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# Regular extremal black hole created by acceleration

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## Abstract

The generation of an extremal black hole by acceleration is proposed in this paper. A direct connection between the black hole mass and the constant acceleration is established. We study initially the extremal Reissner-Nordstrom black hole (which is singular at the origin) and then a regularized version of the Schwarzschild spacetime. The radial acceleration of a static observer has a signature flip when the horizon is crossed, being negative inside and positive outside the black hole. The junction conditions at the horizon are investigated, leading to a zero surface energy density and a positive surface pressure on the horizon  $r = r_H$ .

## 1 Introduction

A lot of investigations have been done on quantum aspects of black holes (BHs) since Hawking discovered that BHs decay by emission of thermal radiation. The purpose is to improve our understanding of a complete model of Quantum Gravity (QG). Hawking himself was the first who emphasized that the information can be lost as a pure quantum state collapses into a BH and then evaporates into a mixed state [1]. A final answer to this puzzle requires a consistent theory of QG [2].

The extremal or near extremal BHs represent, somehow, a special situation [3, 4, 5]. Actually, for a near-extremal Reissner-Nordstrom (RN) black hole, the temperature depends on the BH mass and that questions an exact analytical framework. Fabbri et al. [2] showed that one can get exact quantum results of the evolution of the extremal RN black holes, studying the near-horizon region. Bonanno and Reuter [6] investigated the quantum gravitational effects in spherical-symmetry BH geometry and constructed, by "renormalization group improving", the Schwarzschild (KS) metric. They argued that the BH evaporation stops when its mass  $M$  approaches some critical mass  $M_{cr}$  of the order

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of the Planck mass, so forming a soliton-like remnant with the near-horizon geometry  $AdS_2 \times S^2$ . Due to quantum effects, the singularity at  $r = 0$  is removed (a smooth deS core appears).

Hod [7] studied the stability of the extremal RN BH to charged perturbations. He showed that, for the above BH, the superradiant instability (amplification of trapped modes) is not triggered and so the extremal RN BHs are stable to charged scalar perturbations. Recently similar conclusion reached Beltracchi et al. [8] in their research on surface stress tensor and junction conditions on a null horizon. The equal and opposite surface gravities at the horizon show that equal and opposite radial forces (ingoing from outside, outgoing from inside) balance in a static configuration with no net force. Sofia di Gennaro and Ong [9] used mutual information optimization (introduced by Kim and Wen [10] in Hawking evaporation process, with a tendency to extremality for charged BHs) and found that it violates the cosmic censorship conjecture because extremal BHs are singular at the origin of coordinates. Quite surprisingly, di Gennaro and Ong concluded that, if there is a particle with  $q/m < 1$ , even in the presence of other particles with  $q/m > 1$ , the extremality may be reached.

The paper is organized as follows. In Sec.2 we present the extremal RN geometry and its  $AdS_2 \times S^2$  near-horizon form, emphasizing the role played by the acceleration of a static observer. We consider in Sec.3 the Israel junction conditions at the hypersurface  $r = m$ , finding that the jump of the extrinsic curvature is nonzero, leading to a positive pressure at the horizon. The regular extremal BH is introduced in Sec.4, where one shows that the near-horizon metric is of Rindler-type, in the first order approximation. The constant acceleration  $g$  of the Rindler observer (located very close to the horizon) generates a regular extremal BH of radius  $r = 2m/e$ . We state our conclusions and comments in Sec.5, giving an overall view of the physical system studied.

The geometrical units  $c = G = 4\pi\epsilon_0 = 1$  are used, unless otherwise specified.

## 2 Extremal RN black hole

In spherical coordinates  $(t, r, \theta, \phi)$  the Reissner-Nordstrom line element is given by

$$ds^2 = - \left( 1 - \frac{2m}{r} + \frac{q^2}{r^2} \right) dt^2 + \frac{dr^2}{1 - \frac{2m}{r} + \frac{q^2}{r^2}} + r^2 d\Omega^2, \quad (2.1)$$

where  $m$  and  $q$  are, respectively, the BH mass and its charge and  $d\Omega^2$  stands for the metric on the unit 2-sphere. If  $m > q$ , the metric coefficient  $f(r) \equiv 1 - \frac{2m}{r} + \frac{q^2}{r^2}$  has two roots

$$r_{\pm} = m \pm \sqrt{m^2 - q^2}, \quad m \geq q, \quad (2.2)$$

where  $r_+$  refers to the event horizon and  $r_-$  represents the Cauchy horizon. Our interest is to study the extremal situation:  $m = q$ , when the two horizons

coincide at  $r = m$

$$ds^2 = -\left(1 - \frac{m}{r}\right)^2 dt^2 + \frac{dr^2}{\left(1 - \frac{m}{r}\right)^2} + r^2 d\Omega^2. \quad (2.3)$$

It is worth noticing that there is no signature flip when the horizon  $r = m$  is crossed, contrary to the KS case. Therefore, the geometry inside the horizon is static. The source generating the above geometry is the well-known electrostatic energy-momentum tensor, with the nonzero components

$$\rho = -p_r = p_\theta = p_\phi = \frac{m^2}{8\pi r^4}, \quad (2.4)$$

where  $\rho$  is the electrostatic energy density,  $p_r$  is the radial pressure and  $p_\theta, p_\phi$  are the transversal pressures. As for the KS geometry, the geometry (2.3) has a singularity at  $r = 0$ , where the Kretschmann scalar diverges. A static observer with the velocity vector  $u^a = (1/(1-m/r), 0, 0, 0)$  has the covariant acceleration

$$a^b = \left(0, \frac{m}{r^2} \left(1 - \frac{m}{r}\right), 0, 0\right). \quad (2.5)$$

For the proper acceleration, (2.5) gives us  $\sqrt{a^b a_b} = m/r^2$  and the surface gravity  $\kappa$  is vanishing, that means the Hawking temperature is zero. One observes that  $a^r$  changes its sign at the horizon. In other words, there are equal and opposite radial forces (ingoing from outside, outgoing from inside) that lead to a static configuration with no net force. Moreover, at  $r = 3m/2$ ,  $a^r$  reaches its maximum value  $4/27m$  and it becomes minus infinity at the origin.

Let us take an accelerating observer  $O$ , moving with a constant rest-system acceleration  $g$ . We relate  $g$  to the value of the proper acceleration at the horizon, i.e.  $\sqrt{a^b a_b}|_H = 1/m \equiv g$ . Our main proposal is to consider that an extremal BH is generated by the agent who sets  $O$  in accelerated motion. During its motion,  $O$  remains always very close to the horizon  $r = m$  (the distance to the horizon is  $d \ll m$ , outside of it). Note that the BH is always at rest w.r.t. the accelerating observer, just like the Rindler horizon w.r.t. the same observer. In addition, it does exist only for the observer  $O$ , an inertial observer sees nothing, like a freely-falling observer in the gravitational field of a BH, who does not feel, locally, its horizon.

It is surprising that the generated extremal BH mass is proportional with  $1/g$ . Consequently, a small  $g$  creates a BH of a bigger mass. A similar result has been obtained in [11] (see also [12]) for the energy stored on the Rindler horizon. Taking, for example,  $g = 10^3 \text{ cm/s}^2$ , one obtains  $m = c^4/gG \approx 1.2 \cdot 10^{46} g$ , with the radius  $c^2/g \approx 10^{18} \text{ cm}$ . An estimation of the electrostatic energy density (2.4), on the horizon, gives  $\rho = g^2/8\pi G = 5.6 \times 10^9 \text{ ergs/cm}^3$ , which is much less than, for example, the total energy density of one  $\text{cm}^3$  of water, that is  $9 \times 10^{20} \text{ ergs/cm}^3$ . Near the horizon, the metric coefficient becomes  $f(r) \approx (r - m)^2/m^2$  and, with  $r^* \equiv r - m > 0$ , the metric becomes

$$ds^2 = -\frac{(r^*)^2}{m^2} dt^2 + \frac{m^2}{(r^*)^2} d(r^*)^2 + m^2 d\Omega^2. \quad (2.6)$$

We recognize above the Robinson-Bertotti metric for the product of a two-dimensional  $AdS_2$  with a two-sphere,  $AdS_2 \times S^2$  [6, 14].

Our next goal is to find the Komar energy [13, 14, 15] corresponding to the geometry (2.3), which is static and asymptotically flat. We have

$$W = 2 \int (T_{ab} - \frac{1}{2}g_{ab}T^c_c)u^a u^b N \sqrt{\gamma} d^3x, \quad (2.7)$$

that is measured by a static observer with the velocity field  $u^a$ .  $N = 1 - m/r$  in (2.7) is the lapse function and  $\sqrt{\gamma} = r^2 \sin\theta / (1 - m/r)$  is the square root of the determinant of the spatial 3-metric. From (2.3) and (2.4) we get

$$W(r) = \int_m^\infty \frac{2m^2}{8\pi r^4} N \sqrt{\gamma} dr d\theta d\phi = m - \frac{m^2}{r} \quad (2.8)$$

which is just the ADM mass when the limit  $r \rightarrow \infty$  is taken.

### 3 Horizon stress tensor

Let us find now the expression of the surface energy-momentum tensor on the horizon  $r_H = m$ . We must have a nonzero energy on this hypersurface due to the jump of its extrinsic curvature  $K_{ab}$  when the horizon is crossed. To see this, we write down the expression of the extrinsic curvature tensor [16, 14]

$$K_{ab} = -\frac{f'}{2\sqrt{f}}u_a u_b + \frac{\sqrt{f}}{r}q_{ab}, \quad (3.1)$$

where  $f' \equiv df(r)/dr$ ,  $u_a = (\sqrt{f}, 0, 0, 0)$  (obtained from (2.3)) is the normal to  $t = \text{const.}$  hypersurface,  $h_{ab} = g_{ab} - n_a n_b$  represents the induced metric on a  $r = \text{const.}$  surface with  $n_a = (0, 1/\sqrt{f}, 0, 0)$  its normal vector and  $q_{ab} = h_{ab} + u_a u_b$  the induced metric on the 2-surface of constant  $t$  and  $r$  [16]. Eq. (3.1) yields

$$K_{tt} = -\frac{f'}{2}\sqrt{f}, \quad K_{\theta\theta} = \frac{K_{\phi\phi}}{\sin^2\theta} = r\sqrt{f}, \quad K \equiv h^{ab}K_{ab} = \frac{f'}{2\sqrt{f}} + \frac{2\sqrt{f}}{r} \quad (3.2)$$

We suppose the horizon is a membrane with a surface stress tensor  $S_{ab}$  of a perfect fluid form

$$S_{ab} = \rho_s u_a u_b + p_s q_{ab}, \quad (3.3)$$

with  $\rho_s$  - the surface energy at  $r = r_H$  and  $p_s$  - the surface pressure.  $S_{ab}$  is obtained from the Lanczos equation

$$8\pi S_{ab} = [K]h_{ab} - [K_{ab}], \quad (3.4)$$

where  $[K_{ab}] = K_{ab}^+ - K_{ab}^-$  is the jump of the extrinsic curvature when the horizon is crossed, from  $r > r_H$  to  $r < r_H$ .

The first Israel junction condition is automatically satisfied as we use same coordinates on either side of the horizon  $r_H$ . In addition, the metric on the

horizon is simply [14]  $ds_H^2 = r_H^2 d\Omega^2$ , when we set  $r = r_H$  in (2.3). We shall determine  $\rho_s$  and  $p_s$  from (3.3) so that the second junction conditions be obeyed. The nonzero surface tensor comes from the fact that the ratio  $f'/\sqrt{f}$ , with  $f(r) = (1 - m/r)^2$  from (3.2) changes its sign when the horizon is crossed. From (3.2) one obtains

$$K_{tt}^+ = -K_{tt}^- = -\frac{m}{r^2} \left(1 - \frac{m}{r}\right)^2, \quad K_{\theta\theta}^+ = -K_{\theta\theta}^- = r \left(1 - \frac{m}{r}\right), \quad (3.5)$$

evaluated at  $r = m$ . For the mean extrinsic curvatures, (3.5) yields

$$K^+ = \frac{2}{r} \left(1 - \frac{m}{r}\right), \quad K^- = \frac{2}{r} \left(1 - \frac{3m}{2r}\right), \quad (3.6)$$

evaluated at  $r = m$ . One observes that at  $r = m$ , only the mean curvatures show a jump when the horizon is crossed, with  $K^+ = -K^- = 1/m$ . Therefore, the jump will be  $[K] = 2/m$ . We are now in a position to compute the surface energy density and the surface pressure from the Lanczos equation. From (3.4) we get

$$\rho_s = 0, \quad p_s = \frac{1}{4\pi m}. \quad (3.7)$$

## 4 Regular extremal black hole

As we already noticed, the Kretschmann scalar is divergent at  $r = 0$ . Same is valid for the metric coefficients and the components of the stress tensor from (2.4). A black hole generated by acceleration and with a singularity at the origin is far from being realistic. Therefore, in this section we shall look for a regularized spacetime, with no divergencies whatsoever. A good candidate for that purpose is given by the geometry [14, 15, 17, 18, 19, 20, 21]

$$ds^2 = - \left(1 - \frac{2m}{r} e^{-\frac{2m}{er}}\right) dt^2 + \frac{1}{1 - \frac{2m}{r} e^{-\frac{2m}{er}}} dr^2 + r^2 d\Omega^2, \quad (4.1)$$

with a horizon at  $r_H = 2m/e$ . We have always  $f(r) \equiv -g_{tt} > 0$  (no signature flip) and, in addition,  $f(r)$  tends to unity both when  $r \rightarrow 0$  and  $r \rightarrow \infty$ . As we already mentioned in [14, 15], all curvature invariants and the components of the source stress tensor leading to (4.1) are finite, both at infinity and at  $r = 0$ . The acceleration 4-vector of a static observer is given by [14]

$$a^b = \left(0, \frac{m(1 - \frac{r_H}{r})}{r^2} e^{-\frac{r_H}{r}}, 0, 0, \right). \quad (4.2)$$

One sees that  $a^r$  is vanishing when  $r \rightarrow 0$  and at the horizon  $r_H = 2m/e$ . It is negative for  $r < r_H$  and positive for  $r > r_H$ . That means the gravitational field is repulsive for  $r < r_H$  and attractive for  $r > r_H$ . That means an observer located on the horizon is in equilibrium, being geodesic. For a negative acceleration (a decreasing velocity), the observer is located inside de BH (the

horizon is in front of him, who is attracted by it). In contrast, for a positive acceleration, the observer finds himself outside the BH (the horizon is in the rear, in the direction of the force of inertia).

We now suppose that the extremal BH (4.1) is generated by acceleration, where the constant acceleration  $g$ , as independent parameter, equals the horizon value of the proper acceleration of a static observer in the spacetime (4.1) [14]

$$g = \sqrt{a^b a_b}|_H = \frac{e\sqrt{2}}{4m}. \quad (4.3)$$

We find now the near-horizon version of the metric (4.1), in first order in  $m/r$  (the second order form has been given in [14], Eq. 6.4). One obtains

$$f(r) = 1 - \frac{2m}{r} e^{-\frac{2m}{er}} \approx 1 - \frac{2m}{er} \approx \frac{er - 2m}{2m}. \quad (4.4)$$

A change of the radial variable, namely  $\bar{r} = 2\sqrt{r_H(r - r_H)}$  transforms (4.1) as

$$ds^2 = -\frac{g^2}{2}\bar{r}^2 dt^2 + d\bar{r}^2 + r_H^2 d\Omega^2, \quad (4.5)$$

with  $g = \frac{e\sqrt{2}}{4m}$  and  $r_H = 2m/e$ . We recognize (4.5) as the Rindler version (the two-dimensional  $t - r$  space) of the near-horizon metric (4.1), in the first order of  $m/r$ , with the horizon located at  $\bar{r} = 0$  or  $r = 2m/e$ . It is clear that this version is much more realistic than the RN extremal metric, due to the lack of singularities and the analogy with the KS geometry where the near-horizon metric is also of Rindler type. An evaluation of the energy density close to the horizon (see [14], Eqs. 3.1), gives us  $\rho \approx g^2/4\pi$ , i.e., the energy density is proportional to the gravitational field squared, as in Newtonian gravity or electrostatics.

## 5 Conclusions

A direct relation between an extremal black hole and the creation of inertial forces in accelerated reference systems is conjectured in this paper. We firstly analysed the extremal (when  $m = q$ ) Reissner-Nordstrom black hole, with its  $AdS_2 \times S^2$  Robinson-Bertotti form close to the horizon, and the singularity at the origin of coordinates. The corresponding Komar energy  $W(r)$  becomes the ADM mass at infinity. From the junction conditions at the horizon  $r = m$  we found that the surface energy density  $\rho_s$  is vanishing but the surface pressure  $p_s$  is positive and proportional to  $1/m$ . However, the singularity at  $r = 0$  of the Reissner-Nordstrom line element makes it inappropriate for our situation. Therefore, we looked for a better model by means of a regularized Schwarzschild geometry with no singularity anywhere. The independent quantity - the constant acceleration  $g$  - is directly related to the regular black hole mass  $m$ .

## References

- [1] S. W. Hawking, Phys. Rev. D14, 2460 (1976)
- [2] A. Fabbri, D. Navarro and J. Navarro-Salas, Nucl. Phys. B595, 381 (2001) (arXiv: hep-th/0006035).
- [3] A. Strominger and C. Vafa, Phys. Lett. B379, 99 (1996) (arXiv: hep-th/9601029).
- [4] C. Callan and J. Maldacena, Nucl. Phys. B472, 591 (1996) (arXiv: hep-th/9602043).
- [5] J. Maldacena and A. Strominger, Phys. Rev. Lett.77, 428 (1996) (arXiv: hep-th/9603195).
- [6] A. Bonanno and M. Reuter, Phys. Rev. D62, 043008 (2000) (arXiv: hep-th/0002196).
- [7] S. Hod, Phys. Lett.B713, 505 (2012).
- [8] P. Beltracchi, P. Gondolo and E Mottola, arXiv: 2103.05074.
- [9] Sofia di Gennaro and Y. C. Ong, arXiv: 2103.05516.
- [10] K. K. Kim and W.-Y. Wen, Phys. Lett. B731, 307 (2014) (arXiv: 1311.1656).
- [11] H. Culetu, Int. J. Mod. Phys. D15, 2177 (2006) (arXiv: hep-th/0607049).
- [12] R. Kerner and R. Mann, Phys.Rev.D73, 104010 (2006) (arXiv: 0603019).
- [13] T. Padmanabhan, Class. Quantum Grav. 21, 4485 (2004) (arXiv: gr-qc/0308070).
- [14] H. Culetu, arXiv: 1305.5964.
- [15] H. Culetu, Int. J. Theor. Phys. 54, 2855 (2015) (arXiv: 1408.3334).
- [16] S. Kolekar and T. Padmanabhan, Phys. Rev. D85, 024004 (2011) (arXiv: 1109.5353 [gr-qc]); S. Kolekar, D. Kothawala and T. Padmanabhan, Phys. Rev. D85, 064031 (2012) (arXiv: 1111.0973).
- [17] T. Berry et al., Phys. Rev. D 102, 064054 (2020) (arXiv: 2008.07046); arXiv: 1911.01020.
- [18] A. Simpson and M. Visser, arXiv: 1911.01020.
- [19] M. Rodrigues et al., Phys. Rev. D 102, 084038 (2020) (arXiv: 2010.09490)
- [20] Y. Khan et al., Eur. Phys. J. Plus 135, 620 (2020) (arXiv: 2010.00005).
- [21] E. Junior et al., Nucl. Phys. B 961, 115244 (2020) (arXiv: 2002.04410).