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Sophie Thery, Eric Blayo, Florian Lemarié. Schwarz algorithms for ocean-atmosphere coupled problems including turbulent boundary layer parameterizations. DD26 2020 - 26th International Domain Decomposition Conference, Dec 2020, Hong Kong, China. pp.1-34. hal-03173266

HAL Id: hal-03173266

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Submitted on 18 Mar 2021

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Schwarz algorithms for ocean-atmosphere coupled problems including turbulent boundary layer parameterizations

Sophie THERY

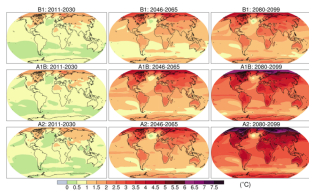
Univ. Grenoble Alpes, Lab. Jean Kuntzmann, Inria AIRSEA team

DD26, December 9th 2020

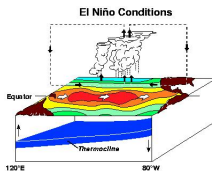
PhD under the supervision of Eric Blayo and Florian Lemarié

Application of ocean-atmosphere coupling

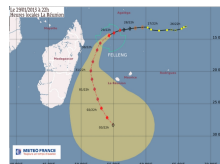
Various physical phenomena are governed by the ocean-atmosphere coupling: long term predictions to short term predictions.



climate modeling



seasonal forecasts

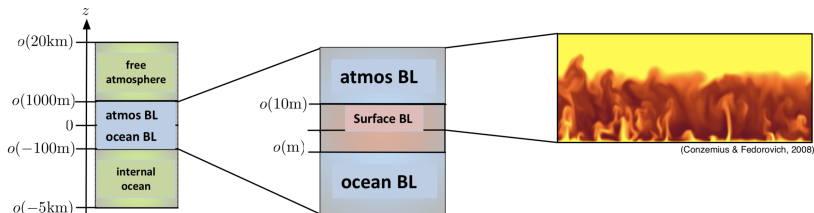


short term predictions

Improve the representation of ocean-atmosphere interactions

Complexity of ocean-atmosphere fluxes

- Turbulente Boundary layer \rightarrow complex parametrization.

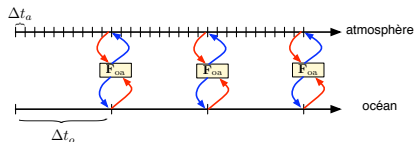


- Near the interface : fluxes estimated by (complicated) formulas depending on the jump of the solution.

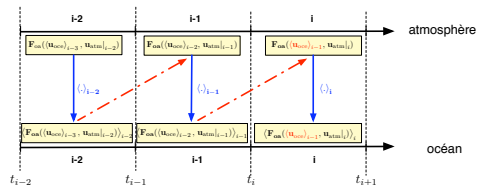
The ocean-atmosphere coupling algorithms

Two current approaches, both mathematically unsatisfactory:

Synchronous coupling at the time step
(local in time)



Asynchronous coupling by time windows
(global in time)



- a lot of communication \Rightarrow inefficient implementations
- physical validity and numerical stability issues **Lemarié & al. (2015)**, **Beljaars & al. (2017)**

- balance of the average flows over each time window
- synchronization problem

Motivations

Practical implementations for ocean-atmosphere coupling algorithms are mathematically unsatisfactory.

Objectives : improve the mathematical coupling methods

Motivations

Practical implementations for ocean-atmosphere coupling algorithms are mathematically unsatisfactory.

Objectives : improve the mathematical coupling methods

A numerical method that would solve these problems

⇒ Schwarz algorithms

French COCOA ANR Project: Study of an iterative process on ocean-atmosphere coupling. In particular

- Implementation of Schwarz algorithms in realistic climate models
- Theoretical work on these algorithms in this context.

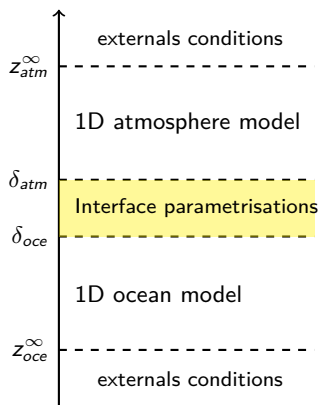
A 1D Simplified coupled ocean-atmosphere model

Hypotheses :

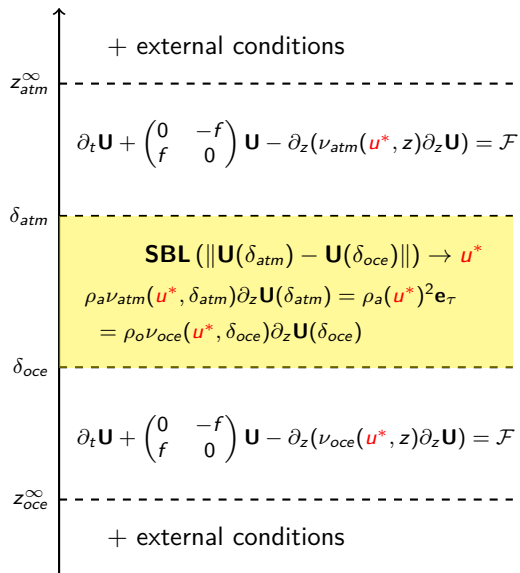
- Focus on the dynamical part
- Physical restriction (1D)
- Taking into account turbulent parametrisations

A reasonably realistic model on $\mathbf{U} = (u, v)^T$ horizontal ocean/atmosphere currents.

Interface is a buffer zone with its own parameterization.



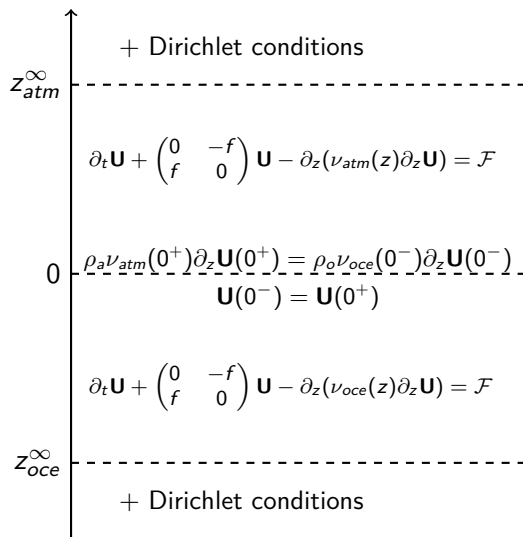
Non-linear coupled ocean-atmosphere model



ocean atmosphere specificities :

- Coriolis effect
- non constants viscosities
- non linear equation
- non linear interface condition

Linear coupled ocean-atmosphere model



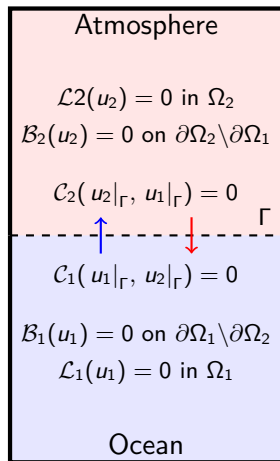
\Rightarrow **Ekman Problem**
 widely used by physicists:
Ekman (1905),
Madsen (1977),
Grisogono (1995),
Lewis & Belcher (2004)

Linear problem specificities :

- Coriolis effect
- non constants viscosities

- 1 Ocean atmosphere coupling
- 2 Schwarz algorithm and OA specificities
- 3 Impact of Coriolis effect
- 4 Impact of non-constants viscosities
- 5 Particular case OA coupling
- 6 Current and futur work

Schwarz algorithm



first guess u_2^0 then

$$\left\{ \begin{array}{ll} \mathcal{L}_1 u_1^n = \mathcal{F}_1 & \text{on } \Omega_1 \times]0, T[\\ \mathcal{B}_1 u_1^n = \mathcal{G}_1 & \text{on } \partial\Omega_1^{\text{ext}} \times]0, T[\\ u_1^n(t=0) = u_0 & \text{on } \Omega_1 \\ \mathcal{C}_{1,1} u_1^n = \mathcal{C}_{1,2} u_2^{n-1} & \text{on } \Gamma \end{array} \right.$$

$$\left\{ \begin{array}{ll} \mathcal{L}_1 u_2^n = \mathcal{F}_2 & \text{on } \Omega_2 \times]0, T[\\ \mathcal{B}_2 u_2^n = \mathcal{G}_2 & \text{on } \partial\Omega_2^{\text{ext}} \times]0, T[\\ u_2^n(t=0) = u_0 & \text{on } \Omega_2 \\ \mathcal{C}_{2,2} u_2^n = \mathcal{C}_{2,1} u_1^n & \text{on } \Gamma \end{array} \right.$$

Convergence factor for linear problems

$$\rho^{obs} = \frac{\|e_j^n(z=0)\|_2}{\|e_j^{n-1}(z=0)\|_2} \quad e_j^n = u_j^n - u^{exact}$$

- 1D Stationary case : solve the equation for each iteration
- 1D Non-stationary case : use Fourier transform in time

$$\Rightarrow \rho(\omega) = \frac{|\widehat{e}_j^n(\omega, 0)|}{|\widehat{e}_j^{n-1}(\omega, 0)|}$$

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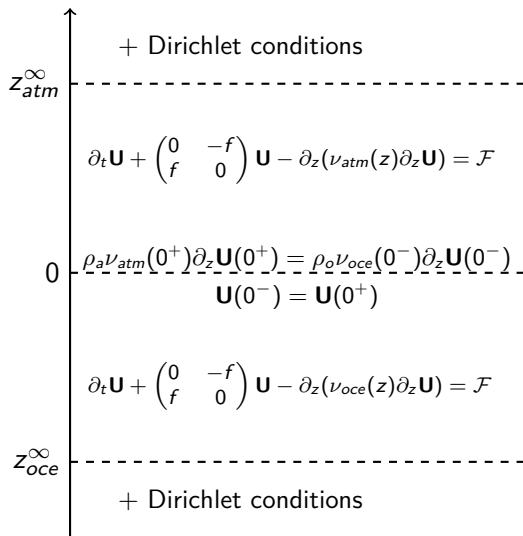
- 1D Stationary case : solve the equation for each iteration
- 1D Non-stationary case : use Fourier transform in time

$$\Rightarrow \rho(\omega) = \frac{|\widehat{e}_j^n(\omega, 0)|}{|\widehat{e}_j^{n-1}(\omega, 0)|}$$

$$\min_{\omega_{min} \leq |\omega| \leq \omega_{max}} \rho(\omega) \leq \rho^{obs} \leq \max_{\omega_{min} \leq |\omega| \leq \omega_{max}} \rho(\omega)$$

$$\text{Discretized-in-time algorithm: } \omega_{max} = \frac{\pi}{\Delta t} \text{ and } \omega_{min} = \frac{\pi}{T}$$

Coupled Ekman problems



⇒ Specificities :

- Coriolis effect (\mathbf{U} components are coupled)
- Non constant diffusion coefficients with interface discontinuity

State of the art

		Constante diffusion	Variable in space diffusion
Stationary		<i>adv-diff 2D</i> Japhet et al., 2001 <i>eq. Helmholtz</i> Dubois, 2007 Magoulès et al., 2004	<i>eq. de diffusion</i> Lions, 1990
Nonstationary	without Coriolis	<i>heat equation</i> Gander and Halpern, 2003 <i>reaction-reaction-diff 2D</i> Bennequin et al., 2016 and Gander et al., 2007	<i>diffusion 1D eq.</i> Lemarié et al., 2013
	with Coriolis	<i>2D shallow water</i> Martin, 2003 <i>primitives eq. 3D</i> Audusse et al., 2009	<i>diffusion 1D eq. + Coriolis</i> They et al., 2020

Impact of the Coriolis effect : example with constant viscosities

- Coriolis effect \rightarrow coupling $\mathbf{U} = \begin{pmatrix} u \\ v \end{pmatrix}$ components

$$\left\{ \begin{array}{l} \partial_t \mathbf{U}_j + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \mathbf{U}_j - \nu_j \partial_z^2 \mathbf{U}_j = \mathcal{F}_j \\ + \text{Dirichlet external conditions} \\ + \text{Initial conditions} \\ \mathbf{U}_1(0^-) = \mathbf{U}_2(0^+) \\ \nu_1(0^-) \partial_z \mathbf{U}_1 = \nu_2(0^+) \partial_z \mathbf{U}_2 \end{array} \right.$$

Impact of the Coriolis effect : example with constant viscosities

- Coriolis effect \rightarrow coupling $\mathbf{U} = \begin{pmatrix} u \\ v \end{pmatrix}$ components
- Study of the convergence with change of variable $\varphi = u + iv$.

$$\left\{ \begin{array}{l} \partial_t \varphi_j + i f \varphi_j - \nu_j \partial_z^2 \varphi_j = \mathcal{F}_{\varphi_j} \\ + \text{Dirichlet external conditions} \\ + \text{Initial conditions} \\ \varphi_1(0^-) = \varphi_2(0^+) \\ \nu_1(0^-) \partial_z \varphi_1 = \nu_2(0^+) \partial_z \varphi_2 \end{array} \right.$$

Impact of the Coriolis effect : exemple with constant viscosities

- Coriolis effect \rightarrow coupling $\mathbf{U} = \begin{pmatrix} u \\ v \end{pmatrix}$ components
- Study of the convergence with change of variable $\varphi = u + iv$.
- Study the convergence on the error e_j

$$\left\{ \begin{array}{l} \partial_t e_j + i f e_j - \nu_j \partial_z^2 e_j = 0 \\ + \text{Dirichlet external conditions} \\ + \text{Null initial condition} \\ e_1(0^-) = e_2(0^+) \\ \nu_1 \partial_z e_1 = \nu_2 \partial_z e_2 \end{array} \right.$$

Impact of the Coriolis effect : example with constant viscosities

- Coriolis effect \rightarrow coupling $\mathbf{U} = \begin{pmatrix} u \\ v \end{pmatrix}$ components
- Study of the convergence with change of variable $\varphi = u + iv$.
- Study the convergence on the error e_j
- Fourier Transform \Rightarrow frequencies shifted by f :

$$\begin{cases} i(\omega + f)\hat{e}_j - \nu_j \partial_z^2 \hat{e}_j = 0 \\ + \text{Dirichlet external conditions} \\ \hat{e}_1(0^-) = \hat{e}_2(0^+) \\ \nu_1 \partial_z \hat{e}_1 = \nu_2 \partial_z \hat{e}_2 \end{cases}$$

With Dirichlet-Neumann interface conditions

$$\begin{cases} \hat{e}_1^n(\omega, 0) = \hat{e}_2^{n-1}(\omega, 0) \\ \nu_2 \partial_z \hat{e}_2^n(\omega, 0) = \nu_1 \partial_z \hat{e}_1^n(\omega, 0) \end{cases}$$

Convergence factor

- Infinite domains:

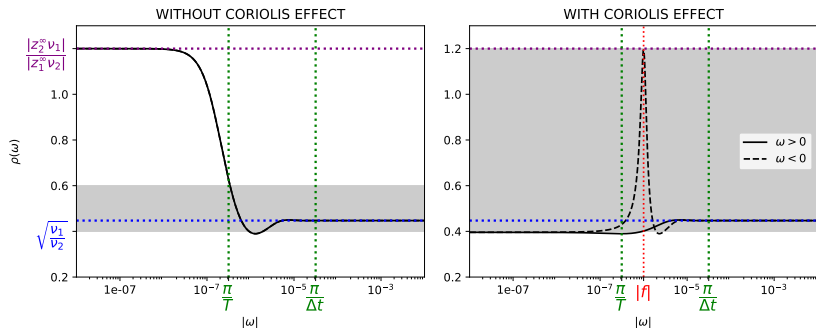
$$\rho_{DN}^{cst}(\omega) = \sqrt{\frac{\nu_1}{\nu_2}}$$

Independent of time frequency

- Finite domains:

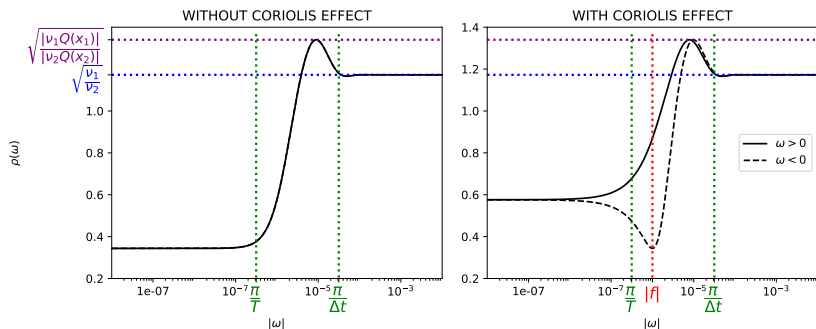
$$\rho_{DN}^{cst}(\omega) = \sqrt{\frac{\nu_1}{\nu_2}} \left| \frac{\tanh \left(z_2^\infty \sqrt{i \frac{\omega + f}{\nu_2}} \right)}{\tanh \left(z_1^\infty \sqrt{i \frac{\omega + f}{\nu_1}} \right)} \right|$$

Case 1 : $|z_2^\infty \sqrt{\nu_1}| > |z_1^\infty \sqrt{\nu_2}|$



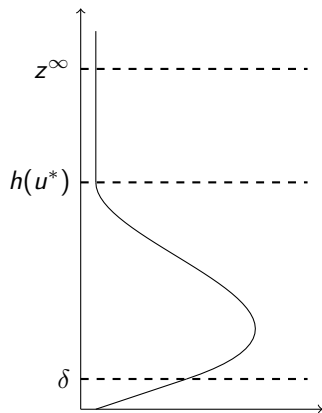
- Convergence factor behavior : $|\rho(\omega)| < \frac{z_2^\infty \nu_1}{z_1^\infty \nu_2}$ and $\rho(\omega) \xrightarrow{|\omega| \rightarrow \infty} \sqrt{\frac{\nu_1}{\nu_2}}$
- Impact of **Coriolis** : shifts the local maximum and non symmetric graph

Case 2 : $|z_2^\infty \sqrt{\nu_1}| < |z_1^\infty \sqrt{\nu_2}|$



- Convergence factor behavior : $|\rho(\omega)| < \sqrt{\frac{\nu_1 Q(x_1)}{\nu_2 Q(x_2)}}$ and $\rho(\omega) \xrightarrow{|\omega| \rightarrow \infty} \sqrt{\frac{\nu_1}{\nu_2}}$ with $Q(x) = |\tanh((1+i)x)|$ and x_1, x_2 solution of the transcendante equation
- Impact of **Coriolis** : shifts the local minimum and non symmetric graph

The effect of turbulence



Parametrisation of turbulence \Rightarrow non constant viscosity

In ocean-atmosphere context :
KPP viscosity (O'Brien, 1970)

- affine profile close to the surface
- parabolic or cubic profile in the turbulent zone
- constant profile in free zone

\Rightarrow convergence for variable viscosity profile

Convergence factor with non-constants viscosities

$$i(f + \omega)\hat{e}_j(z, t) - \partial_z(\nu_j(z)\partial_z\hat{e}_j(z, t)) = 0$$

Mathematical tools to calculate converge :

- with $\nu_j(z) = a_j z + b_j \rightarrow$ Bessel's functions
- with $\nu_j(z) = a_j z^2 + b_j z + c_j \rightarrow$ Legendre polynomials.

\Rightarrow The convergence factor depends on the global viscosity profile

Convergence factor with non-constants viscosities

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Dirichlet-Neumann interface conditions

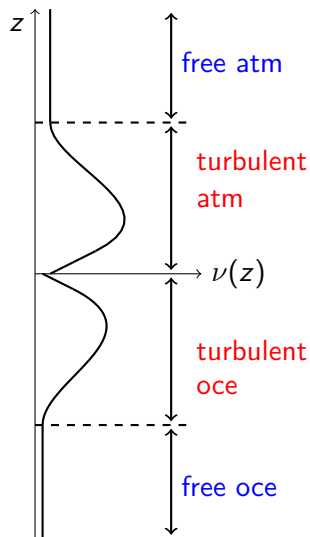
$$\rho_{DN}(\omega) \xrightarrow{|\omega| \rightarrow \infty} \sqrt{\frac{\nu_1(0)}{\nu_2(0)}} \quad \text{for all viscosities profiles}$$

$$\rho_{DN}^{aff}(\omega) \leq \frac{\rho_{DN}^{cst}(\omega)}{\mu_1 \ln(1 + 1/\mu_1)}$$

$$\rho_{DN}^{par}(\omega) \leq \frac{\rho_{DN}^{cst}(\omega)}{4\mu_1 \arccos(\sqrt{1 + 4\mu_1})}$$

$$\text{with } \mu_1 = \left| \frac{\nu_1(0)}{\partial_z \nu_1(0) z_1^\infty} \right|$$

The particular case ocean-atmosphere coupling



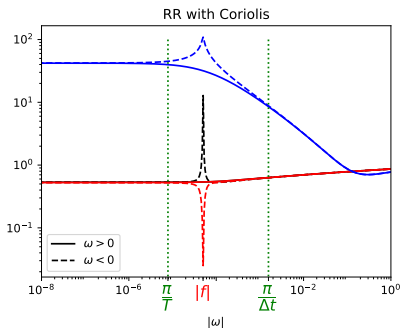
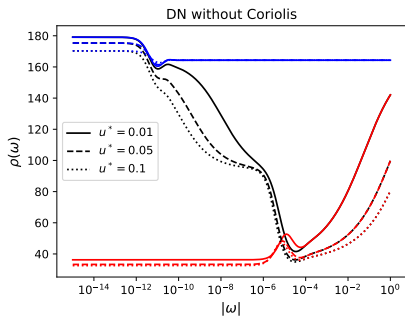
Stationary case for any interface condition :

- without Coriolis \rightarrow **free zones** have a huge influence
- with Coriolis \rightarrow **turbulent zones** have a bigger influence

The particular case ocean-atmosphere coupling

Non-Stationary case for any interface condition :

- $|\omega + f| < 10^{-11}$ → influenced by **free zone**
- $|\omega + f| > 10^{-5}$ → influenced by **turbulent zone**



Example : convergence for all frequencies except frequencies close to $-f$

Conclusion for linear problems

- Impact of Coriolis effect : shift of the graph
⇒ perturbation of the algorithm's behavior
 - Convergence factor depends of the global viscosities profile
⇒ mathematical tools to calculate the convergence factor (Bessel and Legendre functions)
⇒ simplification must be made with caution
- ⇒ Results are shown in : *S. Thery, C. Pelletier, F. Lemarié and Blayo E., 2020: Coupling two Ekman layers with a Schwarz algorithm. under review*

Conclusion for linear problems

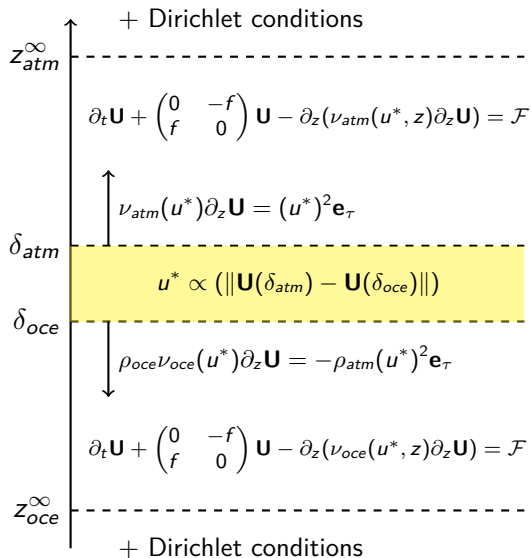
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Ocean-atmosphere coupling

The Coriolis effect and the turbulence zones have a big impact on the convergence

The non-linear model



Application of Schwarz algorithms on non-linear model

- Study of the well posedness of the problem
- Study of convergence of the algorithm

Current work on the non-linear model

Stationary case : using tools from linear problem

- unique solution consistent with the physical constraints
- without Coriolis effect : fast divergence & free zones have a huge influence
- with Coriolis effect : fast convergence & turbulent zones have a bigger influence

Current work on the non-linear model

Stationary case : using tools from linear problem

- unique solution consistent with the physical constraints
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Non stationary case :

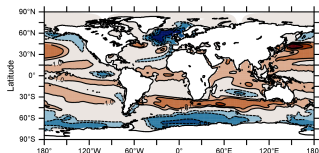
No theoretical method for solve this problem

⇒ experimental results ⇒ similarities with linear problem behavior :

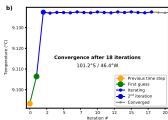
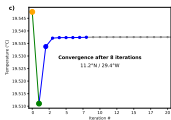
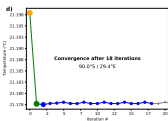
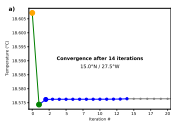
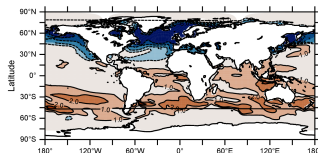
- unique solution consistent with the physical constraints
- if $\omega_{\min} \leq |f| \leq \omega_{\max} \rightarrow$ divergence
- if $|f| \leq \omega_{\min}$ or $\omega_{\max} \leq |f| \rightarrow$ convergence

Test on a real model

a) January SST (CM6-SW-LR minus CM5A2)



b) July SST (CM6-SW-LR minus CM5A2)



IPSL-CM (3D)

- test on a climate model (3D)
- Convergence in two iterations for 90% points

Marti, O., Nguyen, S., Braconnot, P., Valcke, S., Lemarié, F., and Blayo, E.: A Schwarz iterative method to evaluate ocean-atmosphere coupling schemes. Implementation and diagnostics in IPSL-CM6-SW-VLR, *Geosci. Model Dev. Discuss.*, <https://doi.org/10.5194/gmd-2020-307>, 2020

Thank you