



**HAL**  
open science

# Migration, Remittances and Accumulation of Human Capital with Endogenous Debt Constraints

Nicolas Destrée, Karine Gente, Carine Nourry

► **To cite this version:**

Nicolas Destrée, Karine Gente, Carine Nourry. Migration, Remittances and Accumulation of Human Capital with Endogenous Debt Constraints. *Mathematical Social Sciences*, 2021, 112, pp.38-60. 10.1016/j.mathsocsci.2021.03.008 . hal-03170022

**HAL Id: hal-03170022**

**<https://hal.science/hal-03170022>**

Submitted on 31 Jan 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NonCommercial - NoDerivatives 4.0 International License

# Migration, remittances and accumulation of human capital with endogenous debt constraints<sup>☆</sup>

Nicolas Destrée<sup>a,1</sup>, Karine Gente<sup>b,\*</sup>, Carine Nourry<sup>b,2</sup>

<sup>a</sup> *EconomiX, UPL, Univ. Paris Nanterre, CNRS, F92000 Nanterre, France*

<sup>b</sup> *Aix Marseille Univ., CNRS, AMSE, Marseille, France*

## Abstract

This paper studies the impact of migration and workers' remittances on human capital and economic growth when young individuals face debt constraints to finance education. We consider an overlapping generations model *à la* de la Croix and Michel (2007). In this no-commitment setting, education is the engine of growth. Individuals may choose to default on their debt and be excluded from the asset market. We show that remittances tend to tighten the borrowing constraints for a given level of interest rate, but may enhance growth at the equilibrium. The model replicates both negative and positive impacts of migration and remittances on economic growth underlined by the empirical literature. We calibrate the model for 30 economies.

**Keywords:** Migration Remittances, Overlapping generations, Human capital, Borrowing constraints, Indeterminacy

## 1. Introduction

With globalization, both migration and flows of workers' remittances are increasing. In 2019, there were 272 million of migrants in the world and global remittances accounted for 653 billion of dollars.<sup>3</sup> In 2020, the COVID crisis has generated a cut of these remittances for around 20% at the world level. These dramatic changes in such capital inflows really point out the need to understand the impact of such inflows on economic growth.

According to the literature, the effect of migration on economic growth is not clear-cut. Brain drain exerts a negative effect on human capital stock (among others Grubel and Scott (1966), Bhagwati and Hamada (1974) or Wong and Yip (1999)), whereas foreign opportunities for workers constitute an incentive to educate (see Mountford (1997), Vidal (1998) and Beine

et al. (2001, 2008) for instance). Studies focusing on remittances also provide mixed evidence. Their impact may be negative (see Chami et al. (2005, 2008), Karagöz (2009), Le (2009) or Nwosa and Akinbobola (2016)) or positive (see León-Ledesma and Piracha (2004), Faini (2007), Vargas-Silva (2007), Pradhan et al. (2008), Nsiah and Fayissa (2013) and Imai et al. (2014) for instance). In addition, some studies underline that the impact of remittances differs across countries or regions (Jawaid and Raza (2012), Siddique et al. (2012) or Cazachevici et al. (2020)). An increase in wealth due to remittances may reduce labor supply (see Acosta (2006), Amuedo-Dorantes and Pozo (2006, 2012)) and both the incentive to save and to invest (Athukorala and Sen (2004), Hossain (2014) or Yiheyis and Woldemariam (2016)). As underlined by Chami et al. (2008), the low volatility of remittances, which decreases the uncertainty, can explain the decrease in the incentive to save. According to Amuedo-Dorantes and Pozo (2004), remittances appreciate the real exchange rate ("Dutch disease") which generates a decrease in international competitiveness and may decrease economic growth. However, Edwards and Ureta (2003), Calero et al. (2009) and Zhunio et al. (2012) show that, empirically, remittances have a positive effect on education – with an increase in children school attendance or in the length of education – which tends to stimulate economic growth. Remittances may also improve the countries' creditworthiness, which will attract foreign investors (see Ratha (2005a,b, 2007)). In addition, Catrinescu et al. (2009) explain how the development of institutions may make remittances growth improving, especially bringing remittances towards productive spending. For example, this is the case of the *tres por uno* program in Mexico which

<sup>☆</sup> This work was supported by the French National Research Agency Grant ANR-17-EURE-0020, and by the Excellence Initiative of Aix-Marseille University - A\*MIDEX. We thank two anonymous referees, Johanna Etner, Miguel León-Ledesma, Thomas Seegmuller, Bertrand Wigniolle and participants to the ADRES Doctoral Conference (Lyon, 2020) and EconomiX Lunch Seminar (Nanterre, 2020) for helpful advice.

\* Correspondence to: 5 Boulevard Bourdet - CS 50498 - 13205 Marseille Cedex 01, France.

E-mail addresses: nicolas.destree@parisnanterre.fr (N. Destrée), karine.gente@univ-amu.fr (K. Gente).

<sup>1</sup> 200 avenue de la République - 92001 Nanterre Cedex 1, France.

<sup>2</sup> In memoriam: to our beloved friend, colleague and Ph.D. supervisor Carine Nourry. Her lectures and research inspired many vocations.

<sup>3</sup> These two figures are provided respectively by the United Nations [2020] and the World Bank [2020].

provides funds for public works and infrastructure (see [Khan and Merritt \(2020\)](#)). In addition to these facts, the impact of remittances can differ according to the level of financial development. When financial development is low, agents have difficulties to borrow for investment and they use remittances to invest. When financial development is high, the increase in wealth does not necessarily imply an increase in investment since borrowing is easier. This is shown by [Giuliano and Ruiz-Arranz \(2009\)](#) or [Sobiech \(2019\)](#).

The objective of this paper is to develop a theoretical setting able to explain this mixed evidence. We consider an overlapping generations model with endogenous growth based on human capital accumulation. In this setting, agents face endogenous debt constraints to finance their education in the home country. However, they receive remittances from their emigrated children. This model is an extension of [de la Croix and Michel \(2007\)](#) in which we introduce growth of births, migration and remittances. The specificity of this setting lies in the no-commitment framework.

[Kehoe and Levine \(1993, 2001\)](#) develop such a no-commitment framework in which agents may choose to refund or not their loan but support a penalty in case of default, losing the access to the asset market.<sup>4</sup> This assumption makes the borrowing limit endogenous: the maximal amount agents can borrow is such that the utility of refund is not lower than the utility of default. [Andolfatto and Gervais \(2006\)](#) study the role of endogenous constraints when they affect education funding. Assuming that wages and interest rates are exogenous, they consider that young agents face debt constraints to finance human capital investment. They find a negative impact of the conventional policies – defined by education subsidies, income tax and pensions – on welfare. Actually higher pensions make savings less useful, give less incentive to participate in the asset market and therefore raise the utility of default. [Azariadis and Lambertini \(2003\)](#) consider a three-period overlapping-generations endowment economy model in which the presence of endogenous debt constraints introduces complex equilibrium dynamics with multiple steady states and indeterminacy.<sup>5</sup>

Using a similar setting in which they focus on growth based on human capital accumulation, [de la Croix and Michel \(2007\)](#) consider that young agents borrow in order to educate but cannot commit to refund their loan.<sup>6</sup> They show that, on one side, a too low interest rate is detrimental for growth since it discourages agents to save and give them incentive to default. On the other side, a too high interest rate is also detrimental for growth since credit becomes too expensive. In addition, they underline that when the incentive to save relaxes the constraint and allows more education, endogenous credit constraints imply global indeterminacy of balanced growth paths. An equilibrium such that agents are unconstrained on the amount they borrow for education may coexist with an equilibrium where agents are credit-constrained to finance education.

We extend this model assuming that, during the childhood, agents may migrate to another country and send remittances

<sup>4</sup> [Kehoe and Levine \(1993, 2001\)](#) consider infinite-horizon general equilibrium models with individual rationality constraints. If agents default on a contract, they can be excluded from future contingent claims market since creditors can seize defaulting debtors' assets for the payment of past debt. However private endowments cannot be seized and agents can always engage in spot markets. When information is complete, agents cannot enter into a contract in which they would have an incentive to not repay the loan.

<sup>5</sup> [Azariadis and Lambertini \(2003\)](#) argue that hump-shaped endowment profiles imply that young agents want to borrow while middle-aged agents want to lend. Hence for hump-shaped endowment profiles young agents face endogenous debt constraints rationing their current consumption.

<sup>6</sup> [de la Croix and Michel \(2007\)](#) consider that agents' human capital cannot be alienated. Hence they are allowed to borrow up to it is in their interest to repay the loan.

(while young workers) to their family stayed at home. Since migrants neither work nor educate in the home country, we consider the decision of migration exogenous like in [Cassin \(2020\)](#) in order to focus on the role of endogenous borrowing constraints and their interaction with remittances. In the home country, non-migrating agents borrow to finance their education but cannot commit to refund their loans.<sup>7</sup> Nevertheless, if they want to participate in the asset markets, they need to refund their loans. Thus, the non-migrating agents decide not only the amount they want to borrow, the amount they save but also if they refund or not their loans. The decision of refund is based on their incentive to save which depends on their time preference, the time profile of their income, the education productivity and what they expect for the interest rate. Since saving is a non monotonic function of the interest rate whereas optimal education is decreasing with the interest rate, the relationship between effective borrowing and interest rate is hump-shaped. As in [de la Croix and Michel \(2007\)](#), we show that these mechanisms may generate multiple equilibria. Finally, the nature (constrained/unconstrained) and the number of equilibria depends on the productivity of education. When productivity of education is high enough, agents have incentive to save *a priori*, and may be less constrained: both constrained and unconstrained equilibria co-exist. What agents will finally save depends on their interest rate expectations. When they expect a high interest rate, the amount they can borrow is high but the borrowing cost is high as well, they expect high remittances and net income (of cost of refund) available for saving is low: savings are finally low and the interest rate is high, which corresponds to the unconstrained case. Conversely, when they expect a low interest rate, the amount they can borrow is low but the borrowing cost is low as well, they expect low remittances and net income available for saving is high: savings are finally high and the interest rate is low, which corresponds to the constrained case. When the productivity of education is lower, the utility of refund is *a priori* low and there may exist two constrained equilibria where one is locally stable.

As in [Chami et al. \(2008\)](#), in this setting, remittances may exert a negative effect on savings, reinforcing the constraint since the penalty of being excluded from the asset market is less damaging. This is especially true at a partial equilibrium level. However, the global effect of remittances on economic growth depends on the reaction of the interest rate too. It happens that this latter effect – which influences strongly the borrowing constraint – may offset the negative effect on the borrowing constraint and thus increase both the borrowing limit and education. As in [Cassin \(2020\)](#), the overall effect of migration and remittances on human capital is not clear-cut. We calibrate this model for 30 recipient countries<sup>8</sup> and show that the global effect of migration and remittances on education and growth is positive for the majority of countries in our sample. This result implies that remittances may have a positive impact on borrowing as underlined by [Aggarwal et al. \(2011\)](#).

In this model, remittances exert potentially a destabilizing effect in the sense that, as any type of third period of life's income, these transfers offer the opportunity to agents to default on their loan. This generates a friction on the financial market, increases the utility of default, reinforcing endogenously the borrowing

<sup>7</sup> We assume to simplify that – contrary to the non-migrating agents – the migrating agents can commit to refund their loans and face an *ad hoc* borrowing constraint to educate in the host country.

<sup>8</sup> We calibrate the model for Algeria, Benin, Bangladesh, Burkina Faso, Cabo Verde, China, Egypt, Eswatini (Swaziland), Guinea-Bissau, India, Indonesia, Kenya, Mali, Mexico, Morocco, Myanmar, Namibia, Pakistan, Papua New Guinea, Philippines, Senegal, Slovak Republic, Solomon Islands, Sri Lanka, Syrian Arab Republic, Tanzania, Uganda, Vietnam, West Bank and Gaza (Palestinian territories) and Zambia.

constraints. This is especially true in developing countries where, due to the lack of health insurance and retiring system, there may be almost no third period of life's income. In this case, without remittances, the agents' utility of refunding their loans would always be greater than the utility of default and there would be a unique long-run equilibrium. Thus, remittances are especially destabilizing in countries where "old" agents receive no income of any type. In order to limit frictions and the destabilizing effect of remittances, the policy maker may give incentive for agents to participate in the asset markets, for example improving the education productivity. A noticeable feature of this model is that paradoxically when migrants face more stringent financial constraints in the host country, they are able to send less remittances and this generates less frictions in the home economy. In economies where there is a third period labor income, there may already exist multiple equilibria even without remittances. The calibration shows that the gap between economic growth at each equilibrium is lower with remittances. If we consider this gap as a proxy for volatility, we could argue that remittances may reduce volatility of GDP in recipient countries. This result is in accordance with [Chami et al. \(2008\)](#). [Ahamada and Coulibaly \(2011\)](#) show that a high level of financial development allows remittances to stabilize the economy. In our setting, migration and remittances improve economic growth in constrained economies in 70% of the considered countries but increase financial frictions.<sup>9</sup> Finally, the model shows that the equilibrium growth rate is firstly increasing and then decreasing with the interest rate. It is then possible for the policy maker to improve growth of human capital guiding expectations towards a higher (resp. lower) interest rate when it is too low (resp. too high).

The paper is organized as follows. Section 2 deals with the theoretical model and the long-run equilibrium. Section 3 focuses on equilibrium dynamics and presents how migration and remittances could affect growth through their impact on liquidity constraints. In Section 4, we calibrate the model and show the predicted impact of migration and remittances on growth for 30 recipient countries. Section 5 contains concluding remarks.

## 2. The model

The model is a variant of [de la Croix and Michel \(2007\)](#) where accumulation of human capital is growth enhancing. We consider an overlapping generations economy with endogenous credit constraints: households can borrow to finance human capital accumulation. We introduce growth of births, migration and remittances in this framework. When young, agents can migrate to another country. The migrants can not only borrow in the host country to finance their education but also face borrowing constraints. Then they work and send remittances to old parents stayed in the home country.

### 2.1. Production

We consider a representative firm producing  $Y_t$  in period  $t$  using only human capital  $H_t$ . We assume that the production function is linear:

$$Y_t = H_t \quad (1)$$

Thus, marginal productivity of human capital is paid at the wage  $w_t$ :  $w_t = 1$ .

<sup>9</sup> We observe that with remittances only 1% of the countries keep an unconstrained steady state.

### 2.2. The non-migrating households

In period  $t - 1$ , we consider that  $N_{t-1}$  agents are born, living for three periods: each agent is young, then young worker and then old worker.

**Assumption 1.** When young, an agent born in period  $t - 1$  can migrate to a foreign country with the exogenous probability  $p \in [0, 1]$ .

We follow [Mountford \(1997\)](#), [Vidal \(1998\)](#), [Beine et al. \(2001\)](#) and [Cassin \(2020\)](#) and we assume that a constant proportion of the young population  $p$  migrates at each period. We do not study the migration decisions since migrants do not take any decision impacting the home country except for the amount of money they decide to send to their parents. After migration, agents can study in the foreign country and begin to work when they are middle-aged. Following [Assumption 1](#), a fraction  $1 - p$  of young agents stays at home. They study when they are young and work when they are middle-aged and old.

**Assumption 2.** Births grow at the rate  $n > 0$

According to [Assumption 2](#), each non-migrating agent has  $1+n$  children. Following [Assumption 1](#), there are  $p(1+n)$  children in each family who are successfully migrating per period. The evolution of population across periods satisfies:

$$N_t = (1 - p)(1 + n)N_{t-1} \quad (2)$$

Agents born in period  $t - 1$  who have not migrated draw utility from consumption  $c_t$  when they are young workers and  $d_{t+1}$  when they are old workers.

**Assumption 3.** We consider a Cobb–Douglas utility function:

$$U(c_t, d_{t+1}) = c_t^\beta d_{t+1}^{1-\beta} \quad (3)$$

with  $0 < \beta < 1$ .

We assume that young non-migrating agents can borrow<sup>10</sup> an amount  $b_{t-1}$  in period  $t - 1$  for education funding. This allows them to generate their level of human capital for the two periods of work.

**Assumption 4.** A young worker's human capital  $h_t$  at time  $t$  becomes  $\delta h_t$  at time  $t + 1$ .

Parameter  $\delta > 0$  reflects the change in income over life that may be related to health condition ( $\delta < 1$ ) or experience ( $\delta > 1$ ) for instance. Therefore,  $\delta$  represents the steepness of labor income across time.

As [de la Croix and Michel \(2007\)](#), we assume a constant returns to scale human capital accumulation function:

$$h_t = Ab_{t-1}^\lambda h_{t-1}^{1-\lambda} \quad (4)$$

with  $A > 0$ , the productivity of education and  $0 < \lambda < 1$ , the elasticity of human capital to investment on education. The evolution of human capital depends not only on the individual investment  $b_{t-1}$  dedicated to education but also on the human capital of the previous generation  $h_{t-1}$ . This can be viewed as inherited human capital or as an externality which makes more productive the investment in education in a country where the level of education is higher.

<sup>10</sup> In such a framework, the loanable funds are provided by the savers.

**Assumption 5.** Each non-migrating agent born in period  $t - 1$  receives in period  $t + 1$  the following amount from her children who have migrated:

$$f_{t+1} = p(1+n)\gamma h_t \quad (5)$$

Appendix A.1 gives microfoundations for  $\gamma$  - where it depends on migrant's altruism<sup>11</sup>  $\phi$ , the productivity of education in the foreign country  $A_*$ , the income growth abilities of migrants  $\delta_*$ , the elasticity of migrant's human capital to investment on education  $\lambda_*$  and the foreign interest rate  $R_*$ . Moreover, less financial frictions in the host country (higher  $\eta_*$ ) would increase the migrants' level of education and finally  $\gamma$ . Hereafter, we will consider  $\gamma$  as a parameter to simplify.

According to Eq. (5), remittances are increasing not only with the number of children leaving the country (through  $p$ , the probability to emigrate, and  $1+n$ , the number of children the agent has), but also with the level of human capital of parents  $h_t$ . This has to be seen as inter-generational transmission of human capital. As for children who have not migrated, the human capital of each migrant depends on the human capital of parents in addition to individual education. Since migrants send an amount of money which depends on their income (which depends on human capital of their parents), remittances received by old workers in period  $t + 1$  depend on their level of human capital in period  $t$ .

Hereafter, we describe the choices of an agent stayed in the home country who receives remittances from her children. We proceed in three steps: (i) choices under commitment (ii) choices in a no-commitment setting (iii) effective decisions.

### 2.3. Optimal decisions under a commitment setting

In a commitment setting, at period  $t$ , young agents born in period  $t - 1$  refund their loan  $b_{t-1}$  at the interest rate  $R_t$ . Then, their first period labor income  $h_t$  allows them to consume  $c_t$ , repay the loan  $R_t b_{t-1}$ , and save  $s_t$ . When old, their total income is constituted by the wage  $\delta h_t$ , the return on savings  $R_{t+1} s_t$ , and remittances received from emigrated children  $f_{t+1}$ . Using this total income, they choose their second period consumption  $d_{t+1}$ . Therefore, the budget constraints are given by:

$$c_t = h_t - s_t - R_t b_{t-1} \quad (6)$$

$$d_{t+1} = \delta h_t + R_{t+1} s_t + f_{t+1} \quad (7)$$

Let us denote by  $\omega_{1t}$  the first period income net of loan repayment and by  $\omega_{2t+1}$  the second period income such that under equation (5):

$$\omega_{1t} = h_t - R_t b_{t-1} \quad (8)$$

$$\omega_{2t+1} = \delta h_t + p(1+n)\gamma h_t \quad (9)$$

Optimal decisions of agents concern education  $b_{t-1}$  and savings  $s_t$ . Agents choose education in order to maximize their life-cycle income. They choose savings in order to allocate this life-cycle income over the two last periods of life. Following [Gente et al. \(2015\)](#), we proceed in two steps: first, we compute the optimal savings for a given level of education and second, we determine the optimal level of education.

Savings maximize the utility of agents for a given level of education. Substituting Eqs. (8) and (9) into (6) and (7), and then

into (3) the maximization problem of the agent in a commitment setting is given by:

$$\text{Max}_{s_t} (\omega_{1t} - s_t)^\beta (\omega_{2t+1} + R_{t+1} s_t)^{1-\beta}$$

The first order condition (FOC) of this program is:

$$\beta (\omega_{2t+1} + R_{t+1} s_t^*) = (1 - \beta) R_{t+1} (\omega_{1t} - s_t^*)$$

This FOC determines the optimal level of savings:

$$s_t^* = (1 - \beta) \omega_{1t} - \frac{\beta \omega_{2t+1}}{R_{t+1}} \quad (10)$$

Notice that optimal savings in period  $t$  are increasing with the first period income  $\omega_{1t}$  and the interest rate  $R_{t+1}$  and are decreasing with  $\beta$  and the second period income  $\omega_{2t+1}$ . Therefore, optimal savings are decreasing with the cost of debt  $R_t$  and with both  $p$  and  $\gamma$ . Migrations and remittances reduce the incentive to save in partial equilibrium.

Let us now underline the optimal level of education. To facilitate the algebra, we use  $e_{t-1}$  which represents the level of education spending per unit of human capital and  $x_t$  the repayment share of the loan relative to the first period (of work) income:

$$e_{t-1} \equiv \frac{b_{t-1}}{h_{t-1}} \quad (11)$$

$$x_t \equiv \frac{R_t b_{t-1}}{h_t} = \frac{R_t}{A} e_{t-1}^{1-\lambda} \quad (12)$$

Using Eq. (12), we can rewrite the per-period income as:

$$\omega_{1t} = h_t (1 - x_t) \quad (13)$$

$$\omega_{2t+1} = h_t (\delta + p(1+n)\gamma) \quad (14)$$

We are looking for the optimal level of education per unit of human capital  $e_{t-1}$  such that the one that maximizes the life-cycle income in period  $t - 1$ :

$$\omega_{1t} + \frac{\omega_{2t+1}}{R_{t+1}} \quad (15)$$

Substituting the expression of human capital given by Eq. (4), and  $b_{t-1} = e_{t-1} h_{t-1}$  from Eq. (11), into the life-cycle income, the non-migrant agent solves the following program:

$$\text{Max}_{e_{t-1}} A e_{t-1}^\lambda h_{t-1} \left( 1 + \frac{\delta + p(1+n)\gamma}{R_{t+1}} \right) - R_t e_{t-1} h_{t-1} \quad (16)$$

The FOC of this program is:

$$A \lambda e_{t-1}^{\lambda-1} \left( 1 + \frac{\delta + p(1+n)\gamma}{R_{t+1}} \right) = R_t$$

which finally gives the optimal level of education per unit of human capital:

$$e_{t-1}^* = \left( \frac{A \lambda \left( 1 + \frac{\delta + p(1+n)\gamma}{R_{t+1}} \right)}{R_t} \right)^{\frac{1}{1-\lambda}} \quad (17)$$

Notice that this optimal amount of education is increasing with  $A$ ,  $\lambda$ ,  $\delta$ ,  $p$ ,  $n$  and  $\gamma$  and is decreasing with the interest rates  $R_t$  and  $R_{t+1}$ .

**Lemma 2.1.** Under Assumptions 1–5, the optimal share of debt repayment relative to the first period income, denoted by  $x_t^* \equiv x^*(R_{t+1}, p, \gamma)$ , and which maximizes the agent's life-cycle income is given by:

$$x_t^* = \lambda \left( 1 + \frac{\delta + p(1+n)\gamma}{R_{t+1}} \right) \quad (18)$$

<sup>11</sup> Eq. (5) reflects the ascendant altruism of emigrated children towards parents stayed in the home country. For instance [Lucas and Stark \(1985\)](#), [Chami et al. \(2005\)](#), [Rapoport and Docquier \(2006\)](#), [Chami et al. \(2008\)](#) or [Melkonyan and Grigorian \(2012\)](#) study the determinants of decisions to remit. They underline that altruism and self interest exchange are the main determinants for sending money back, altruism being the most important.

**Proof.** Using Eq. (12) we have:

$$x_t^* = \frac{R_t}{A} e^{s_t^* - \lambda}$$

Substituting Eq. (17) into the above expression, we get equation (18).  $\square$

The optimal levels of education and the share of debt repayment depend on  $p$  and  $\gamma$ , since both  $p$  and  $\gamma$  increase the return on education. A rise in  $R_t$  decreases the optimal level of education since the cost of borrowing increases. A rise in  $R_{t+1}$  decreases both the optimal level of education and the optimal share of debt repayment since it makes the return on savings higher.

#### 2.4. Optimal decisions under a no-commitment setting

In a no-commitment setting, non-migrating agents may choose to refund or not their loan. In case of repayment, agents can participate in the asset market – and therefore save – and their budget constraints are similar to the case with commitment. When they choose to default, since their assets could be seized to repay the outstanding debt,<sup>12</sup> they cannot participate in the asset market. In that case, budget constraints become:

$$c_t = h_t \quad (19)$$

$$d_{t+1} = \delta h_t + f_{t+1} \quad (20)$$

The loans are provided by savings. This framework entails an endogenous constraint on the borrowed amount. We then have to impose two individual rationality constraints to guarantee that the non-migrating agent has incentive to refund her loan:

1. IRC old-age: The middle-aged agents are not allowed to borrow because they would rationally never reimburse their debt when old. Hence savings should be nonnegative:

$$s_t \geq 0 \quad (21)$$

2. IRC middle-age: The utility of savings and repaying the debt has to be higher than the utility of no repaying. This implies under Eqs. (5)–(7) and (19)–(20):

$$\begin{aligned} \max_{s_t} \quad & U(h_t - s_t - R_t b_{t-1}, h_t (\delta + p(1+n)\gamma) + R_{t+1} s_t) \\ & \geq U(h_t, h_t (\delta + p(1+n)\gamma)) \end{aligned} \quad (22)$$

with  $R_t$  observed,  $R_{t+1}$  is expected and  $h_t$  is predetermined.

Combining Eqs. (21) and (22), we get a unique incentive rationality constraint.

$$\begin{aligned} \max_{s_t \geq 0} \quad & U(h_t - s_t - R_t b_{t-1}, h_t (\delta + p(1+n)\gamma) + R_{t+1} s_t) \\ & \geq U(h_t, h_t (\delta + p(1+n)\gamma)) \end{aligned} \quad (23)$$

The problem of the consumer is to maximize the utility function (3) under the human capital accumulation (4), the budget constraints in the two periods of work (6)–(7) and the individual rationality constraint (23).

In a no-commitment setting, investors will accept to lend only if agents have incentive to repay the loan that is under both IRC old-age and IRC middle-age. This allows us to define the constrained level of debt repayment  $\bar{x}_t$  such that the utility of repaying the loan is equal to the utility of default. The effective savings and share of debt repayment – defined respectively by  $\hat{s}_t$  and  $\hat{x}_t$  – are obtained by comparing the optimal levels with the constrained levels.

**Proposition 2.1.** *Let us consider:*

$$R_{min} = \frac{\beta (\delta + p(1+n)\gamma)}{1 - \beta} \quad (24)$$

*Under Assumptions 1–5, if  $R_{t+1} \leq R_{min}$  then  $s_t^* \leq 0$  and  $\hat{x}_t = \hat{s}_t = 0$ . Otherwise,  $\hat{s}_t = s_t^* > 0$  and the effective level of borrowing satisfies:*

$$\hat{x}_t = \min \{x_t^*, \bar{x}_t\} \quad (25)$$

where:

$$\bar{x}_t = 1 - \frac{(\delta + p(1+n)\gamma)^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta} R_{t+1}^{1-\beta}} + \frac{\delta + p(1+n)\gamma}{R_{t+1}} \equiv \bar{x}(R_{t+1}, p, \gamma) \quad (26)$$

and  $\bar{x}_t \in [0, 1]$ .

**Proof.** See Appendix A.2.  $\square$

If  $R_{t+1} \leq R_{min}$ , we have  $s_t^* \leq 0$  and the IRC old-age is binding which implies that  $\hat{s}_t = 0$  and  $\hat{x}_t = 0$ . In that case, since agents do not have incentive to save, they cannot borrow. For  $R_{t+1} > R_{min}$ , Eq. (26) determines the upper bound on borrowing, since the utility of default is higher than the utility of repaying if  $x_t > \bar{x}_t$ . Therefore, the effective level of debt repayment is  $x_t^*$  only if  $x_t^* < \bar{x}_t$ . Otherwise, the constraint binds and the effective level of debt repayment is  $\bar{x}_t$ . Notice that the constraint  $\bar{x}_t$  is increasing with  $R_{t+1}$ . A higher return on savings makes the utility of default decreasing other things being equal, since the incentive to save increases. This tends to raise the amount the agents can borrow.

**Corollary 1.** *Under Assumptions 1–5, when migrations ( $p$ ) or remittances ( $\gamma$ ) increase:*

- agents have more incentive to educate since the return on education is higher due to higher remittances they will receive when old ( $x_t^*$  is increasing with  $p$  and  $\gamma$ )
- agents have less incentive to save since they have higher remittances to consume when old ( $\bar{x}_t$  is decreasing with  $p$  and  $\gamma$ )
- there are fewer agents borrowing in the economy ( $R_{min}$  is increasing with  $p$  and  $\gamma$ )

**Proof.** See Appendix A.3.  $\square$

We notice that  $R_{min}$  is increasing with  $p$  and  $\gamma$ , implying that migrations and remittances affect the threshold determining whether agents will be educated or not. However, parameter  $p$  has an impact on  $R_{min}$  only through the remittance mechanisms. Notice that when  $\gamma = 0$ , migrations have no impact on this threshold since, without remittances, migrations do not affect the individual level of savings.

For a given level of interest rate, migrations and remittances increase the optimal amount of education. However, they decrease the amount that a non-migrating agent can borrow to finance education. An increase in remittances implies less incentive to save in order to finance consumption when old, and thus an increase in the incentive to default. Therefore, remittances affect negatively  $\bar{x}_t$  for a given interest rate. Here again, migrations have an impact on  $\bar{x}_t$  only through the remittance mechanisms. For a given level of remittances per emigrated child, an increase in  $p$  decreases the need to save since it increases the total amount of remittances. However, if  $\gamma = 0$ , an increase in  $p$  does not affect the constraint since agents do not receive money from emigrated children.

For simplicity, let us now denominate the optimal share of debt repayment as  $x^*(R_{t+1})$  and the constrained share of debt

<sup>12</sup> Following Kehoe and Levine (1993) and de la Croix and Michel (2007), we consider that assets can be seized. However, human capital cannot be alienated.

repayment as  $\bar{x}(R_{t+1})$ . Hence, for a given level of interest rate  $R_{t+1} = R$ , the effective level of borrowing – denoted by  $\hat{x}(R)$  – satisfies  $\hat{x}(R) = \min \{x^*(R), \bar{x}(R)\}$ . A comparison between  $\bar{x}(R)$  and  $x^*(R)$  will give us the values of  $R$  for which the optimal choice is constrained.

**Proposition 2.2.** *Under Assumptions 1–5, there exists a unique interest factor  $\check{R} > R_{\min}$  such that the optimal and the constrained level of borrowing coincide:  $x^*(\check{R}) = \bar{x}(\check{R}) \equiv \check{x}$ .*

1. If  $R < \check{R}$ , the borrowing constraint binds and restricts the optimal choice:  $\bar{x}(R) < x^*(R)$ .
2. If  $R > \check{R}$ , the optimal choice is unconstrained:  $x^*(R) < \bar{x}(R)$ .

$\check{R}$  is increasing with  $p$  and  $\gamma$ .

**Proof.** See Appendix A.4.  $\square$

Fig. 2.1 depicts the optimal repayment share  $x^*(R)$  and the threshold  $\bar{x}(R)$ , according to the interest rate  $R$ . The highlighted part of the curves represents the effective repayment share. When  $p$  and  $\gamma$  increase, this highlighted range on interest rate, such that education choice is constrained, increases too.

**Corollary 2.** *Under Assumptions 1–5, there is a corresponding level of constrained education associated to the borrowing limit  $\bar{x}_t$  such that:*

- If  $R_{t+1} \leq R_{\min}$  and  $R_t \leq R_{\min}$ , then  $\bar{e}_{t-1} = \bar{x}_t = 0$ .
- If  $R_{t+1} > R_{\min}$  and  $R_t > R_{\min}$ , then

$$\bar{e}_{t-1} = \left( \frac{A\bar{x}_t}{R_t} \right)^{\frac{1}{1-\lambda}} \equiv \bar{e}(R_{t+1}, R_t, p, \gamma) \quad (27)$$

**Proof.** We get  $\bar{e}_{t-1}$  using Eq. (12) which relies  $x_t$  and  $e_{t-1}$ .  $\square$

The effective level of education per unit of human capital is:

$$\hat{e}_{t-1} = \min \{e_{t-1}^*, \bar{e}_{t-1}\} \quad (28)$$

where  $e_{t-1}^*$  and  $\bar{e}_{t-1}$  are given by Eqs. (17) and (27).

Let us now analyze the impact of migrations and remittances on the effective level of education when we consider a given level of interest rate  $R$ .

**Corollary 3.** *Under Assumptions 1–5, and for a given interest rate  $R$ , the optimal and constrained levels of education per unit of human capital are given respectively by:*

$$e^* = \left( \frac{Ax^*(R)}{R} \right)^{\frac{1}{1-\lambda}} \equiv e^*(R) \quad (29)$$

$$\bar{e} = \left( \frac{A\bar{x}(R)}{R} \right)^{\frac{1}{1-\lambda}} \equiv \bar{e}(R) \quad (30)$$

Optimal education  $e^*$  is decreasing with  $R$  while constrained education  $\bar{e}$  increases when  $R_{\min} < R < \check{R}$  and then decreases; When  $R = \check{R}$ , the optimal and the constrained levels of education coincide:  $e^* = \bar{e}$ .

Moreover,  $e^*$  is increasing with  $p$  and  $\gamma$  and  $\bar{e}$  is decreasing with  $p$  and  $\gamma$ .

**Proof.** See Appendix A.5.  $\square$

While the constrained repaying share  $\bar{x}(R)$  is increasing with  $R$ , the constrained education  $\bar{e}(R)$  is firstly increasing and then decreasing. The constrained education reflects the arbitrage of agents between the default on their loan and their participation to the asset markets. A high interest rate means a high cost of borrowing but as well a high return on savings. A high cost of

borrowing increases the utility to default – exerting a negative effect on the amount agents can borrow – whereas a high return on savings increases the utility of refund – exerting a positive effect on the amount agents can borrow. Therefore, when the interest rate  $R$  is low, the positive effect dominates and an increase in  $R$  implies that agents can borrow more. Whereas when  $R$  is high, the negative effect dominates and an increase in  $R$  implies that agents can borrow less. Thus, there exists a threshold in  $R$  after which lenders prefer to reduce the amount of the loan to ensure that the weight of loan repayment is not too high. The effects of parameters  $p$  and  $\gamma$  on  $e^*(R)$  and  $\bar{e}(R)$  are qualitatively identical to the ones on  $x^*(R)$  and  $\bar{x}(R)$ . Fig. 2.2 represents the optimal and the constrained levels of education per unit of human capital:  $e^*(R)$  and  $\bar{e}(R)$ . The highlighted parts of each graph represent the effective level of education per unit of human capital. As explained by de la Croix and Michel (2007), a too high interest rate may be detrimental to education. Notice that the maximum level of constrained education  $e(\check{R})$  is attainable only if  $\check{R} < \check{R}$ . This case is represented by the right part of Fig. 2.2.

**Corollary 4.** *Under Assumptions 1–5, and for a given interest rate  $R$ , there exists a bound<sup>13</sup>  $\bar{\lambda} \in [0, 1]$ :*

- if  $\lambda < \bar{\lambda}$ , the maximum level of effective education is such that  $R = \check{R}$ ,
- if  $\lambda > \bar{\lambda}$ , the maximum level of effective education is such that  $R = \bar{R}$ .

Migrations and remittances do not affect  $\bar{\lambda}$ , which depends only on the agent's preferences ( $\beta$ ).

**Proof.** See Appendix A.6.  $\square$

If  $\lambda$ , the elasticity of human capital with respect to education, is sufficiently high ( $\lambda > \bar{\lambda}$ ), then the maximum level of education occurs in the constrained regime (right part of Fig. 2.2). Let us also notice that  $\bar{e}(\bar{R}) \geq \bar{e}(\check{R})$ . Hence, the maximum level of education is higher if the return on education is sufficiently high. Since migrations and remittances do not affect  $\bar{\lambda}$ , they do not change the two configurations shown in Fig. 2.2. Notice that migrations and remittances change only the level of effective education since they affect both  $e^*(R)$  and  $\bar{e}(R)$ .

## 2.5. Migration, remittances and economic growth

We examine the relationship between migrations, remittances and economic growth in partial equilibrium. Since the production depends on human capital, the output growth depends on both education and population growth. Using Eqs. (4) and (11) the growth of human capital for each agent between period  $t - 1$  and  $t$  satisfies:

$$\frac{h_t}{h_{t-1}} = Ae_{t-1}^\lambda \quad (31)$$

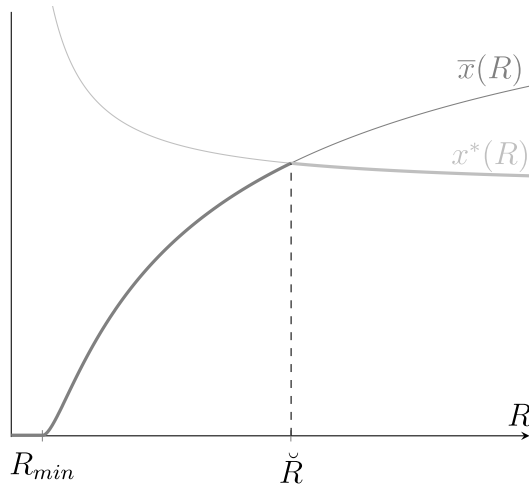
From Eq. (12), we have  $e_{t-1} = \left( \frac{x_{t-1}A}{R_t} \right)^{\frac{1}{1-\lambda}}$ . Using Eq. (25), the growth of human capital per worker between period  $t - 1$  and  $t$  satisfies for a given level of interest rate  $R_t$ :

$$\frac{h_t}{h_{t-1}} = A\hat{e}_{t-1}^\lambda \text{ with}$$

$$\hat{e}_{t-1} = \min \left\{ \left( \frac{Ax^*(R_{t+1})}{R_t} \right)^{\frac{1}{1-\lambda}}, \left( \frac{A\bar{x}(R_{t+1})}{R_t} \right)^{\frac{1}{1-\lambda}} \right\}$$

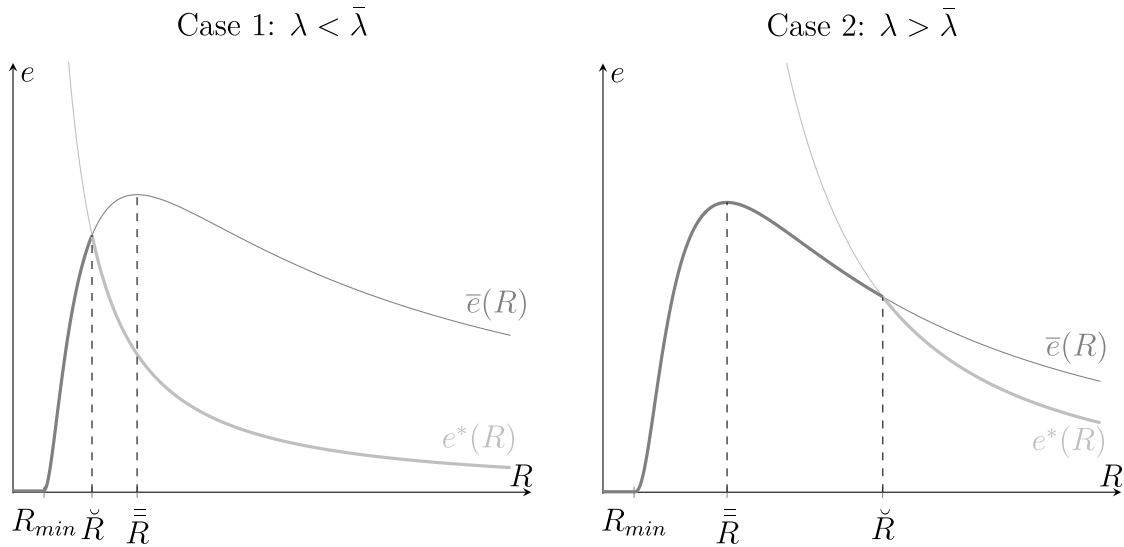
As a consequence, the education decisions are at the core of the growth mechanism in this model. At a given level of  $R_t$ , the value of  $R_{t+1}$  allows us to distinguish three cases:

<sup>13</sup> See Appendix A.6 for the value of  $\bar{\lambda}$ .



Note: The effective share of debt repayment satisfies  $\hat{x}(R) = \min \{x^*(R), \bar{x}(R)\}$ . If  $R < \check{R}$ , the borrowing constraint binds and restricts the choice of education.

**Fig. 2.1.** A representation of  $x^*(R)$  and  $\bar{x}(R)$ .



Note: The effective level of education relative to human capital is:  $\hat{e}(R) = \min \{e^*(R), \bar{e}(R)\}$ . If  $R < \check{R}$ , the borrowing constraint binds and restricts the choice of education.

**Fig. 2.2.** Two representations of  $e^*(R)$  and  $\bar{e}(R)$  depending on  $\lambda$ .

1.  $R_{t+1} < R_{min}$ : young agents cannot borrow to finance education. In that case, the interest rate is too low, which discourages middle-aged agents to save. Since there are no savings in the economy, there are no funds available for borrowing.
2.  $R_{min} < R_{t+1} < \check{R}$ : the interest rate is high enough to allow borrowing, education is positive but constrained. In this case, the return on savings is relatively low (which decreases the incentive to save) but the discounted third period income is relatively high (which increases incentive to educate). Hence  $\bar{x}(R_{t+1})$  is low and  $x^*(R_{t+1})$  is high.
3.  $R_{min} < \check{R} < R_{t+1}$ : the interest rate is so high that the economy is unconstrained, thus  $\bar{x}(R_{t+1}) > x^*(R_{t+1})$ . In this case, the return on savings is high: agents have a high

incentive to participate in the asset markets ( $\bar{x}(R_{t+1})$  is high) but the discounted third period income is low and the optimal share of repayment  $x^*(R_{t+1})$  is low. Thus, the level of growth of human capital is not systematically higher when the country is unconstrained.

As shown in [Proposition 2.2](#), migrations and remittances increase the range on interest rate such that the agent is constrained  $[0, \check{R}]$ . This is due to the fact that remittances and migrations increase optimal education but decrease the incentive to save. However the global impact of remittances and migrations on whether an economy is constrained or unconstrained is ambiguous. Let us consider four examples assuming a given interest rate  $R_{it+1}$  for country  $i$ ,  $i = 1 \dots 4$ .



- As  $R_{min}$  is increasing with  $p$  and  $\gamma$ , remittances could bring the economy into a poverty trap. Let  $R_{1t+1}$  be the interest rate of economy 1 such that without remittances<sup>14</sup>  $R_{1t+1} > R_{min}$ , the human capital in economy 1 is growing. Remittances are a substitute for old-age income and thus decrease the incentive to save. It may happen that remittances increase so much  $R_{min}$  that it becomes higher than the interest rate  $R_{1t+1}$ : in that case, economy 1 is stuck into a poverty trap in which there are neither loans nor education.
- Let  $R_{2t+1}$  be the interest rate of economy 2 such that  $R_{min} < R_{2t+1} < \bar{R}$ . Economy 2 is initially constrained; remittances strengthen the constraint and thus may decrease human capital accumulation.
- Let  $R_{3t+1}$  be the interest rate of economy 3 such that  $\check{R} < R_{3t+1}$ , economy 3 is initially unconstrained and may become constrained with remittances since  $d\check{R}/dp > 0$ . In this case, remittances may be detrimental to human capital accumulation.
- Finally, let  $R_{4t+1}$  be the interest rate of economy 4 which is unconstrained even with remittances, such that  $R_{4t+1} > \bar{R}$ . In economy 4, increasing remittances will increase the effective level of borrowing, since agents are unconstrained and the optimal level of borrowing is increasing with  $p$  and  $\gamma$ .

**Remark 2.1.** Without migration ( $p = 0$ ) or without remittances ( $\gamma = 0$ ), if  $\delta = 0$ , then  $\bar{x}_t = 1$  and  $x_t^* = \lambda$ . Therefore, the effective share of repayment,  $\hat{x}_t$ , is equal to  $\lambda$  and does not depend on  $R_{t+1}$ . Without migration/remittances and without last period labor income, the agent is never constrained since she is forced to save in order to consume the last period. However, with remittances, even if  $\delta = 0$ , the agent may be constrained (according to [Proposition 2.2](#) and [Corollary 1](#)) since remittances allow her to consume the last period without saving.

These mechanisms are true in partial equilibrium. Let us consider now how  $R_{t+1}$  changes at the macroeconomic equilibrium.

## 2.6. The macroeconomic equilibrium

At each period, the macroeconomic equilibrium satisfies:

1. The labor market equilibrium:

$$H_t = [(1-p)(1+n)h_t + \delta h_{t-1}](1-p)N_{t-2} \quad (32)$$

The level of human capital in the economy in period  $t$  is equal to the human capital of old workers born in period  $t-2$  who have not migrated plus the human capital of young workers born in period  $t-1$  who have not migrated.

2. The asset market equilibrium:

$$(1-p)N_{t-1}b_{t-1} = (1-p)N_{t-2}s_{t-1} \quad (33)$$

The left hand side represents the borrowed amount of young agents who have not migrated and the right hand side represents the total amount of savings of middle-aged agents who have not migrated. Using [Eq. \(2\)](#), the asset market equilibrium equation simplifies to:

$$(1-p)(1+n)b_{t-1} = s_{t-1} \quad (34)$$

Substituting [Eqs. \(10\)](#), [\(13\)](#) and [\(14\)](#) into [Eq. \(34\)](#), we obtain:

$$(1-p)(1+n)\frac{b_{t-1}}{h_{t-1}} = (1-\beta)(1-x_{t-1}) - \frac{\beta(\delta+p(1+n)\gamma)}{R_t} \quad (35)$$

Combining [Eqs. \(11\)](#) and [\(35\)](#), we get an expression for education compatible with the asset market equilibrium:

$$e_{t-1} = \frac{1-\beta}{(1-p)(1+n)} \left( 1 - x_{t-1} - \frac{\beta(\delta+p(1+n)\gamma)}{(1-\beta)R_t} \right) \quad (36)$$

Substituting this expression of  $e_{t-1}$  into [Eq. \(12\)](#) and replacing  $x_t$  and  $x_{t-1}$  by the effective repayment shares  $\hat{x}_t$  and  $\hat{x}_{t-1}$  we get the effective debt repayment share given by the asset market equilibrium:

$$\hat{x}_t = \frac{R_t}{A} \left[ \frac{1-\beta}{(1-p)(1+n)} \left( 1 - \hat{x}_{t-1} - \frac{\beta(\delta+p(1+n)\gamma)}{(1-\beta)R_t} \right) \right]^{1-\lambda} \quad (37)$$

Under [Eq. \(25\)](#), we know that  $\hat{x}_t$  is equal to either  $x_t^*$  or  $\bar{x}_t$  which - according to [Eqs. \(18\)](#) and [\(26\)](#) - both depend on  $R_{t+1}$ . Therefore, we know that  $\hat{x}_{t-1}$  depends on  $R_t$  and we can define the following functions:

$$\hat{x}_t = \min \{x^*(R_{t+1}), \bar{x}(R_{t+1})\} \equiv \hat{x}(R_{t+1}) \quad (38)$$

$$\hat{x}_{t-1} = \min \{x^*(R_t), \bar{x}(R_t)\} \equiv \hat{x}(R_t) \quad (39)$$

This allows us to get the equation describing the dynamics of the interest rate:

$$\hat{x}(R_{t+1}) = \frac{R_t}{A} \left[ \frac{1-\beta}{(1-p)(1+n)} \times \left( 1 - \hat{x}(R_t) - \frac{\beta(\delta+p(1+n)\gamma)}{(1-\beta)R_t} \right) \right]^{1-\lambda} \quad (40)$$

[Eq. \(40\)](#) defines a relationship between  $R_t$  and  $R_{t+1}$  that holds for any  $t \geq 0$ . For simplicity of writing, let us define:

$$\begin{aligned} \Theta(R_t, \hat{x}(R_t)) \\ \equiv R_t \left[ \frac{1-\beta}{(1-p)(1+n)} \left( 1 - \hat{x}(R_t) - \frac{\beta(\delta+p(1+n)\gamma)}{(1-\beta)R_t} \right) \right]^{1-\lambda} \end{aligned} \quad (41)$$

Thus [Eq. \(40\)](#) can be written as  $\hat{x}(R_{t+1}) = \Theta(R_t, \hat{x}(R_t))/A$ . Therefore, the dynamics are characterized by a first-order difference equation with no predetermined variable (since there is no first-period consumption). Interest rate is a forward variable and initial interest rate  $R_0$  should satisfy  $R_0 \leq \frac{\bar{x}h_0}{b_{-1}}$  which guarantees that the first generation of young adult refunds her loan. Any steady state  $R$  will give a stationary value  $x$  and the couple  $(R, x)$  has to satisfy:

$$x = \frac{R}{A} \left[ \frac{1-\beta}{(1-p)(1+n)} \left( 1 - x - \frac{\beta(\delta+p(1+n)\gamma)}{(1-\beta)R} \right) \right]^{1-\lambda} \quad (42)$$

We use [Eq. \(42\)](#) to implicitly express a function  $\tilde{x}(R, p, \gamma)$  defining the combinations  $x$  and  $R$  compatible with a steady state, which depends on parameters  $p$  and  $\gamma$ .

**Lemma 2.2.** Under [Assumptions 1-5](#), the function  $\tilde{x}(R, p, \gamma)$ , goes from 0 (with a positive slope) to 1 when  $R$  increases from  $R_{min}$  to  $+\infty$ , is increasing and concave. Moreover,  $\tilde{x}(R, p, \gamma)$  is decreasing with  $\gamma$ . This function is increasing with  $p$  if:

$$x < 1 - \frac{\beta(\delta+(1+n)\gamma)}{(1-\beta)R}$$

**Proof.** See [Appendix A.7](#).  $\square$

To determine the existence of steady states, we use a graphical approach. We analyze the constrained repayment share  $\bar{x}(R)$ ,

<sup>14</sup> Notice that if  $\gamma = 0$ , the expression of  $R_{min}$  becomes:  $\beta\delta/(1-\beta)$ .

the optimal repayment share  $x^*(R)$  and the steady-state curve  $\tilde{x}(R, p, \gamma)$ , hereafter denominated  $\tilde{x}(R)$ . There exists one trivial steady state such that  $\tilde{x} = 0$  if interest rate is too low (since  $\tilde{x}(R_{\min}) = \bar{x}(R_{\min}) = 0$ ). Let us now focus on the non-trivial steady states.

**Proposition 2.3.** *Under Assumptions 1–5, there may exist multiplicity of long-run equilibria. Let us consider:*

$$\check{A} \equiv \frac{\Theta(\check{R}, \check{x})}{\check{x}} \quad (43)$$

1. If  $A > \check{A}$  (Case 1, Fig. 2.3), there is an unconstrained steady state  $R_u > \check{R}$  which coexists with a constrained steady state  $R_c < \check{R}$ .
2. If  $A < \check{A}$ , there is no unconstrained steady state (since  $R_u < \check{R}$ ) and there may exist:
  - no constrained steady state (Case 2, Fig. 2.3).
  - two constrained steady states  $R_{c1}$  and  $R_{c2}$  such that  $R_{c1} < R_{c2} < R_u < \check{R}$  (Case 3, Fig. 2.3).

**Proof.** See Appendix A.8.  $\square$

The specificity of this model with the no-commitment setting is the fact that in order to save, agents have to refund their loans. Then, the tightness of the endogenous constraint depends on the agents' willingness to save. Therefore, both the willingness to refund the loan and the incentive to save depend on the expected interest rate  $R$ , the productivity  $A$  and the parameters linked to the return on education ( $\gamma$ ,  $\delta$  and  $p$ ). As a result, the relationship between savings and  $R$  is not very intuitive: a higher  $R$  increases the return on savings but increases as well the cost of refunding the loan. This explains why we may have lower savings when agents expect higher  $R$ .

Proposition 2.3 states that there exist three possible configurations of equilibria. These configurations depend on where the steady state curve  $\tilde{x}(R)$  and the optimal share of repayment  $x^*(R)$  intersect, which finally changes with the level of productivity  $A$ . In presence of migrations ( $p > 0$ ), remittances ( $\gamma > 0$ ) and a third period income ( $\delta > 0$ ), there may exist multiple steady states and the number of constrained/unconstrained steady state depends on both the level of productivity  $A$  and the expected interest rate  $R$ . We may distinguish the two cases: (i) When productivity of education is high ( $A > \check{A}$ ), the agents have large incentive to save (other things being equal). This means that the utility of default is low. In this case, there exist two steady states: an unconstrained one and a constrained one. When agents expect  $R$  to be large enough, their willingness to save is high. Thus, they can borrow as much as they want, educate optimally. Since they expect large remittances and a high cost of refunding the loan, they do not save a lot finally. This situation refers to the unconstrained steady state,  $R_u$ , on Fig. 2.3: the share of debt repayment is high, saving is low and the unconstrained interest rate is high. When agents expect a low  $R$ , it makes saving less attractive, agents are constrained, may invest less in education, expect less remittances but a low cost of refunding and finally save more: the constrained steady state is  $R_c$  with a lower interest rate.

(ii) When productivity of education is low ( $A < \check{A}$ ), the agents have low incentive to save whatever they expect for  $R$  and thus low incentive to refund their loans. When it exists, the steady state is constrained and the level of  $R$  depends on what agents expect. If they expect a higher  $R$ , their willingness to save is higher, the amount they are allowed to borrow may be higher, they expect higher remittances but a higher cost of refunding and finally have lower savings: the constrained steady state is at  $R_{c2}$  instead of  $R_{c1}$ . This gives the intuition for having two constrained steady states with low interest rates.

**Remark 2.2.** Without migration ( $p = 0$ ) or without remittances ( $\gamma = 0$ ), if  $\delta = 0$ , the utility of default is zero and then there exists a unique unconstrained equilibrium such that  $\tilde{x}(R) = \lambda$ . Therefore, without labor income in last period, migration and remittances entail multiple equilibria.

In the next section, we deal with local stability of these equilibria.

### 3. The equilibrium dynamics

In the case where both the current interest rate and the expected one are lower than  $R_{\min}$ , agents neither save nor borrow and the equilibrium is self-fulfilled.

We now focus on the non-trivial steady states. For simplicity, we follow de la Croix and Michel (2007) and we do not consider the dynamics including regime shifts. Therefore, we study the dynamics of the constrained and unconstrained regime separately.

Let us first studying the unconstrained regime on the interval  $[\check{R}, +\infty[$ . In that case,  $\tilde{x}(R_{t+1}) = x^*(R_{t+1})$ . Therefore, dynamics satisfy:

$$x^*(R_{t+1}) = \frac{R_t}{A} \left[ \frac{1 - \beta}{(1 - p)(1 + n)} \times \left( 1 - x^*(R_t) - \frac{\beta(\delta + p(1 + n)\gamma)}{(1 - \beta)R_t} \right) \right]^{1 - \lambda} \quad (44)$$

Eq. (44) can be written as:

$$x^*(R_{t+1}) = \frac{1}{A} \Theta(R_t, x^*(R_t))$$

where the function  $\Theta(\cdot)$  comes from Eq. (41). As  $x^*(\cdot)$  is invertible on the interval  $]\lambda, +\infty[$ , this equation allows us to define the function:

$$R_{t+1} = \psi^*(R_t)$$

where:

$$\psi^*(R_t) = x^{*-1} \left( \frac{1}{A} \Theta(R_t, x^*(R_t)) \right)$$

The function  $\psi^*(R_t)$  is defined if:

$$\frac{1}{A} \Theta(R_t, x^*(R_t)) > \lambda \quad \forall R_t \in [\check{R}, +\infty[$$

Since on the interval  $[\check{R}, +\infty[$ , the function  $\Theta(R, x^*(R))$  is positive and increasing (see Appendix A.9), the function  $\psi^*(R)$  is defined on the whole interval  $[\check{R}, +\infty[$  if and only if  $\Theta(\check{R}, \check{x})/A > \lambda$ . This is equivalent to:

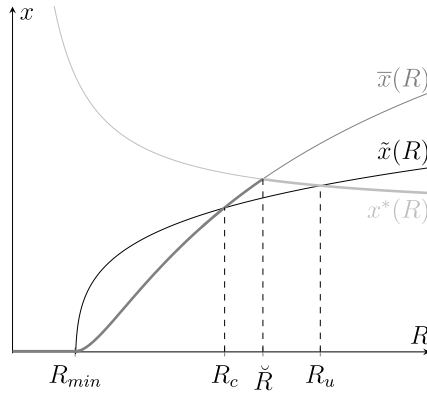
$$A < \frac{\Theta(\check{R}, \check{x})}{\lambda} \equiv A_\lambda \quad (45)$$

**Proposition 3.1.** *Under Assumptions 1–5, and if  $A > \check{A}$ , the unconstrained steady state  $R_u > \check{R}$  is locally unstable.*

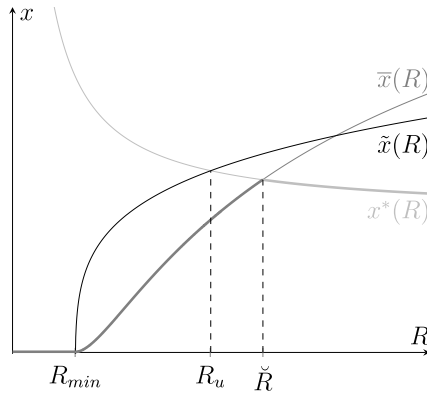
**Proof.** See Appendix A.10.  $\square$

Notice that if the condition given by Eq. (45) is not satisfied, the unconstrained dynamics may be not defined for some values of interest rate close to  $\check{R}$ . Hence, dynamics may not stay in the unconstrained regime and there may be a regime shift. We now focus on the constrained regime on the interval  $[R_{\min}, \check{R}]$ . In that case,  $\tilde{x}(R_{t+1}) = \bar{x}(R_{t+1})$ . Therefore, dynamics satisfy:

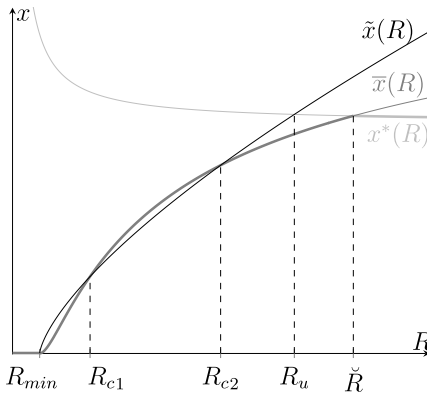
$$\bar{x}(R_{t+1}) = \frac{1}{A} \Theta(R_t, \bar{x}(R_t)) \quad (46)$$



Case 1 ( $A > \check{A}$ ): In this configuration, there are two non-trivial steady states: a constrained one  $R_c$  and an unconstrained one  $R_u$ .



Case 2 ( $A < \check{A}$ ): In this configuration, there is no non-trivial steady state, and  $R_u$  is not an equilibrium.



Case 3 ( $A < \check{A}$ ): In this configuration, there are two non-trivial constrained steady states:  $R_{c1}$  and  $R_{c2}$ .  $R_u$  is not an equilibrium.

Fig. 2.3. Representations of  $\bar{x}$ ,  $x^*$  and  $\tilde{x}$ .

where the function  $\Theta(\cdot)$  comes from Eq. (41). As  $\bar{x}(\cdot)$  is invertible on the interval  $]0, 1[$ , we define:

$$R_{t+1} = \bar{\psi}(R_t)$$

with:

$$\bar{\psi}(R_t) = \bar{x}^{-1} \left( \frac{1}{A} \Theta(R_t, \bar{x}(R_t)) \right)$$

The function  $\bar{\psi}(R)$  is defined if:

$$0 < \frac{1}{A} \Theta(R, \bar{x}(R)) < 1 \quad \forall R \in [R_{min}, \check{R}]$$

Since  $\Theta(R, \bar{x}(R))$  is positive and increasing on  $[R_{min}, \check{R}]$  (see Appendix A.11), the function  $\bar{\psi}(R)$  is defined on the whole interval  $[R_{min}, \check{R}]$  if and only if  $\Theta(\check{R}, \check{x})/A < 1$ . This is equivalent to:

$$A > \Theta(\check{R}, \check{x}) \equiv A_1 \quad (47)$$

**Proposition 3.2.** Under Assumptions 1–5, and if  $A > \check{A}$  the constrained equilibrium  $R_c < \check{R}$  is locally stable. In the configuration where  $A < \check{A}$  and where there exist two constrained steady states, the first  $R_{c_1}$  is locally stable while the second  $R_{c_2}$  is locally unstable.

**Proof.** See Appendix A.12.  $\square$

Notice that if the condition given by Eq. (47) is not satisfied, the constrained dynamics may be not defined for some values of interest rate close to  $\check{R}$ .

In the previous propositions, we have defined a threshold on the productivity of education  $\check{A}$ . If  $A > \check{A}$ , there exist two non trivial steady states  $R_c$  and  $R_u$ . The first  $R_c$  is constrained and locally stable. There is an infinite number of trajectories converging to this steady state meaning local indeterminacy. The second  $R_u$  is unconstrained and locally unstable meaning that the economy stays at  $R_u$  if the expected interest rate is  $R_u$  at the initial period. In this configuration, there is indeterminacy as the interest rate at the equilibrium could satisfy  $R_{min}$  or a trajectory leading to one of the two non trivial steady states. If  $A < \check{A}$ , there may exist zero constrained steady states or two. The economy does not grow in the non trivial steady state. Otherwise the first constrained steady state  $R_{c_1}$  is locally stable – implying that there exists an infinite number of trajectories leading to this steady state – and the second  $R_{c_2}$  is locally unstable – meaning that there exists a unique trajectory conducting to select  $R_{c_2}$  from the initial period. As a consequence, there is global indeterminacy. Each steady state is an equilibrium.

Focusing on the impact of migration and remittances on the non-trivial steady states, the effect is ambiguous. Each of this phenomenon may exert a positive or a negative effect on the repayment share,  $x$  for a given steady state. Moreover, we know that an increase in  $x$  is not necessarily associated to an increase in education  $e$ , since an increase in  $x$  affects the interest rate which may either decrease or increase education. Moreover due to multiple equilibria, both migration and remittances may bring the economy to another steady state. Finally,  $p$  and  $\gamma$  have an impact on  $\check{A}$ , meaning that migration and remittances may affect the existence of steady states in addition to the convergence towards one steady state or another. In other words, they may bring an economy from one case of Fig. 2.3 to another. For these reasons it is difficult to theoretically predict an impact of migration and remittances in this framework. This model is able to explain both negative and positive impact of remittances. In the next section, we illustrate the impact of migration and remittances for some recipient countries.

#### 4. Calibration

We assume that each period lasts for 25 years. We calibrate the model for 30 recipient countries over the period 1970–2017 considering the average of the data over the entire period: Algeria, Benin, Bangladesh, Burkina Faso, Cabo Verde, China, Egypt, Eswatini (Swaziland), Guinea-Bissau, India, Indonesia, Kenya, Mali, Mexico, Morocco, Myanmar, Namibia, Pakistan, Papua New Guinea, Philippines, Senegal, Slovak Republic, Solomon Islands, Sri Lanka, Syrian Arab Republic, Tanzania, Uganda, Vietnam, West Bank and Gaza (Palestinian territories) and Zambia. We calibrate parameters in order to make the steady state, with migrations and remittances, matching the data. Through this calibration exercise, we want to explore the effect of migrations and remittances on the long-run economic growth. We compare for each country the three situations: the economy with both migration and remittances considered as the benchmark, the economy with migration but no remittances and finally the economy without migration or remittances.

At this stage, we have to determine for each country the suitable numerical values of parameters  $n, p, \delta, \lambda, \gamma, A$  and  $\beta$ . We first explain the calibration of  $n, p, \delta$  and  $\lambda$  which result directly from the data. The remaining parameters – which will influence remittances over GDP, the long-run growth of GDP *per capita* and the long-run interest rate – are the solutions of a 3-dimension system.

In the model, parameter  $p$  corresponds to the proportion of young agents leaving the country per period and parameter  $n$  represents the growth rate of births. We use the World Bank data on migration,<sup>15</sup> population size and annual population growth. We calibrate  $p$  dividing the yearly average number of migrations over the long period by the average size of population. We convert this proxy of annual rate of migration to a 25-year rate. Then, parameter  $n$  is calibrated to match the average growth rate of population observed over 25 years. We calibrate  $\delta$  – the ratio between human capital of the old workers and human capital of the young workers – as the ratio of workers older than 50 years over the working population aged between 25 and 50, using ILO data.<sup>16</sup> To calibrate parameter  $\lambda$  we follow Gente et al. (2015) and we choose  $\lambda = 0.45$  for all countries.<sup>17</sup>

For the remaining parameters ( $\gamma, A$  and  $\beta$ ), we proceed in two steps. First, we determine values of these parameters which fit the data simultaneously for remittances over GDP, growth rate of GDP *per capita* and long-run interest rate. Second, we determine whether the observed long-run interest rate corresponds to a constrained or an unconstrained steady state of the model.

We first express the total amount of remittances entering in the country at period  $t$  as  $F_t = (1 - p)N_{t-2}f_t$ . Then we compute the amount of remittances relative to GDP in period  $t$ , defined by  $F_t/Y_t$ . Using Eqs. (1), (2), (5) and (32), this expression can be written as:

$$\frac{F_t}{Y_t} = \frac{p(1+n)\gamma}{(1-p)(1+n)\frac{h_t}{h_{t-1}} + \delta}$$

Using Eq. (31) and the relation between  $e$  and  $x$  resulting from Eq. (12), the ratio of remittances to GDP in the long run can be written:

$$\frac{F}{Y} = \frac{p(1+n)\gamma}{(1-p)(1+n)A\left(\frac{A\hat{x}(R)}{R}\right)^{\frac{\lambda}{1-\lambda}} + \delta} \quad (48)$$

Then, we determine the growth rate of GDP *per capita* for a 25-year period, denoted by  $G$ . Using Eqs. (1) and (32), knowing that in period  $t$  the number of workers (young and old) is equal to  $[(1-p)(1+n)+1](1-p)N_{t-2}$  and using the expression  $h_t = Ae^{\lambda}h_{t-1}$  coming from Eqs. (4) and (11), we can express the growth rate of GDP *per capita* between period  $t-1$  and  $t$  as:

$$G_{t-1} = \frac{h_{t-1}[(1-p)(1+n)A\hat{e}_{t-1}^{\lambda} + \delta]}{h_{t-2}[(1-p)(1+n)A\hat{e}_{t-2}^{\lambda} + \delta]} - 1$$

Since at steady state, the effective education per unit of human capital is constant ( $\hat{e}_{t-1} = \hat{e}_{t-2}$  because  $R$  is constant), we get  $G = A\hat{e}(R)^{\lambda} - 1$ . Hence, using Eq. (12) we can express the annual growth rate of GDP *per capita* in the long-run equilibrium, denoted by  $g$  as:

$$g = \left( A \left( \frac{A\hat{x}(R)}{R} \right)^{\frac{\lambda}{1-\lambda}} \right)^{\frac{1}{25}} - 1 \quad (49)$$

<sup>15</sup> In these data, we have figures every 5 years, during 45 years.

<sup>16</sup> ILO data only covers the period 1990–2018.

<sup>17</sup> Gente et al. (2015) also calibrate their model with  $\lambda = 0.35$  and  $\lambda = 0.55$ .

**Table 1**  
Calibrated parameters for each recipient country.

| Country            | $n$    | $p$    | $\delta$ | $\beta$ | $\gamma$ | $A$     |
|--------------------|--------|--------|----------|---------|----------|---------|
| Algeria            | 0.8007 | 0.0354 | 0.19     | 0.6233  | 0.5843   | 4.8070  |
| Bangladesh         | 0.7616 | 0.0714 | 0.30     | 0.5907  | 1.1406   | 4.3220  |
| Benin              | 1.0303 | 0.0065 | 0.33     | 0.5821  | 7.9734   | 3.3632  |
| Burkina Faso       | 1.0639 | 0.0811 | 0.27     | 0.6382  | 0.8130   | 4.5863  |
| Cabo Verde         | 0.8605 | 0.2186 | 0.26     | 0.5215  | 1.7711   | 7.1658  |
| China              | 0.3389 | 0.0045 | 0.31     | 0.7821  | 2.7511   | 26.6500 |
| Egypt              | 0.7671 | 0.0286 | 0.26     | 0.5955  | 5.3188   | 5.1905  |
| Eswatini           | 0.9613 | 0.0792 | 0.19     | 0.6768  | 1.2786   | 5.5047  |
| Guinea-Bissau      | 0.9579 | 0.1533 | 0.32     | 0.5412  | 0.2697   | 2.9109  |
| India              | 0.6004 | 0.003  | 0.33     | 0.6838  | 18.6272  | 6.4864  |
| Indonesia          | 0.5760 | 0.0113 | 0.34     | 0.7085  | 1.5110   | 6.6489  |
| Kenya              | 1.1817 | 0.0006 | 0.25     | 0.6843  | 37.6215  | 4.2018  |
| Mali               | 1.0077 | 0.0990 | 0.26     | 0.6047  | 0.6165   | 4.4760  |
| Mexico             | 0.7628 | 0.0785 | 0.30     | 0.6456  | 0.3214   | 4.1757  |
| Morocco            | 0.6808 | 0.0887 | 0.32     | 0.5750  | 1.3459   | 4.4743  |
| Myanmar            | 0.5570 | 0.0638 | 0.25     | 0.6639  | 0.7285   | 11.0463 |
| Namibia            | 0.8917 | 0.0168 | 0.24     | 0.6583  | 0.3115   | 3.4474  |
| Pakistan           | 0.9279 | 0.0145 | 0.33     | 0.5800  | 6.6625   | 4.8178  |
| Papua New Guinea   | 0.8666 | 0.0026 | 0.32     | 0.6182  | 1.3215   | 3.3955  |
| Philippines        | 0.8538 | 0.0476 | 0.32     | 0.5569  | 2.4804   | 3.9139  |
| Senegal            | 1.1258 | 0.0617 | 0.27     | 0.5716  | 1.0709   | 2.9915  |
| Slovak Republic    | 0.1144 | 0.0118 | 0.29     | 0.6521  | 2.9231   | 5.7350  |
| Solomon Islands    | 1.1296 | 0.0496 | 0.31     | 0.6130  | 0.4226   | 3.3468  |
| Sri Lanka          | 0.4822 | 0.0971 | 0.36     | 0.5693  | 1.5106   | 5.5876  |
| Syrian Arab Rep.   | 1.1635 | 0.1895 | 0.21     | 0.6030  | 0.2456   | 5.9688  |
| Tanzania           | 1.1325 | 0.0035 | 0.32     | 0.7089  | 2.1385   | 5.3075  |
| Uganda             | 1.2380 | 0.0146 | 0.25     | 0.6495  | 5.5037   | 5.8774  |
| Vietnam            | 0.5721 | 0.0312 | 0.29     | 0.6432  | 6.4611   | 8.9925  |
| West Bank and Gaza | 1.2850 | 0.0398 | 0.18     | 0.5445  | 7.0118   | 5.3207  |
| Zambia             | 1.1091 | 0.0059 | 0.28     | 0.6200  | 0.7370   | 3.2075  |

Finally, from Eq. (40), if a long-run equilibrium  $R$  exists, it satisfies:

$$\hat{\kappa}(R) = \frac{R}{A} \left[ \frac{1 - \beta}{(1 - p)(1 + n)} \left( 1 - \hat{\kappa}(R) - \frac{\beta(\delta + p(1 + n)\gamma)}{(1 - \beta)R} \right) \right]^{1 - \lambda} \quad (50)$$

The numerical values of  $\gamma$ ,  $A$  and  $\beta$  are determined using Eqs. (48)–(50) and data given by the World Bank:

- The value of  $\gamma$  is such that remittances to GDP at steady state ( $F/Y$ ) fit the average observed percentage of remittances relative to GDP.
- The value of  $A$  is such that the annual growth rate of GDP *per capita* at steady state ( $g$ ) matches the observed annual growth rate of GDP *per capita*.
- The value of  $\beta$  is such that the observed 25-year average real interest rate is a solution of Eq. (50).

Substituting the expression of  $\hat{\kappa}(R)$  and the numerical values of  $n$ ,  $p$ ,  $\delta$ ,  $\lambda$  and  $R$  in the 3 considered equations, we solve the system twice to get the values  $\gamma$ ,  $A$  and  $\beta$ . Once, substituting  $\hat{\kappa}_t = x^*(R)$  from Eq. (18), and a second time substituting  $\hat{\kappa}_t = \bar{x}(R)$  from Eq. (26). This allows us to determine whether there exists a constrained steady state fitting the data ( $R_{c1}$  or  $R_{c2}$  if  $A < \bar{A}$ , or  $R_c$  if  $A > \bar{A}$ ) or an unconstrained one ( $R_u$  if  $A > \bar{A}$ ).<sup>18</sup> Parameter values are collected in Table 1. For each country, Table 3 (in Appendix A.13) gives the corresponding annual values of parameters, or the variables, resulting from the calibration.

Having identified  $R_c$ ,  $R_{c1}$ ,  $R_{c2}$  or  $R_u$  in the benchmark case, we now compute annual growth rates for the two other configurations: the one with migration only and the one without migration or remittances. As there is multiplicity of equilibria, we

<sup>18</sup> Notice that we consider only the combinations of parameters in which  $\beta > 0.5$ . This implies that, according to Eq. (3), agents give more importance to consumption in first period of work than consumption in second period of work (preference for present).

**Table 2**  
The possible impact of migration and remittances on economic growth.

| Country                      | Migration | Migration and remittances |
|------------------------------|-----------|---------------------------|
| Algeria                      | ?         | ?                         |
| China                        | ?         | ?                         |
| Guinea-Bissau                | +         | +                         |
| Myanmar                      | ?         | ?                         |
| Pakistan                     | ?         | ?                         |
| Syrian Arab Rep.             | ?         | ?                         |
| Uganda                       | ?         | ?                         |
| Vietnam                      | ?         | ?                         |
| West Bank and Gaza           | ?         | ?                         |
| Zambia                       | ?         | ?                         |
| Other countries <sup>a</sup> | ?         | +                         |

<sup>a</sup>Other countries are: Benin, Bangladesh, Burkina Faso, Cabo Verde, Egypt, Eswatini, India, Indonesia, Kenya, Mali, Mexico, Morocco, Namibia, Papua New Guinea, Philippines, Senegal, Slovak Republic, Solomon Islands, Sri Lanka, and Tanzania.

compare the growth rate in the benchmark with the growth rates predicted by the model in each configuration. Figs. 4.1 and 4.2 show the computed growth rate in each case. We can conclude that while migration alone has an ambiguous effect in almost all countries considered<sup>19</sup> (except in Guinea-Bissau), migration and remittances affect positively the growth rate in the majority of countries. Table 2 summarizes the main results and underlines the countries in which the effect of migration and remittances may differ from the others.

The economy with the highest average growth rate for the period 1970–2017 is China. In this economy, we do not find a suitable solution of the 3-dimension system where  $\hat{\kappa}_t = x^*(R)$  but we do find a suitable solution of the 3-dimension system where  $\hat{\kappa}_t = \bar{x}(R)$ .<sup>20</sup> The equilibrium depicted by the data corresponds

<sup>19</sup> The isolated impact of migration is ambiguous due to multiple equilibria. However, this impact is positive if the steady state with migration is the same as the steady state without migration.

<sup>20</sup> Parameter  $\beta$  would be negative in the first case, which is not realistic.

**Table 3**  
Annual values of main variables for each country.

| Country            | Demographic growth | Emigration rate | Remittances over GDP | Real interest rate | Growth rate of GDP p. c. |
|--------------------|--------------------|-----------------|----------------------|--------------------|--------------------------|
| Algeria            | 2.23%              | 0.14%           | 1.39%                | -2.22%             | 1.45%                    |
| Bangladesh         | 1.99%              | 0.30%           | 4.58%                | 6.07%              | 2.22%                    |
| Benin              | 2.85%              | 0.03%           | 3.72%                | 2.90%              | 0.86%                    |
| Burkina Faso       | 2.59%              | 0.34%           | 4.07%                | 2.81%              | 1.95%                    |
| Cabo Verde         | 1.51%              | 0.98%           | 14.94%               | 8.23%              | 4.68%                    |
| China              | 1.16%              | 0.02%           | 0.18%                | 2.04%              | 7.89%                    |
| Egypt              | 2.18%              | 0.12%           | 7.38%                | 2.82%              | 2.75%                    |
| Eswatini           | 2.39%              | 0.33%           | 5.41%                | 3.61%              | 2.66%                    |
| Guinea-Bissau      | 2.04%              | 0.66%           | 3.53%                | 1.24%              | 0.70%                    |
| India              | 1.89%              | 0.01%           | 2.10%                | 6.13%              | 3.67%                    |
| Indonesia          | 1.79%              | 0.05%           | 0.64%                | 5.77%              | 3.70%                    |
| Kenya              | 3.37%              | 0.002%          | 1.51%                | 6.15%              | 1.30%                    |
| Mali               | 2.40%              | 0.42%           | 3.98%                | 0.73%              | 1.79%                    |
| Mexico             | 1.96%              | 0.33%           | 1.52%                | 3.47%              | 1.94%                    |
| Morocco            | 1.72%              | 0.37%           | 6.24%                | 4.57%              | 2.58%                    |
| Myanmar            | 1.52%              | 0.26%           | 1.46%                | -0.36%             | 4.80%                    |
| Namibia            | 2.51%              | 0.07%           | 0.39%                | 5.53%              | 0.85%                    |
| Pakistan           | 2.60%              | 0.06%           | 5.18%                | 1.86%              | 2.19%                    |
| Papua New G.       | 2.52%              | 0.01%           | 0.24%                | 5.11%              | 0.94%                    |
| Philippines        | 2.30%              | 0.19%           | 7.08%                | 4.52%              | 1.82%                    |
| Senegal            | 2.80%              | 0.25%           | 5.59%                | 3.30%              | 0.47%                    |
| Slovak Rep.        | 0.39%              | 0.05%           | 1.21%                | 6.35%              | 3.93%                    |
| Solomon Isl.       | 2.86%              | 0.20%           | 1.62%                | 4.96%              | 0.76%                    |
| Sri Lanka          | 1.17%              | 0.41%           | 6%                   | 2.87%              | 3.63%                    |
| Syrian Arab Rep.   | 2.27%              | 0.84%           | 2.99%                | -1.34%             | 2.38%                    |
| Tanzania           | 3.06%              | 0.01%           | 0.40%                | 6.28%              | 2.21%                    |
| Uganda             | 3.21%              | 0.06%           | 4.32%                | 1.89%              | 2.32%                    |
| Vietnam            | 1.70%              | 0.13%           | 5.91%                | 3.62%              | 4.93%                    |
| West Bank and Gaza | 3.19%              | 0.16%           | 14.70%               | 3.89%              | 2.59%                    |
| Zambia             | 3.01%              | 0.02%           | 0.37%                | 0.12%              | 0.19%                    |

to the constrained equilibrium  $R_{c1} < \check{R}$  in our model. Without remittances or without migration, the long-run solution of the model is still two constrained steady states  $R_{c1}$  and  $R_{c2}$ . Without remittances, the equilibrium  $R_{c1}$  would correspond to an average growth rate evaluated at 7.82% per year. Without remittances or migration, this rate would be 7.80%. However, if the long-run equilibrium is  $R_{c2}$ , the annual growth is 9.16% without remittances and 9.15% without migration. Hence, the impact of both migration and remittances is positive if the equilibrium stays in  $R_{c1}$ . Otherwise, the impact of migration or/and remittances is ambiguous. Notice that an unconstrained steady state is not possible without remittances since  $R_u < \check{R}$  for the given parameters. We have similar qualitative results for Zambia.

The case of Egypt is different since the configuration of equilibria changes when we remove remittances. There are two constrained steady states  $R_{c1}$  and  $R_{c2}$  in the benchmark, whereas there are a unique constrained steady state  $R_c$  and an unconstrained steady state  $R_u > \check{R}$  without remittances (and without migration). As for the above cited countries, we do not find a combination of parameters for an unconstrained steady state to exist with remittances. However, without remittances, we get  $A > \check{A}$  implying both a unique constrained and a unique unconstrained steady state. If the equilibrium is the constrained one,  $R_c$ , the growth rate is 1.80% per year without remittances and 1.72% per year without migration. If the equilibrium is the unconstrained one,  $R_u$ , the model predicts a growth rate of 2.73% without remittances and 2.68% without migration. Therefore, the overall impact of migration and remittances is positive (+59.73% or +2.70%). Although migration and remittances make the constraint binding, they increase education and growth. We can also notice that the impact of migration alone is positive only if migration does not imply a jump from  $R_u$  to  $R_c$ . Finally, the model predicts a positive impact of remittances (2.75% > 1.8% and 2.75% > 2.73%). We have the same qualitative results for Sri Lanka, Eswatini, Morocco, Bangladesh, Burkina Faso, Philippines and Senegal: the overall impact of migration and remittances is positive but migration

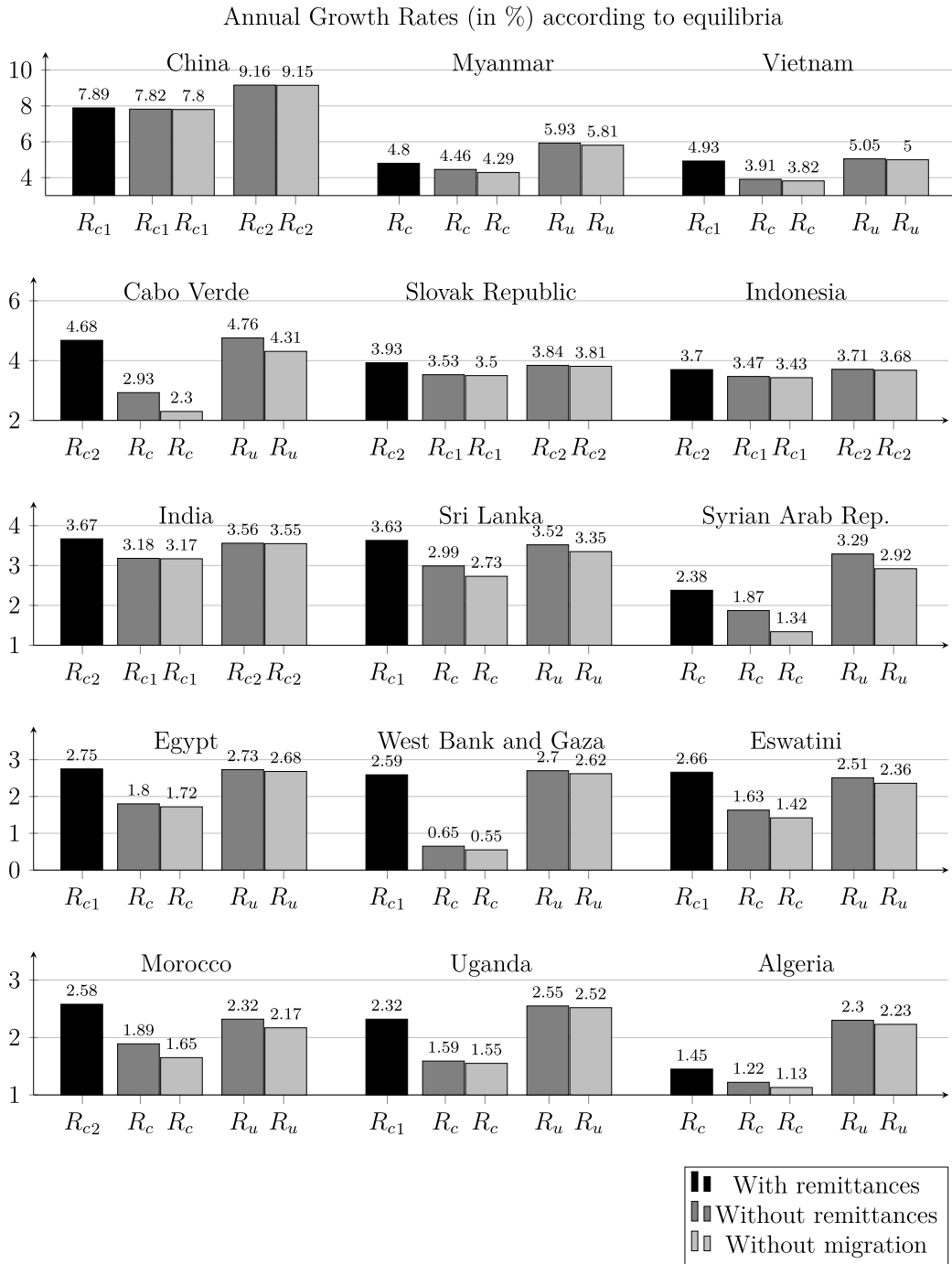
(without remittances) may have a negative impact if it entails a jump from  $R_u$  to  $R_c$ . The configuration in Vietnam, West Bank and Gaza, Uganda and Pakistan is similar with two constrained steady states with remittances and a unique constrained steady state without remittances. However, the overall impact of migration and remittances may be negative in these countries when they are unconstrained without migration or remittances. In addition, in Cabo Verde and in Mali, the overall impact of migration and remittances is necessarily positive, but the isolated impact of migration may be negative as the isolated impact of remittances.

In Guinea-Bissau, the configuration of steady states depends on whether there is migration or not. There are two constrained steady states with migration (and with remittances) but there are a unique constrained steady state and an unconstrained steady state without migration. According to the model, the impact of migration alone is necessarily positive and the isolated impact of remittances as well; so does the overall impact of migration and remittances.

In Myanmar, Syrian Arab Republic and Algeria, there is a constrained steady state which coexists with an unconstrained steady state in each configuration. The one fitting the data with remittances is constrained. The impact of migration and/or remittances is positive if they do not imply a jump from  $R_u$  to  $R_c$ . In others words, except for the configuration of steady states with remittances, the impact of migration and remittances is qualitatively the same as in Vietnam, West Bank and Gaza, Uganda and Pakistan: the overall impact of migration and remittances may be negative.

In other countries,<sup>21</sup> the configuration of steady states does not change with migration and remittances, and their overall impact is always positive. For instance, in Mexico the model is calibrated to depict an economic growth of 1.94% per year. The model predicts that the overall impact of migration and

<sup>21</sup> As for other countries in the sample, we do not find any realistic solution for the 3-dimension system for an unconstrained equilibrium.



Source: Authors' computations.

Note: The first bar of each graph depicts the growth rate for the steady state which fits the data with remittances. The other bars depict the predicted growth rates without remittances and without migration for each possible equilibrium of the model.

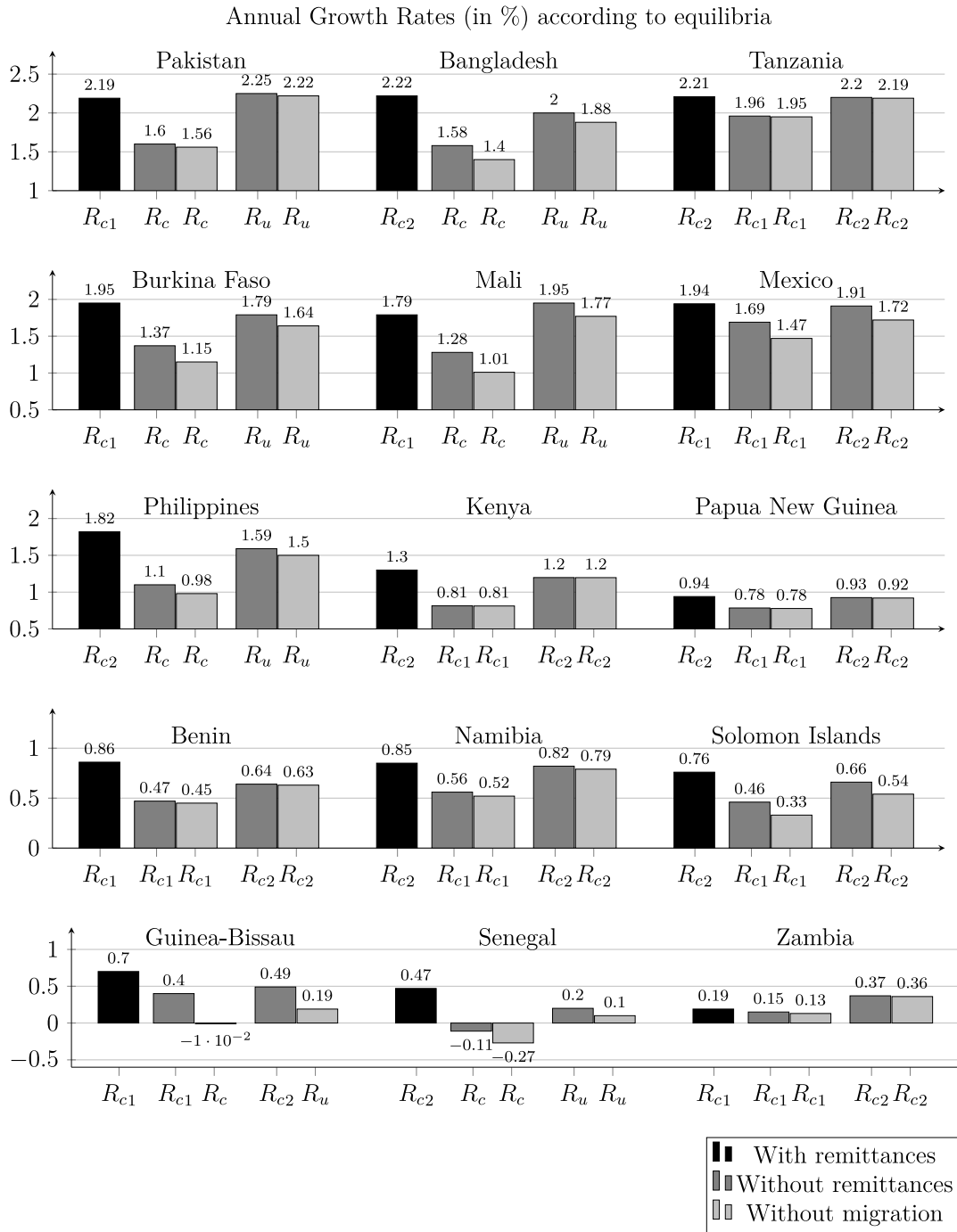
Fig. 4.1. Comparison of growth rate of GDP per capita with remittances, without remittances and without migration.

remittances is always positive. The isolated impact of remittances is also positive (1.94% > 1.69% and 1.94% > 1.91%) while the impact of migration is ambiguous due to the possibility of a jump of steady state.

Using Eq. (49), we compute the spread between the lowest and the highest growth rates associated to each type of existing equilibrium (among  $R_{c1}$ ,  $R_{c2}$ ,  $R_c$  and  $R_u$ ) which satisfies Eq. (50) for each country. We find that the spread in growth rates is always lower in an economy with remittances. For example for China,

there is a potential spread between growth rates of 1.3 with remittances, against 1.34 without remittances, and 1.35 without migrations. We observe exactly the same result for all other countries. According to this, we argue that remittances may be good to reduce growth volatility between potential equilibria. This result is in line with the literature by Chami et al. (2008).

As explained in the previous sections, remittances, as any type of income occurring during the third period of life, give the opportunity to agents to choose between defaulting on their loans



Source: Authors' computations.

Note: The first bar of each graph depicts the growth rate for the steady state which fits the data with remittances. The other bars depict the predicted growth rates without remittances and without migration for each possible equilibrium of the model.

Fig. 4.2. Comparison of growth rate of GDP *per capita* with remittances, without remittances and without migration.

or participating in the asset market. In other words, remittances increase the utility of default, and finally makes the endogenous constraint more stringent. This means that, in an economy with remittances, an unconstrained steady state is less likely to occur than in an economy without remittances – or without remittances or migrations. It is interesting to notice in Figs. 4.1 and

4.2 that only three countries, Algeria, Myanmar and Syrian Arab Republic, keep an unconstrained steady state with remittances.

Finally, the model predicts a positive overall impact of migration and remittances on the growth rate of human capital in the majority of countries in the sample. Nevertheless, due to indeterminacy, the separated impact of migration is ambiguous.



This can explain the difficulty for empirical studies to find a robust effect of separated impact.

We have seen that remittances tend to reinforce the constraint giving less incentive for agents to refund their loan. This *ex-ante* negative effect is true at a partial equilibrium level (or if we consider an exogenous interest rate). Here,  $R$  is endogenous and finally the global effect of remittances on economic growth depends as well on the reaction of  $R$  and its impact on education.

The reaction of  $R$  to a shock on remittances depends on the pre-shock level of  $R$ . For example, the calibration shows that in countries where there are two constrained steady states with and without remittances, remittances make  $R_{c1}$  higher and  $R_{c2}$  lower. These interest rate reactions come from the fact that the relationship between savings and interest rate is non-monotonic<sup>22</sup>:

(i) when  $R$  is low ( $R_{c1}$ ), saving is increasing with  $R$ . The rise in remittances tends to increase the borrowing on one side and to decrease saving on the other side. Thus, to clear the asset market, the interest rate  $R_{c1}$  increases.

(ii) when  $R$  is high ( $R_{c2}$ ), saving is decreasing with  $R$ . The rise in remittances tends to increase the borrowing on one side and to decrease savings on the other side. Thus, to clear the asset market, the interest rate  $R_{c2}$  decreases.

We have that  $R_{c1}$  ( $R_{c2}$ ) belongs to the area where constrained education is increasing (decreasing) with  $R$  (see the right part of Fig. 2.2). Both the rise in  $R_{c1}$  and the drop in  $R_{c2}$  increases the level of effective education, and finally may offset the initial *ex-ante* negative effect of remittances. This is the case for instance in Solomon Islands (as shown by Fig. 4.3) or in Benin, India, Kenya, Namibia, Mexico, Papua New Guinea, Slovak Republic and Tanzania. Moreover, in these countries the education is higher with remittances even if the interest rate jumps from one steady state to the other due to remittances. In China and Zambia, the positive effect of the increase in  $R_{c1}$  dominates the *ex-ante* negative effect of remittances if the economies stay at  $R_{c1}$ . However, the effect is negative if these economies were at  $R_{c2}$  without remittances. In that case, the decrease in interest rate from  $R_{c2}$  to  $R_{c1}$  would be too important and the education would be lower at  $R_{c1}$ . Similar explanations hold for other configurations of steady states. Finally, this calibration exercise shows that migration and remittances improve economic growth in 70% of the 30 countries considered. This result suggests that a remittance and migration shock as the COVID-crisis one, may have dramatic effects on economic growth and human capital accumulation in low income countries. Regarding the policy recommendation, Figs. 2.2 and 4.3 show that the policy maker should guide expectations on interest rate towards a level close to the one which maximizes economic growth.

## 5. Concluding remarks

For the last fifty years, the literature studies the impact of emigration on human capital. Even if the first brand of literature reveals a negative impact through the emigration of educated agents, more recently, the literature has shown a possible positive effect. The literature focusing on the impact of remittances on growth also reveals an ambiguous impact. Some studies find a positive impact while others find a negative impact.

Relative to these two topics, this paper aims at providing a theoretical framework in order to explain the impact of both migration and remittances on human capital. More precisely, this paper focuses on their impact on the asset market which allows agents to finance education.

<sup>22</sup> As shown in Section 2, in order to be able to participate in the asset markets, agents have first to refund their loan. Therefore, when  $R$  is low, the cost of refunding is low and saving increases with  $R$ ; whereas when  $R$  is high, the cost of refunding is high and saving decreases with  $R$ .

Following de la Croix and Michel (2007), we consider a model where human capital is growth enhancing and education is financed through loans towards young agents. As a consequence, the growth of human capital is related to the education financed by borrowing. In this model, borrowing constraints are endogenous and depend on the incentive of agents to repay their loan. If they do not repay, they are excluded from the asset market. In this no-commitment setting, agents can borrow as soon as their utility of default is lower than their utility of refunding the loan.

We show that remittances decrease the need to save for the last period of life. As a consequence, agents have less incentive to repay the loan. Therefore, remittances reduce the maximal amount an agent can borrow for a given level of interest rate. However, remittances may increase education at the macroeconomic equilibrium through changes in interest rate. Moreover, migration reduces the number of agents who borrow in the economy. For a given amount of savings, the amount of funds available for each young agent is therefore higher than without migration. At the equilibrium, the impact of migration and remittances is ambiguous. The model is able to predict both positive and negative impacts.

## Appendix

### A.1. Program of the migrant

The migrant's program allows us to give micro-foundations for  $\gamma$  (in Assumption 5).

#### A.1.1. The migration

When young, an agent born in period  $t - 1$  can migrate to a foreign country with the exogenous probability  $p \in [0, 1]$ . Then, she can borrow an amount  $b_{t-1*}$  in the host country to finance her education but faces an *ad hoc* borrowing constraint. We consider a constant returns to scale technology for human capital accumulation according to:

$$h_{t*} = A_* b_{t-1*}^{\lambda_*} h_{t-1}^{1-\lambda_*} \quad (51)$$

where  $A_* > 0$  is the productivity of education,  $0 < \lambda_* < 1$  is the elasticity of human capital to investment in education in the host country and  $h_{t-1}$  is the parents' human capital (in the home country). We assume that moral values transmitted in the early childhood affect the rhythm of human capital accumulation for children. This assumption is empirically consistent (see Black et al. (2005)) and explains why the human capital accumulation function in the foreign country still depends on the level of human capital of parents stayed in the home country.

Similarly to the home country, we consider that a young worker's human capital  $h_{t*}$  at time  $t$  in the foreign country becomes  $\delta_* h_{t*}$  at time  $t + 1$ . Parameter  $\delta_* > 0$  reflects the income growth ability over the migrant's life.

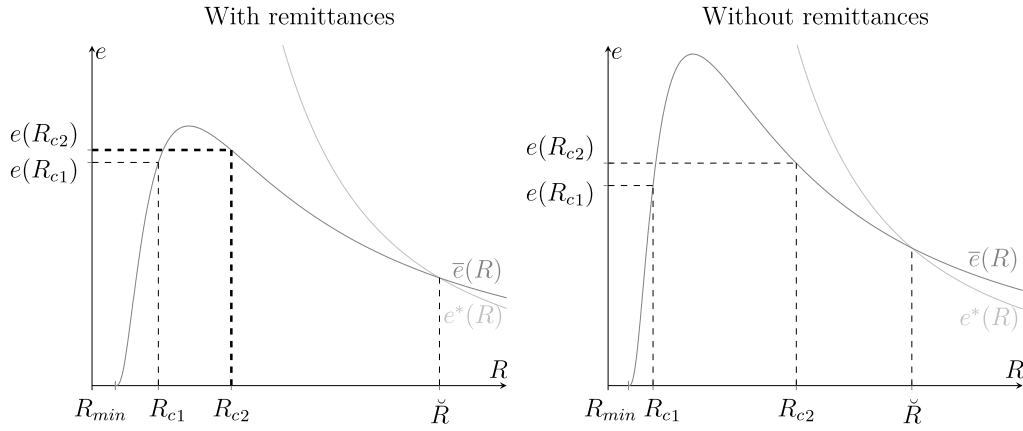
#### A.1.2. The migrants' decisions

Agents born in period  $t - 1$  who have migrated draw utility from consumption  $c_{t*}$  when they are young workers and  $d_{t+1*}$  when they are old workers. They also draw utility from remittances,  $f_{t*}$ , allotted to their old parent. In that case, migrants altruistically send remittances to family. Let us assume the following utility function:

$$U(c_{t*}, f_{t*}, d_{t+1*}) = c_{t*}^\alpha f_{t*}^\phi d_{t+1*}^{1-\alpha-\phi} \quad (52)$$

where  $0 < \alpha < 1$  and  $0 < \phi < 1$ .

We assume that the migrants born in period  $t - 1$  take their decisions in a commitment setting and then always refund their loan  $b_{t-1*}$  during period  $t$  at the exogenous interest rate  $R_*$ .



Source: Authors' computations.

Note: The highlighted dashed lines represent the equilibrium which matches the data with remittances.

Fig. 4.3. The levels of education at each equilibrium in Solomon Islands.

The first period labor income  $h_{t*}$  allows them to consume  $c_{t*}$ , repay the loan  $R_*b_{t-1*}$ , remit  $f_{t*}$  and save  $s_{t*}$ . When they are old workers the labor income  $\delta_*h_{t*}$  and the return on savings  $R_*s_{t*}$  make consumption  $d_{t+1*}$  possible. Therefore, the migrants' budget constraints in the two periods of work are given by:

$$c_{t*} = h_{t*} - s_{t*} - R_*b_{t-1*} - f_{t*} \quad (53)$$

$$d_{t+1*} = \delta_*h_{t*} + R_*s_{t*} \quad (54)$$

We consider that in period  $t - 1$  the exogenous borrowing constraint in the foreign country has the following form:

$$b_{t-1*} \leq \eta_* h_{t-1} \quad \text{where } 0 < \eta_* < \left( \frac{A_* \left( 1 + \frac{\delta_*}{R_*} \right)^{\frac{1}{1-\lambda_*}}}{R_*} \right) \quad (55)$$

This constraint implies that the parents' income affects the amount that migrants can borrow in the host country. The upper bound on  $\eta_*$  guarantees that the repayment of the loan is not so high and that the migrant's life-cycle income<sup>23</sup> (net of the loan repayment) given by  $h_{t*} - R_*b_{t-1*} + \delta_*h_{t*}/R_*$  is positive.

The problem of the consumer is to maximize the utility function (52) under the human capital accumulation function (51), the budget constraints in the two periods (53) and (54) and the borrowing constraint (55).

For simplicity we also assume that the constraint always binds. This implies that the migrant always wants to borrow more for education in the foreign country.<sup>24</sup> Using Eqs. (52)–(54) savings and remittances for a given level of education are the solution of the following program:

$$\text{Max}_{s_{t*}, f_{t*}} (h_{t*} - s_{t*} - R_*b_{t-1*} - f_{t*})^\alpha (f_{t*})^\phi (\delta_*h_{t*} + R_*s_{t*})^{1-\alpha-\phi}$$

The first order conditions (FOC) of this program are:

$$\alpha (\delta_*h_{t*} + R_*s_{t*}) = (1 - \alpha - \phi) R_* (h_{t*} - s_{t*} - R_*b_{t-1*} - f_{t*})$$

$$\alpha f_{t*} = \phi (h_{t*} - s_{t*} - R_*b_{t-1*} - f_{t*})$$

These FOC give us the optimal savings and remittances for a given level of education:

$$s_{t*} = (1 - \alpha - \phi) (h_{t*} - R_*b_{t-1*}) - \frac{(\alpha + \phi) \delta_* h_{t*}}{R_*} \quad (56)$$

<sup>23</sup> The migrant's life-cycle income comes from Eqs. (53) and (54).

<sup>24</sup> We have also solved the migrant's program without the assumption that the constraint always binds. We get similar qualitative results for the expression of remittances. The computations are available upon request.

$$f_{t*} = \phi \left( h_{t*} - R_*b_{t-1*} + \frac{\delta_* h_{t*}}{R_*} \right) \quad (57)$$

Hence, each migrating agent born in period  $t - 1$  sends to her old parent in period  $t$  a proportion  $\phi$  of her life-cycle income. For a given level of income, the higher the ascendant altruism, the higher the remittances are.

Since the constraint binds, we have  $b_{t-1*} = \eta_* h_{t-1}$ . Knowing that  $h_{t*} = A_* \eta_*^{\lambda_*} h_{t-1}$  from Eq. (51), the amount of money sent by a migrant in period  $t$  to her old parent is:

$$f_{t*} = \phi \eta_*^{\lambda_*} \left( A_* \left( 1 + \frac{\delta_*}{R_*} \right) - R_* \eta_*^{1-\lambda_*} \right) h_{t-1}$$

This expression is positive under the restrictions on  $\eta_*$ . Let us now define:

$$\gamma \equiv \phi \eta_*^{\lambda_*} \left( A_* \left( 1 + \frac{\delta_*}{R_*} \right) - R_* \eta_*^{1-\lambda_*} \right) > 0 \quad (58)$$

We can express  $f_{t*}$  as  $f_{t*} = \gamma h_{t-1}$ . Similarly, in period  $t + 1$ , the amount sent by each migrating child is  $f_{t+1*} = \gamma h_t$ . Knowing that among the agent's children in home country, there are  $p(1+n)$  children who migrate, we argue that the amount of remittances received by each old agent in the home country in period  $t + 1$ , denoted by  $f_{t+1}$ , is:

$$f_{t+1} = p(1+n)\gamma h_t$$

Eq. (58) gives the expression and micro-foundations of  $\gamma$  defined by Assumption 5:  $\gamma \equiv \gamma(\phi, A_*, \delta_*, R_*, \lambda_*, \eta_*)$ . Notice that  $\gamma(\phi, A_*, \delta_*, R_*, \lambda_*, \eta_*)$  is increasing with  $\phi$ ,  $A_*$ ,  $\delta_*$ ,  $\lambda_*$  and  $\eta_*$  if  $\eta_*$  is not too high. Moreover,  $\gamma(\phi, A_*, \delta_*, R_*, \lambda_*, \eta_*)$  is decreasing with  $R$ .

#### A.2. Proof of Proposition 2.1

Substituting Eqs. (13) and (14) into Eq. (10), savings become:

$$s_t^* = (1 - \beta) (1 - x_t) h_t - \frac{\beta (h_t (\delta + p(1+n)\gamma))}{R_{t+1}} \quad (59)$$

In the case of default ( $x_t = 0$ ), the optimal of saving would be:

$$s_t^* = (1 - \beta) h_t - \frac{\beta h_t (\delta + p(1+n)\gamma)}{R_{t+1}} \quad (60)$$

Then, in the case of default, the level of interest rate  $R_{min}$  such that  $s_t^* = 0$  is given by Eq. (24).

If  $x_t = 0$ , then  $s_t^* \leq 0 \Leftrightarrow R_{t+1} \leq R_{min}$ . Since  $s_t^*$  is decreasing with  $x_t$ , we still have  $s_t^* \leq 0$  when  $x_t > 0$  and  $R_{t+1} \leq R_{min}$ . If the agent has no incentive to save ( $s_t^* \leq 0$ ), then under IRC old-age, given by Eq. (21), we get  $\hat{s}_t = 0$ . Knowing that the amounts for the loan are provided by savers, the maximum level of borrowing is  $\bar{x}_t = 0$  and thus  $\hat{x}_t = 0$ .

For  $R_{t+1} > R_{min}$ , since  $s_t^*$  is decreasing with  $x_t$ , there is a maximal bound  $x_{max} > 0$  such that  $s_t^* \geq 0$  if  $x_t \leq x_{max}$ . More precisely,

$$x_{max} = 1 - \frac{\beta(\delta + p(1+n)\gamma)}{(1-\beta)R_{t+1}} \quad (61)$$

Therefore, the IRC old-age binds when:

- $R_{t+1} \leq R_{min}$ .
- $R_{t+1} \geq R_{min}$  and  $x_t \geq x_{max}$ .

In those cases, this constraint implies that  $\hat{s}_t = 0$ .

When  $R_{t+1} > R_{min}$ , the IRC middle-age given by Eq. (22) implies that:

$$\begin{aligned} U(h_t - s_t - R_t b_{t-1}, h_t(\delta + p(1+n)\gamma) + R_{t+1}s_t) \\ \geq U(h_t, h_t(\delta + p(1+n)\gamma)) \end{aligned}$$

Under Assumption 3, substituting  $R_t b_{t-1} = x_t h_t$ , this IRC middle-age becomes:

$$\begin{aligned} (h_t - s_t - x_t h_t)^\beta (h_t(\delta + p(1+n)\gamma) + R_{t+1}s_t)^{1-\beta} \\ \geq (h_t)^\beta (h_t(\delta + p(1+n)\gamma))^{1-\beta} \end{aligned}$$

Substituting Eq. (59), we have:

$$\begin{aligned} \beta^\beta (1-\beta)^{1-\beta} R_{t+1}^{1-\beta} \left(1 - x_t + \frac{\delta + p(1+n)\gamma}{R_{t+1}}\right) \\ \geq (\delta + p(1+n)\gamma)^{1-\beta} \end{aligned} \quad (62)$$

As the left hand side of the above equation is decreasing with  $x_t$ , we can define  $\bar{x}_t$  as the maximum level of borrowing such that:

$$\begin{aligned} (h_t - s_t - \bar{x}_t h_t)^\beta (h_t(\delta + p(1+n)\gamma) + R_{t+1}s_t)^{1-\beta} \\ = (h_t)^\beta (h_t(\delta + p(1+n)\gamma))^{1-\beta} \end{aligned} \quad (63)$$

We finally get:

$$\bar{x}_t = 1 - \frac{(\delta + p(1+n)\gamma)^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta} R_{t+1}^{1-\beta}} + \frac{\delta + p(1+n)\gamma}{R_{t+1}} \quad (64)$$

We can show that  $\bar{x}_t < x_{max}$  for  $R_{t+1} > R_{min}$ . Therefore,  $x_t$  has to satisfies  $x_t < \bar{x}_t$ .

Moreover:

$$\begin{aligned} \frac{\partial \bar{x}_t}{\partial R_{t+1}} = \frac{(\delta + p(1+n)\gamma)^{1-\beta}}{\beta^\beta (1-\beta)^{-\beta} R_{t+1}^{2-\beta}} - \frac{\delta + p(1+n)\gamma}{R_{t+1}^2} \\ \geq 0 \Leftrightarrow R_{t+1} \geq R_{min} \end{aligned}$$

As  $\lim_{R_{t+1} \rightarrow +\infty} \bar{x}_t = 1$  and  $\lim_{R_{t+1} \rightarrow R_{min}} \bar{x}_t = 0$ , then  $\bar{x}_t \in [0, 1]$ .

### A.3. Proof of Corollary 1

Using Eqs. (18) and (24), we argue that both  $x_t^*$  and  $R_{min}$  are increasing with  $p$ ,  $n$  and  $\gamma$ . Using the expression of  $\bar{x}_t$  given by Eq. (26), we compute:

$$\begin{aligned} \frac{\partial \bar{x}_t}{\partial p} &= -\frac{(1+n)\gamma(\delta + p(1+n)\gamma)^{-\beta}}{\beta^\beta (1-\beta)^{-\beta} R_{t+1}^{1-\beta}} + \frac{(1+n)\gamma}{R_{t+1}} \\ \frac{\partial \bar{x}_t}{\partial n} &= -\frac{p\gamma(\delta + p(1+n)\gamma)^{-\beta}}{\beta^\beta (1-\beta)^{-\beta} R_{t+1}^{1-\beta}} + \frac{p\gamma}{R_{t+1}} \\ \frac{\partial \bar{x}_t}{\partial \gamma} &= -\frac{p(1+n)(\delta + p(1+n)\gamma)^{-\beta}}{\beta^\beta (1-\beta)^{-\beta} R_{t+1}^{1-\beta}} + \frac{p(1+n)}{R_{t+1}} \end{aligned}$$

When  $R_{t+1} \geq R_{min}$ , we have:  $\frac{\partial \bar{x}_t}{\partial p} \leq 0$ ,  $\frac{\partial \bar{x}_t}{\partial n} \leq 0$  and  $\frac{\partial \bar{x}_t}{\partial \gamma} \leq 0$ .

### A.4. Proof of Proposition 2.2

Under Assumptions 1–5,  $x_t^*$  is decreasing with  $R_{t+1}$ , is convex and tends to  $\lambda$  when  $R_{t+1}$  tends to  $+\infty$ :

$$\frac{\partial x_t^*}{\partial R_{t+1}} = \frac{-\lambda(\delta + p(1+n)\gamma)}{R_{t+1}^2} < 0$$

$$\frac{\partial^2 x_t^*}{\partial R_{t+1}^2} = \frac{2\lambda(\delta + p(1+n)\gamma)}{R_{t+1}^3} > 0$$

$$\lim_{R_{t+1} \rightarrow 0} x_t^* = +\infty \quad \lim_{R_{t+1} \rightarrow +\infty} x_t^* = \lambda$$

In Appendix A.2 we have shown that  $\bar{x}_t$  is increasing from 0 to 1 when  $R_{t+1}$  increases from  $R_{min}$  to  $+\infty$  and its slope is equal to 0 at  $R_{min}$ . Moreover, we can argue that  $\bar{x}_t$  is firstly convex and then concave:

$$\frac{\partial^2 \bar{x}_t}{\partial R_{t+1}^2} = \frac{(\beta-2)(\delta + p(1+n)\gamma)^{1-\beta}}{\beta^\beta (1-\beta)^{-\beta} R_{t+1}^{3-\beta}} + \frac{2(\delta + p(1+n)\gamma)}{R_{t+1}^3}$$

$$\frac{\partial^2 \bar{x}_t}{\partial R_{t+1}^2} \geq 0 \Leftrightarrow R_{t+1} \leq \frac{\beta(\delta + p(1+n)\gamma)}{1-\beta} \left(\frac{2}{2-\beta}\right)^{\frac{1}{\beta}}$$

Since

$$\left(\frac{2}{2-\beta}\right)^{\frac{1}{\beta}} > 1$$

Then

$$\frac{\beta(\delta + p(1+n)\gamma)}{1-\beta} \left(\frac{2}{2-\beta}\right)^{\frac{1}{\beta}} > R_{min}$$

The increasing curve  $\bar{x}(R)$  meets the decreasing curve  $x^*(R)$  at a unique point  $\check{R}$ . If  $R \leq \check{R}$ , then  $\bar{x}(R) \leq x^*(R)$ . The impact of parameters  $p$  and  $\gamma$  on  $\bar{x}(R)$  and  $x^*(R)$  has been shown in Corollary 1. To prove that  $\check{R}$  is increasing with  $p$  and  $\gamma$ , let us define:

$$\check{X}(\bar{x}(\check{R}), x^*(\check{R})) \equiv \bar{x}(\check{R}) - x^*(\check{R}) = 0$$

We argue that:

$$\frac{\partial \check{X}(\bar{x}(\check{R}), x^*(\check{R}))}{\partial p} < 0 \quad \frac{\partial \check{X}(\bar{x}(\check{R}), x^*(\check{R}))}{\partial R} > 0$$

As a consequence, the implicit function theorem implies:

$$\frac{d\check{R}}{dp} > 0$$

The proof is similar for the impact of the parameter  $\gamma$  on  $\check{R}$  and we get the same qualitative result.

### A.5. Proof of Corollary 3

Eqs. (29) and (30) are derived from Eq. (12) when we consider  $R_{t+1} = R_t = R$ . Using Eq. (18) we get:

$$e^*(R) = \left( A\lambda \left( \frac{1}{R} + \frac{\delta + p(1+n)\gamma}{R^2} \right) \right)^{\frac{1}{1-\lambda}}$$

We then compute:

$$\begin{aligned} \frac{\partial e^*(R)}{\partial R} &= -\frac{A\lambda}{1-\lambda} \left( \frac{1}{R^2} + \frac{2(\delta + p(1+n)\gamma)}{R^3} \right) \\ &\quad \times \left( A\lambda \left( \frac{1}{R} + \frac{\delta + p(1+n)\gamma}{R^2} \right) \right)^{\frac{\lambda}{1-\lambda}} < 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial^2 e^*(R)}{\partial R^2} &= \frac{A^2 \lambda^3}{(1-\lambda)^2} \left( \frac{1}{R^2} + \frac{2(\delta + p(1+n)\gamma)}{R^3} \right)^2 \\ &\quad \times \left( A\lambda \left( \frac{1}{R} + \frac{\delta + p(1+n)\gamma}{R^2} \right) \right)^{\frac{2\lambda-1}{1-\lambda}} \\ &\quad + \frac{A\lambda}{1-\lambda} \left( \frac{2}{R^3} + \frac{6(\delta + p(1+n)\gamma)}{R^4} \right) \\ &\quad \times \left( A\lambda \left( \frac{1}{R} + \frac{\delta + p(1+n)\gamma}{R^2} \right) \right)^{\frac{\lambda}{1-\lambda}} > 0 \end{aligned}$$

Hence  $e^*(R)$  is decreasing and convex. Moreover,

$$\lim_{R \rightarrow 0} e^*(R) = +\infty \quad \lim_{R \rightarrow +\infty} e^*(R) = 0$$

Regarding  $\bar{e}(R)$ , we know that if  $R \leq R_{min}$ , we have  $\bar{e}(R) = 0$  since  $\bar{x}(R) = 0$ . Moreover, as  $\lim_{R \rightarrow +\infty} \bar{x}(R) = 1$ , we have  $\lim_{R \rightarrow +\infty} \bar{e}(R) = 0$ . Then, using Eq. (30) we get:

$$\frac{\partial \bar{e}(R)}{\partial R} = \left( \frac{1}{(1-\lambda)R} \right) \left( \frac{\frac{\partial \bar{x}(R)}{\partial R} R}{\bar{x}(R)} - 1 \right) \left( \frac{A\bar{x}(R)}{R} \right)^{\frac{1}{1-\lambda}}$$

Therefore,  $\bar{e}(R)$  is increasing only if the elasticity of repayment share with respect to interest rate is high enough:

$$\frac{\partial \bar{x}(R)}{\partial R} \frac{R}{\bar{x}(R)} > 1 \quad (65)$$

Let us now define:

$$\Omega(R) \equiv \frac{(2-\beta)(\delta + p(1+n)\gamma)^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta}} R^\beta - R - 2(\delta + p(1+n)\gamma) \quad (66)$$

Solving inequation (65) is equivalent to solve  $\Omega(R) > 0$ . We provide a graphical argument. The function  $\Omega(R)$  is such that:  $\Omega(0) < 0$ ,  $\Omega'(R) = \frac{(2-\beta)(\delta + p(1+n)\gamma)^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta}} \beta R^{\beta-1} - 1 \geq 0$  for  $R \leq \underline{R}$  with  $\underline{R} = \frac{\beta}{1-\beta} (2-\beta)^{\frac{1}{1-\beta}} (\delta + p(1+n)\gamma)$  and  $\Omega''(R) = \frac{(2-\beta)(\delta + p(1+n)\gamma)^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta}} \beta(\beta-1)R^{\beta-2} < 0$ . Finally,  $\Omega(R)$  tends to  $-\infty$  when  $R$  tends to  $+\infty$ . As a consequence, the equation  $\Omega(R) = 0$  has either 0, 1 or 2 solutions on  $\mathbb{R}_+^*$ . This implies that  $\bar{e}(R)$  would be either strictly decreasing, decreasing or decreasing then increasing and decreasing on  $\mathbb{R}_+^*$ . Nevertheless, we know that at the point  $R = R_{min}$ :

$$\frac{\partial \bar{x}(R_{min})}{\partial R} = 0 \quad \text{and} \quad \bar{x}(R_{min}) = \bar{e}(R_{min}) = 0$$

Hence, we can check that  $R = R_{min}$  is a solution to  $\Omega(R) = 0$ . As  $\bar{x}(R) > 0$  if  $R > R_{min}$ , the equation  $\Omega(R) = 0$  has necessarily 2 solutions. The point  $R = R_{min}$  is the first solution and there exists a point  $R = \bar{R} > R_{min}$  which is the second solution. Therefore, the function  $\bar{e}(R)$  is firstly increasing and then decreasing with  $R$  on the interval  $]R_{min}, +\infty[$ , and  $R = \bar{R}$  is the maximum of  $\bar{e}(R)$ .

For  $R \in \mathbb{R}_+^*$ , we know that there exists a unique interest rate such that the optimal and the constrained education per unit of capital coincide since the optimal and the constrained share of loan repayment coincide only if  $R = \check{R}$ :  $e^*(\check{R}) = \bar{e}(\check{R}) \Leftrightarrow x^*(\check{R}) = \bar{x}(\check{R})$ .

According to the relation between  $x$  and  $e$ , let us notice that for a given  $R$ , the impact of  $p$  and  $\gamma$  on  $e^*(R)$  and  $\bar{e}(R)$  is qualitatively similar to the impact of  $p$  and  $\gamma$  on  $x^*(R)$  and  $\bar{x}(R)$ . Hence,  $e^*(R)$  is increasing with  $p$  and  $\gamma$  and  $\bar{e}(R)$  is decreasing with  $p$  and  $\gamma$ .

## A.6. Proof of Corollary 4

Since  $x^*(R)$  is decreasing,  $\bar{x}(R)$  is increasing and  $x^*(\check{R}) = \bar{x}(\check{R})$ , we argue that  $\check{R} < \bar{R} \Leftrightarrow x^*(\bar{R}) < \bar{x}(\bar{R})$ . Using Eqs. (18) and (26) for a given level of interest rate, we get  $x^*(\bar{R}) < \bar{x}(\bar{R}) \Leftrightarrow \lambda < \bar{\lambda}$  where:

$$\bar{\lambda} = \frac{1 - \frac{(\delta + p(1+n)\gamma)^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta} \bar{R}^{1-\beta}} + \frac{\delta + p(1+n)\gamma}{\bar{R}}}{1 + \frac{\delta + p(1+n)\gamma}{\bar{R}}}$$

and  $\bar{R}$  is the maximum of the constrained level of education, such that  $\Omega(\bar{R}) = 0$  (see Eq. (66) in Appendix A.5). Using the above expression, we know that  $\bar{\lambda} < 1$ . Then, we solve  $\bar{\lambda} > 0 \Leftrightarrow \bar{R} > R_{min}$ . Since we have shown in Appendix A.5 that  $\bar{R} > R_{min}$ , we get  $\bar{\lambda} \in ]0, 1[$ .

From the expression of  $\bar{\lambda}$  we derive:

$$\frac{d\bar{\lambda}}{d\gamma} = \frac{\partial \bar{\lambda}}{\partial \gamma} + \frac{\partial \bar{\lambda}}{\partial \bar{R}} \times \frac{\partial \bar{R}}{\partial \gamma}$$

where:

$$\begin{aligned} \frac{\partial \bar{\lambda}}{\partial \gamma} &= \frac{p(1+n) \left[ \beta \left( \bar{R} + \delta + p(1+n)\gamma \right) - \bar{R} \right] \bar{R}^\beta}{\beta^\beta (1-\beta)^{1-\beta} \left( \bar{R} + \delta + p(1+n)\gamma \right)^2 (\delta + p(1+n)\gamma)^\beta} \\ \frac{\partial \bar{\lambda}}{\partial \bar{R}} &= - \frac{(\delta + p(1+n)\gamma)^{1-\beta} \left[ \beta \left( \bar{R} + \delta + p(1+n)\gamma \right) - \bar{R} \right]}{\beta^\beta (1-\beta)^{1-\beta} \left( \bar{R} + \delta + p(1+n)\gamma \right)^2 \bar{R}^{1-\beta}} \\ \frac{\partial \bar{R}}{\partial \gamma} &\equiv \frac{d\bar{R}}{d\gamma} \\ &= \frac{p(1+n) \left[ 2\beta^\beta (1-\beta)^{1-\beta} - (1-\beta)(2-\beta)(\delta + p(1+n)\gamma)^{-\beta} \bar{R}^\beta \right]}{\beta(2-\beta)(\delta + p(1+n)\gamma)^{1-\beta} \bar{R}^{\beta-1} - \beta^\beta (1-\beta)^{1-\beta}} \end{aligned}$$

Then we solve,

$$\frac{d\bar{\lambda}}{d\gamma} \geq 0 \Leftrightarrow \Omega(R) \geq 0$$

In Appendix A.5, we have shown that  $\bar{R}$  is such that  $\Omega(\bar{R}) = 0$ . Hence we argue that

$$\frac{d\bar{\lambda}}{d\gamma} = 0$$

The proofs are similar for the impact of  $p$  and  $\delta$  on  $\bar{\lambda}$ , and results are the same. As a consequence,  $\bar{\lambda}$  only depends on  $\beta$ .

## A.7. Proof of Lemma 2.2

We solve Eq. (42) with a graphical argument by representing each side of this equation with respect to  $x \in [0, 1]$ . If  $x$  is a steady state of Eq. (40), then Eq. (42) is equivalent to:

$$xA = \Theta(R, x) \quad (67)$$

where  $\Theta(R, x)$  comes from Eq. (41). This function is defined if  $x \leq x_{max}$  with  $x_{max}$  given by Eq. (61) in Appendix A.2. We know that  $x_t < x_{max}$  under the IRC constraints. Since  $x_{max} \geq 0$  if  $R \geq R_{min}$ , the function  $\Theta(R, x)$  is defined if  $R \geq R_{min}$ . Let us now compute:

$$\begin{aligned} \frac{\partial \Theta(R, x)}{\partial x} &= \frac{-R(1-\lambda)(1-\beta)}{(1-p)(1+n)} \\ &\quad \times \left[ \frac{1-\beta}{(1-p)(1+n)} \left( 1-x - \frac{\beta(\delta + p(1+n)\gamma)}{(1-\beta)R} \right) \right]^{-\lambda} \\ &< 0 \text{ if } R > R_{min} \end{aligned}$$

$$\begin{aligned} & \frac{\partial^2 \Theta(R, x)}{\partial x^2} \\ &= \frac{-R\lambda(1-\lambda)(1-\beta)^2}{(1-p)^2(1+n)^2} \\ & \times \left[ \frac{1-\beta}{(1-p)(1+n)} \left( 1-x - \frac{\beta(\delta+p(1+n)\gamma)}{(1-\beta)R} \right) \right]^{-\lambda-1} \\ & < 0 \text{ if } R > R_{min} \end{aligned}$$

The left hand side of Eq. (67) is increasing with  $x$  and belongs to  $[0, A]$  while the right hand side of Eq. (67) is decreasing monotonically with  $x$  from  $\Theta(R, 0) > 0$  if  $R > R_{min}$  to  $\Theta(R, x_{max}) = 0$ . Hence, there is a unique  $x$  for a given interest rate  $R > R_{min}$ . Moreover, if  $R = R_{min}$ , savings are equal to 0 implying that  $x = 0$  and then  $\Theta(R_{min}, 0) = 0$ . Therefore, Eq. (67) is satisfied if  $R = R_{min}$  as well. These results imply that there exists a function  $\tilde{x}(R, p, \gamma)$  which defines the combinations  $x$  and  $R$  compatible with a steady state on the interval  $[R_{min}, +\infty[$  and such that  $\tilde{x}(R_{min}, p, \gamma) = 0$ . To characterize the function  $\tilde{x}(R, p, \gamma)$ , let us compute:

$$\begin{aligned} & \frac{\partial \Theta(R, x)}{\partial R} \\ &= \left[ \frac{1-\beta}{(1-p)(1+n)} \left( 1-x - \frac{\beta(\delta+p(1+n)\gamma)}{(1-\beta)R} \right) \right]^{1-\lambda} \\ & + \frac{(1-\lambda)\beta(\delta+p(1+n)\gamma)}{(1-p)(1+n)R} \\ & \times \left[ \frac{1-\beta}{(1-p)(1+n)} \left( 1-x - \frac{\beta(\delta+p(1+n)\gamma)}{(1-\beta)R} \right) \right]^{-\lambda} \\ & > 0 \text{ if } R > R_{min} \end{aligned}$$

$$\begin{aligned} & \frac{\partial^2 \Theta(R, x)}{\partial R^2} \\ &= \frac{-\lambda(1-\lambda)(\beta(\delta+p(1+n)\gamma))^2}{(1-p)^2(1+n)^2 R^3} \\ & \times \left[ \frac{1-\beta}{(1-p)(1+n)} \left( 1-x - \frac{\beta(\delta+p(1+n)\gamma)}{(1-\beta)R} \right) \right]^{-\lambda-1} \\ & < 0 \text{ if } R > R_{min} \end{aligned}$$

Therefore,  $\Theta(R, x)$  is increasing and concave with  $R$ , for  $R > R_{min}$ . As a consequence,  $\tilde{x}(R, p, \gamma)$  is increasing and concave with  $R$ . Indeed, when  $R$  increases,  $\Theta(R, x)$  increases and cuts the line  $xA$  for a greater  $x$ . But the more  $R$  increases, the less  $\Theta(R, x)$  increases. Therefore, this function cuts the line  $xA$  for a less and less higher  $x$ . Fig. A.1 illustrates this point. There is a unique  $x$  for a given  $R$ , and  $x$  is increasing with  $R$  but with a decreasing rate. Using Eq. (42) and knowing that  $\Theta(R, x)$  is increasing with  $R$ , we argue that:

$$\lim_{R \rightarrow +\infty} \tilde{x}(R, p, \gamma) = 1$$

Then, to see if  $\tilde{x}(R, p, \gamma)$  is increasing at the point  $R = R_{min}$ , let us rewrite Eq. (67) as:

$$\begin{aligned} \tilde{X}(x, R) &\equiv x^{\frac{1}{1-\lambda}} + x \left( \frac{1-\beta}{(1-p)(1+n)} \left( \frac{R}{A} \right)^{\frac{1}{1-\lambda}} \right) \\ & - \left( \frac{1-\beta}{(1-p)(1+n)} \left( \frac{R}{A} \right)^{\frac{1}{1-\lambda}} \right) \\ & \times \left( 1 - \frac{\beta(\delta+p(1+n)\gamma)}{(1-\beta)R} \right) = 0 \end{aligned}$$

We can compute:

$$\begin{aligned} \left. \frac{\partial \tilde{X}(x, R)}{\partial x} \right|_{R=R_{min}} &= \frac{1-\beta}{(1-p)(1+n)} \left( \frac{R}{A} \right)^{\frac{1}{1-\lambda}} > 0 \\ \left. \frac{\partial \tilde{X}(x, R)}{\partial R} \right|_{R=R_{min}} &= -\frac{\beta(\delta+p(1+n)\gamma)}{(1-p)(1+n)R^2} \left( \frac{R}{A} \right)^{\frac{1}{1-\lambda}} < 0 \end{aligned}$$

Using the implicit function theorem:

$$\left. \frac{d\tilde{x}}{dR} \right|_{R=R_{min}} = \frac{\beta(\delta+p(1+n)\gamma)}{(1-\beta)R^2} > 0$$

Hence, the slope of  $\tilde{x}(R, p, \gamma)$  is positive at the point  $R = R_{min}$  and  $\tilde{x}(R, p, \gamma)$  is increasing on the interval  $[R_{min}, +\infty[$ .

To prove the second part of Lemma 2.2, let us compute:

$$\begin{aligned} \frac{\partial \Theta}{\partial \gamma} &= -\frac{(1-\lambda)\beta p}{1-p} \left[ \frac{1-\beta}{(1-p)(1+n)} \right. \\ & \times \left. \left( 1-x - \frac{\beta(\delta+p(1+n)\gamma)}{(1-\beta)R} \right) \right]^{-\lambda} < 0 \text{ if } R > R_{min} \end{aligned}$$

The left hand side of Eq. (67) does not depend on  $\gamma$  while the right hand side is decreasing with  $\gamma$ . Therefore using Fig. A.1, we can argue that  $\frac{\partial \tilde{x}}{\partial \gamma} < 0$ . Finally:

$$\begin{aligned} \frac{\partial \Theta}{\partial p} &= \frac{R(1-\lambda)(1-\beta)}{1-p} \\ & \times \left( \frac{1-x}{1-p} - \frac{\beta(\delta+p(1+n)\gamma)}{(1-p)(1-\beta)R} - \frac{\beta(1+n)\gamma}{(1-\beta)R} \right) \\ & \times \left[ \frac{1-\beta}{1-p} \left( 1-x - \frac{\beta(\delta+p(1+n)\gamma)}{(1-\beta)R} \right) \right]^{-\lambda} \end{aligned}$$

We solve:

$$\frac{\partial \Theta}{\partial p} > 0 \Leftrightarrow x < 1 - \frac{\beta(\delta+(1+n)\gamma)}{(1-\beta)R} < x_{max}$$

The left hand side of Eq. (67) does not depend on  $p$ . Using Fig. A.1 and the above result, we argue that  $\frac{\partial \tilde{x}}{\partial p} < 0$  if:

$$x > 1 - \frac{\beta(\delta+(1+n)\gamma)}{(1-\beta)R}$$

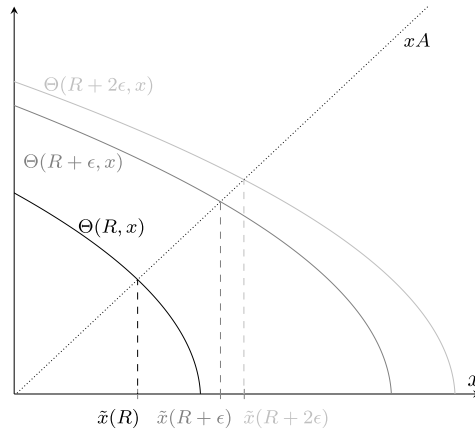
#### A.8. Proof of Proposition 2.3

According to Lemma 2.1,  $x^*(R)$  is decreasing from  $+\infty$  to  $\lambda$  when  $R$  goes from 0 to  $+\infty$ . According to Lemma 2.2,  $\tilde{x}(R)$  is increasing from 0 to 1 when  $R$  goes from  $R_{min}$  to  $+\infty$ . Hence, there is always a unique point  $R_u > R_{min}$ , such that  $\tilde{x}(R_u) = x^*(R_u)$ . However, an unconstrained steady state  $R_u$  has to satisfy  $x^*(R_u) < \tilde{x}(R_u)$  which implies, under Proposition 2.2, that  $R_u > \check{R}$  (see Fig. 2.1). Therefore, an unconstrained equilibrium occurs only if  $\tilde{x}(\check{R}) < \check{x}$  where  $\check{x} = \tilde{x}(\check{R}) = x^*(\check{R})$ . We know that  $\tilde{x}(R)$  is such that,  $x$  and  $R$  satisfy Eq. (42). We have  $\tilde{x}(\check{R}) = x^*(\check{R}) = \check{x}$  which is equivalent to  $R_u = \check{R}$  if:

$$\check{x}A = \Theta(\check{R}, \check{x}) \Leftrightarrow A = \check{A}$$

where the function  $\Theta(\cdot)$  is given by Eq. (41) and  $\check{A}$  is given by Eq. (43). We also know that  $x^*(\check{R})$  and  $\tilde{x}(\check{R})$  do not depend on  $A$ . However, following the proof of Lemma 2.2 and Fig. A.1 (see Appendix A.7) we have that  $\tilde{x}(\check{R}) \equiv \tilde{x}(\check{R}, A)$  and  $\frac{\partial \tilde{x}}{\partial A} < 0$ . Therefore, if  $A > \check{A}$ , we get  $\tilde{x}(\check{R}) < x^*(\check{R})$  which is equivalent to  $R_u > \check{R}$ . Hence there is an unconstrained steady state if and only if  $A > \check{A}$ .

At the point  $R = R_{min}$ , the slope of  $\tilde{x}(R)$  is positive whereas the slope of  $\bar{x}(R)$  is equal to 0. Therefore,  $\tilde{x}(R) > \bar{x}(R)$  in the



Note: There is a unique intersection, which guarantees that there exists a unique value  $\tilde{x}$  for each value of  $R$ .

**Fig. A.1.** A representation of Eq. (67).

neighborhood of  $R_{min}$ . Since  $\tilde{x}(\check{R}) < \bar{x}(\check{R}) = x^*(\check{R})$  if  $A > \check{A}$  we can argue that  $\tilde{x}(R)$  meets  $\bar{x}(R)$  between  $R_{min}$  and  $\check{R}$  and there is necessarily another steady state  $R_c$  which is constrained.

When  $A < \check{A}$ , we have  $\tilde{x}(\check{R}) > \bar{x}(\check{R})$  and therefore  $R_u < \check{R}$ . This point  $R_u$  is not an equilibrium as  $x^*(R_u) > \bar{x}(R_u)$ . Since  $\tilde{x}(R) > \bar{x}(R)$  in the neighborhood of  $R_{min}$ , there may exist no constrained steady state if  $\tilde{x}(R) > \bar{x}(R)$  on the whole interval  $]R_{min}, \check{R}[$ . However, when  $A < \check{A}$  there may also exist two constrained steady states  $R_{c1}$  and  $R_{c2}$  such that  $R_{c1} < R_{c2} < R_u < \check{R}$ . This may occur since  $\bar{x}(R)$  is increasing convex and then concave,  $\tilde{x}(R)$  is increasing and concave,  $\tilde{x}(R) > \bar{x}(R)$  in the neighborhood of  $R_{min}$  and  $\tilde{x}(\check{R}) > \bar{x}(\check{R})$ . Therefore, we may have  $\tilde{x}(R) < \bar{x}(R)$  for some points between  $R_{min}$  and  $\check{R}$ . The three configurations are represented by Fig. 2.3.

#### A.9. Study of $\Theta(R, x^*(R))$

Replacing  $x^*(R)$  by its expression given by Eq. (18) in  $\Theta(R, x^*(R))$ , we get:

$$\Theta(R, x^*(R)) = R \left[ \frac{1 - \beta}{(1 - p)(1 + n)} \left( 1 - \lambda \left( 1 + \frac{\delta + p(1 + n)\gamma}{R} \right) - \frac{\beta(\delta + p(1 + n)\gamma)}{(1 - \beta)R} \right) \right]^{1 - \lambda}$$

At the point  $\check{R}$ , we have  $\Theta(\check{R}, x^*(\check{R})) = \Theta(\check{R}, \bar{x}(\check{R}))$ . Using the expression of  $\bar{x}(R)$  given by Eq. (26), we can write:

$$\Theta(\check{R}, x^*(\check{R})) = \check{R}^\lambda \left[ \frac{1 - \beta}{(1 - p)(1 + n)} \left( \frac{\check{R}^\beta (\delta + p(1 + n)\gamma)^{1 - \beta}}{\beta^\beta (1 - \beta)^{1 - \beta}} - \frac{\delta + p(1 + n)\gamma}{1 - \beta} \right) \right]^{1 - \lambda}$$

Then we deduce that  $\Theta(\check{R}, x^*(\check{R})) > 0$  since  $\check{R} > R_{min}$ .

Regarding the points  $R > \check{R} > R_{min}$ , in Appendix A.7, we show that the function  $\Theta(R, x)$  is increasing with  $R$  and decreasing with  $x$ . In Lemma 2.1, we show that  $x^*(R)$  is decreasing with  $R$ . Therefore:

$$\frac{d\Theta(R, x^*(R))}{dR} = \frac{\partial\Theta(R, x)}{\partial R} + \frac{\partial\Theta(R, x)}{\partial x^*(R)} \times \frac{\partial x^*(R)}{\partial R} > 0$$

Hence the function  $\Theta(R, x^*(R))$  is positive and increasing on  $[\check{R}, +\infty[$ .

#### A.10. Proof of Proposition 3.1

At the point  $R = \check{R}$  we have,

$$x^*(\psi^*(\check{R})) = \frac{\Theta(\check{R}, \check{x})}{A} = \frac{x^*(\check{R})\Theta(\check{R}, \check{x})}{x^*(\check{R})A} = \frac{x^*(\check{R})\check{A}}{A}$$

Hence,

$$\psi^*(\check{R}) > \check{R} \Leftrightarrow x^*(\psi^*(\check{R})) < x^*(\check{R}) \Leftrightarrow \frac{x^*(\check{R})\check{A}}{A} < x^*(\check{R}) \Leftrightarrow A > \check{A}$$

We thus have  $\psi^*(\check{R}) > \check{R}$ . To determine if  $\psi^*(R_t)$  is increasing or decreasing, let us start from Eq. (44). Knowing the expressions of  $x^*(R_{t+1})$  and  $x^*(R_t)$ , this equation can be rewritten as:

$$\lambda \left( 1 + \frac{\delta + p(1 + n)\gamma}{R_{t+1}} \right) - \frac{R_t}{A} \left[ \frac{1 - \beta}{(1 - p)(1 + n)} \left( 1 - \lambda \left( 1 + \frac{\delta + p(1 + n)\gamma}{R_t} \right) - \frac{\beta(\delta + p(1 + n)\gamma)}{(1 - \beta)R_t} \right) \right]^{1 - \lambda} = 0$$

$$\equiv X^*(R_{t+1}, R_t) = 0$$

We have shown that  $\Theta(R, x^*(R))$  is increasing with  $R$  and  $x^*(R)$  is decreasing with  $R$ . We argue that:

$$\frac{\partial X^*(R_{t+1}, R_t)}{\partial R_t} < 0 \quad \frac{\partial X^*(R_{t+1}, R_t)}{\partial R_{t+1}} < 0$$

As a consequence, the implicit function theorem implies for all  $R_{t+1} \in ]\check{R}, +\infty[$  and  $R_t \in ]\check{R}, +\infty[$ :

$$\frac{dR_{t+1}}{dR_t} < 0$$

Therefore,  $\psi^*(R_t)$  is decreasing. We solve:

$$\frac{dR_{t+1}}{dR_t} \Big|_{R=R_u} < -1 \Leftrightarrow \frac{(1 - \beta)(1 - \lambda)}{A(1 - p)(1 + n)} \left( \frac{\lambda(\delta + p(1 + n)\gamma)}{R_u} + \frac{\beta(\delta + p(1 + n)\gamma)}{(1 - \beta)R_u} \right)$$

$$> \frac{-\lambda}{R_u} \left[ \frac{1-\beta}{(1-p)(1+n)} \left( 1 - \lambda \left( 1 + \frac{\delta + p(1+n)\gamma}{R_u} \right) - \frac{\beta(\delta + p(1+n)\gamma)}{(1-\beta)R_u} \right) \right]^\lambda$$

Since the left hand side of the previous inequation is positive and the right hand side is negative, we notice that  $\left. \frac{dR_{t+1}}{dR_t} \right|_{R=R_u} < -1$ .

If  $A > \check{A}$ , then  $\psi^*(\check{R}) > \check{R}$ . The function  $\psi^*(R_t)$  is decreasing on  $]\check{R}, +\infty[$  and cuts the line  $R_{t+1} = R_t$ . At the intersection, the slope is lower than  $-1$ . Hence, the unconstrained steady state  $R_u$  is locally unstable.

#### A.11. Study of $\Theta(R, \bar{x}(R))$

We know that:

$$\Theta(R, \bar{x}(R)) = R^\lambda \left[ \frac{1-\beta}{(1-p)(1+n)} \left( \frac{R^\beta (\delta + p(1+n)\gamma)^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta}} - \frac{\delta + p(1+n)\gamma}{1-\beta} \right) \right]^{1-\lambda}$$

This implies that  $\Theta(R, \bar{x}(R)) \geq 0$  if  $R \geq R_{min}$ .

We compute:

$$\begin{aligned} \frac{\partial \Theta(R, \bar{x}(R))}{\partial R} &= \lambda R^{\lambda-1} \left[ \frac{1-\beta}{(1-p)(1+n)} \left( \frac{R^\beta (\delta + p(1+n)\gamma)^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta}} - \frac{\delta + p(1+n)\gamma}{1-\beta} \right) \right]^{1-\lambda} \\ &+ \left( \frac{R^{\beta-1+\lambda} \beta^{1-\beta} (1-\beta)^\beta (\delta + p(1+n)\gamma)^{1-\beta}}{(1-p)(1+n)} \right) \\ &\times \left[ \frac{1-\beta}{(1-p)(1+n)} \left( \frac{R^\beta (\delta + p(1+n)\gamma)^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta}} - \frac{\delta + p(1+n)\gamma}{1-\beta} \right) \right]^{-\lambda} \\ &\geq 0 \text{ if } R \geq R_{min} \end{aligned}$$

Hence  $\Theta(R, \bar{x}(R))$  is positive and increasing on  $[R_{min}, \check{R}]$ .

#### A.12. Proof of Proposition 3.2

We know that if  $A > \check{A}$  there also exists a steady state  $R_c < \check{R}$ . To determine if  $\bar{\psi}(R_t)$  is increasing or decreasing on  $[R_{min}, \check{R}]$ , let us start from Eq. (46). Knowing the expressions of  $\bar{x}(R_{t+1})$  and  $\bar{x}(R_t)$ , this equation can be rewritten as:

$$\begin{aligned} 1 - \frac{(\delta + p(1+n)\gamma)^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta} R_{t+1}^{1-\beta}} + \frac{\delta + p(1+n)\gamma}{R_{t+1}} \\ - \frac{R_t^\lambda}{A} \left[ \frac{1-\beta}{(1-p)(1+n)} \left( \frac{R_t^\beta (\delta + p(1+n)\gamma)^{1-\beta}}{\beta^\beta (1-\beta)^{1-\beta}} - \frac{\delta + p(1+n)\gamma}{1-\beta} \right) \right]^{1-\lambda} &= 0 \end{aligned}$$

$$\equiv \bar{X}(R_{t+1}, R_t) = 0$$

We have shown that  $\Theta(R, \bar{x}(R))$  is increasing with  $R$  and  $\bar{x}(R)$  is increasing with  $R$ . We argue that

$$\frac{\partial \bar{X}(R_{t+1}, R_t)}{\partial R_t} < 0 \quad \frac{\partial \bar{X}(R_{t+1}, R_t)}{\partial R_{t+1}} > 0$$

As a consequence, the implicit function theorem implies for all  $R_{t+1} \in [R_{min}, \check{R}]$  and  $R_t \in [R_{min}, \check{R}]$ :

$$\frac{dR_{t+1}}{dR_t} > 0$$

Therefore,  $\bar{\psi}(R_t)$  is increasing with  $t$ . For the point  $R_{min}$ , we have:

$$\lim_{R \rightarrow R_{min}} \frac{\partial \bar{X}(R_{t+1}, R_t)}{\partial R_t} \Big|_R = -\infty \quad \lim_{R \rightarrow R_{min}} \frac{\partial \bar{X}(R_{t+1}, R_t)}{\partial R_{t+1}} \Big|_R = 0$$

Therefore, this equilibrium is locally unstable.

$$\lim_{R \rightarrow R_{min}} \frac{dR_{t+1}}{dR_t} \Big|_R = +\infty$$

At the point  $R = \check{R}$ ,

$$\bar{x}(\psi^*(\check{R})) = \frac{\Theta(\check{R}, \check{x})}{A} = \frac{x^*(\check{R})\Theta(\check{R}, \check{x})}{x^*(\check{R})A} = \frac{x^*(\check{R})\check{A}}{A}$$

Hence,

$$\bar{\psi}(\check{R}) < \check{R} \Leftrightarrow \bar{x}(\psi^*(\check{R})) < \bar{x}(\check{R}) \Leftrightarrow \frac{x^*(\check{R})\check{A}}{A} < x^*(\check{R}) \Leftrightarrow A > \check{A}$$

At the neighborhood of  $R_{min}$ , we argue that  $\bar{\psi}(R) > R$ . As  $\bar{\psi}(\check{R}) < \check{R}$  this function necessarily cuts the 45 degrees line (at least once). At the largest intersection defined by  $R_c$ , the slope is lower than one. This implies that  $R_c$  is locally stable.

If  $A < \check{A}$  it may exist 0 steady state or 2 steady states (we do not exclude other even number). We know that at the neighborhood of  $R_{min}$ ,  $\bar{\psi}(R) > R$ . However,  $\bar{\psi}(\check{R}) > \check{R}$  since  $A < \check{A}$ . Therefore, the first non-trivial steady state  $R_{c1}$  is locally stable (the slope of  $\bar{\psi}(R)$  is lower than 1) and the second  $R_{c2}$  is locally unstable (the slope of  $\bar{\psi}(R)$  is greater than 1). More generally, the odd steady states are stable and the even are unstable.

#### A.13. Calibration

See Table 3.

#### References

- Acosta, P., 2006. Labor supply, school attendance, and remittances from international migration: The case of el salvador. The World Bank.
- Aggarwal, R., Demirgüç-Kunt, A., Pería, M.S.M., 2011. Do remittances promote financial development?. J. Dev. Econ. 96 (2), 255–264.
- Ahamada, I., Coulibaly, D., 2011. How does financial development influence the impact of remittances on growth volatility?. Econ. Model. 28 (6), 2748–2760.
- Amuedo-Dorantes, C., Pozo, S., 2004. Workers' remittances and the real exchange rate: A paradox of gifts. World Dev. 32 (8), 1407–1417.
- Amuedo-Dorantes, C., Pozo, S., 2006. Migration, remittances, and male and female employment patterns. The American Economic Review 96 (2), 222–226.
- Amuedo-Dorantes, C., Pozo, S., 2012. Remittance income volatility and labor supply in Mexico. Southern Economic Journal 79 (2), 257–276. <http://www.jstor.org/stable/41638875>.
- Andolfatto, D., Gervais, M., 2006. Human capital investment and debt constraints. Rev. Econ. Dyn. 9 (1), 52–67.
- Athukorala, P.-C., Sen, K., 2004. The determinants of private saving in India. World Dev. 32 (3), 491–503.
- Azariadis, C., Lambertini, L., 2003. Endogenous debt constraints in lifecycle economies. Rev. Econom. Stud. 70 (3), 461–487.
- Beine, M., Docquier, F., Rapoport, H., 2001. Brain drain and economic growth: theory and evidence. J. Dev. Econ. 64 (1), 275–289.
- Beine, M., Docquier, F., Rapoport, H., 2008. Brain drain and human capital formation in developing countries: Winners and losers. Econ. J. 118 (528), 631–652.
- Bhagwati, J., Hamada, K., 1974. The brain drain, international integration of markets for professionals and unemployment: A theoretical analysis. J. Dev. Econ. 1 (1), 19–42.
- Black, S.E., Devereux, P.J., Salvanes, K.G., 2005. Why the apple doesn't fall far: Understanding intergenerational transmission of human capital. Amer. Econ. Rev. 95 (1), 437–449.

- Calero, C., Bedi, A.S., Sparrow, R., 2009. Remittances, liquidity constraints and human capital investments in Ecuador. *World Dev.* 37 (6), 1143–1154.
- Cassin, L., 2020. The effects of migration and pollution on cognitive skills in Caribbean economies: a theoretical analysis. *Environ. Dev. Econ.* 25 (6), 657–686.
- Catrinescu, N., Leon-Ledesma, M., Piracha, M., Quillin, B., 2009. Remittances, institutions, and economic growth. *World Dev.* 37 (1), 81–92.
- Cazachevici, A., Havranek, T., Horvath, R., 2020. Remittances and economic growth: A meta-analysis. *World Dev.* 134.
- Chami, R., Barajas, A., Cosimano, T., Fullenkamp, C., Gapen, M., Montiel, P., 2008. *Macroeconomic consequences of remittances*. Washington, DC: International Monetary Fund.
- Chami, R., Fullenkamp, C., Jahjah, S., 2005. Are immigrant remittance flows a source of capital for development?. *IMF Staff papers* 52 (1), 55–81.
- de la Croix, D., Michel, P., 2007. Education and growth with endogenous debt constraints. *Econom. Theory* 33 (3), 509–530.
- Edwards, A.C., Ureta, M., 2003. International migration, remittances, and schooling: evidence from El Salvador. *J. Dev. Econ.* 72 (2), 429–461.
- Faini, R., 2007. Migrations et transferts de fonds. Impact sur les pays d'origine. *Revue d'économie du développement* 15 (2), 153–182.
- Gente, K., León-Ledesma, M.A., Nourry, C., 2015. External constraints and endogenous growth: Why didn't some countries benefit from capital flows?. *J. Int. Money Financ.* 56, 223–249.
- Giuliano, P., Ruiz-Arranz, M., 2009. Remittances, financial development, and growth. *J. Dev. Econ.* 90 (1), 144–152.
- Grubel, H.B., Scott, A.D., 1966. The international flow of human capital. *Amer. Econ. Rev.* 56 (1/2), 268–274.
- Hossain, D., 2014. Differential impacts of foreign capital and remittance inflows on domestic savings in developing countries: A dynamic heterogeneous panel analysis. *Econ. Rec.* 90, 102–126.
- Imai, K.S., Gaiha, R., Ali, A., Kaicker, N., 2014. Remittances, growth and poverty: New evidence from Asian countries. *Journal of Policy Modeling* 36 (3), 524–538.
- Jawaid, S.T., Raza, S.A., 2012. Workers' remittances and economic growth in China and Korea: an empirical analysis. *Journal of Chinese Economic and Foreign Trade Studies* 5 (3), 185–193.
- Karagöz, K., 2009. Workers' remittances and economic growth: evidence from Turkey. *Journal of Yaşar University* 4 (13), 1891–1908.
- Kehoe, T.J., Levine, D.K., 1993. Debt-constrained asset markets. *Rev. Econom. Stud.* 60 (4), 865–888.
- Kehoe, T.J., Levine, D.K., 2001. Liquidity constrained markets versus debt constrained markets. *Econometrica* 69 (3), 575–598.
- Khan, S., Merritt, D.M., 2020. *Remittances and International Development: The Invisible Forces Shaping Community*. Routledge.
- Le, T., 2009. Trade, remittances, institutions, and economic growth. *International Economic Journal* 23 (3), 391–408.
- León-Ledesma, M., Piracha, M., 2004. International migration and the role of remittances in eastern Europe. *International Migration* 42 (4), 65–83.
- Lucas, R.E., Stark, O., 1985. Motivations to remit: Evidence from Botswana. *J. Polit. Econ.* 93 (5), 901–918.
- Melkonyan, T.A., Grigorian, D.A., 2012. Microeconomic implications of remittances in an overlapping generations model with altruism and a motive to receive inheritance. *J. Dev. Stud.* 48 (8), 1026–1044.
- Mountford, A., 1997. Can a brain drain be good for growth in the source economy?. *J. Dev. Econ.* 53 (2), 287–303.
- Nsiah, C., Fayissa, B., 2013. Remittances and economic growth in Africa, Asia, and Latin American-Caribbean countries: a panel unit root and panel cointegration analysis. *Journal of Economics and Finance* 37 (3), 424–441.
- Nwosa, P.I., Akinbobola, T.O., 2016. Capital inflows and economic growth in Nigeria: The role of macroeconomic policies. *Afr. Dev. Rev.* 28 (3), 277–290.
- Pradhan, G., Upadhyay, M., Upadhyaya, K., 2008. Remittances and economic growth in developing countries. *The European Journal of Development Research* 20 (3), 497–506.
- Rapoport, H., Docquier, F., 2006. The economics of migrants' remittances. In: Kolm, S.-C., Ythier, J.M. (Eds.), *Handbook of the Economics of Giving, Altruism and Reciprocity: Applications*, Vol. 2. Elsevier, pp. 1135–1198.
- Ratha, D., 2005a. Leveraging remittances for international capital market access. World Bank.
- Ratha, D., 2005b. Workers' remittances: An important and stable source of external development finance. *Remittances: Dev. Impact Future Prospect.* 19–51.
- Ratha, D., 2007. Leveraging remittances for development. *Policy Brief* 3 (11).
- Siddique, A., Selvanathan, E.A., Selvanathan, S., 2012. Remittances and economic growth: Empirical evidence from Bangladesh, India and Sri Lanka. *Journal of Development Studies* 48 (8), 1045–1062.
- Sobiech, I., 2019. Remittances, finance and growth: Does financial development foster the impact of remittances on economic growth?. *World Dev.* 113, 44–59.
- Vargas-Silva, C., 2007. Measuring the macroeconomic impact of workers' remittances in a data-rich environment. *Applied Financial Economics Letters* 3 (6), 359–363.
- Vidal, J.-P., 1998. The effect of emigration on human capital formation. *J. Popul. Econ.* 11 (4), 589–600.
- Wong, K.-y., Yip, C.K., 1999. Education, economic growth, and brain drain. *J. Econom. Dynam. Control* 23 (5–6), 699–726.
- Yiheis, Z., Woldemariam, K., 2016. The effect of remittances on domestic capital formation in select African countries: A comparative empirical analysis. *J. Int. Dev.* 28 (2), 243–265.
- Zhunio, M.C., Vishwasrao, S., Chiang, E.P., 2012. The influence of remittances on education and health outcomes: a cross country study. *Appl. Econ.* 44 (35), 4605–4616.