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Schwarz algorithms for ocean-atmosphere coupled problems including turbulent boundary layer parameterizations

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DD26, December 9th 2020

PhD under the supervision of Eric Blayo and Florian Lemarié
Application of ocean-atmosphere coupling

Various physical phenomena are governed by the ocean-atmosphere coupling: long term predictions to short term predictions.

climate modeling  seasonal forecasts  short term predictions

Improve the representation of ocean-atmosphere interactions
Complexity of ocean-atmosphere fluxes

- Turbulente Boundary layer $\rightarrow$ complex parametrization.

- Near the interface: fluxes estimated by (complicated) formulas depending on the jump of the solution.
The ocean-atmosphere coupling algorithms

Two current approaches, both mathematically unsatisfactory:

- Synchronous coupling at the time step (local in time)
  - a lot of communication ⇒ inefficient implementations
  - physical validity and numerical stability issues Lemarié & al. (2015), Beljaars & al. (2017)

- Asynchronous coupling by time windows (global in time)
  - balance of the average flows over each time window
  - synchronization problem
Motivations

Practical implementations for ocean-atmosphere coupling algorithms are mathematically unsatisfactory.

**Objectives**: improve the mathematical coupling methods
Motivations

Practical implementations for ocean-atmosphere coupling algorithms are mathematically unsatisfactory.

**Objectives** : improve the mathematical coupling methods

A numerical method that would solve these problems

⇒ Schwarz algorithms

**French COCOA ANR Project**: Study of an iterative process on ocean-atmosphere coupling. In particular

- Implementation of Schwarz algorithms in realistic climate models
- Theoretical work on these algorithms in this context.
A 1D Simplified coupled ocean-atmosphere model

Hypotheses:
- Focus on the dynamical part
- Physical restriction (1D)
- Taking into account turbulent parametrisations

A reasonably realistic model on $U = (u, v)^T$ horizontal ocean/atmosphere currents.

Interface is a buffer zone with its own parameterization.
Non-linear coupled ocean-atmosphere model

\[ \partial_t \mathbf{U} + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \mathbf{U} - \partial_z (\nu_{atm}(u^*, z) \partial_z \mathbf{U}) = \mathcal{F} \]

\[ SBL (\| \mathbf{U}(\delta_{atm}) - \mathbf{U}(\delta_{oce}) \|) \rightarrow u^* \]

\[ \rho_a \nu_{atm}(u^*, \delta_{atm}) \partial_z \mathbf{U}(\delta_{atm}) = \rho_a (u^*)^2 e_T \]

\[ = \rho_o \nu_{oce}(u^*, \delta_{oce}) \partial_z \mathbf{U}(\delta_{oce}) \]

Ocean atmosphere specificities:
- Coriolis effect
- Non constants viscosities
- Non linear equation
- Non linear interface condition

+ External conditions
Linear coupled ocean-atmosphere model

\[ \partial_t u + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} u - \partial_z (\nu_{atm}(z) \partial_z u) = F \]

\[ \rho_a \nu_{atm}(0^+) \partial_z u(0^+) = \rho_o \nu_{oce}(0^-) \partial_z u(0^-) \]

\[ u(0^-) = u(0^+) \]

\[ \partial_t u + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} u - \partial_z (\nu_{oce}(z) \partial_z u) = F \]

\[ + \text{ Dirichlet conditions} \]

\[ \Rightarrow \text{Ekman Problem} \]

widely used by physicists:

- \( \text{Ekman (1905)} \)
- \( \text{Madsen (1977)} \)
- \( \text{Grisogono (1995)} \)
- \( \text{Lewis & Belcher (2004)} \)

Linear problem specificities:

- Coriolis effect
- non constants viscosities
1. Ocean atmosphere coupling
2. Schwarz algorithm and OA specificities
3. Impact of Coriolis effect
4. Impact of non-constants viscosities
5. Particular case OA coupling
6. Current and future work
Schwarz algorithm

\[ \begin{align*}
L_2(u_2) &= 0 \text{ in } \Omega_2 \\
B_2(u_2) &= 0 \text{ on } \partial\Omega_2 \setminus \partial\Omega_1 \\
C_2(u_2|_\Gamma, u_1|_\Gamma) &= 0 \\
\Gamma \\
C_1(u_1|_\Gamma, u_2|_\Gamma) &= 0 \\
B_1(u_1) &= 0 \text{ on } \partial\Omega_1 \setminus \partial\Omega_2 \\
L_1(u_1) &= 0 \text{ in } \Omega_1
\end{align*} \]

First guess \( u_2^0 \) then

\[ \begin{align*}
L_1 u_1^n &= F_1 & \text{on } \Omega_1 \times ]0, T[ \\
B_1 u_1^n &= G_1 & \text{on } \partial\Omega_1^{\text{ext}} \times ]0, T[ \\
u_1^n(t = 0) &= u_0 \\
C_{1,1} u_1^n &= C_{1,2} u_2^{n-1} & \text{on } \Gamma
\end{align*} \]

\[ \begin{align*}
L_2 u_2^n &= F_2 & \text{on } \Omega_2 \times ]0, T[ \\
B_2 u_2^n &= G_2 & \text{on } \partial\Omega_2^{\text{ext}} \times ]0, T[ \\
u_2^n(t = 0) &= u_0 \\
C_{2,2} u_2^n &= C_{2,1} u_1^n & \text{on } \Gamma
\end{align*} \]
Convergence factor for linear problems

\[ \rho^{obs} = \frac{\| e_j^n(z = 0) \|_2}{\| e_j^{n-1}(z = 0) \|_2} \quad e_j^n = u_j^n - u^{exact} \]

- 1D Stationary case: solve the equation for each iteration
- 1D Non-stationary case: use Fourier transform in time

\[ \Rightarrow \rho(\omega) = \frac{\left| \hat{e}_j^n(\omega, 0) \right|}{\left| \hat{e}_j^{n-1}(\omega, 0) \right|} \]
Schwarz algorithm and OA specificities

Reminder on Schwarz algorithm

Convergence factor for linear problems

\[ \rho^{obs} = \frac{\|e^n_j(z = 0)\|_2}{\|e^n_{j-1}(z = 0)\|_2} \quad e^n_j = u^n_j - u^{exact} \]

- 1D Stationary case: solve the equation for each iteration
- 1D Non-stationary case: use Fourier transform in time

\[ \Rightarrow \rho(\omega) = \frac{|\hat{e}^n_j(\omega, 0)|}{|\hat{e}^{n-1}_j(\omega, 0)|} \]

\[ \min_{\omega_{\min} \leq |\omega| \leq \omega_{\max}} \rho(\omega) \leq \rho^{obs} \leq \max_{\omega_{\min} \leq |\omega| \leq \omega_{\max}} \rho(\omega) \]

Discretized-in-time algorithm: \( \omega_{\max} = \frac{\pi}{\Delta t} \) and \( \omega_{\min} = \frac{\pi}{T} \)
Coupled Ekman problems

\[ \partial_t U + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} U - \partial_z (\nu_{atm}(z) \partial_z U) = F \]

\[ \rho_{atm}(0^+) \partial_z U(0^+) = \rho_{oce}(0^-) \partial_z U(0^-) \]

\[ U(0^-) = U(0^+) \]

\[ \partial_t U + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} U - \partial_z (\nu_{oce}(z) \partial_z U) = F \]

\[ \Rightarrow \text{Specificities:} \]

- Coriolis effect (\(U\) components are coupled)
- Non constant diffusion coefficients with interface discontinuity

+ Dirichlet conditions
# State of the art

<table>
<thead>
<tr>
<th></th>
<th>Constante diffusion</th>
<th>Variable in space diffusion</th>
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<td><strong>Stationary</strong></td>
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|                | *adv-diff 2D* Japhet et al., 2001  
*eq. Helmholtz* Dubois, 2007 Magoulès et al., 2004 | *eq. de diffusion* Lions, 1990 |
| **Nonstationary** | *heat equation* Gander and Halpern, 2003  
*reaction-reaction-diff 2D* Bennequin et al., 2016 and Gander et al., 2007 | *diffusion 1D eq.* Lemarié et al., 2013 |
| without Coriolis | *2D shallow water* Martin, 2003 *primitives eq.* 3D Audusse et al., 2009 |                            |
| with Coriolis    |                     | *diffusion 1D eq. + Coriolis* Thery et al., 2020 |
Impact of the Coriolis effect: example with constant viscosities

- Coriolis effect → coupling \( \mathbf{U} = \begin{pmatrix} u \\ v \end{pmatrix} \) components

\[
\begin{align*}
\partial_t \mathbf{U}_j + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} \mathbf{U}_j - \nu_j \partial_z^2 \mathbf{U}_j &= \mathbf{F}_j \\
\text{+ Dirichlet external conditions} \\
\text{+ Initial conditions} \\
\mathbf{U}_1(0^-) &= \mathbf{U}_2(0^+) \\
\nu_1(0^-) \partial_z \mathbf{U}_1 &= \nu_2(0^+) \partial_z \mathbf{U}_2
\end{align*}
\]
Impact of the Coriolis effect: example with constant viscosities

- Coriolis effect → coupling $\mathbf{U} = \begin{pmatrix} u \\ v \end{pmatrix}$ components
- Study of the convergence with change of variable $\varphi = u + iv$.

$$\begin{cases}
\partial_t \varphi_j + if \varphi_j - \nu_j \partial_z^2 \varphi_j = \mathcal{F}_\varphi_j \\
\text{+ Dirichlet external conditions} \\
\text{+ Initial conditions} \\
\varphi_1(0^-) = \varphi_2(0^+) \\
\nu_1(0^-) \partial_z \varphi_1 = \nu_2(0^+) \partial_z \varphi_2
\end{cases}$$
Impact of the Coriolis effect: example with constant viscosities

- Coriolis effect $\rightarrow$ coupling $\mathbf{U} = \begin{pmatrix} u \\ v \end{pmatrix}$ components
- Study of the convergence with change of variable $\varphi = u + iv$.
- Study the convergence on the error $e_j$

\[
\partial_t e_j + if e_j - \nu_j \partial_z^2 e_j = 0
\]

+ Dirichlet external conditions
+ Null initial condition
\[
e_1(0^-) = e_2(0^+)
\]
\[
\nu_1 \partial_z e_1 = \nu_2 \partial_z e_2
\]
Impact of the Coriolis effect: example with constant viscosities

- Coriolis effect → coupling $\mathbf{U} = \begin{pmatrix} u \\ v \end{pmatrix}$ components
- Study of the convergence with change of variable $\varphi = u + iv$.
- Study the convergence on the error $e_j$
- Fourier Transform $\Rightarrow$ frequencies shifted by $f$:

$$i(\omega + f)\hat{e}_j - \nu_j \partial_z^2 \hat{e}_j = 0$$
+ Dirichlet external conditions

$$\hat{e}_1(0^-) = \hat{e}_2(0^+)$$
$$\nu_1 \partial_z \hat{e}_1 = \nu_2 \partial_z \hat{e}_2$$
With Dirichlet-Neumann interface conditions

\[
\begin{align*}
\hat{e}_1^n(\omega, 0) &= \hat{e}_{n-1}^n(\omega, 0) \\
\nu_2 \partial_z \hat{e}_2^n(\omega, 0) &= \nu_1 \partial_z \hat{e}_1^n(\omega, 0)
\end{align*}
\]

Convergence factor

- **Infinite domains:**
  \[
  \rho_{cst}^{DN}(\omega) = \sqrt{\frac{\nu_1}{\nu_2}}
  \]
  Independent of time frequency

- **Finite domains:**
  \[
  \rho_{cst}^{DN}(\omega) = \sqrt{\frac{\nu_1}{\nu_2}} \left| \frac{\tanh \left( z_2^\infty \sqrt{i \frac{\omega + f}{\nu_2}} \right)}{\tanh \left( z_1^\infty \sqrt{i \frac{\omega + f}{\nu_1}} \right)} \right|
  \]
Case 1: $|z_2^\infty \sqrt{\nu_1}| > |z_1^\infty \sqrt{\nu_2}|$

- Convergence factor behavior: $|\rho(\omega)| < \frac{z_2^\infty \nu_1}{z_1^\infty \nu_2}$ and $\rho(\omega) \xrightarrow{|\omega| \to \infty} \sqrt{\frac{\nu_1}{\nu_2}}$

- Impact of Coriolis: shifts the local maximum and non symmetric graph
Case 2 : $|z_2^\infty \sqrt{\nu_1}| < |z_1^\infty \sqrt{\nu_2}|$

**Convergence factor behavior** : $|\rho(\omega)| < \sqrt{\frac{\nu_1}{\nu_2}} \frac{Q(x_1)}{Q(x_2)}$ and $\rho(\omega) \rightarrow \sqrt{\frac{\nu_1}{\nu_2}}$ as $|\omega| \rightarrow \infty$

with $Q(x) = |\tanh((1 + i)x)|$ and $x_1, x_2$ solution of the transcendental equation

**Impact of Coriolis** : shifts the local minimum and non symmetric graph
The effect of turbulence

Parametrisation of turbulence $\Rightarrow$ non constant viscosity

In ocean-atmosphere context:
KPP viscosity (O’Brien, 1970)
- affine profile close to the surface
- parabolic or cubic profile in the turbulent zone
- constant profile in free zone
$\Rightarrow$ convergence for variable viscosity profile
Convergence factor with non-constants viscosities

\[ i(f + \omega)\hat{e}_j(z, t) - \partial_z(\nu_j(z)\partial_z\hat{e}_j(z, t)) = 0 \]

Mathematical tools to calculate converge:

- with \( \nu_j(z) = a_j z + b_j \) \( \rightarrow \) Bessel’s functions
- with \( \nu_j(z) = a_j z^2 + b_j z + c_j \) \( \rightarrow \) Legendre polynomials.

\( \Rightarrow \) The convergence factor depends on the global viscosity profile
Impact of non-constants viscosities

Convergence factor with non-constants viscosities

\[ i(f + \omega)\hat{e}_j(z, t) - \partial_z(\nu_j(z)\partial_z\hat{e}_j(z, t)) = 0 \]

Mathematical tools to calculate converge:

- with \( \nu_j(z) = a_jz + b_j \) → Bessel’s functions
- with \( \nu_j(z) = a_jz^2 + b_jz + c_j \) → Legendre polynomials.

⇒ The convergence factor depends on the global viscosity profile

Dirichlet-Neumann interface conditions

\[
\rho_{DN}(\omega) \xrightarrow{|\omega|\to\infty} \sqrt{\frac{\nu_1(0)}{\nu_2(0)}}
\]

for all viscosities profiles

\[
\rho_{DN}^{aff}(\omega) \leq \frac{\rho_{DN}^{cst}(\omega)}{\mu_1 \ln(1 + \frac{1}{\mu_1})}
\]

\[
\rho_{DN}^{par}(\omega) \leq \frac{\rho_{DN}^{cst}(\omega)}{4\mu_1 \arccos(\sqrt{1 + 4\mu_1})}
\]

with \( \mu_1 = \left| \frac{\nu_1(0)}{\partial_z\nu_1(0)z_1^\infty} \right| \)
The particular case ocean-atmosphere coupling

Stationary case for any interface condition:

- without Coriolis $\rightarrow$ free zones have a huge influence
- with Coriolis $\rightarrow$ turbulent zones have a bigger influence
The particular case ocean-atmosphere coupling

Non-Stationary case for any interface condition:
- $|\omega + f| < 10^{-11} \rightarrow$ influenced by free zone
- $|\omega + f| > 10^{-5} \rightarrow$ influenced by turbulent zone

Example: convergence for all frequencies except frequencies close to $-f$
Conclusion for linear problems

- Impact of Coriolis effect: shift of the graph
  ⇒ perturbation of the algorithm’s behavior
- Convergence factor depends of the global viscosities profile
  ⇒ mathematical tools to calculate the convergence factor (Bessel and Legendre functions)
  ⇒ simplification must be made with caution
⇒ Results are shown in: S. Thery, C. Pelletier, F. Lemarié and Blayo E., 2020: Coupling two Ekman layers with a Schwarz algorithm. under review
Conclusion for linear problems

- Impact of Coriolis effect: shift of the graph
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Ocean-atmosphere coupling

The Coriolis effect and the turbulence zones have a big impact on the convergence
The non-linear model

\[ \partial_t U + \begin{pmatrix} 0 & -f \\ f & 0 \end{pmatrix} U - \partial_z (\nu_{atm}(u^*, z) \partial_z U) = \mathcal{F} \]

Application of Schwarz algorithms on non-linear model
- Study of the well posedness of the problem
- Study of convergence of the algorithm
Current work on the non-linear model

Stationary case: using tools from linear problem

- unique solution consistent with the physical constraints
- without Coriolis effect: fast divergence & free zones have a huge influence
- with Coriolis effect: fast convergence & turbulent zones have a bigger influence
Current work on the non-linear model

Stationary case: using tools from linear problem

- unique solution consistent with the physical constraints
- without Coriolis effect: fast divergence & free zones have a huge influence
- with Coriolis effect: fast convergence & turbulent zones have a bigger influence

Non stationary case:

No theoretical method for solve this problem
⇒ experimental results ⇒ similarities with linear problem behavior:

- unique solution consistent with the physical constraints
- if $\omega_{\text{min}} \leq |f| \leq \omega_{\text{max}} \rightarrow$ divergence
- if $|f| \leq \omega_{\text{min}}$ or $\omega_{\text{max}} \leq |f| \rightarrow$ convergence
Test on a real model

IPSLS-CM (3D)

- test on a climate model (3D)
- Convergence in two iterations for 90% points

Thank you