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A causal mixture model decomposition for root cause identification

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Abstract: Multivariate statistical process monitoring methods usually assume the Gaussianity of data. However, in practice, data are multi-modal. Therefore, it is not always reasonable and enough to use methods that only deal with the data overall covariance matrix. As the latter may wrap less information compared to the data distribution. Also, such prior assumption is prejudicial to the estimation of the data's structure and the causal direction of variables. An interesting challenge would then be the development of relevant metrics to monitor variables and address their causal nature in the context of the non-Gaussianity of the data. Therefore, adequate parametric tests are required to ensure an acceptable and adjustable compromise between false positives and false negatives. In this paper, a new statistical approach is introduced to root cause and fault path propagation analysis. The obtained results demonstrate that the proposed method performs better than the existing methods.

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Keywords: statistical process monitoring, root cause identification, fault path propagation, Gaussian mixture models

1. INTRODUCTION

The statistical process monitoring notion relies on the statistical theory that data variations are similar unless a fault happens in the process. A fault is defined as an abnormal process condition that can be characterized by its data variations Atoui and Cohen (2021). It implies that each fault can have its data variations features. These features may differentiate it from other process's operating conditions.

Nowadays, a large amount of data are collected from modern complex industrial processes. These data are multivariate and rich in information. Multivariate Statistical Process Control (MSPC) methods can be used to reduce the data dimension or extract useful information and latent knowledge (Yin et al., 2014; Woodall, 2017; Cohen and Atoui, 2020) to guarantee high operating efficiency and stable product quality. They are usually supported by quadratic statistics. These metrics are compared to their statistical limits to decide whether the system is under Normal Operating Conditions (NoC). These methods have been extensively developed and extended, such as Principal Component Analysis (PCA), Partial Least Squares (PLS), Canonical correlation analysis (CCA), subspace methods (Xu et al., 2020; Atoui and Cohen, 2020; Chen et al., 2017; Woodall, 2017; Ding, 2014; Peng et al., 2015; Atoui et al., 2014; Yin et al., 2014).

Another challenging task when a fault is detected aims to identify the root cause of the present fault. This task is in continuous development and requires further investigation and enhancement. Still, several attempts have been

proposed to tackle it. Among the identification methods, MSPC approaches like The Mason, Young and Tracy (MYT) approach, multivariate contribution plots, reconstruction analysis, nonnegative garrote-based were proposed in Wang et al. (2020); Yan et al. (2018, 2017); Kuang et al. (2015); Liu et al. (2014); He et al. (2012); Yue and Qin (2001); Westerhuis et al. (2000); Mason et al. (1995). MYT decomposition, unlike most of these approaches, holds solid statistical properties and foundations. The required sizable number of decompositions makes it less attractive in practice.

The uncertainty of the measured data quantity, data acquisition accuracy, and uncertainty of the mapping relationship between variables and faults may strain the capacity of correctly make decisions (Wang et al., 2018). Furthermore, most of the proposed identification approaches rely upon the assumption that the normal operating data follow a multivariate Gaussian distribution. However, in practice, such prior requirement may not be possible as complex systems works at different operating steady modes. Therefore, the gathered data likely follow a non-Gaussian distribution. A few trials have been proposed in the literature to extend traditional statistical process monitoring approaches to multi modes process root cause identification.

Density-based Gaussian mixture model (GMM) method, has shown strong ability to monitor systems with multiple operating modes. It has been applied to various continuous or batch processes Ma et al. (2018). It decomposes data into K Gaussian components, in which each Gaussian component corresponds to an individual operating mode. Each mode, can be then monitored individually or associated to

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the other modes through a Bayesian inference Yu (2013). In this work, a new identification causal mixture model-based method is proposed. The new proposed metric outperforms the state of the arts metrics proposed in the same context.

The rest of the paper is organized as follows. Sections 2 and 3 provide the needed theoretical basis. Section 4 presents the related work and introduces the new approach for root cause identification. In Section 5, the performance of the proposed method is compared with the existing methods on the Tennessee Eastman process (TEP), a well-known Benchmark. Finally, the conclusions are summarized in Section 6.

2. PRELIMINARIES

2.1 Causal models

A Directed Acyclic Graph (DAG) \mathbf{G} is set of random variables $\mathbf{X} = \mathbf{X}_1, \dots, \mathbf{X}_m$, nodes, connected by arcs Atoui et al. (2019); Lou et al. (2020). These arcs describe the dependencies between variables. These dependencies are governed by the Markov conditions; variables are independent of their non-descendants given their parents. The conditional independence relationships of DAG allow, by applying the chain rule, the decomposition of the joint distribution of \mathbf{X} as a set of independent conditional probabilities. The relationship between node A , direct ascendant, in a DAG, to B can be interpreted as A "causes" B . These dependencies between variables represent a causal model. They can be set up based on experts or learned from data. These causal relationships between variables can be summarized in a matrix, named adjacency matrix, W .

2.2 Causal direction of variables

Consider a Gaussian node \mathbf{X}_i with Gaussian parents \mathbf{X}_{ρ_i} , with ρ_i is the set of variables influencing on the variable \mathbf{X}_i . The parametric form of the conditional probability distribution of the node \mathbf{X}_i given its parents \mathbf{X}_{ρ_i} follows also a Gaussian distribution with mean $\mu_{\mathbf{X}_i \cdot \mathbf{X}_{\rho_i}}$ and variance $\sigma_{\mathbf{X}_i \cdot \mathbf{X}_{\rho_i}}$

$$p(\mathbf{X}_i \cdot \mathbf{X}_{\rho_i}) = \mathcal{N}(\mu_{\mathbf{X}_i \cdot \mathbf{X}_{\rho_i}}, \sigma_{\mathbf{X}_i \cdot \mathbf{X}_{\rho_i}}) \quad (1)$$

The full joint distribution of a DAG composed only of Gaussian nodes follows then a multivariate Gaussian model Atoui et al. (2015), $p(\mathbf{X}) = \prod_{i=1}^m p(\mathbf{X}_i \cdot \mathbf{X}_{\rho_i})$. In Gaussian models, the covariance matrix, in instance, is enough to encompasses all the conditional correlations between variables. Learning the causal model inherent to a set of a multivariate Gaussian process is easily done using a series of statistical significance tests of conditional independence (e.g. the Fisher's z-transform for the estimation of partial correlations). More details about the structure learning algorithms for Gaussian models can be found in Barber (2012); Peters et al. (2017).

In practice systems are usually non-Gaussians/ multimodals which makes the Gaussian assumption a bit too general and sometimes not efficient. Therefore, assuming the variables are non-Gaussian can be actually more realistic and useful. Also, it is emphasis in Shimizu and

Bollen (2014) that algorithms used under the Gaussian assumption and based only on second-order statistics are generally not able to discern the full causal structure in most cases. Thus, to discover the direction of causality it is more compelling to assume non-Gaussianity of data.

To estimate a causal structure from continuous data, non Gaussian acyclic models, can be used when data are not following Gaussian distributions. Several computationally efficient algorithms can be found in Guo et al. (2020); Peters et al. (2017); Shimizu (2014). Causality analysis techniques as Granger causality test Cheng et al. (2019) and Transfer entropy Luo et al. (2020) can be used as well. The choice of the adequate structure is out of the scope of this paper.

3. QUADRATIC STATISTIC DECOMPOSITION

Quadratic statistics are widely used to monitor quality characteristics. In multivariate processes, the generalized likelihood-ratio or quadratic statistics, Δ , can be used to monitor changes in the behavior of the system's variables Chen et al. (2017).

Let the new multivariate observation X of \mathbf{X} . Its quadratic statistic, Δ_X , $\Delta_X = \|X - \mu\|_{\Sigma}$, compared to its limit decide about the system's state: normal or faulty operating conditions. μ and Σ are the parameters of the distribution of \mathbf{X} . The distribution of Quadratic statistics when multiplied by the constant $\frac{m(N^2-1)}{N(N-m)}$ follows a Fisher distribution, $F_{m, N-m}$, with m and $N-m$ degrees of freedom, where N is the number of the considered Normal Operating Conditions (NoC) samples.

To cope with co-linearity Yu (2013), it is possible to use regularized forms of Δ_X can. For instance, Σ^{-1} is replaced by $(\Sigma + \epsilon I)^{-1}$, where ϵ is a small value used to remove the potential ill-condition of the covariance matrix. Thus, the distribution of Δ_X is approximated by $g\chi_h^2$ distribution, with $g = \frac{(\text{tr}(\Sigma(\Sigma + \epsilon I)^{-1}))^2}{(\text{tr}(\Sigma(\Sigma + \epsilon I)^{-1}))}$ and $h = \frac{[\text{tr}(\Sigma(\Sigma + \epsilon I)^{-1})]^2}{(\text{tr}(\Sigma(\Sigma + \epsilon I)^{-1}))^2}$.

3.1 Orthogonal-based decomposition

These statistical tests can efficiently detect faults but are not able explicitly to indicate which variables take the responsibility of their presence. Although one can monitor every single variable, when the variables are not independent and their number is too large. This is too expensive and time consuming. A popular solution to handle such a problem was proposed in Mason et al. (1995); often called the MYT decomposition.

MYT decomposition allows ($m!$) writing of the quadratic statistic in forms of m different orthogonal quadratic statistics. For a given decomposition two types of terms can be distinguished, non-conditional terms and conditional terms. The conditional term associated to a quadratic statistic Δ_X is the statistic associated to the regression of a set of variables A on B . The non-conditional term is simply the statistic associated to variables not influenced by any other variables. This independent statistic multiplied by the coefficient $\frac{N(N-(c+1)-1)}{(N+1)(N-1)}$ follows as well a Fisher distribution $F_{1, N-c-d}$. If the statistic is a

conditional term, c is equal to the number of conditioned variables, otherwise $c = 0$.

Although this method is useful to identify the variables responsible of the faulty operating conditions, $m \times 2^{m-1}$ terms need to be examined. It is challenging to go through all of them regarding their numbers, even sorted Mason et al. (1997). This approach reaches its limits when m is large, which is most likely in practice.

3.2 Causation-based decomposition

An alternative to MYT was proposed by Li et al. (2008). The authors showed a causation-based quadratic statistic decomposition. Their approach denotes each variable and its causal relationships explicitly under a DAG, and then investigates how to trace backward from certain condition to the variables that are the responsible. The authors increased the identification performances and reduced the number of checked terms by the MYT approach.

Authors distinguish two new types of terms in the MYT quadratic statistic decomposition: Type A and B. The type B decomposition of Δ_X , in the context of Gaussian models, come together under a same writing which can be deduced directly from the relations of dependencies between variables and given by the decomposition $\Delta_X = \sum_{i=1}^m \Delta(X_i \cdot \mathbf{X}_{\rho_i})$, where the quadratic term $\Delta(X_i \cdot \mathbf{X}_{\rho_i})$ is characterized Mason et al. (1997) by the mean $\mu_{\mathbf{X}_i \cdot \mathbf{X}_{\rho_i}}$ and the variance $\sigma_{\mathbf{X}_i \cdot \mathbf{X}_{\rho_i}}$.

It has been proven that it is enough to focus only on the one of the type B decomposition than going over all of the $m!$ MYT decomposition.

The main drawback of this approach is that data from multivariate industrial systems are assumed to follow a Gaussian distribution, whereas this is not always the case in practice. In the following, a new scheme to deal with data non-Gaussianity and multi-modality is proposed to detect faults and identify root-cause variables explaining faulty operating conditions.

4. FINITE GAUSSIAN MIXTURE MODEL FOR MONITORING

4.1 Finite Gaussian mixture model

Assume an m -dimension sample X from a non-Gaussian/multi-modal process. The marginal probability of X represented then by a Gaussian Mixture Model (GMM) can be written as following

$$p(X|\Gamma) = \sum_{k=1}^K \omega_k p(X|\Gamma_k), \Gamma = (\Gamma_1, \dots, \Gamma_K) \quad (2)$$

where ω_k represents the weight of the i th Gaussian component and K represents the number of modes. Γ_i represents the parameters of the i th mode. Here, each mode is considered to follow a multivariate Gaussian distribution. Therefore, Γ_i represents the parameters of the i th Gaussian component. The integral of the weighted sum of all Gaussian components should satisfy

$$\int_{\mathbb{R}^m} p(\mathbf{X}|\Gamma) = \sum_{k=1}^K \omega_k \int_{\mathbb{R}^m} p(\mathbf{X}|\Gamma_k) = 1 \quad (3)$$

since by definition $\int_{\mathbb{R}^m} p(\mathbf{X}|\Gamma_k) = 1$ and ω_k reflects the prior probability that any sampling point belongs to the i th mode, M_k , $0 \leq \omega_k \leq 1$ and $\sum_{k=1}^K \omega_k = 1$.

The parameters of the GMM model need to be estimated. The parameters are estimated with the F-J algorithm, similarly to Yu (2013), mainly for comparison purposes. Other estimations algorithms can be used as well. Through repeated calculation of the expectation step (E-step) and maximization step (M-step), the F-J algorithm estimates the corresponding distribution parameters until the convergence criterion of logarithmic likelihood function is met to achieve iteration. The algorithm can start from any given number of Gaussian components. Then, it adjusts the number of Gaussian components automatically by deleting components with unimportant weights Yang et al. (2012). The training data set consists of N samples X^n of \mathbf{X} . E-step and M-step are alternated such as

- E-step

$$p(M_k|X^n) = \frac{\omega_k^{(s)} p(X^n|\mu_k^{(s)}, \Sigma_k^{(s)})}{\sum_{v=1}^K \omega_v^{(s)} p(X^n|\mu_v^{(s)}, \Sigma_v^{(s)})} \quad (4)$$

- M-step

$$\mu_k^{(s+1)} = \frac{\sum_{n=1}^N p^{(s)}(M_k|X^n) X^n}{\sum_{n=1}^N p^{(s)}(M_k|X^n)} \quad (5)$$

$$\Sigma_k^{(s+1)} = \frac{\sum_{n=1}^N p^{(s)}(M_k|X^n) \|X^n - \mu_k^{(s+1)}\|^2}{\sum_{n=1}^N p^{(s)}(M_k|X^n)} \quad (6)$$

$$\omega_k^{(s+1)} = \frac{\max(0, \sum_{n=1}^N p^{(s)}(M_k|X^n) - \frac{V}{2})}{\sum_{k=1}^K \max(0, \sum_{n=1}^N p^{(s)}(M_k|X^n) - \frac{V}{2})} \quad (7)$$

where $\mu_k^{(s+1)}$, $\Sigma_k^{(s+1)}$ and $\omega_k^{(s+1)}$ are the mean value, covariance, and prior probability, respectively, of M_k at the $(s+1)$ th iteration. V is the number of scalar parameters of each specified mode.

Here, a new Bayesian inference based Gaussian mixture decomposition method is developed to identify the leading faulty variables that most likely cause the faulty operation conditions.

4.2 Bayesian inference-based GMM contribution

GMM can be used for fault detection without preliminary data projection. In Yu and Qin (2008) authors proposed a Bayesian inference-based probability index, named NG .

$$NG = \sum_{k=1}^K p(M_k|X) p(\Delta_X^k \leq L_{\Delta^k}) \quad (8)$$

The statistical test based on quadratic statistics, $p(\Delta_X^k \leq L_{\Delta^k})$, represents the chance the observations in the k -th operating mode have the shorter distance from the mode center than the new observation X . $p(M_k|X)$ represents the posterior probability of the k -th mode in respect to X . So, NG term corresponds to the overall probability that the observations in all different modes have the shorter distance from the corresponding mode centers than the monitored sample X .

In Yu (2013), a Bayesian inference based Gaussian mixture contribution method for a comprehensive fault diagnosis with process multi-modality embedded is presented. They proposed a measure to deal with all the possible operating modes and diagnose the leading faulty variables that most likely cause the faulty operation conditions. The proposed scaled Contribution Metric (CM_i) of the i th variable to the faulty operating conditions is given by

$$\tilde{CM}_i = \frac{CM_i}{\sum_{j=1}^K CM_j} \quad (9)$$

where CM_i is the overall contribution metric. It is written as following

$$\begin{aligned} CM_i &= \sum_{k=1}^K CM_i(k) \\ &= \sum_{k=1}^K p(M_k|X)\{CM_i(k)|X \in M_k\} \end{aligned} \quad (10)$$

with $CM_i(k)$ the sum of the contribution metric of the i th variable under the assumption that X comes from the k th mode with the posterior probabilities as the weighting factors. $\{CM_i(k)|X \in M_k\}$ equals to $\|\mathbf{v}^i \Sigma_{\mathbf{X}}^{-0.5}(X - \mu_{\mathbf{X}})\|^2$ with \mathbf{v}^i is the unit vector of the i th variable.

Based on the values of \tilde{CM}_i , a decision is made. The variable i with the largest metric is considered as carrying faults. Similar metrics based on the same formalism were proposed in Ma et al. (2018).

4.3 Bayesian causal-based GMM decomposition

Although, contribution plots-based methods are very popular to identify faulty variables, they suffer from some smearing effects and may cause misleading diagnosis results in some cases Rashidi and Zhao (2020).

Metrics, such the one seen above, are built upon a probabilistic framework and have been considered as an alternative to contribution plots-based approaches to improve the root cause variables analysis accuracy. They take into account the modality of data to identify responsible faults accordingly. However, they don't respect a false alarm rate. Responsible variables are assigned based on the maximum rule, thresholds are not fixed. In addition, the benefits of causality are not addressed to correctly reveal the exact roots and the propagation path. These metrics are not sensitive to changes, under each mode, in the process correlation.

Therefore, a new interesting approach is proposed. The quadratic statistic of each mode is decomposed, based on the causal relationships between variables, on independent orthogonal statistics. Each reflects the contribution of faults on an individual variable. The presented approach can be seen as an extension to the MYT approach to deal with multi-modality Mason et al. (1997) but with much lower computational cost. Furthermore, it can also be considered as the decomposition of the metric NG presented in (8).

Recall ρ_i is the set of variables influencing each variable \mathbf{X}_i . These connections described by ρ can be deduced, for instance, from the adjacency matrix of a non-Gaussian

DAG. Thus, the density function of \mathbf{X} can be decomposed as below

$$\begin{aligned} p(\mathbf{X}|\Gamma) &= \sum_{k=1}^K \omega_k \prod_{i=1}^m p(\mathbf{X}_i \cdot \mathbf{X}_{\rho_i} | \Gamma_k) \\ &= \sum_{k=1}^K \omega_k \prod_{i=1}^m \mathcal{N}(\mu_{k\mathbf{X}_i \cdot \mathbf{X}_{\rho_i}}, \sigma_{k\mathbf{X}_i \cdot \mathbf{X}_{\rho_i}}) \end{aligned} \quad (11)$$

Since the quadratic statistic of X in regards to a mode k can be decomposed as $\Delta_X^k = \sum_{i=1}^m \Delta^k(X_i \cdot \mathbf{X}_{\rho_i})$, the posterior probability weighted index of the variable i in respect to their ascendants and the component k is formulated as

$$MCD_i^k = p(M_k|X_i \cdot \mathbf{X}_{\rho_i})p(\Delta^k(X_i \cdot \mathbf{X}_{\rho_i}) \leq L_{\Delta^k(\mathbf{X}_i \cdot \mathbf{X}_{\rho_i})}) \quad (12)$$

where $p(M_k|X_i \cdot \mathbf{X}_{\rho_i})$ represents the posterior probability of the component M_k in respect to $\mathbf{X}_i \cdot \mathbf{X}_{\rho_i}$, and given by

$$p(M_k|X_i \cdot \mathbf{X}_{\rho_i}) = \frac{\omega_k p(X_i | \mu_{k\mathbf{X}_i \cdot \mathbf{X}_{\rho_i}}, \sigma_{k\mathbf{X}_i \cdot \mathbf{X}_{\rho_i}})}{\sum_{v=1}^K \omega_v p(X_i | \mu_{v\mathbf{X}_i \cdot \mathbf{X}_{\rho_i}}, \sigma_{v\mathbf{X}_i \cdot \mathbf{X}_{\rho_i}})} \quad (13)$$

Therefore, the overall possibility that the i -th variable is the responsible of the process faulty conditions across all the K modes can be deduced as below

$$\begin{aligned} MCD_i &= \sum_{k=1}^K MCD_i^k \\ &= \sum_{k=1}^K p(M_k|X_i \cdot \mathbf{X}_{\rho_i})p(\Delta^k(X_i \cdot \mathbf{X}_{\rho_i}) \leq L_{\Delta^k(\mathbf{X}_i \cdot \mathbf{X}_{\rho_i})}) \end{aligned} \quad (14)$$

Here the Mahalanobis distance-based probability represents the probability that the observations of a variable i , in respect to its ascendant and the k -th component, have the shorter distance from the component center than the new observation X . It is then enough to compare each MCD_i to $1 - \frac{1}{m}\alpha$ to identify root cause variables and their propagation path through the causal connections.

4.4 Summary

The step-by-step procedure of the Bayesian causal-based GMM decomposition method to fault detection and root cause identification is summarized below.

- (1) Collect process historical data as training set that includes a sufficient number of observations gathering different normal operating modes;
- (2) Search for the optimal number of Gaussian clusters and further estimate the corresponding mean, covariance and prior probability of each component k ;
- (3) Learn the causal relationships between variables; This can be accomplished by experts or/and using a non-Gaussian approach for estimating causal structures Guo et al. (2020);
- (4) For the observation X , calculate its corresponding quadratic statistic and its posterior probability, conditionally to every Gaussian component;
- (5) Deduce the fault detection index value (see equation (8));
- (6) If the detection index value is less than $1 - \alpha$, the corresponding observation belongs to normal

operation conditions. Otherwise, the sample is faulty and go to next step for further investigations;

- (7) In respect to the faulty observation, for each variable i compute the identification term MCD_i (see equation (14));
- (8) The variables with the metrics significantly larger than the statistically threshold are treated as faulty and responsible for process abnormality. Next investigate following the identified propagation path deduced from the variables' causal relationships.

5. CASE STUDY

In this section, the diagnosis results of the proposed method are illustrated by a well-known benchmark. It simulates a complex process and has been widely used to verify the effectiveness of fault monitoring methods Yin et al. (2014). The process has 12 manipulated variables and 41 measured variables consisting of 22 continuous process variables and 19 composition measurements. It admits several operating modes.

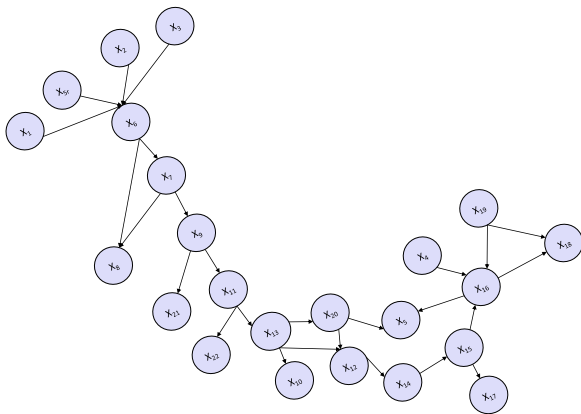


Fig. 1. The causal network associated to the considered variables

In this work, mode 1, mode 3 and only the continuous process variables are selected. To evaluate the performance of the proposed method, two faults' scenarios are considered, faults 11 and 14. Fault 14 represents a strike while Fault 11 induces a random variation, large oscillation. Fault 11 enables variables to behave similarly as NoC and remain around the set-points. As both faults influence the same component, they can be observed by the values of the same variable. Despite the different nature of these faults, both share the same root variable, X_9 .

Granger causality and process knowledge are combined to determine the causal relationship of the 22 continuous variables of the process. These relationship are presented in Figure 1. The parameters of each one of these variables are deduced based on NoC training data, 2000 samples.

To evaluate the proposed root cause identification approach, a comparison with other recent approaches is made. One is the GMM contribution metric used in Yu (2013). The other one is the Gaussian model decomposition, denoted CD_i . These approaches are evaluated with 2000 samples from each fault. Note that the identification procedure proceeds with only faulty declared samples.

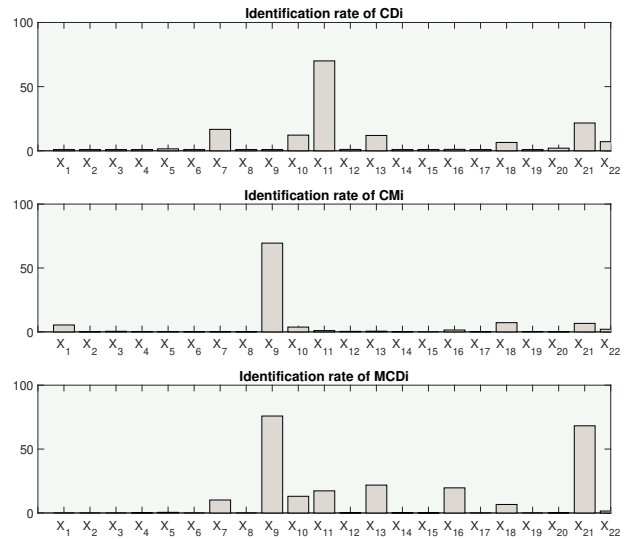


Fig. 2. The identification rates of fault 11

Figures 2 and 3 presents the overall root cause identification results obtained by each one of the considered approaches in regards to Faults 11 and 14. The variable X_9 is diagnosed the most frequently, which is consistent with the expectations that came with the tested data. From those figures, it is clear that the proposed method performs better than the other approaches. CD totally misidentify the root cause in both scenarios. Here, one can note the interest of identifying the system's modes for an accurate root cause identification. CD is a powerful metric but not suited to multi-modes systems. The new proposed metric MCD outperforms the metric CM in these both scenarios. In fault 11 scenario, MCD identify 1517 detected samples with X_9 as the root cause, while only 1389 samples were correctly identified by CM. Similarly, in fault 14 scenario, 1714 detected samples were well identified by MCD. CM was able to correctly decide about 1161 samples.

In addition, other variables are also identified by MCD (whether simultaneously with X_9 or not) as the root cause. Mainly, in both fault scenarios, X_{21} are put in front by the approach. This actually corresponds to a propagation path (see Figure 4). The variable X_9 dilute its effect by spreading it to its descendant. Thus, under abnormal conditions, the deviation of X_9 from its normal behavior is able to spread through causality, thereby affecting its child nodes.

Therefore, the TEP case study fairly demonstrates the potential and the ability of our approach to monitor complex processes dealing with multivariate variables in complex industrial systems. It also shows the interest of the new proposed identification metric over the well-known CM. MCD can be seen as a serious alternative to CM to improve fault identification in multimode processes.

6. CONCLUSIONS

In this paper, a causal-based GMM decomposition for root cause identification is presented. The new metric with statistical foundations optimizes and overcomes the

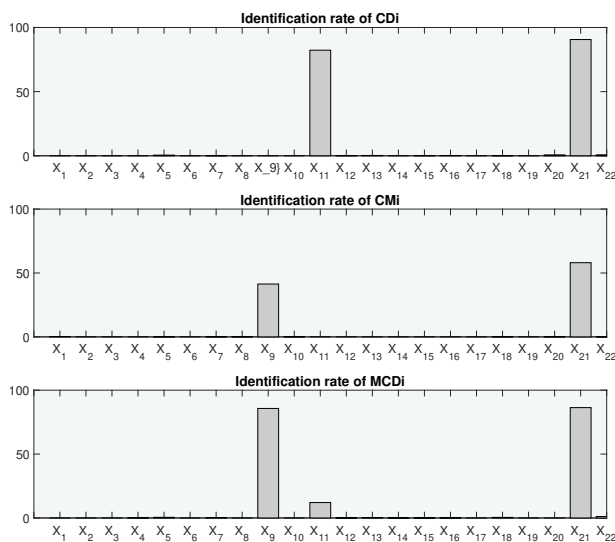


Fig. 3. The identification rates of fault 14

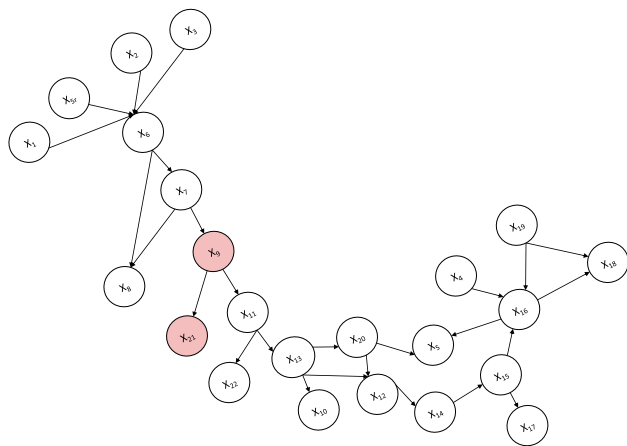


Fig. 4. The causal network associated to the considered variables with the faults path propagation

drawbacks of the most recent and well-known works. The proposed strategy is tested using a well-known benchmark to demonstrate its interest and practical feasibility.

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