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Shrödinger's Cat: From false Paradox to genuine Paradigm

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Abstract

The famous school case of Schrödinger is quickly reviewed and evaluated in the light of recent experimental and formal developments, and conclusions are proposed at theoretical, epistemological and philosophical points of view.

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I. INTRODUCTION

Since its discovery and completion around 1925, Quantum Mechanics (QM) has never ceased to lend itself to endless debates. Not questioning its efficiency, ever: Quite on the contrary, the set of QM theoretical predictions is so supported by generations of experimental tests that no doubt exists today concerning the extraordinary relevance of QM to the description of the microphysical world.

It is true that the concurrent theory of Bohm-de Broglie is able to reproduce the results of the QM standard version. Now, the price to be paid is ‘a spooky action at a distance’ that prevents it from the possibility of acquiring the status of a relativistic theory. Equally important in this respect is the experimental refutation of the famous Couder’s experiment which seemed to provide the Bohm-de Broglie theory with a renewed support [1]. QM standard version, instead, can be endowed with several relativistic extensions, among which Quantum Field Theories ($QFTs$) can certainly be thought of as the most elaborated and fecund realizations, those which have given rise to the *Standard Model of Elementary Particles*, recently confirmed at the LHC experimental runs of CERN [2].

At a formal level, it is remarkable that QM amounts to just a few axioms which, if not complying with ordinary intuition, are simple enough to be the matter of an undergraduate course. Moreover, contrary to what was first thought - that QM laws would apply in a statistical sense to a large number of *identically prepared systems* - it is known at present that elementary quantum systems are also perfectly described by the laws of Quantum Mechanics. Some authors consider this state of affairs as a kind of second quantum revolution, initiated in the last quarter of the XXth century [3]. As for the QM statistical aspects, moreover, deep theorems obtained at a later time in 1967, 2006 and 2009, have elucidated a QM peculiarity which definitely lacks any classical equivalent [4]. In effect, the statistical character of a quantum system is *inherent* and manifest in the case of a single and simple quantum entity, in contradistinction to the classical situation where it always reflects a lack of knowledge on a system endowed with a large number of degrees of freedom.

In this way, the fundamental character of quantum laws became unquestionable. Not only because of the formal simplicity of the theory, the accuracy of its predictions, elementary systems included, but also because of an aspect of the quantum theory which is barely evoked, that is, .. *the amazing fecundity of a formalism allowing one to anticipate on*

experiments in a successful way, as well as to think of new realizations, one wouldn't have thought of otherwise. At the theoretical, experimental and even technological levels, it is by carrying out a reflexion within the formalism itself, that creativity often shows up in the realm of microphysics [5].

Now, this is also notable that the classical physics we started from doesn't present the same fundamental character as *QM*, and should rather be derived from the latter. As is well known, this point is in itself a major issue, that of the classical/quantum frontier that will be coped with elsewhere [6]. The difficulty, as it appears, is in the definition of such a frontier; the *criteria* that would be necessary for such a definition are never quite universal enough. Right from the onset, and with good reasons, one thinks of the size of the physical system. However, quickly, counter-examples accumulate, and that, even beyond the size of the intermediate systems, known as *mesoscopic*. In the end, the system's size, the number of particles or *quanta* they are composed of, turn out to be quite deceptive criteria.

Admittedly, there is neither definition nor any criterium that would be so universal as to encompass the variety of boundary relations that classical and quantum physics have one to the other. In the scientific literature, this persisting and frustrating situation finds various expressions, some of them in a mild form [7],

'This experience suggests that the problems belong to physics and that there is little chance of providing very general solutions using sophisticated mathematical machinery or philosophical considerations.'

While others may be more extreme or provocative, as for instance [8],

'Existe-t-il une frontière classique/quantique?'

In the same line and even more recently, one may quote this passage of a book precisely devoted to the grasp of this divide [9],

'After all, it is quantum mechanics, not classical mechanics that is generally thought to be the most fundamental theory. So, perhaps, the real question is not so much about the location of the quantum/classical divide rather than about whether there even is a divide.'

That is, from the difficulty of defining a classical/quantum frontier in a universal enough manner, one arrives at questioning the very existence of such a frontier. However, who could seriously doubt of the striking difference separating the classical and quantum worlds? Shouldn't this be taken, rather, as an indication that something essential is still escaping us,

some *quantum enigma* [10]? To some extent at least, this is what we will try to apprehend and to initiate on the basis of the following famous example, to begin with, before proceeding with further and deeper analyses in Refs. [6, 11, 12].

In section II, the paradox is quickly recalled, within its historical cradle, the so-called Copenhagen interpretation of Quantum Mechanics which the paradox was intended to test. Section III begins with a reminder that this interpretation fails to provide the paradox with any clue, while, through the introduction of *Von Neumann chains* more recent treatments have taken the original Schrödinger cat paradox to the archetypical discussion of a classical/quantum divide. The issues of the paradox are evaluated, and Section IV gathers conclusions that can be drawn from these. To make contact with the main text, a short Appendix summarizes the basic formal properties of entanglement.

II. SCHRÖDINGER'S CAT PARADOX

The famous Schrödinger's cat paradox fits in this classical/quantum problematic, even beyond Schrödinger's original purposes [13]. In 1935, when E. Schrödinger proposes his case, the canonical interpretation of *QM* - the Copenhagen interpretation - is a debated subject in itself. This interpretation, in effect, is not so well defined and makes up a rather motley ensemble. Year after year, though, it has been basically preserved until today [16], even if, sometimes, with serious provisos, as it can consistently be qualified as a 'non-interpretation', to quote A. Legget.

One of the striking principles posited by the Copenhagen interpretation is that there is no other reality than classical, and that nothing at all can be stated concerning the microphysical world. The question would even be totally irrelevant [14],

'There is no quantum world. There is only an abstract physical description. It is wrong to think that the task of physics is to find out how Nature is. Physics is concerned with what one can say about Nature.'

According to this view, and let alone which theory is put forward, observing or measuring would make the only reality, classical, emerge out of an objectively undetermined microphysical situation. It is in order to probe the coherence of this vision of things, that Schrödinger has enunciated the school case of a cat locked in a box together with a flask of poison. Through the expediency of some evil device involving a hammer, the flask can

or cannot be broken based on whether a radio-active atom has decayed or not, after some duration of time, one hour for example.

In order to revisit this paradox, it may be helpful to recall a few standard elements of *linear algebra* that are at the origin of the paradox formulation, and which also shed light on some aspect of its resolution. These elements are summarized in the Appendix.

The paradox is ordinarily given the following expression. One considers that after one hour there is equal chances for the radio-active atom to have either decayed, and be therefore in its fundamental level, $|\Psi_1\rangle$, or to have stayed in its excited state $|\Psi_2\rangle$. The wave function of the full system, ‘cat plus atom’, therefore reads as,

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \{ a |\Psi_1\rangle \otimes |dead\rangle + b |\Psi_2\rangle \otimes |alive\rangle \} \quad (1)$$

where a and b , complex numbers, are the process *probability amplitudes*. This state, $|\Psi\rangle$ represents a possible state of the full system ‘atom plus cat’. It is a vector of the linear Hilbert space $H_a \otimes H_c$ obtained by forming the tensorial product of H_a and H_c , the linear Hilbert spaces of atom and cat states respectively.

With supporting graphical illustrations, literature has often seized this wave function to represent a cat in a sort of a mixed or fuzzy state, as if the animal was wandering about in *the quantum limbos* between life and death [13]. Now, it is true that Schrödinger himself was using similar terms,

‘In it (the wave function), the dead and alive cat are (if I dare to say so), mixed up or scrambled in equal proportions.’

It is by observation alone, that is by opening the box [15], that the animal would escape an uncertainty which would be fixed then, once and for all, in either of the two possible states, dead or alive.

III. MODERN APPROACHES : FROM PARADOX TO PARADIGM

A. Copenhagen Interpretation

It is interesting to begin with the Copenhagen Interpretation of the paradox because, as quoted above, this interpretation furnishes the basic interpretation of Quantum Mechanics [16] (never forgetting that the famous paradox was inspired by the Copenhagen interpretation itself).

The system ‘atom+cat’ being a quantum system, the issue of knowing in which state is the cat, before the box is unlocked, is not a point. This is simply because, according to this interpretation, ‘*There is no quantum world!*’. The only one is classical.

As a matter of fact, it is only by opening the box that the cat’s final state is eventually *measured* or *decided*, whereas, so long as the box remains locked, the cat’s state remains perfectly undetermined. What is worth emphasizing here is a singular consequence of this point of view. The box in effect, acts as if it were some ‘*de-realizing*’ machine : While the entering cat is a plain real being at the beginning, after one hour of entanglement to the radio-active atom’s states, the animal has become phantom-like, as if it had passed from this real world to the limbos of that *quantum non-world*. An odd consequence indeed, rarely put forth though worth of consideration.

B. Von Neumann chains

Now, it is important to point out that what is contained in the wave function (1) is not a mixture of cat’s states $|alive\rangle$ and $|dead\rangle$, but, explicitly, a mixture (a linear combination) of two states of the product space $H_a \otimes H_c$, the states $|\Psi_1\rangle \otimes |dead\rangle$ and $|\Psi_2\rangle \otimes |alive\rangle$. These two states are *factorized states* (Cf. Appendix), but their sum isn’t, and is an entangled state by definition : It cannot be written as a product of a state in H_a by a state in H_c . As a result, what is to be considered as ‘mixed up’, if ever, is not the dead cat with the live cat, but instead the ‘decayed atom and dead cat’ with the ‘non-decayed atom and live cat’. This remark, due to Ref.[13], has two main virtues :

- First, it prevents us from considering, in an inappropriate way, any mixture of only the states ‘dead cat’ and ‘alive cat’.
- And secondly, it puts the focus on the real issue of the cat paradox, which is that of a paradigm as we will see.

In order to not miss the point, it is helpful to observe that a Hilbert space, H , which is a linear space over the complex field, cannot be used to describe the possible states of a cat, supposing that such description could make sense over the real field. To our best knowledge, the difference, complex versus real field, is never really appreciated and to our knowledge, it is in Ref.[17] only that an interesting discussion of the point can be found. Though intuitively obvious [9], the latter statement will be provided with a more systematic

foundation in Refs.[6]. For now, suffices it to say that this statement alone would invalidate the whole affair of the Schrödinger's cat paradox, so long as it is formulated the standard way of Equation (1). Now, based on the fact that all of the experiment's protagonists are made out of *quanta*, this objection is circumvented in the following manner.

A refined formulation of the paradox argues for the need of a closer look at the experiment, one which considers the possible states of the Geiger counter, the $|triggered\rangle$ and $|non - triggered\rangle$ states. Then also the hammer states, $|fallen\rangle$ and $|non - fallen\rangle$ must be considered, and the flask states, $|broken\rangle$ and $|non - broken\rangle$, etc. . That is, to Equation (1), one should rather substitute something like [9],

$$|\Psi\rangle = \frac{1}{\sqrt{2}} \left\{ |not - decayed\rangle |not - triggered\rangle |not - fallen\rangle |alive\rangle + |decayed\rangle |triggered\rangle |fallen\rangle |dead\rangle \right\}, \quad (2)$$

and of course, for the sake of completeness, a whole series of similar decompositions should be considered. These are the so-called *Von Neumann chains*, which come about as infinite regressions, due to the huge set of all the possible physical entanglements stretching from the atom to the cat states.

However surrealistic the enumeration of all of the entangled mediations may appear (they can be *innumerable*), it is important to realize that it suffers from no *obstruction of principle*. Moreover, as compared to Equation (1), Equation (2) can be considered as exhibiting a deeper degree of compliance to the physical reality of the experiment, supposing again that Equation (2) could even be written in an exhaustive and complete form.

This Von Neumann regression accounting for the full wave-function of the experiment, is now to yield one of either two expected results for the cat, dead or alive. The outcome having to be generated *via* some sort of *wave-function collapse* [11], and/or measurement procedure.

Now, this is what is impossible. The experiment state vector so conceived never collapses [9] to a unique result, and this is precisely the gist of the paradox. When, where and how does the wave function reduction operate?

Here is the *real issue* of the Schrödinger cat paradox, the archetype of the classical/quantum divide, that is indeed a *paradigm*.

Comments are in order.

(i) Besides the incompleteness of the vector state enunciation, this impossibility has to do with a measuring process. The Von Neumann model on which one can rely when it comes to measuring a quantum system, offers an interesting starting point, but in its current state, is unable to account for the emergence of a unique result for the measure. That is to say, so long as the state vector evolution is driven linearly by the Schrödinger equation, measurement doesn't yield a unique result.

This is why, over the past five decades, non-linear extensions to this equation have been considered so as to remedy this impediment. These attempts, whose inspirations are not so different from Bohm's original goal are illustrated through the GRW theory [18] and the series of completions brought to it, gravitation and the role of the observer's mind included. Now according to the experts themselves, a new universal constant would be necessary to account for a complete theory of quantum measurement (See Ref.[19], where an interesting and exhaustive review of these attempts and several others are given).

These tentative solutions to the wave function collapse are worth being compared to another approach, recently drawn from a sound experimental *savoir faire* [20], [21]. Relying on a series of indirect *quantum non-demolition* measurements, one is able to proof the convergence of the series toward the collapse of the wavefunction, such as assumed by the axioms of quantum mechanics for direct measurements [22]. In doing so, it is of utmost interest to note that there is no need for considerations other than those of repeated probe-system interactions within a pure quantum mechanical framework. Alternatively, taking things the other way round, the infinite series of indirect quantum non-demolition measurements could be viewed as building/defining an adequate measurement apparatus [22].

Ahead of all of these speculations, it will be seen elsewhere that a cogent metaphysical argument can help us evaluate how pertinent these attempts may or may not be [11].

(ii) Second, at this level of complexity (and even before indeed), entanglement has escaped any possible form of theoretical control (See Appendix) and conceivable detection by means of *witness operators* [23] or recent analyses of *tensor-stable positive maps* [24]. So that in their practice, in order to appreciate a degree of entanglement, physicists indeed rely on guesses, and this for much smaller systems, genuinely quantum.

(iii) In order to get a unique result out of a given Schrödinger cat experiment, it is customary to base some hopes on the *decoherence* phenomenon [25]: As a matter of fact, in a

series of experimental tests carried out on ‘Schrödinger kitties’^[1], linear superpositions have been proven to exist over very short lapses, τ_{dec} [26], provided that the quantum systems are isolated enough from their environments. The problem, though, is that decoherence transforms original linear superpositions only into so-called *improper mixtures*, and the purpose of generating a unique experimental result is still not met [27]. More to the point, and moving upstream, a corollary of the analysis in [23] is that even though a Schrödinger cat state vector would stand in a superposition, before decoherence takes place, no witness operator would have a substantial chance to reveal it. This is because this impossibility is already effective at the much simpler level of an idealized ‘kitty system’, perfectly isolated, and involving just a few *q-bits* of information. Eventually, the impossibility in question, theoretically estimated in [23], is joined also by technical limits such as the finite resolution power of experimental devices [25].

(iv) Along the same line, an *a fortiori*-type of argument can be drawn out of the Badurek experiment [27]. Before even considering the case of a cat, it is instructive to remark that a simple classical field, as is the electromagnetic field cannot get entangled to a quantum system. The experiment consists in a spin-up neutron interferometer. In each of the branches of the interferometer, there is a constant magnetic field B_0 , along the direction Oz , and a field of radio-frequency $B_1(t)$, polarized along the direction Ox . The latter is oscillating at a frequency close to the Larmor frequency ω_0 which makes the spins flip. Now, if the classical field $B_1(t)$ ever got entangled to the neutron’s spins, the environment would keep track of the neutron’s trajectories in either of the two arms of the interferometer, and as a result, interference fringes would fade away.

But they don’t. Formally, it is possible to show [28] that the interaction of spins with the classical field, as described by a *coherent state*, preserves the coherence of the spin states $|up\rangle_1$ and $|up\rangle_2$, so that interference fringes are maintained, as experiment shows.

Now, what is to be noted is that the coherent (or *quasi-classical*) state which accounts for the classical field and plays the role of the Schrödinger cat, is by construction as close as possible to a quantum reality, an eigenstate of the *quantum annihilation operator*,

$$a|z\rangle = \alpha|z\rangle, \quad \alpha \in \mathcal{C} \quad (3)$$

This coherent state is made out of a very large number of quantum states, and it is precisely

[1] ‘Kitties’, because made out of only 8 intricated photons in a cavity [26].

this limit that makes the coherent state a classical field which does *not* get entangled to the neutron's spin states. Needless to insist at this stage that one is extremely far from the level of complexity of a real cat, and it is really surprising that such a significant case has so poorly been taken into account in Schrödinger's cat analyses. Equation (2) and related issues, appear then as one and the first instance of Everett parallel worlds, that is a mere academic problem, deprived of sound physical content [2].

In the end, the state of the cat is still unknown, and the box has to be opened in order to know it. That is, not much has really been improved on the previous case of interpretation A. For this and other similar reasons, the famous Copenhagen interpretation, with appropriate adaptations, has been able to survive until today [16].

C. Everett parallel worlds

Everett's interpretation of Quantum Mechanics seems to have the favor of a certain number of *cosmologists* whose considerations bear on the Universe wave-function. Admittedly, though, this approach meets quite serious, yet unsolved difficulties [19].

Everett's interpretation takes seriously the existence of Von Neumann chains, and posits that they furnish a direct and sound representation of the physical reality [19]. As can be found in a number of specialised articles, the entanglement mechanism is assumed to extend, step by step, to the whole environment [7]; up to, and included, the experimenters themselves appear to be as entangled as is the cat of the famous Schrödinger case.

To sum up, Von Neumann chains ramify and suffuse infinitely. These chains lead to a multitude of different Universes, that one is bound to think of as being parallel to our Universe. No need for any wave-function collapse mechanism that would fix the experimental result in one of two final possibilities, because all intermediate possibilities get their own Universe. They all get realised and the Von Neumann chain is not interrupted. In the end, there are as many Universes, parallel the ones to the others, as ramifications undergone by the wave-function.

A variant of this interpretation posits that the state vector ramifications are no longer

[2] Opposite points of view exist, taking Everett parallel worlds seriously. An example can be found in [29], positing Universes made out of pure mathematical relations deprived of any *substratum* on which the relations to hinge.

realised in a ‘multi-world’ production, but are realized in the observers own minds, while observers own states themselves get entangled to the other states of the Von Neumann chain. Within one and the same Everett interpretation of Quantum Mechanics, these two variants may be regarded as reminiscent of the long *epistemological-vs-ontological* issue concerning the wave-function status [11].

Now, what does all this amount to, in consideration of the cat’s affairs? Clearly, one cannot help agreeing that Everett’s interpretation is most appealing to the animal, which can freely escape to a better world in which radio-active atoms remain excited, while hammers fall down on flasks without breaking them, not to speak of entanglement to tortuous physicists minds. With Everett, one can therefore hope that the cat of Mr. Schrödinger is now purring in one of these better worlds.

However if one opens the box, either a dead or a live cat is found, and this, whatever the number of times the experiment is run. And in this way one can see dramatically enough that Everett’s parallel worlds do not seem to offer a safe escape from the terrible alternative.

One may object that within this interpretation, in another world corresponding to another ramification of the state vector (the chain), the radio-active atom did not decay, and accordingly, neither did the cat.

Now, which cat? After one hour, opening the box, a cat is found, the one that entered the box. Has the other state vector ramification generated the exact copy of Mr. Schrödinger’s cat in an alternate Universe? Were it alone, a famous theorem of quantum physics would object to that [30]. Are the state vector ramifications accompanied with new worlds spontaneous creation?.. in our immediate neighborhood?.. without us perceiving the slightest echo of those creations?

IV. CONCLUSIONS

Since it was launched by E. Schrödinger in the 1930s, the cat paradox has been the matter of an important number of publications and speculations, and this until nowadays [9, 25]. Moreover, one is now able to prepare ‘kitties’ and ‘squids’ (*superconducting quantum interference devices* involving the mesoscopic scale of some 10^6 electrons) which, though bigger and bigger, still manifest quantum behaviors such as linear superposition, entanglement and interferences [3]. E. Schrödinger himself considered entanglement the most salient feature

of quantum physics, whereas it should rather be viewed a consequence of the *superposition principle*, the only *mystery* of quantum physics, to quote R.P. Feynman's own words [9].

- At a theoretical level, it is usually believed that a resolution of generic Schrödinger cat situations would require that a theory of quantum measurement be sufficiently achieved in order to end up with some definite and unique result.

Decades of tremendous developments, though, relying on all sorts of extensions to the original linear problem, and including gravitation, have not produced convincing and universal solutions. It is then not surprising that, in a recurrent fashion in quantum physics, in addition to gravity whose unification to quantum physics is not yet available, some authors go back again to the observer's conscience in order to find a solution [31].

In view of these 'over-speculative' considerations, one may think instead that the resolution of the quantum enigma is not to be looked for in these directions, and that it may stand closer to the quantum object itself [32].

- At an epistemological level now, what seems important to notice is that entanglement displays a capacity proper to a quantum entity, to bear the *potentiality* of a property in common with another quantum entity. The notion of potentiality must be used to recall that a quantum entity does not possess its properties in an actual manner. The result of a measure doesn't pre-exist the measuring procedure, or, in other words, is not revealed by the measure as in the classical case; rather it is induced through and along the measure itself [11]. And this, the quantum entities considered being entangled or not. As is well known, and has been the matter of a great number of experiments carried out since the mid-seventies, one may think of two *elementary particles* bearing together the potentiality of a measurable degree of freedom such as the spin or the polarization. At this point, however, care must be taken not to be fooled by words.

Elementary particles are indeed plain quantum entities [12], and they do not have real and actual proper existences, aside from the experimental protocols which define them as such [33, 34]. Strictly. This is even more blatant with particles in entangled states. When their proper reality is considered, in a much deeper understanding, they must be referred to *that particular state of the quantum field associated to the particles* [35], which accounts for all of the subsequent measurements that will be described in terms of elementary particles in order to stick to a more intuitive scheme [12].

As for the cat paradox, things unravel as soon as they are taken from a perspective along

which it is the *nature* of those particles (deeply: the nature of the quantum fields associated to them) which displays a property, entanglement, to which linear algebra offers a simple and accurate mathematical formulation, so long as a few degrees of freedom are involved. As bigger numbers of degrees of freedom get involved, possibly infinite, the formalism of relativistic quantum field theories must be used, in particular that of C^* -operator algebras [36]. Quantum fields entanglement, then, reveals to be an even more significant property than that of smaller systems, ordinarily described within the standard formalism of Quantum Mechanics [36].

From a historical point of view, of course, it is true that the notion of entanglement came out of the Quantum Mechanics formalism in the first place, and was experimentally confirmed afterwards [37]. As evoked in the Introduction, this is an amazing instance of the fecundity of the formalism, an interpretation of which will be proposed elsewhere. Fecund and a place for anticipation/creativity, the formalism of Quantum Mechanics has no doubt proven to be; but not to the point of positing that the formalism would precede reality, in a somewhat platonic way.

As stated in [25] ‘.. *nothing in the QM formalism prevents one from applying it to macroscopical objects.*’, and this is what has been done with the cat in the famous paradox. But it has been done recklessly. Contrarily to quantum entities, a cat possesses the plain actuality of its determinations and there is no need to substitute operators and state vectors for numbers and functions in order to describe a cat.

Much less than a classical magnetic field to neutron’s spins, the animal state gets entangled to the radio-active atom states. The only issue for this system, considering the atom state’s superposition, is that after a given duration of time, there is a certain probability for the animal to be either dead or alive, and this is in no way indicative that the cat would have passed from a supposedly entangled state (would this be possible) to a factorized one. Final state statistics are the same in either cases [38], a quite generic situation of Quantum Mechanics, known under the name of *non-uniqueness of the preparation*.

The school case of Schrödinger, though designed as ‘perfectly burlesque’ from the onset, entails two points which are fundamental and stand at the core of the quantum enigma.

A first point is that by enunciating such a case, one relies on a hidden implicit assumption, that of an unlimited principle of *reductionism*. Clearly, equation (2) is based on this assumption. This is also stated explicitly in the citation given above, that ‘.. nothing in

the Quantum Mechanics formalism prevents one from applying it to macroscopical objects.’ That new levels of reality may have emerged between the atom and the cat, is simply not envisaged [39].

A second point has to do with three facts the paradox confronts us with. As coherence holds for the radio active atom, the atom *is* in a superposition of two states, decayed and non-decayed (f1). As coherence is over, and some state vector reduction has taken place, the atom *is* decayed (f2). As the atom has decayed, the cat *is* dead (f3). The three facts are all three *real* facts, and it is one of the merits of the Schrödinger school case to force us to understand that ‘to be’ in (f1) is as real as ‘to be’ in the second (f2) and third ones (f3). In other words, the Schrödinger’s cat paradox works as a paradigm of what is at stake with quantum reality: How is it that something real, something ‘that is’, is in the manner of a superposition? The cat is not the problem, for we know for sure that it is either alive or dead. The atom is the problem, and the formalism of Quantum Mechanics conveys this problem in the most revealing manner. It is in the very nature of a quantum reality to possess the real property of being potentially many orthogonal actualities.

Having said that, Schrödinger’s cat paradox appealed to all from the beginning because it linked a quantum reality to a classical one, pretending that since the former exerted a causality on the state of the latter, they were entangled as every other entangled quantum systems. The cat was supposed to inherit a quantum nature by the mere fact of its dependence upon a quantum behaviour. Once placed in the box, it was no more a common cat, which must be either dead or alive, but it entered a new mode of being where cats are dead and alive at the same time, in the same manner as quantum entities *are* in a state of superposition.

This of course cannot stand. However this common sense observation was obfuscated when the paradox was formulated because of the condition that the box would remain sealed for a certain period of time.

Being in a state of linear superposition is a strange mode of being compared to the actual mode of being of cats, and we must not escape the question this strange mode of being raises. It is not everyday that physics gets the privilege to confront us with such a puzzle, which is metaphysical by essence.

Acknowledgments

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Appendix A: Entanglement at the formal level

This appendix is but an adapted translation in english of a passage of a remarkable book which can be found online [17] and which is reproduced here for the only sake of convenience.

In classical physics, linear superpositions have to do with *waves* only, whereas tensorial product doesn't play any particular role. In quantum physics, just the opposite holds. The superposition principle and the tensorial product are the core of Quantum Mechanics, and beyond, of relativistic quantum theories of fields.

In classical physics a *material point*, a paradigm of classical physic, is described by a vector endowed with 6 components, 3 for the position and 3 for the velocity, and this vector belongs to the so-called *phase space* attached to this very particle. For a system comprising N particles the phase space of the global system, \mathcal{V} , say, is the *direct sum* of each of the N particles phase spaces, the \mathcal{V}_i s,

$$\mathcal{V} = \mathcal{V}_1 \oplus \mathcal{V}_2 \oplus \cdots \oplus \mathcal{V}_N, \quad (\text{A1})$$

and one infers from this that if a vector attached to a given system has $6M$ coordinates, then necessarily this system is made out of M particles.

In Quantum Mechanics instead, the tensorial product, \otimes , is substituted for the direct sum, \oplus . A system 'comprising' N particles is described in a Hilbert space \mathcal{H} given by the tensorial product of the N subspaces \mathcal{H}_i attached to each of the composing particles,

$$\mathcal{H} = \mathcal{H}_1 \otimes \mathcal{H}_2 \otimes \cdots \otimes \mathcal{H}_N, \quad (\text{A2})$$

and while one has $\dim \mathcal{V} = \sum_{i=1}^N \dim \mathcal{V}_i$, in the classical case, in Quantum Mechanics the dimension of \mathcal{H} reads $\dim \mathcal{H} = \prod_{i=1}^N \dim \mathcal{H}_i$, a number which can be much bigger.

The notion of tensorial product is at the origin of one of the most intriguing aspect of the quantum theory, or, taking things the other way round, accounts for this intriguing aspect. The state of a composed quantum system does not define the state of its components. This

comes in contradistinction to the classical case where a $6M$ component vector allows one to define the positions and velocities of any of its components. In the quantum theory, the vector state of a composed system is described by a vector of the space $\mathcal{H} \otimes \mathcal{H}'$, tensorial product of the spaces of states \mathcal{H} and \mathcal{H}' of the two composing sub-systems. Now in most cases, a vector $\omega \in \mathcal{H} \otimes \mathcal{H}'$ cannot be written as $v \otimes v'$ with $v \in \mathcal{H}$ and $v' \in \mathcal{H}'$, that is into a *factorised* form.

The consequence is that knowing ω , the composed system state, does not allow one to associate well defined states of the two composing sub-systems.

The necessity of considering the composed system globally corresponds to a deep and intrinsic coupling of the sub-systems which is a characteristic of their quantum nature, irrespective of any particular form of interaction between them. This is sometimes called *quantum non-separability*, it is experimentally verified and it is remarkable that the tensorial product mathematics allows one to provide this fact with a simple and cogent expression.

Let there be \mathcal{H} et \mathcal{H}' two vectorial spaces and $\mathcal{H} \otimes \mathcal{H}'$ their tensorial product. It is endowed with the following properties. It is a vectorial space whose dimension is NN' . If $\{|i_H\rangle\}$ et $\{|j_{H'}\rangle\}$ are two orthonormal bases of \mathcal{H} and \mathcal{H}' , the couples of vectors $|i_H\rangle \otimes |j_{H'}\rangle := |i_H \otimes j_{H'}\rangle$ form an orthonormal basis of $\mathcal{H} \otimes \mathcal{H}'$.

Considering $|\varphi\rangle \in \mathcal{H}$ and $|\chi\rangle \in \mathcal{H}'$, then $|\varphi\rangle = \sum_{i=1}^N c_i |i_H\rangle$ and $|\chi\rangle = \sum_{j=1}^{N'} c_j |j_{H'}\rangle$, and their tensorial product reads,

$$|\varphi\rangle \otimes |\chi\rangle = \sum_{i,j} c_i c_j |i_H \otimes j_{H'}\rangle \quad (\text{A3})$$

and this state is of course a factorised state, by construction. Now, given ω , a state in $\mathcal{H} \otimes \mathcal{H}'$, one has,

$$|\omega\rangle = \sum_{i,j} b_{ij} |i_H \otimes j_{H'}\rangle \quad (\text{A4})$$

and in the general case there does not exist series of numbers, c_i and c_j , $1 \leq i \leq N$, $1 \leq j \leq N'$ which would allow one to write $|\omega\rangle$ as the product $\sum_i c_i |i_H\rangle \otimes \sum_{j=1}^{N'} c_j |j_{H'}\rangle$. For this to be possible in effect certain relations between these series of numbers must be satisfied. For example at $N = N' = 2$ a necessary and sufficient condition is that the relation $b_{11}b_{22} - b_{12}b_{21} = 0$ be satisfied. More generally, if $v = \sum_n c_n |u_n\rangle$ and $v' = \sum_l c'_l |u_l\rangle$, then $v \otimes v' = \sum_{n,l} c_n c'_l |u_n \otimes u'_l\rangle$. Let us consider another couple of vectors w and w' and their tensorial product $w \otimes w' = \sum_{n,l} d_n d'_l |u_n \otimes u'_l\rangle$. The sum of these two tensorial products is a

vector which reads as $\sum_{n,l}(c_n c'_l + d_n d'_l)|u_n \otimes u'_l\rangle$, and the point is that its coefficients cannot be factorised, $c_n c'_l + d_n d'_l \neq f_n f'_l$.

The factorised vectors are a small part only of $\mathcal{H} \otimes \mathcal{H}'$. Factorised vectors in effect are defined by their components, N for the part in \mathcal{H} and N' for the part in \mathcal{H}' , that is $N + N'$ parameters define a factorised vector, instead of $N \times N'$ in the general case of a vector in $\mathcal{H} \otimes \mathcal{H}'$. Even less than $N + N'$ indeed, as the example of $N = N' = 2$ has shown, since not all of the $N + N'$ components are independent. In this very case in effect, the factorised vectors belong to a space of dimension $2 + 2 - 1 = 3 (= N + N' - 1)$, which is not a linear space but a *manifold* (as we have just seen in the general case, the sum of two factorised vectors is not a factorised vector).

An interesting enough theorem allows one to *quantify* how much a given vector of $\mathcal{H} \otimes \mathcal{H}'$ differs from a factorised vector.

Theorem (Schmidt basis) Any vector $|\omega\rangle \in \mathcal{H} \otimes \mathcal{H}'$ determines two orthonormal bases, $\{|\omega_k\rangle, k = 1, \dots, N\}$ in \mathcal{H} and $\{|\omega'_l\rangle, l = 1, \dots, N'\}$ in \mathcal{H}' , such that the decomposition of ω on the tensorial basis $\{|\omega_k \otimes \omega'_l\rangle\}$ is diagonal,

$$|\omega\rangle = \sum_{k=1}^{\nu} d_k |\omega_k\rangle \otimes |\omega'_k\rangle, \quad d_k \in \mathbb{R}^+, \quad \nu \leq \min(N, N') \quad (\text{A5})$$

If the vector is normed, one has also, $\sum_{k=1}^{\nu} d_k^2 = 1$. Ordering the values of the components d_k in a decreasing way, $d_1 \geq d_2 \geq \dots \geq d_{\nu}$, one can see that the factorised vector $d_1 |\omega_1 \otimes \omega'_1\rangle$ represents the best approximation of ω by a factorised vector, and in view of the relation $1/\nu \leq d_1^2 \leq 1$, it is possible to define *the degree of entanglement* of a given vector, by introducing the number,

$$\delta(|\omega\rangle) = \frac{1 - d_1^2}{1 - 1/\nu} \quad (\text{A6})$$

This degree varies between 0 for a factorised vector, and 1 for a *maximally entangled* vector for which one has $d_1^2 = d_2^2 = \dots = d_{\nu}^2 = 1/\nu$. The Schmidt basis is unique only if the entanglement coefficients, d_k have unequal absolute values.

Unfortunately the theorem above does not really extend beyond the case of the product of two Hilbert spaces, and such a convenient measure of entanglement as the one offered by (A6) is missing in more involved situations. This is why physicists have devised other, more sophisticated ways of appreciating how much a given state vector may be entangled in more involved quantum systems [23, 24]. In these latter instances, though, entanglement cannot

be measured as accurately as it is measured by (A6) in the case of tensorial products of only two finite dimensional quantum sub-systems.

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