch-
29 erson *et al.*, 2012](#); [Mish and Devenport, 2006a](#); [Paterson and Amiet, 1976](#); [Roger and Moreau,
30 2010](#)). [Paterson and Amiet \(1976\)](#) theoretically and experimentally investigated the noise and un-
31 steady surface pressure of a NACA0012 airfoil in a grid generated turbulence. They considered
32 a free stream Mach number range of 0.1 to 0.5. Unsteady airfoil surface pressure spectra and

33 surface pressure cross-spectra were obtained in both spanwise and chordwise distribution in an
34 anechoic chamber that surrounded the tunnel open jet test section. A reasonable agreement be-
35 tween measured and predicted surface pressure spectra at zero angle of attack was achieved. A
36 good agreement between Amiet's far-field predictions and far field measurements was also ob-
37 served (Amiet, 1975). Mish (2001); Mish and Devenport (2003) investigated the response of an
38 airfoil to the incoming turbulence. The mean loading effects on the response of the airfoil encoun-
39 tering turbulence was experimentally studied (Mish and Devenport, 2006a). Measured pressure on
40 the surface of a NACA 0015 immersed in grid turbulence indicated a reduction of up to 5 dB with
41 increasing angle of attack for reduced frequencies less than 5. It was demonstrated that Amiet's
42 method not only works for zero angle of attack but also it can be used for predicting the airfoil
43 response at small angle of attack.

44 A detailed measurements of turbulent flow generated by either a grid or a rod interacting with
45 NACA 0012 airfoil was conducted by a de Santana *et al.* (2015). They used stereoscopic particle
46 image velocimetry as well as hot wire anemometry in the experiment. The focus of the study was
47 on quantification of turbulence distortion. This was done by measuring the turbulence properties
48 in the vicinity of the leading edge. It was found that even when the flow in the spanwise direc-
49 tion is almost homogenous, in order to improve the sound prediction small variations have to be
50 considered. Hutcheson *et al.* (2012) experimentally studied the radiated leading edge noise of an
51 airfoil immersed in nearly isotropic turbulence. It was shown that at zero angle of attack, the air-
52 foil's thickness and chord play an important role in controlling the spectral peak level. Further, an
53 increase in turbulence intensity and integral scale resulted in a uniform increase of the noise levels.
54 Previously proposed analytical models in the literature to predict the broadband noise generated

55 by thin airfoils in a turbulent flow were reviewed by [Roger and Moreau \(2010\)](#). Three generat-
56 ing mechanisms were examined, the turbulence-interaction noise (leading edge noise), turbulent
57 boundary layer noise (trailing edge noise), and vortex shedding noise (near wake of a thick trailing
58 edge). The models for noise prediction in all these cases were experimentally validated for thin
59 airfoils and the limitations and assumptions behind the development of the models were discussed.

60 Most of the previous studies focused on noise prediction of a rigid airfoil in turbulent flow dis-
61 regarding elasticity of the structure ([Amiet, 1975](#); [de Santana *et al.*, 2015](#); [Mish, 2001](#)). However,
62 in practice airfoils are made of elastic materials, therefore the surface pressure jump can excite the
63 airfoil and cause vibration and this vibration radiates noise to the far field. Neglecting this radiated
64 noise might be justifiable in air, as the aeroelastic coupling between the airfoil and air is quite
65 weak. However, for an airfoil in water, where the fluid-structure coupling is strong, vibroacous-
66 tic contribution to the radiated noise can be significant. To study this vibroacoustic contribution,
67 element-based vibroacoustic models can be employed to estimate the radiated noise by the elas-
68 tic structure under incoming turbulence excitation. The main challenge of this framework is that
69 the element-based models require deterministic loads whereas the incoming turbulence is non-
70 deterministic by its nature and described by stochastic models. In other words, due to the random
71 nature of incoming turbulence, the surface pressure can be characterized by statistical parameters
72 such as auto spectrum density (ASD) or cross spectrum density (CSD) that can be obtained using
73 stochastic models. However, in an element-based model such as the finite element model, air-
74 foil surface is discretized by finite elements and deterministic nodal forces are required to obtain
75 the vibroacoustic response. To overcome this challenge, we have proposed an efficient numeri-
76 cal technique to synthesize realizations of deterministic surface pressure produced by incoming

77 turbulence. The proposed technique can be easily implemented in any finite element or boundary
78 element software.

79 The uncorrelated wall plane wave (UWPW) technique has recently been used to synthesize the
80 wall pressure field underneath a turbulent boundary layer by some of the authors ([Karimi *et al.*,
81 2020, 2019a,b; Maxit, 2016](#)). This method which is based on representing a stochastic field by
82 superposition of uncorrelated wall plane waves is extended here to synthesize the surface pressure
83 from incoming turbulence. This approach is conceptually similar to stochastic noise generation
84 and radiation (SNGR) method ([Bechara *et al.*, 1994; Kraichnan, 1970](#)) which is one of the promis-
85 ing approaches often applied in synthetic turbulence modeling. The SNGR method expresses the
86 turbulence velocity field as a superposition of time-harmonic plane waves (Fourier modes), where
87 mode amplitudes and random phases are chosen in such a way to match the properties of the tur-
88 bulent flow. Employing the same concept herein, a formulation has been proposed to simulate the
89 pressure fluctuations on an airfoil interacting with an incoming turbulent flow. Realizations of the
90 unsteady surface pressure jump from incoming turbulence are expressed as a set of uncorrelated
91 plane waves traveling in the spanwise direction. The amplitudes of the waves are defined such that
92 the CSD of the UWPWs approximates the CSD of the pressure field induced by the turbulence in-
93 gestion. Therefore, the amplitudes of the waves become dependent on the power spectrum density
94 function of the incoming upwash velocity fluctuation and the airfoil aeroacoustic transfer func-
95 tion. To generate uncorrelated waves, the phases of the plane waves are randomly chosen from
96 a uniform distribution between 0 and 2π . The predicted auto spectrum of the surface pressure
97 jump is then obtained by ensemble average over different realizations. The effect of the number
98 of realizations and cut-off wavenumber on the predicted surface pressure jump is investigated.

99 Numerical results are compared with available experimental data from literature, showing very
100 good agreement. Although the focus of the current study is on simulation of the surface pressure
101 jump and demonstration of validity of the proposed method, to show how the deterministic surface
102 pressure jump can be employed to perform further analysis, the unsteady force acting on an airfoil
103 is also predicted using the UWPW technique, and numerical results are verified against analytical
104 results. To the best knowledge of the authors, this is the first time that the surface pressure jump
105 induced by airfoil-turbulence interaction is deterministically synthesized.

106 II. AIRFOIL-TURBULENCE INTERACTION MODEL

107 [Amiet \(1975\)](#) developed a model to describe airfoil-turbulence interaction and the resulting
108 noise production. Amiet's model is based on linearized thin-airfoil theory, and the airfoil is repre-
109 sented by a flat plate of a zero thickness and assumed to be at zero angle of attack ([Lysak, 2001](#)).
110 The model is based on several assumptions:

- 111 • The incoming turbulence fluctuation is considered to be small compared to the mean flow
112 velocity. This enables us to employ the small perturbation theory and to linearize the aero-
113 dynamic response.
- 114 • The turbulent flow is assumed to be inviscid so that the problem is reduced to solving lin-
115 earized Euler equations.
- 116 • The turbulence is frozen so that turbulent gust properties are not altered when it is convected
117 by the mean flow.

118 Amiet's formulation is summarized in this section for an airfoil of chord $2b$ and span $2d$ in a turbu-
 119 lent flow with a mean flow U_∞ in the x -direction, as shown in Figure 1. The airfoil is located in the
 120 plane $z = 0$. The y -coordinate extends in the spanwise direction, and the origin of the coordinate
 121 system is at the leading edge of the airfoil. To analyze the gust-airfoil problem, the Helmholtz

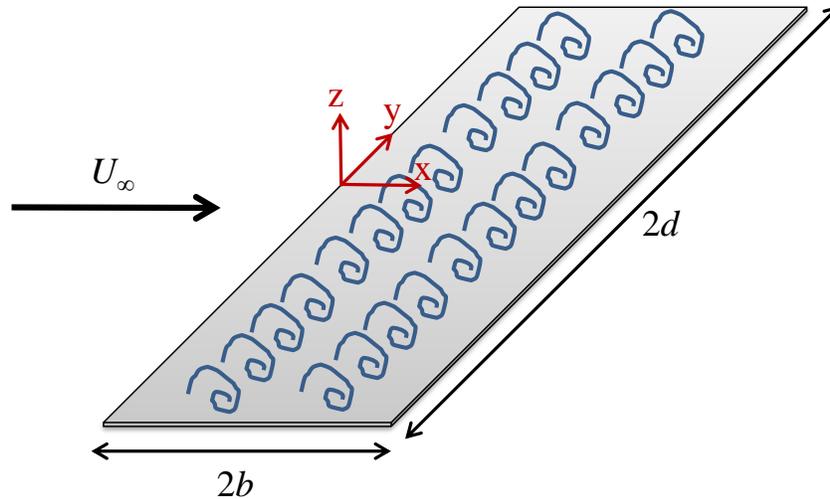


FIG. 1. (color online) Schematic diagram of airfoil-gust interaction.

122
 123

124 equation along with three boundary conditions need to be solved. To deal with this problem, an
 125 iterative procedure was proposed by Amiet (Amiet, 1975, 1976b). In this three-step procedure
 126 each boundary equation is solved in each iteration. First, the Helmholtz equation is solved for the
 127 aerodynamic gust interacting with an infinite flat plate with a non-penetration boundary condition.
 128 The non-penetration boundary condition and a zero flow potential condition are then imposed up-
 129 stream of the leading edge and the problem is solved using the Schwarzschild theorem. In the final
 130 iteration, this theorem is used to find a solution for the problem by considering a zero pressure
 131 jump and the Kutta conditions as well as non-penetration boundary equation downstream of the
 132 trailing edge.

133 The upwash velocity perturbation is convected with the uniform velocity U_∞ , and can thus be
 134 written as a spatial Fourier decomposition as follows (Amiet, 1975; de Santana *et al.*, 2015)

$$w(x, y, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{w}(k_x, k_y) e^{i(k_x(x-U_\infty t)+k_y y)} dk_x dk_y, \quad (1)$$

135 where $i = \sqrt{-1}$ is the imaginary unit, and k_x, k_y are the wavevector components in the streamwise
 136 and spanwise directions, respectively. The Fourier components $\tilde{w}(k_x, k_y)$ are defined as:

$$\tilde{w}(k_x, k_y) = \frac{1}{4\pi^2} \int_{-R}^R \int_{-R}^R w(x, y) e^{i(k_x x + k_y y)} dx dy, \quad (2)$$

137 the turbulence was assumed to be extended in the area of $-R < \{x, y\} < R$. R is not set equal to
 138 infinity because of convergence difficulties if $w(x, y)$ does not go to zero as x and y go to infinity.
 139 The pressure jump at a given point on the airfoil due to all wavenumber components is given
 140 by (Amiet, 1975; de Santana *et al.*, 2015; Santana *et al.*, 2016)

$$\Delta p(x, y, t) = 2\pi\rho U_\infty \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \tilde{w}(k_x, k_y) g(x, k_x, k_y) e^{i(k_x(x-U_\infty t)+k_y y)} dk_x dk_y, \quad (3)$$

141 where ρ is fluid density and $g(x, k_x, k_y)$ is the transfer function between the incoming gust velocity
 142 perturbation $\tilde{w}(k_x, k_y)$ and the airfoil pressure jump at the chordwise coordinate x . The Fourier
 143 transform with respect to time of Equation (3) yields (Amiet, 1975; de Santana *et al.*, 2015)

$$\Delta p(x, y, \omega) = 2\pi\rho \int_{-\infty}^{\infty} \tilde{w}(K_x, k_y) g(x, K_x, k_y) e^{i(k_y y)} dk_y, \quad (4)$$

144 where $K_x = \omega/U_\infty$ is the convective wavenumber and $\omega = 2\pi f$ is the angular frequency. For
 145 broadband noise problems, $\hat{w}(K_x, k_y)$ is considered as a non-deterministic quantity (Amiet, 1975,

146 1978). The single-sided CSD of the pressure jump at two points on the airfoil surface is given by
 147 (de Santana *et al.*, 2015; Mish and Devenport, 2006a)

$$S_{\Delta p \Delta p}(x, x', y, y', f) = 16\pi(\pi\rho)^2 U_\infty \int_0^\infty \phi_{ww}(K_x, k_y) g^*(x, K_x, k_y) g(x', K_x, k_y) e^{i(k_y(y-y'))} dk_y, \quad (5)$$

148 ϕ_{ww} is the single-sided power spectrum density of the incoming upwash velocity fluctuation and
 149 based on the von Karman isotropic model can be written as (Amiet, 1975; de Santana *et al.*, 2015)

150

$$\phi_{ww}(K_x, k_y) = \frac{4u'^2}{9\pi k_e^2} \frac{K_x^2 + k_y^2}{(1 + \frac{K_x^2}{k_e^2} + \frac{k_y^2}{k_e^2})^{7/3}}, \quad (6)$$

151 where u' is turbulence intensity, $\underline{k} = k/k_e$, $k_e = (\sqrt{\pi}/\Lambda_f)\Gamma(5/6)/\Gamma(1/3)$, Λ_f is the turbulence
 152 integral length scale and Γ represents the Gamma function. It has been reported that depending on
 153 the distance to the leading edge, the turbulence spectrum shows two different decay slopes at high
 154 frequencies. For long distances, one can apply von Karman isotropic model given by Equation (6)
 155 (for more details please see Christophe (2011); de Santana *et al.* (2015)). However, for short
 156 distances a modified von Karman spectrum applies which is based on the rapid distortion theory
 157 and is given by (de Santana *et al.*, 2015; Hunt, 1973)

$$\phi_{ww}(K_x, k_y) = \frac{91u'^2}{36\pi k_e^2} \frac{K_x^2 + k_y^2}{(1 + \frac{K_x^2}{k_e^2} + \frac{k_y^2}{k_e^2})^{19/6}}. \quad (7)$$

158 Amiet (1976b) derived solutions for the pressure and lift of a flat plat in incident turbulence by
 159 modifying Filotas' work (Filotas, 1969). This complete solution consists of two solution regions.
 160 One region is valid for small spanwise wavenumbers k_y (super-critical gust) and the other re-
 161 gion for large spanwise wavenumbers (sub-critical gust) giving essentially a complete low to high

162 frequency solution to the problem of a flat-plate airfoil encountering a skewed gust. The airfoil
 163 transfer function g associated with these two solution regions was derived by Mish (Mish, 2001;
 164 Mish and Devenport, 2006a) as follows

$$g(x, K_x, k_y) = -\frac{\Xi(x, K_x, k_y)}{\pi\beta} \left(\pi x \left(\left(\frac{k_y^2}{\beta^2} - \mu^2 \right)^{\frac{1}{2}} + i(\mu M + K_x) \right) \right)^{-\frac{1}{2}} e^{i\mu M x - x \left(\frac{k_y^2}{\beta^2} - \mu^2 \right)^{\frac{1}{2}}}, \quad (8)$$

165 if $k_y \geq (M/\beta)K_x$

$$\Xi(x, K_x, k_y) = 1 - \left(\frac{x}{2b} \right)^{\frac{1}{2}} \left[1 - \operatorname{erf} \left(\left(2b \left(2 - \frac{x}{b} \right) \left(\frac{k_y^2}{\beta^2} - \mu^2 \right)^{\frac{1}{2}} \right)^{\frac{1}{2}} \right) \right], \quad (9)$$

166 if $k_y < (M/\beta)K_x$

$$\Xi(x, K_x, k_y) = 1 - \left(\frac{x}{2b} \right)^{\frac{1}{2}} \left[1 - (1 - i) \mathfrak{F} \left(2i \left(\frac{b}{\pi} \left(2 - \frac{x}{b} \right) \left(-\frac{k_y^2}{\beta^2} + \mu^2 \right)^{\frac{1}{2}} \right)^{\frac{1}{2}} \right) \right]. \quad (10)$$

167 In these equations, M is the Mach number, $\beta = \sqrt{1 - M^2}$, $\mu = MK_x/\beta^2$, $\operatorname{erf}(\cdot)$ is the Error
 168 function, and $\mathfrak{F}(\cdot)$ is a combination of Fresnel integrals function and are given by

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} dt, \quad (11)$$

169

$$\mathfrak{F}(z) = C(z) + iS(z), \quad (12)$$

170 where $C(z)$ and $S(z)$ are respectively Fresnel cosine and sine integral functions

$$C(z) = \int_0^z \cos\left(\frac{\pi}{2}t^2\right)dt, \quad \text{and} \quad S(z) = \int_0^z \sin\left(\frac{\pi}{2}t^2\right)dt, \quad (13)$$

171 assuming $z = 2\left(\frac{b}{\pi}\left(2 - \frac{x}{b}\right)\left(-\frac{k_y^2}{\beta^2} + \mu^2\right)^{\frac{1}{2}}\right)^{\frac{1}{2}}$, the Fresnel integrals in Equation (10) can be written as
 172 follows

$$\mathfrak{F}(iz) = \begin{cases} S(z) + i C(z) & \text{if } z \geq 0, \\ -S(z) - i C(z) & \text{if } z \leq 0, \end{cases}, \quad (14)$$

173 To reduce the computation time, one can avoid evaluating the integrals in the Fresnel functions by
 174 exploiting approximate functions. $C(z)$ and $S(z)$ can be rewritten in terms of auxiliary functions
 175 $\Psi(z)$ and $\Phi(z)$ (Abramowitz and Stegun, 1948)

$$C(z) = \frac{1}{2} + \Psi(z)\sin\left(\frac{\pi}{2}z^2\right) - \Phi(z)\cos\left(\frac{\pi}{2}z^2\right), \quad (15)$$

176

$$S(z) = \frac{1}{2} - \Psi(z)\cos\left(\frac{\pi}{2}z^2\right) - \Phi(z)\sin\left(\frac{\pi}{2}z^2\right), \quad (16)$$

177 where the auxiliary functions can be approximated by

$$\Psi(z) = \frac{1 + 0.926z}{2 + 1.792z + 3.104z^2} + \epsilon(z), \quad |\epsilon(z)| \leq 2 \times 10^{-3}, \quad (17)$$

178

$$\Phi(z) = \frac{1}{2 + 4.142z + 3.492z^2 + 6.670z^3} + \epsilon(z), \quad |\epsilon(z)| \leq 2 \times 10^{-3}, \quad (18)$$

179 $\epsilon(z)$ represents the error in using these approximate functions.

180 III. UNCORRELATED WALL PLANE WAVE TECHNIQUE

181 The uncorrelated wall plane wave technique was initially introduced by Maxit (2016) to sim-
 182 ulate the pressure field beneath a turbulent boundary layer using realizations of UWPWs. This

183 technique has been extended to simulate the pressure jump on the airfoil, which can be expressed
 184 as a set of uncorrelated wall plane waves traveling in the y -direction for the n^{th} realization as
 185 follows

$$(\Delta p^n(x, y, f))_{\text{UWPW}} = \sum_{i=1}^N A(x, k_y^i, f) e^{i(k_y^i y + \varphi_i^n)}, \quad (19)$$

186 where A^i are the stochastic amplitudes of the UWPWs, N represents the number of UWPWs and
 187 φ is a random phase uniformly distributed in $[0, 2\pi]$. It should be noted that these wall plane
 188 waves are surface waves as they are only defined at the surface of the structure. Further, unlike
 189 the original version of the UWPW technique which was developed to synthesize wall pressure
 190 underneath a turbulent boundary layer (Maxit, 2016), the plane waves here are only traveling in
 191 the y -direction and their amplitudes varies along the x -direction. It can be shown that the cross
 192 spectrum of the surface pressure jump is given by

$$(S_{\Delta p \Delta p}(x, x', y, y', f))_{\text{UWPW}} = \mathbf{E} \left[\sum_{i=1}^N A(x, k_y^i, f) e^{i(k_y^i y + \varphi_i^n)} \times \overline{\sum_{j=1}^N A(x', k_y^j, f) e^{i(k_y^j y' + \varphi_j^n)}} \right], \quad (20)$$

193 where $\mathbf{E}[\]$ represents the ensemble average over the realizations and the upper bar denotes the
 194 complex conjugate of the complex number. Equation (20) can be rewritten as follows

$$(S_{\Delta p \Delta p}(x, x', y, y', f))_{\text{UWPW}} = \sum_{i=1}^N \sum_{j=1}^N \mathbf{E} [A(x, k_y^i, f) A^*(x', k_y^j, f)] e^{i(k_y^i y - k_y^j y')} \mathbf{E} [e^{i(\varphi_i^n - \varphi_j^n)}], \quad (21)$$

195 where

$$\mathbf{E} [e^{i(\varphi_i^n - \varphi_j^n)}] = \begin{cases} 1 & i = j \\ 0 & \text{otherwise} \end{cases}, \quad (22)$$

196 therefore

$$(S_{\Delta p \Delta p})_{\text{UWPW}} = \sum_{i=1}^N \mathbb{E} [A(x, k_y^i, f) A^*(x', k_y^i, f)] e^{i(k_y^i(y-y'))}. \quad (23)$$

197 Revisiting Equation (5), the improper integral can be approximated using the rectangular rule by
 198 truncating and regularly sampling the wavenumber space as follows

$$S_{\Delta p \Delta p} \approx 16\pi(\pi\rho)^2 U_\infty \sum_{i=1}^N \phi_{ww}(K_x, k_y^i) g^*(x, K_x, k_y^i) g(x', K_x, k_y^i) e^{i(k_y^i(y-y'))} \delta k_y^i, \quad (24)$$

199 comparing equations (23) and (24), it can be concluded that the CSD of the pressure field by the
 200 turbulence ingestion is approximately equal to the CSD of the UWPWs if the amplitudes of the
 201 UWPWs in Equation (19) are defined by

$$A(x, k_y^i, f) = 4\pi\rho g(x, K_x, k_y^i) \sqrt{\pi U_\infty \phi_{ww}(K_x, k_y^i) \delta k_y^i}. \quad (25)$$

202 In fact, the amplitude of each wall pressure plane wave is defined such that the whole set can repre-
 203 sent the statistical properties of a pressure jump generated by incoming gust. The final expression
 204 of the n^{th} realization of surface pressure jump on the airfoil using the UWPW technique can be
 205 obtained by

$$\Delta p^n(x, y, f) = 4\pi\rho \sum_{i=1}^N g(x, K_x, k_y^i) \sqrt{\pi U_\infty \phi_{ww}(K_x, k_y^i) \delta k_y^i} e^{i(k_y^i y + \varphi_i^n)}, \quad (26)$$

206 The surface pressure jump is varying spatially in the spanwise direction as $e^{i(k_y^i y)}$ while its variation
 207 in the streamwise direction is governed by the transfer function. The auto spectrum density (ASD)
 208 of the surface pressure jump $S_{\Delta p \Delta p}$ due to the incident turbulence is then calculated from the

209 ensemble average of different realizations by

$$S_{\Delta p \Delta p}(x, y, f) = E [\Delta p^n(x, y, f) \Delta p^{*n}(x, y, f)], \quad (27)$$

210 This process is repeated for each frequency to obtain the spectra of the surface pressure jump.

211 Once the surface pressure jump is available, realizations of unsteady force acting on the airfoil

212 surface can also be obtained by integrating the pressure jump over the airfoil surface (represented

213 by a flat plate) as follows

$$F^n(f) = \int_{-d}^d \int_0^{2b} \Delta p^n(x, y, f) dx dy, \quad (28)$$

214 Accordingly, the ASD of the unsteady force S_{FF} is calculated from the ensemble average of

215 different realizations by

$$S_{FF}(f) = E [F^n(f) F^{*n}(f)]. \quad (29)$$

216 It should be noted the surface pressure (p_s) is related to pressure jump by $|p_s| = |\Delta p|/2$. The tech-

217 nique developed in this work is based on the assumption that the surface pressure field induced by

218 incoming turbulence is not altered by the vibration of the airfoil. The proposed method enables us

219 to estimate the unsteady surface pressure jump (loads) on an airfoil due to turbulence interaction

220 as deterministic excitations, these excitations can then be applied to element-based vibroacoustic

221 analysis techniques. The proposed technique allows that the statistical model to describe the sur-

222 face pressure fluctuations to be coupled with deterministic and element-based numerical methods

223 such as the finite element and boundary element methods to investigate the vibroacoustic behavior

224 of airfoils in turbulent flow.

225 **IV. NUMERICAL RESULTS**

226 In this section the UWPW technique has been implemented in Matlab to synthesize the surface
 227 pressure jump for two different airfoil models of varying chord length and flow speed as listed
 228 in Table 1. The first airfoil (Case 1) corresponds to the experiments performed at the Virginia
 229 Tech Stability Wind Tunnel (Mish and Devenport, 2006a,b). The unsteady surface pressure jump
 230 was measured on a NACA0015 airfoil with 0.610 m chord and 1.83 m span at flow speed of 30
 231 m/s. This airfoil is used for all the numerical investigations in this work unless stated otherwise.
 232 The second airfoil (Case 2) is a NACA0012 with a chord of 0.230 m and span of 0.530 m which
 233 was tested by Paterson and Amiet (1976) and it has been used in this work to further validate the
 234 proposed method.

TABLE I. Input parameters used in the prediction code for each case study

Case	Airfoil	Flow speed (m/s)	Integral length scale	Turbulence intensity
			Λ_f (m)	u'/U_∞
1	NACA0015	30	0.0818	0.0398
2	NACA0012	40	0.0302	0.0453
		60	0.0301	0.0392
		90	0.0294	0.0482
		120 and 165	0.0305	0.0414

235

236

237 **A. Cutoff wavenumber and wavenumber resolution**

238 To correctly approximate the improper integral in Equation (5), criteria for choosing the cut-
 239 off wavenumber and wavenumber resolution need to be defined in the spanwise direction. Fig-

240 ure 2 (a) presents the square of the transfer function modulus in the wavenumber domain. The
 241 dashed line corresponds to $k_y = (M/\beta)K_x$ which splits the wavenumber domain into two re-
 242 gions. The region where $k_y < (M/\beta)K_x$ is associated with super-critical gust while the re-
 243 gion where $k_y \geq (M/\beta)K_x$ contains the sub-critical gusts. According to Figure 2 (a) when
 244 the convective wavenumber is almost below 4000 m^{-1} , the highest values of the transfer func-
 245 tion occurs for spanwise wavenumbers $k_y < 200 \text{ m}^{-1}$ and the contributions of both sub-critical
 246 or super-critical gusts are significant. At higher convective wavenumbers (or higher frequencies)
 247 only those wavenumbers corresponding to the super-critical gusts are important and the effect of
 248 sub-critical gust can be neglected. Figure 2 (b) shows the color map of the modified von Karman
 249 spectrum. The values of the function $\phi_{ww}(K_x, k_y)$ is insignificant for convective wavenumbers
 250 larger than 700 m^{-1} . Below this convective wavenumber, the values of the function is consider-
 251 able for spanwise wavenumbers $k_y < 900 \text{ m}^{-1}$. It should be noted that to find the effective range
 252 of wavenumbers for calculation of surface pressure jump we need to inspect the integrand of the
 253 integral in Equation (5). Determination of the cutoff wavenumber can be achieved by plotting
 254 the integrand which is the product of the square of the transfer function modulus (Figure 2 (a))
 255 and von Karman spectrum (Figure 2 (b)). Figure 2 (c) presents the color map of the integrand.
 256 As illustrated in this figure, the Graham's parameter ($\Theta = MK_x/(\beta k_y)$) of less than or greater
 257 than unity corresponds respectively to sub-critical or super-critical gusts (Karimi *et al.*, 2019a).
 258 Figure 2 (c) reveals that at very low frequencies, contributions of sub-critical wavenumbers are
 259 dominant. However, as frequency increases, the super-critical gusts have an increasing contribu-
 260 tion to the incoming turbulence pressure to the point where above $K_x \approx 400 \text{ m}^{-1}$ they become the

261 dominant contribution. The significance of these contributions to the predicted surface pressure
 262 jump over frequency range is further investigated in Section [IV D](#).

263 To take into account the significant contributions of the integrand, the following cut-off
 264 wavenumber is selected: $k_y^{\text{co}}=70 \text{ m}^{-1}$ for $K_x \leq 700 \text{ m}^{-1}$ and $k_y^{\text{co}} = MK_x/\beta$ when $K_x > 700 \text{ m}^{-1}$.
 265 This criterion for cut-off wavenumber was achieved by ensuring that the maximum estimated error
 266 in the calculation of the surface pressure jump associated with the wavenumber domain truncation
 267 was less than 1 dB for the frequency range. A trial and error process was also conducted to find
 268 a wavenumber resolution which correctly represent the spatial variations of the integrand in the
 269 wavenumber space. It was found that a constant wavenumber resolution of $\delta k_y=0.1 \text{ m}^{-1}$ is a good
 270 choice.

272 **B. Number of realizations**

273 The effect of the number of realizations on the accuracy of the UWPW technique was evalu-
 274 ated as follows. The result obtained using 500 realizations was selected as a converged solution
 275 and used as a reference to compute the discrepancy (Δ) between results obtained with different
 276 numbers of realizations. The discrepancy is shown in [Figure 3](#) for 10, 30, 50, 80 and 100 real-
 277 izations. For the current case study, the maximum error using 10, 30, 50, 80 and 100 realizations
 278 over the frequency range are approximately 5.2 dB, 2.1 dB, 1.7 dB, 1.5 dB and 1.4 dB. A suitable
 279 number of realizations can be selected depending on the required accuracy. In this work, for all
 280 the calculations 50 realizations have been considered in the UWPW technique. [Figure 4](#) shows
 281 the ASD of the surface pressure jump as a function of reduced frequency ($\omega_r = \omega b/U_\infty$) at 6%
 282 chord. The spectral level predicted using 50 different realizations is shown in gray lines and the

Simulation of Airfoil Surface Pressure

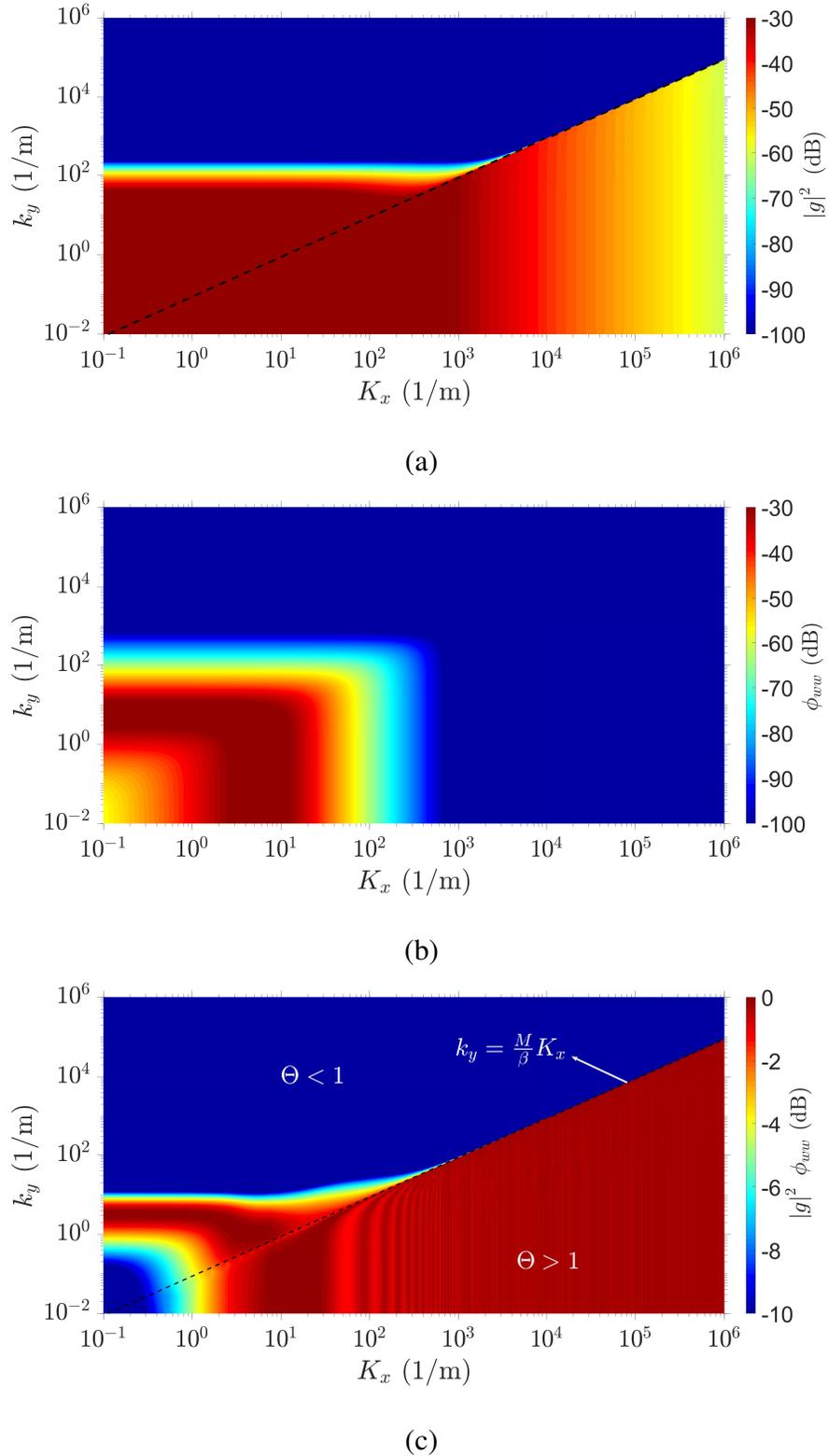


FIG. 2. (color online) Color maps of the (a) airfoil transfer function $|g(x, K_x, k_y)|^2$ (dB, ref. 1), (b) modified von Karman spectrum $\phi_{ww}(K_x, k_y)$ (dB, ref. $1 \text{ m}^2\text{s}^{-2}\text{Hz}^{-1}$), and (c) integrand obtained by the product of (a) and (b) normalized by the maximum value at each convective wavenumber (dB, ref. $1 \text{ m}^2\text{s}^{-2}\text{Hz}^{-1}$). The dashed line in (a) and (c) corresponds to the Graham's parameter showing critical gusts $\Theta = 1$, which separates sub-critical gusts ($\Theta < 1$) from super-critical gusts ($\Theta > 1$).

283 black line represents the predicted results by averaging of 50 realizations. It can be observed from
 284 Figures 3 and 4 that the numerical results become smoother and converge quickly by increasing
 285 the number of realizations.

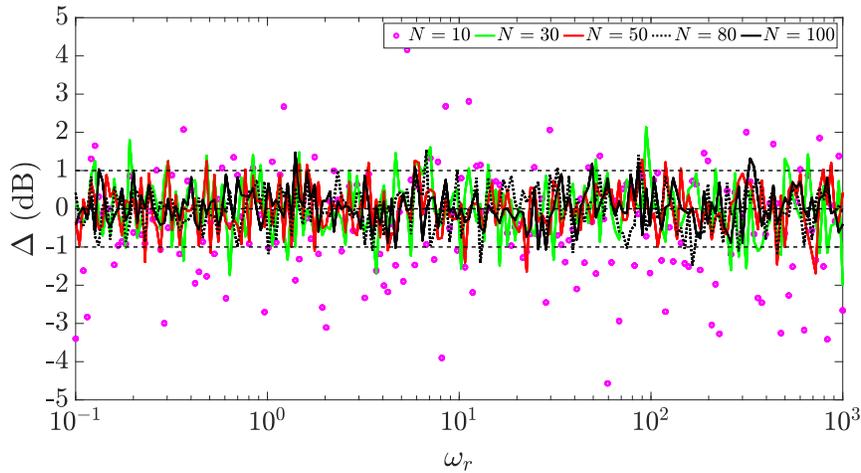


FIG. 3. (color online) Difference in the spectral level of the surface pressure jump for different numbers of realizations using the results obtained with 500 realizations as the reference solution, (dB, ref. $4 \times 10^{-10} \text{Pa}^2/\text{Hz}$).

286

287

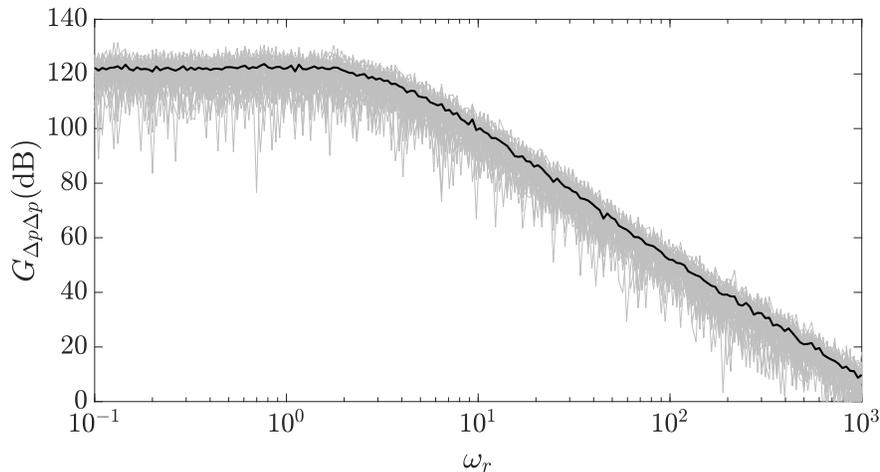


FIG. 4. ASD of the surface pressure jump predicted numerically using the UWPW technique for 50 realizations (gray lines), as well as predicted using the average of 50 realizations (black line), (dB, ref. $4 \times 10^{-10} \text{Pa}^2/\text{Hz}$).

288

289

290 C. Statistics of the synthesized surface pressure jump

291 As stated in Section III, the surface pressure jump can be obtained using the UWPW technique
 292 from summation of uncorrelated plane waves with a random phase uniformly distributed in $[0, 2\pi]$
 293 as given by Equation (19). If the amplitudes of the waves were independent of k_y , each plane wave
 294 would be an independent and identically distributed random variable. In such a case, according
 295 to the central limit theorem (CLT) as N approaches infinity the sum of these independent random
 296 variables tends toward a normal distribution. Moreover, since $E[e^{i(\varphi_i^n)}] = 0$ (see Equation (19)),
 297 the mean value of the plane wave contributions would be zero. However, in the present case,
 298 the wave amplitudes are a function of k_y and the CLT may not be rigorously valid. Figure 5
 299 presents the probability density function (PDF) of the real and imaginary parts of synthesized
 300 surface pressure jump using 500 realizations at $\omega_r = 1$. In spite of the fact that the wave amplitudes
 301 are dependent on k_y , it can be observed that the PDF of the real and imaginary parts of the pressure
 302 jump generated by the UWPW technique follows the normal distribution with a mean value of
 303 zero. Further, variance of the synthesized pressures equals the sum of the variances of the real and
 304 imaginary part of these pressures which corresponds to the targeted ASD of the pressure jump.
 305 Targeted ASD can be obtained from Equation (24) by considering $x = x'$ and $y = y'$. For the
 306 current case shown in Figure 5, variances of the real and imaginary parts of the pressures are
 307 respectively 22.52 Pa^2 and 22.35 Pa^2 . Therefore, variance of the complex synthesized pressures
 308 is 44.87 Pa^2 which corresponds to the targeted ASD of the pressure jump with a value of 45.42
 309 Pa^2/Hz . It should be noted that the estimated ASD of the pressure jump using 500 realizations is
 310 $45.53 \text{ Pa}^2/\text{Hz}$ which is very close to the targeted ASD.

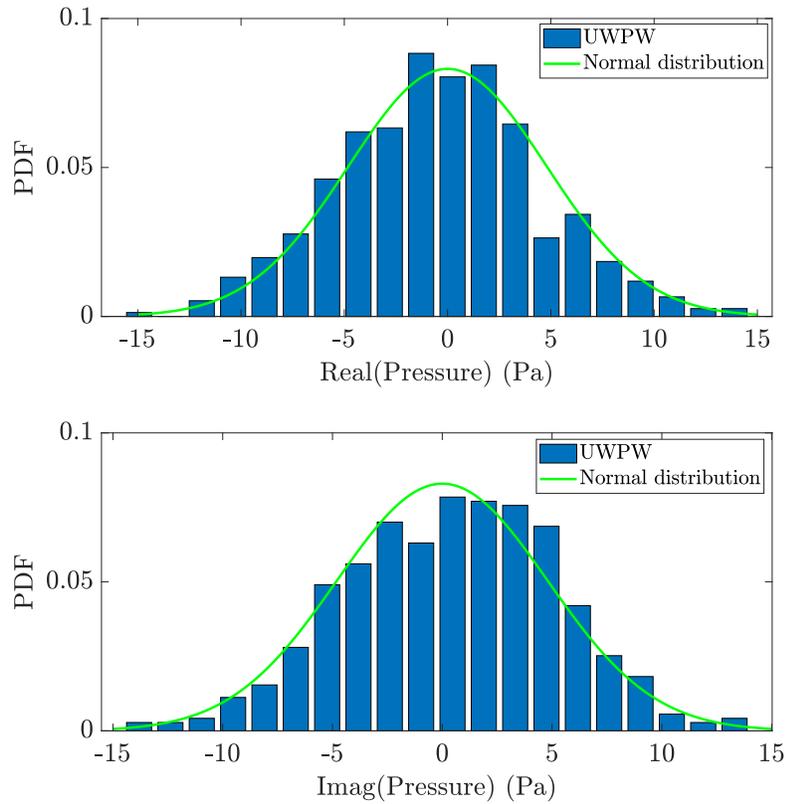


FIG. 5. (color online) Probability density function of the real and imaginary parts of synthesized surface pressure jump using 500 realizations at $\omega_r = 1$.

312 Figure 6 shows the averaged ASD distribution and its standard deviation based on 1000 calcu-
 313 lations for a given number of realization at $\omega_r = 1$. It is evident from Figure 6 that dispersion of
 314 the averaged ASD of the synthesized pressure decreases as the number of realization increases. A
 315 similar trend can be seen for the standard deviation where the higher number of realizations results
 316 in a lower standard deviation. Above 50 realizations, the standard deviation of the ASD level is
 317 lower than 0.6 dB. Assuming a normal distribution for ASD of synthesized pressures, we have
 318 95% confidence interval of ± 1.2 dB which indicates that for 95% of the samples, mean value lies
 319 within two standard deviations of the mean.

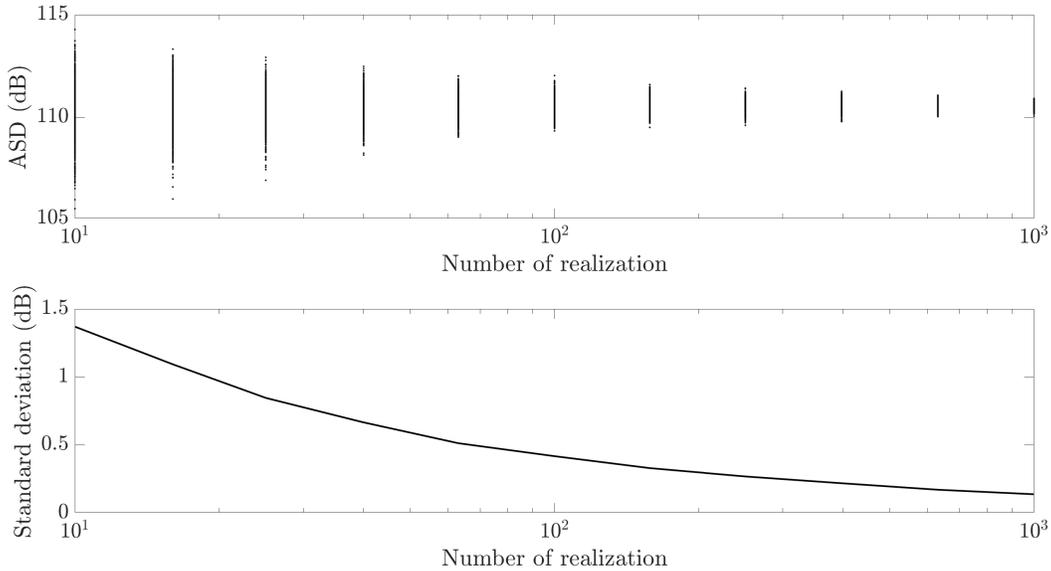


FIG. 6. Averaged ASD distribution (top) and its standard deviation (bottom) as a function of number of realization for 1000 samples (dB, ref. $4 \times 10^{-10} \text{Pa}^2/\text{Hz}$).

320 **D. Sub-critical and super-critical gusts contribution**

321 Gusts can be classified as sub- or super-critical, respectively corresponding to subsonic and
 322 supersonic phase speeds of their trace along the leading/trailing edge with respect to the in-
 323 cident mean flow (Amiet, 1975). For a given flow condition, the Graham's parameter of less
 324 than or greater than unity corresponds respectively to sub-critical or super-critical pressure gusts.
 325 Figure 7 highlights the contributions of sub-critical and super-critical gusts to the surface pres-
 326 sure jump. It can be observed that at low frequencies ($\omega_r \leq 4$) the super-critical gust have al-
 327 most no contribution to the surface pressure jump and it is sufficient to only consider the sub-
 328 critical gust contribution. This fact can also be seen in Figure 2 (c) where at low frequency
 329 ($\omega_r \leq 4$ or $K_x \leq 13 \text{ m}^{-1}$) contribution of super-critical gust is confined to the triangular re-
 330 gion $R_1 = \{(K_x, k_y) \mid 0 \leq K_x \leq 13, 0 \leq k_y \leq (MK_x/\beta)\}$ while this region accounts only for

331 a small portion of the total contribution at this low frequency range in the entire region of $R =$
 332 $\{(K_x, k_y) \mid 0 \leq K_x \leq 13, 0 \leq k_y \leq 10\}$ where most of the wavenumbers are located in the sub-
 333 critical region.

334 According to Figure 2 (c), contributions of both sub-critical and super-critical gusts are equally
 335 important in the frequency range of $4 < \omega_r < 100$. At $\omega_r \approx 30$ or $K_x \approx 100 \text{ m}^{-1}$ they have the
 336 same contributions to the surface pressure field. For $\omega_r \geq 100$ the super-critical gust contributions
 337 dominate and there is no need to include sub-critical gust. This is consistent with the results in
 338 Figure 2 (c) where above this reduced frequency (corresponding to $K_x \approx 328 \text{ m}^{-1}$) the wavenum-
 339 bers located above the dashed line in the region of $\Theta < 1$ have almost zero contribution to the
 340 predicted surface pressure jump.

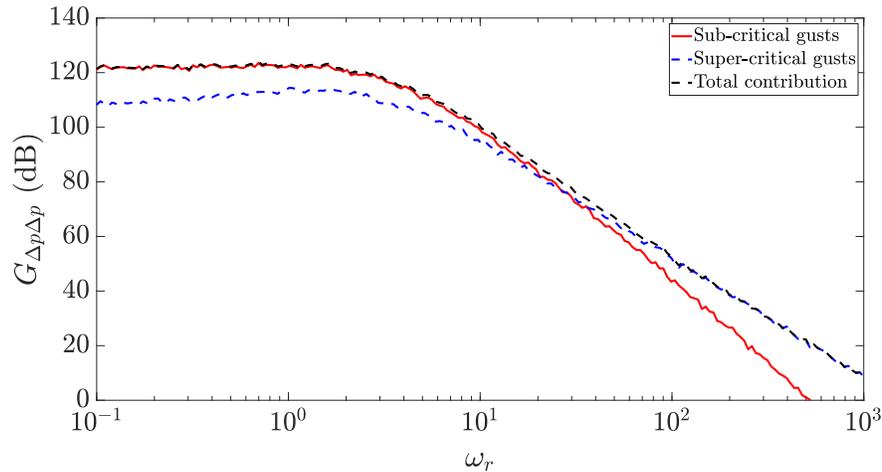


FIG. 7. (color online) Contributions of sub- and super-critical gusts to the surface pressure jump as a function of reduced frequency, (dB, ref. $4 \times 10^{-10} \text{ Pa}^2/\text{Hz}$).

341

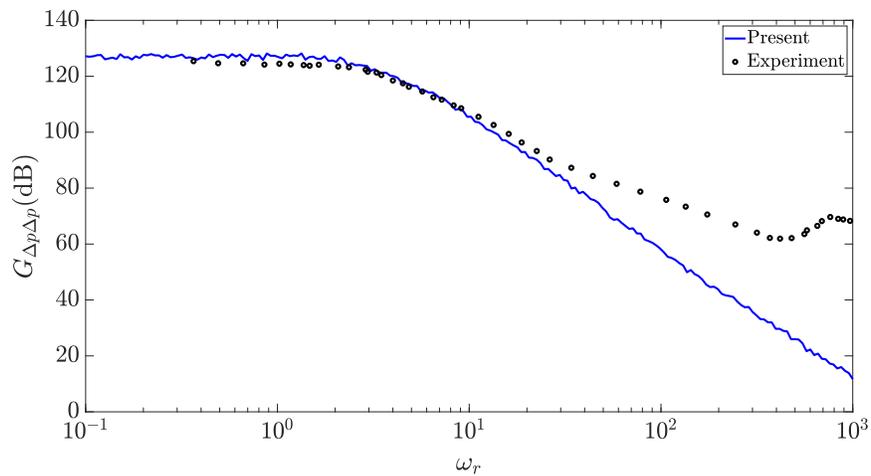
342

343 E. Verification and validation

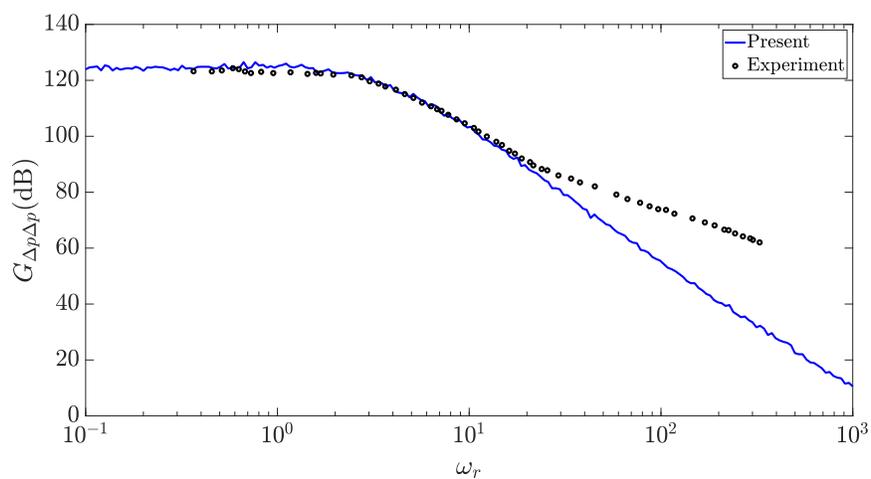
344 To validate the surface pressure jump produced by the proposed method, the predicted pressure
 345 difference spectra at chordwise locations, $x/b = 0.05$, $x/b = 0.08$ and $x/b = 0.12$ are compared
 346 with experimental data measured by [Mish and Devenport \(2006a\)](#) in Figure 8. A NACA0015
 347 with a chord of 0.610 m and span of 1.83 m at zero angle of attack has been considered with the
 348 turbulence parameters given in Table 1. $G_{\Delta p \Delta p}$ is the ASD of the surface pressure jump. It should
 349 be noted that in the work by [Mish and Devenport \(2006a\)](#), $G_{\Delta p \Delta p}$ was normalized by q^2 , where
 350 q is the free-stream dynamic pressure ($q = 1/2\rho U_\infty^2$). Due to the proximity of the measurement
 351 locations to the leading edge, the modified von Karman spectrum has been implemented. Figure 8
 352 shows that the present results are in excellent agreement with the measured shape and level of
 353 the spectra for $\omega_r \leq 20$. It can be observed that at low frequencies ($\omega_r \leq 2$) the spectrum level is
 354 almost constant and has its highest value over the entire frequency range. At these low frequencies,
 355 the sub-critical wavenumbers dominate the surface pressure response (as shown in Figure 2 (c)).
 356 Although sub-critical wavenumbers are often neglected in aeroacoustics, these results demonstrate
 357 that these wavenumbers must be considered when examining the force and vibroacoustic response
 358 of airfoils in turbulent flow.

359 The slope of the measured data decreases for $\omega_r > 20$. The divergence of the predicted results
 360 from the experimental data at high frequencies could be attributed to the pressure signatures of the
 361 large scale eddies convecting past the leading edge. This can be explained by the inner-outer scale
 362 interaction model developed by [Mathis et al. \(2009\)](#) that demonstrates large scale fluctuations in
 363 the outer region of the boundary layer have an amplitude and phase modulating effect on the inner
 364 scale fluctuations, wall shear stress and pressures. This modulation effect was also demonstrated

365 by [Dogan *et al.* \(2016\)](#) to occur for free stream turbulence convecting over turbulent boundary
 366 layers. Further, when the turbulent boundary layer develops over the airfoil surface, it induces
 367 pressure fluctuations which affect the high frequency response. As the boundary layer thickness
 368 increases by moving away from the leading edge, this affect is more profound at larger chordwise
 369 locations as shown in [Figure 8](#). The spike in the measured data at high frequency was reported to
 370 be related to resonance within the microphone mounting hole ([Mish and Devenport, 2006a](#)).



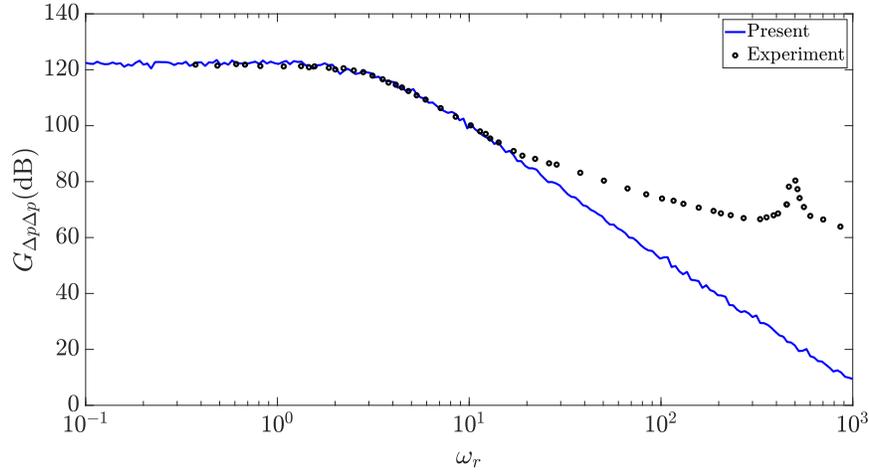
(a)



(b)

371

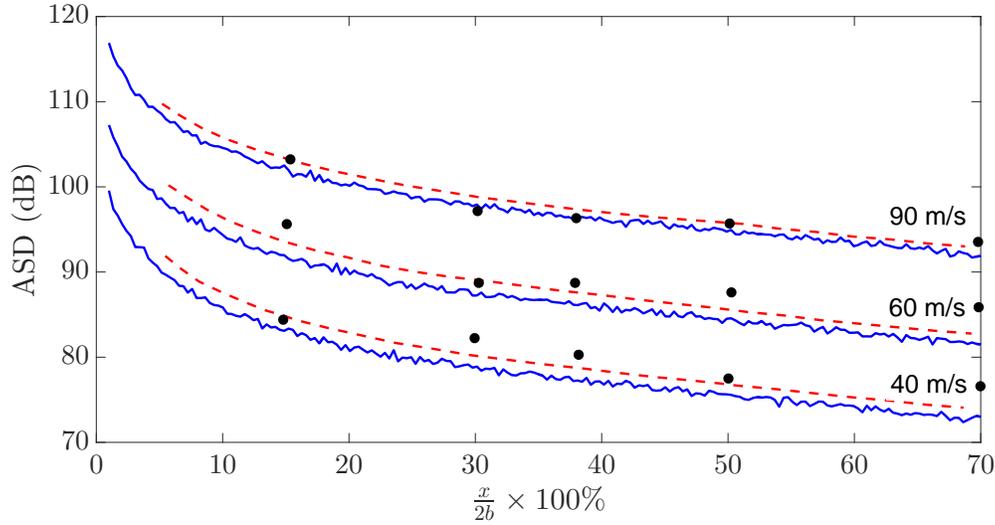
372



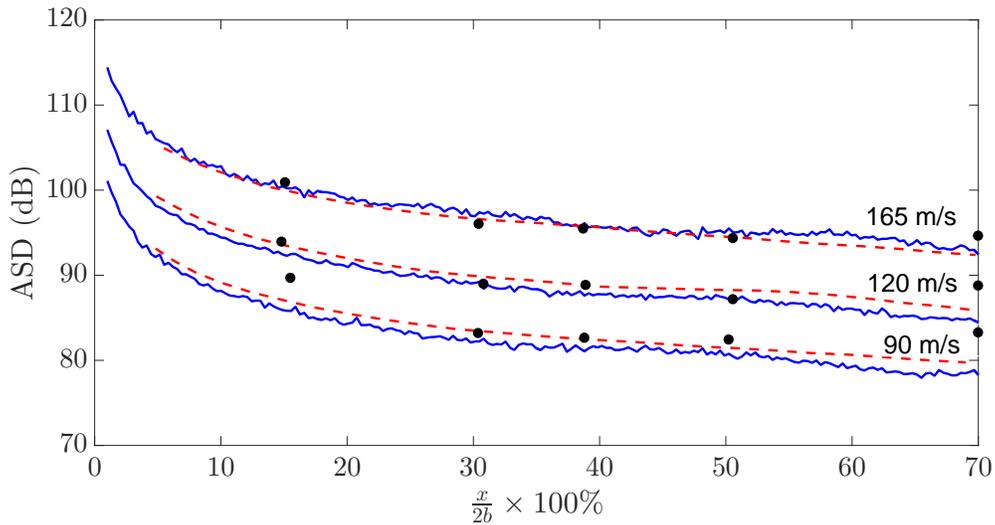
(c)

FIG. 8. (color online) Comparison between measured and predicted pressure difference spectral level for NACA0015 at (a) $x/b = 0.05$, (b) $x/b = 0.08$ and (c) $x/b = 0.12$ (dB, ref. $4 \times 10^{-10} \text{Pa}^2/\text{Hz}$).

373 To further validate the proposed method, the NACA0012 airfoil with a chord of 0.230 m and
 374 span of 0.530 m, which was tested by [Paterson and Amiet \(1976\)](#), has been considered. Figure 9
 375 shows a comparison between numerical results using the UWPW technique and experimental
 376 data for the airfoil chordwise unsteady surface pressure distribution at zero angle of attack. The
 377 prediction has been performed for different flow speeds at 400 Hz and 2000 Hz. The von Karman
 378 spectrum along with the turbulence parameters given in Table 1 were implemented. As can be seen,
 379 numerical results are in very good agreement with experimental data. The theoretical predictions
 380 using Amiet's theory is also included ([Paterson and Amiet, 1976](#)). The results indicate a strong
 381 increase in unsteady pressure near the leading edge. The surface pressure decreases moving away
 382 from the leading edge. It can also be observed that the spectrum level increases with an increase
 383 in flow speed. Figures 8 and 9 show that the proposed method can accurately predict the surface
 384 pressure at different location on the airfoil surface as well as at different frequencies.



(a) 400 Hz



(b) 2000 Hz

FIG. 9. (color online) Comparison between measured and predicted auto spectrum density of surface pressure for a NACA0012 at (a) 400 Hz and (b) 2000 Hz. Present results (solid line), analytical results (dashed line) and experiment (Paterson and Amiet, 1976) (solid circle), (dB, ref. $4 \times 10^{-10} \text{Pa}^2/\text{Hz}$).

386 F. Unsteady force calculation

387 The unsteady force caused by turbulence ingestion has been studied by several investigators in
 388 the past (Anderson *et al.*, 2015; Jiang *et al.*, 1994). In this section unsteady force acting on the

389 NACA0015 airfoil (Case 1) due to the incoming turbulence is computed. A flat plate representing
390 the airfoil was meshed with more grid points close to the leading edge to capture the variation of
391 the surface pressure jump at the locations close to the leading edge. To estimate the unsteady force,
392 realizations of surface pressure jump were initially obtained at each node on the surface. Figure 10
393 shows color maps of a realization of the surface pressure jump at frequencies of 10 Hz, 100 Hz,
394 300 Hz and 500 Hz. As expected, the highest values of the surface pressure occur at the leading
395 edge and the pressure level monotonically decreases for the locations further away from leading
396 edge, this was also observed in Figure 9. Moreover, it can be seen from these color maps that the
397 pressure level reduces as frequency increases. This is also consistent with the results in Figure 8.
398 Once enough realizations of the surface pressure jump are generated, Equation 28 can be used to
399 calculate realizations of unsteady force by integrating the corresponding realization of pressure
400 jump over the surface area. Finally, the unsteady force was computed from ensemble average
401 over different realizations. The predicted force using the UWPW technique is compared with
402 an analytical prediction described in the Appendix. Figure 11 shows that the present prediction is
403 very close to the analytical results. Different realizations of the force are also included in the figure
404 (in gray lines) which shows how a single realization of the response differs significantly from the
405 final response obtained by ensemble average of all the realizations as shown by solid blue line.

Simulation of Airfoil Surface Pressure

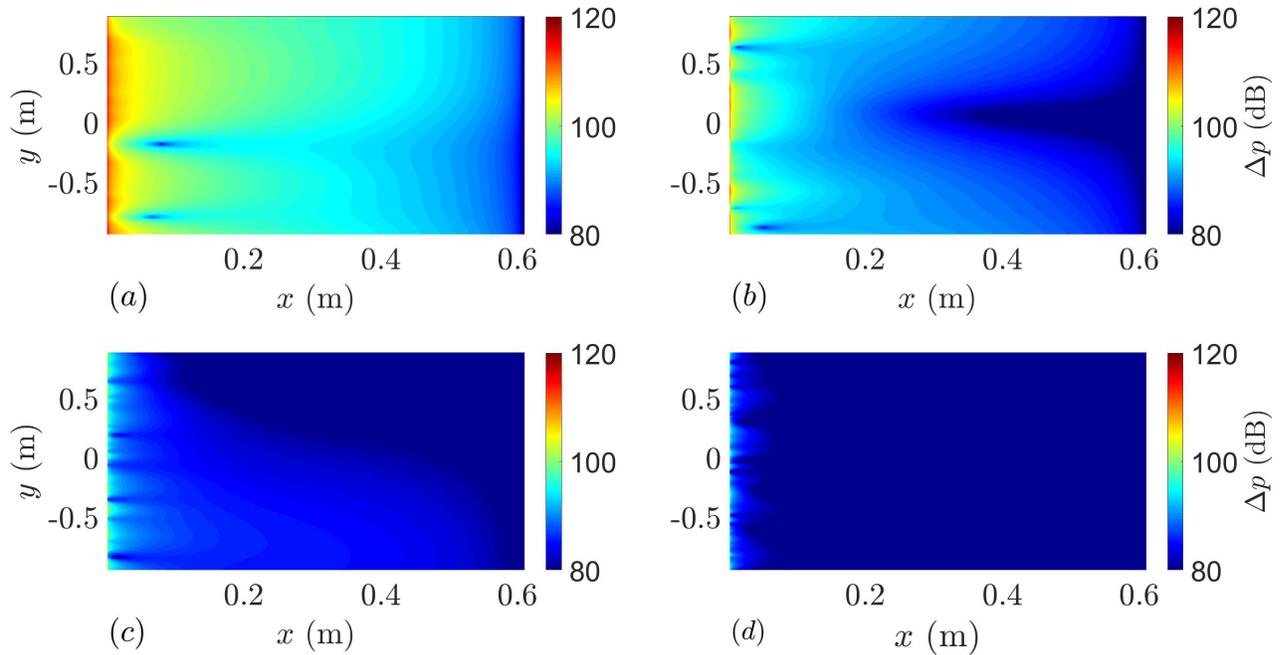


FIG. 10. (color online) A realization of the surface pressure jump on a flat plate representing the NACA0015 airfoil (flow direction is from left to right) at four distinct frequencies of (a) 10 Hz, (b) 100 Hz, (c) 300 Hz and (d) 500 Hz, (dB ref. 2×10^{-5} Pa).

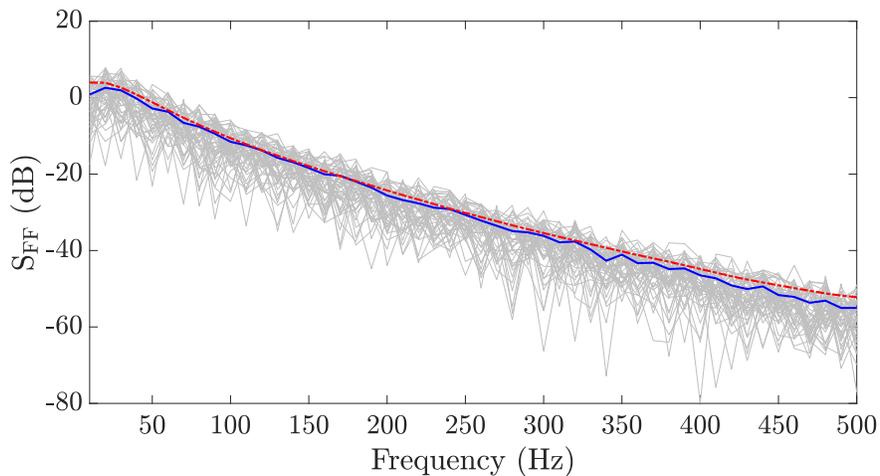


FIG. 11. (color online) ASD of unsteady force predicted using the UWPW technique for 50 realizations (gray lines), using the average of 50 realizations (blue solid line) as well as predicted analytically (red dashed line) for the NACA0015, (dB, ref. 1 N/Hz).

408 **V. CONCLUSIONS**

409 In this work, we proposed a numerical technique to simulate the surface pressure induced by
410 incoming turbulence. The surface pressure jump was expressed as a set of uncorrelated wall pres-
411 sure plane waves with their amplitudes determined by the incoming upwash velocity spectrum
412 and the airfoil aeroacoustic transfer function. To achieve a good estimation of the surface pressure
413 jump, criteria for cut-off wavenumber and wavenumber resolution were established. Contributions
414 of sub- and super-critical gust to the surface pressure were examined. It was found that the sub-
415 critical gust contribution is dominant at low frequency while the super-critical gust contribution
416 dominates at high frequencies. The surface pressure jump predicted from an ensemble average
417 of the different realizations was compared with numerical and experimental results, showing very
418 good agreement. To demonstrate an application of the proposed technique, the unsteady force due
419 to incoming turbulence excitation on an airfoil was calculated. The numerical prediction agreed
420 well with analytical results. One of the main advantages of the proposed technique is that the sur-
421 face pressures simulated by the UWPW technique are deterministic and they can be applied as load
422 to an element-based vibroacoustic solver to obtain structural and acoustic responses of airfoils. In
423 the future study this advantage of the proposed method will be exploited to predict and analyze
424 vibroacoustic response of a structure due to stochastic excitation (incident turbulence) using de-
425 terministic element-based methods such as the finite element and boundary element methods.

426 **ACKNOWLEDGMENTS**

427 The first author gratefully acknowledges financial support from the Australian Government
 428 through the Australian Research Council's Discovery Early Career Project funding scheme
 429 (project DE190101412).

430 **APPENDIX**

431 The analytical model used to estimate the force produced on an airfoil in turbulent flow is
 432 presented here. The analytical model is based on the aeroacoustic scattering model originally
 433 presented by Amiet (Amiet, 1975) and later extended by Roger and Moreau to include trailing
 434 edge backscatter (Roger and Moreau, 2010). The turbulence ingestion noise model derived by
 435 Roger and Moreau is given by:

$$\begin{aligned}
 S_{pp}(\mathbf{x}, \omega) = & \left(\frac{k_a \rho_0 b x_3}{S_0^2} \right)^2 U_0 \int_{-\infty}^{\infty} \Phi_{ww} \left(\frac{\omega}{U_c}, k_y \right) \frac{\sin^2 \left(d \left(\frac{k_a x_2}{S_0} - k_y \right) \right)}{\left(\frac{k_a x_2}{S_0} - k_y \right)^2} \\
 & \times \left| \mathcal{L}_{\Pi} \left(x_1, \frac{\omega}{U_0}, k_y \right) \right|^2 dk_y \quad (30)
 \end{aligned}$$

436 where $S_{pp}(\mathbf{x}, \omega)$ is the power spectral density of the far-field sound at angular frequency ω and
 437 field point \mathbf{x} . k_a is the acoustic wavenumber and ρ_0 is the speed of sound and density of the
 438 fluid at rest. x_i is the i^{th} component of the far-field position vector \mathbf{x} . $S_0^2 = x_1^2 + \beta^2 (x_2^2 + x_3^2)$
 439 is the distance from the coordinate system origin at the leading edge to the far-field location \mathbf{x} ,
 440 with $\beta^2 = 1 - M^2$, M being the Mach number. The airfoil span and chord length are given
 441 respectively by $2d$ and $2b$. $\Phi_{ww} \left(\frac{\omega}{U_c}, k_y \right)$ is the upwash spectrum and is a function of convective
 442 wavenumber $k_c = \frac{\omega}{U_c}$, where U_c is the convection velocity of the upwash disturbance, and the

443 spanwise wavenumber k_y . \mathcal{L}_{TI} is the aeroacoustic transfer function. The present analytical work
 444 uses the von Karman spectrum modified to account for rapid distortion of turbulence structures by
 445 the leading edge to approximate the upwash spectrum.

446 The airfoil can be discretised into a number of strips, N_s , with the total far-field sound over all
 447 strips given by:

$$S_{pp}(\mathbf{x}, \omega) = \sum_{s=1}^{N_s} \left[\left(\frac{k_a \rho_0 b x_{s3}}{S_{s0}^2} \right)^2 U_{s0} \int_{-\infty}^{\infty} \Phi_{ww} \left(\frac{\omega}{U_{s0}}, k_y \right) \frac{\sin^2 \left(d_s \left(\frac{k_a x_{s2}}{S_{s0}} - k_y \right) \right)}{\left(\frac{k_a x_{s2}}{S_{s0}} - k_y \right)^2} \times \left| \mathcal{L}_{\text{TI}} \left(x_{s1}, \frac{\omega}{U_{s0}}, k_y \right) \right|^2 \mathrm{d} k_2 \right] \quad (31)$$

448 where the subscript s denotes the strip-wise value, with d_s representing the half strip span. Equa-
 449 tion (31) is referred to as the direct strip-wise approach and was found by Christophe et al.
 450 (Christophe *et al.*, 2009) to diverge from the correct result as strip size decreases. Christophe
 451 et al. (Christophe *et al.*, 2009) proposed an alternate strategy for strip-wise turbulence ingestion
 452 noise modelling that they called the 'inverse' strip-wise method. The inverse strip-wise technique
 453 calculates the sound from each strip by representing the strip-wise half span d_s as the difference
 454 of two large span values $d_s = d_L - (d_L - d_s)$, $d_L \gg d_s$ to give:

$$S_{pp}(\mathbf{x}, \omega) = \sum_{s=1}^{N_s} \left[\left(\frac{k_a \rho_0 b x_{s3}}{S_{s0}^2} \right)^2 U_{s0} \int_{-\infty}^{\infty} \Phi_{ww} \left(\frac{\omega}{U_{s0}}, k_y \right) \left(\frac{\sin^2 \left(d_L \left(\frac{k_a x_{s2}}{S_{s0}} - k_y \right) \right)}{\left(\frac{k_a x_{s2}}{S_{s0}} - k_y \right)^2} - \frac{\sin^2 \left((d_L - d_s) \left(\frac{k_a x_{s2}}{S_{s0}} - k_y \right) \right)}{\left(\frac{k_a x_{s2}}{S_{s0}} - k_y \right)^2} \right) \times \left| \mathcal{L}_{\text{TI}} \left(x_{s1}, \frac{\omega}{U_{s0}}, k_y \right) \right|^2 \mathrm{d} k_y \right] \quad (32)$$

455 In the current work, d_L is set equal to the total half span d . The inverse strip-wise technique
 456 is often combined with the large aspect ratio approximation to equation (31). For infinite span

457 airfoils, only gusts which are parallel to the leading edge generates pressure waves that radiate to
 458 the far-field as sound. However, the current application is to estimate the total force on the airfoil
 459 and hence evanescent components may be significant. For this reason, equation (32) retains the
 460 integral over spanwise wavenumbers.

461 As the application of this technique is focused on force prediction, the radiation of pressure
 462 waves to the far-field is not required. Propagation of pressure waves from the leading edge to the
 463 far-field is included in equation (32) via a dipole radiation function and by propagation terms in
 464 the transfer function \mathcal{L}_{TI} . The first step in removing the far-field radiation terms from equation
 465 (32) is to extract the dipole radiation function. The far-field sound produced by a unit harmonic
 466 point force oriented in the wall-normal, x_{s3} direction, can be expressed as:

$$S_{pp}(\mathbf{x}, \omega) = \left(\frac{k_a x_{s3}}{4\pi S_0^2} \right)^2 \quad (33)$$

467 Removing this radiation component and aeroacoustic transfer function propagation terms from
 468 equation(32) gives the following expression for the total force on the airfoil due to interaction with
 469 the turbulence stream:

$$S_{ff}(\mathbf{x}, \omega) = \sum_{s=1}^{N_s} \left[(4\pi\rho_0 b)^2 U_{s0} \int_{-\infty}^{\infty} \Phi_{ww} \left(\frac{\omega}{U_{s0}}, k_y \right) \right. \\
\left. \left(\frac{\sin^2 \left(d \left(\frac{k_a x_{s2}}{S_{s0}} - k_y \right) \right)}{\left(\frac{k_a x_{s2}}{S_{s0}} - k_y \right)^2} - \frac{\sin^2 \left((d - d_s) \left(\frac{k_a x_{s2}}{S_{s0}} - k_y \right) \right)}{\left(\frac{k_a x_{s2}}{S_{s0}} - k_y \right)^2} \right) \right. \\
\left. \times \left| \mathcal{L}_{\text{TI}} \left(\frac{\omega}{U_{s0}}, k_y \right) \right|^2 dk_y \right] \quad (34)$$

470 The transfer function $\mathcal{L}_{\text{TI}} = \mathcal{L}_1 + \mathcal{L}_2$, where \mathcal{L}_1 represents the principal scattering from the airfoil
 471 leading edge and \mathcal{L}_2 accounts for back-scattering from the airfoil trailing edge. The transfer func-

472 tions for supercritical wavenumbers derived by Roger and Moreau, with the far-field propagation
 473 terms removed, are given by:

$$\mathcal{L}_1 = -\frac{1}{\pi} \sqrt{\frac{2}{(K_x^* + \beta^2 \bar{\kappa}) \bar{\kappa}}} e^{-i\Theta_2} \mathfrak{F}[2\bar{\kappa}] \quad (35)$$

$$\mathcal{L}_2 = \frac{e^{-i\Theta_2}}{\pi \bar{\kappa} \sqrt{2\pi (K_x^* + \beta^2 \bar{\kappa})}} \times \left(i(1 - e^{2i\bar{\kappa}}) - (1 + i) \left(\mathfrak{F}[4\bar{\kappa}] - e^{2i\bar{\kappa}} \sqrt{2} \mathfrak{F}[2(\bar{\kappa})] \right) \right) \quad (36)$$

474 where $K_x^* = K_x b$, $\bar{\kappa} = \sqrt{\bar{\mu}^2 - \frac{k_y^{*2}}{\beta^2}}$, $\bar{\mu} = \frac{k_a b}{\beta^2} = \frac{K_x^* M_0}{\beta^2}$, $\Theta_2 = \bar{\mu} M_0 - \frac{\pi}{4}$ and $k_y^* = k_y b$. The transfer
 475 functions for subcritical wavenumbers are given by:

$$\mathcal{L}_1 = -\frac{1}{\pi} \sqrt{\frac{2}{(K_x^* + i\beta^2 \bar{\kappa}') i \bar{\kappa}'}} e^{-i\Theta_2} \mathfrak{F}[2i\bar{\kappa}'] \quad (37)$$

$$\mathcal{L}_2 = \frac{e^{-i\Theta_2}}{\pi \bar{\kappa}' \sqrt{2\pi (K_x^* + i\beta^2 \bar{\kappa}')}} \times \left(1 - e^{-2\bar{\kappa}'} - \operatorname{erf}(\sqrt{4\bar{\kappa}'}') + 2e^{-2\bar{\kappa}'} \sqrt{-i} \mathfrak{F}[2(i\bar{\kappa}')] \right) \quad (38)$$

476 where $\bar{\kappa}' = \sqrt{\frac{k_y^{*2}}{\beta^2} - \bar{\mu}^2}$.

477 REFERENCES

478

479 Abramowitz, M., and Stegun, I. A. (1948). *Handbook of mathematical functions with formulas,*
 480 *graphs, and mathematical tables*, **55** (US Government printing office).

481 Amiet, R. . (1976a). "Noise due to turbulent flow past a trailing edge," J. Sound Vib. **47**(3), 387–
 482 393.

- 483 Amiet, R. (1975). “Acoustic radiation from an airfoil in a turbulent stream,” *J. Sound Vib.* **41**(4),
484 407–420.
- 485 Amiet, R. (1978). “Effect of the incident surface pressure field on noise due to turbulent flow past
486 a trailing edge,” *J. Sound Vib.* **57**(2), 305–306.
- 487 Amiet, R. K. (1976b). “High frequency thin-airfoil theory for subsonic flow,” *AIAA J.* **14**(8),
488 1076–1082.
- 489 Anderson, J. M., Catlett, M. R., and Stewart, D. O. (2015). “Modeling rotor unsteady forces and
490 sound due to homogeneous turbulence ingestion,” *AIAA J.* **53**(1), 81–92.
- 491 Ayton, L. J., and Chaitanya, P. (2017). “Analytical and experimental investigation into the effects
492 of leading-edge radius on gust–aerofoil interaction noise,” *J. Fluid Mech.* **829**, 780–808.
- 493 Ayton, L. J., and Peake, N. (2016). “Interaction of turbulence with the leading-edge stagnation
494 point of a thin aerofoil,” *J. Fluid Mech.* **798**, 436–456.
- 495 Bechara, W., Bailly, C., Lafon, P., and Candel, S. M. (1994). “Stochastic approach to noise mod-
496 eling for free turbulent flows,” *AIAA J.* **32**(3), 455–463.
- 497 Brooks, T. (1981). “Trailing edge noise prediction using Amiet’s method,” *J. Sound Vib.* **77**(3),
498 437–439.
- 499 Christophe, J. (2011). “Application of hybrid methods to high frequency aeroacoustics,” Université
500 Libre de Bruxelles .
- 501 Christophe, J., Anthoine, J., and Moreau, S. (2009). “Amiet’s theory in spanwise-varying flow
502 conditions,” *AIAA J.* **47**, 788–790.
- 503 de Santana, L. D., Schram, C., and Desmet, W. (2015). “Airfoil noise prediction from 2D3C PIV
504 data,” 21st AIAA/CEAS Aeroacoustics Conference, Dallas, Texas, 22-26 June **No. 2203**.

- 505 Devenport, W. J., Staubs, J. K., and Glegg, S. A. (2010). "Sound radiation from real airfoils in
506 turbulence," *J. Sound Vib.* **329**(17), 3470–3483.
- 507 Dogan, E., Hanson, R., and Ganapathisubramani, B. (2016). "Interactions of large-scale free-
508 stream turbulence with turbulent boundary layers," *J. Fluid Mech.* 1–29.
- 509 Filotas, L. (1969). "Response of an infinite wing to an oblique sinusoidal gust: a generalization of
510 sears' problem," NASA Special Publication **207**(No. 231).
- 511 Glegg, S. A., and Devenport, W. J. (2010). "Panel methods for airfoils in turbulent flow," *J. Sound
512 Vib.* **329**(18), 3709–3720.
- 513 Hunt, J. (1973). "A theory of turbulent flow round two-dimensional bluff bodies," *J. Fluid Mech.*
514 **61**(4), 625–706.
- 515 Hutcheson, F. V., Brooks, T. F., and Stead, D. J. (2012). "Measurement of the noise resulting from
516 the interaction of turbulence with a lifting surface," *Int. J. Aeroacoust.* **11**(5-6), 675–700.
- 517 Jiang, C., Chang, M., and Liu, Y. (1994). "The effect of turbulence ingestion on propeller broad-
518 band forces," Naval Hydrodynamics, 19th Symposium; 23-28 August 1992; Seoul, Korea. .
- 519 Karimi, M., Croaker, P., Maxit, L., Robin, O., Skvortsov, A., Marburg, S., and Kessissoglou, N.
520 (2020). "A hybrid numerical approach to predict the vibrational responses of panels excited by
521 a turbulent boundary layer," *J. Fluids Struct.* **92**, 102814.
- 522 Karimi, M., Croaker, P., Skvortsov, A., Moreau, D., and Kessissoglou, N. (2019a). "Numerical
523 prediction of turbulent boundary layer noise from a sharp-edged flat plate," *Int. J. Numer. Meth.*
524 *Fl.* **90**, 522–543.
- 525 Karimi, M., Maxit, L., Croaker, P., Robin, O., Skvortsov, A., Marburg, S., Atalla, N., and Kessis-
526 soglou, N. (2019b). "Analytical and numerical prediction of acoustic radiation from a panel

- 527 under turbulent boundary layer excitation,” *J. Sound Vib.* .
- 528 Kraichnan, R. H. (1970). “Diffusion by a random velocity field,” *Phys. Fluids* **13**(1), 22–31.
- 529 Lysak, P. D. (2001). “A model for the broadband unsteady forces in a marine propulsor due to
530 inflow turbulence,” Ph.D. thesis, Pennsylvania State University.
- 531 Mathis, R., Hutchins, N., and Marusic, I. (2009). “Large-scale amplitude modulation of the small-
532 scale structures in turbulent boundary layers,” *J. Fluid Mech.* **628**, 311–337.
- 533 Maxit, L. (2016). “Simulation of the pressure field beneath a turbulent boundary layer using real-
534 izations of uncorrelated wall plane waves,” *J. Acoust. Soc. Am.* **140**(2), 1268–1285.
- 535 Mish, P. F. (2001). “Mean loading and turbulence scale effects on the surface pressure fluctuations
536 occurring on a NACA 0015 airfoil immersed in grid generated turbulence,” Phd thesis, Virginia
537 Tech.
- 538 Mish, P. F., and Devenport, W. J. (2003). “An experimental investigation of unsteady surface
539 pressure on an airfoil in turbulence,” Final report to NASA Langley under grant NAG-1-2272 .
- 540 Mish, P. F., and Devenport, W. J. (2006a). “An experimental investigation of unsteady surface
541 pressure on an airfoil in turbulence-part 1: Effects of mean loading,” *J. Sound Vib.* **296**(3), 417–
542 446.
- 543 Mish, P. F., and Devenport, W. J. (2006b). “An experimental investigation of unsteady surface
544 pressure on an airfoil in turbulence-part 2: Sources and prediction of mean loading effects,” *J.*
545 *Sound Vib.* **296**(3), 447–460.
- 546 Moreau, S., Christopher, J., and Roger, M. (2008). “Les of the trailing-edge flow and noise of a
547 naca0012 airfoil near stall,” in *Proceedings of the Summer Program*, pp. 317–329.

- 548 Moreau, S., and Roger, M. (2007). “Competing broadband noise mechanisms in low-speed axial
549 fans,” *AIAA Journal* **45**(1), 48–57.
- 550 Paterson, R., and Amiet, R. (1976). “Acoustic radiation and surface pressure characteristics of an
551 airfoil due to incident turbulence,” 3rd Aeroacoustics Conference, Palo Alto, California, 20-23
552 July No. 76-571.
- 553 Roger, M., and Moreau, S. (2010). “Extensions and limitations of analytical airfoil broadband
554 noise models,” *Int. J. Aeroacoust.* **9**(3), 273–305.
- 555 Santana, L. D., Christophe, J., Schram, C., and Desmet, W. (2016). “A rapid distortion theory
556 modified turbulence spectra for semi-analytical airfoil noise prediction,” *J. Sound Vib.* **383**, 349–
557 363.