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## Mathematical optimisation of strategies for the realisation of sustainable urban water management

Optimisation mathématique des stratégies pour la réalisation de la gestion durable des eaux urbaines

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### RESUME

Actuellement, le drainage et l'approvisionnement en eau urbains dans les pays développés sont dominés par des systèmes centralisés qui ne sont incontestablement pas conformes aux conditions durables. Si les technologies pour une gestion des eaux de pluies naturelle ou un assainissement décentralisé remplacent au moins partiellement les systèmes existants les travaux de reconstruction intensifs seront essentiels. L'exposé présente le développement et l'implémentation pratique d'un outil mathématique afin d'élaborer une stratégie optimisée pour la réalisation de concepts de drainage et d'assainissement alternatifs et plus décentralisés dans les zones urbaines existantes. La succession des mesures de construction pour la totalité de la période considérée (environ 50 à 100 ans) a été déterminée sur base d'un modèle d'optimisation mathématique à condition que le futur état favorisé soit connu. Le modèle décrit les interdépendances complexes du cycle urbain de l'eau et permet la minimisation des efforts financiers et des impacts écologiques sur le chemin vers l'état futur.

### ABSTRACT

Urban drainage and water supply in developed countries are at present dominated by centralised systems, which unquestionably do not comply with sustainable requirements. If technologies for a natural stormwater management or decentralised sanitation should at least partially replace existing systems, intensive reconstruction work becomes essential. This paper presents the development and practical implementation of a mathematical tool to find an optimised strategy for the realisation of alternative and more decentralised drainage and sanitation concepts in existing urban areas. The succession of construction measures for the whole period of consideration (about 50 to 100 years) is determined based upon a mathematical optimisation model on condition that the favoured future state is known. The model describes the complex interdependencies of the urban water cycle and enables the minimisation of both financial efforts and ecological impacts on the way towards the future state.

### KEYWORDS

Change in urban drainage, cost-benefit consideration, mathematical modelling, optimisation of strategies, sustainable urban water management.

## 1 INTRODUCTION

### 1.1 Background

Urban drainage and water supply in industrialised countries are at present dominated by centralised systems and structures. As these concepts do unquestionably not comply with sustainable requirements, alternative concepts of sustainable drainage and sanitation have become more and more significant in recent years. To revise relying on conventional piped drainage systems in stormwater management has been a recent trend. Developed areas are drained in a more natural way, using the infiltration and storage capacities of semi-natural devices such as infiltration trenches, swales and ponds (Butler and Davies, 2004) – Sustainable Urban Drainage Systems (SUDS). In Germany, in most federal states an obligation to implement SUDS exists for development areas. In existing areas the realisation of SUDS devices will be more difficult. A disconnection of 10 to 15 % of paved areas from the piped systems is regarded as realistic.

In recent years alternatives for the disposal of foul water are also discussed (e.g. Hiessl, 2005; Starkl et al., 2004). Concepts of decentralised sanitation and reuse (DESAR) should close urban water and nutrient cycles and conserve water resources (Lens et al., 2001). Devices for water reuse and decentralised treatment of sanitary wastewater are investigated in numerous field studies. The realisation of such concepts in existing areas would cause extensive financial and constructional efforts and would be more difficult – particularly because of residents' acceptance – than the implementation of sustainable stormwater management.

### 1.2 Open questions and aim of the study

Decision support approaches to the selection of sustainable drainage systems or DESAR are investigated in some studies (e.g. Ellis et al., 2006, Huang et al., 2004). But to find strategies how the future state can be reached in an optimal way is not investigated so far. If devices of SUDS and DESAR should at least partially replace existing centralised systems, intensive reconstruction work becomes essential. The conceptual change is superposed by a high demand for rehabilitation in water supply and sewer networks. An open question is how these two requests can be reconciled under economical and ecological aspects. At all stages of a transition a reliable water supply and disposal of wastewater have to be guaranteed. A conversion can only be realised successively over a long period due to high constructional and financial expenses and requires new strategies for 'hot plug-in'. 'Manually' the optimal strategy to attain the favoured future state is difficult to develop. Therefore, an urgent need for research exists for a strategy for the implementation of decentralised and sustainable drainage and sanitation devices in existing urban areas on condition that the favoured future state is known. The paper will present the development and implementation of a mathematical optimisation tool to determine the succession of the construction of devices under minimal ecological impacts and economical efforts.

## 2 MATERIAL AND METHODS

### 2.1 Mathematical Approach

#### 2.1.1 Optimisation problem

As financial efforts (economical costs) as well as ecological impacts (ecological costs) should be minimised on the way to more sustainable systems, the mentioned problem belongs to the field of multi-criteria optimisation. Essential constraints of the optimisation problem are to ensure the proper functioning of waste water disposal and to meet standards in regulations at any time. Therefore such feasible strategies of

reconstructing measures should be found, which could not be enhanced in both criteria (economical and ecological costs). Generally, not only one solution of the optimisation problem exists but numerous reasonable Pareto-optimal solutions (see e.g. Ehr Gott, 2005). Only the subjective weighting of the different criteria or the discussion of local deciders can lead to the definite choice of solution, namely the strategy of conversion that should be applied.

### 2.1.2 Design of the model

In a first step the functional progression within the optimising procedure was defined and is shown in Figure 1. The mathematical modelling is based upon the scale of subcatchments and simplified networks of drainage elements (functioning network). The subcatchments are connected due to flow directions and all interrelationships of the main elements are represented. This allows besides the temporal succession of appropriate measures a spatial consideration.

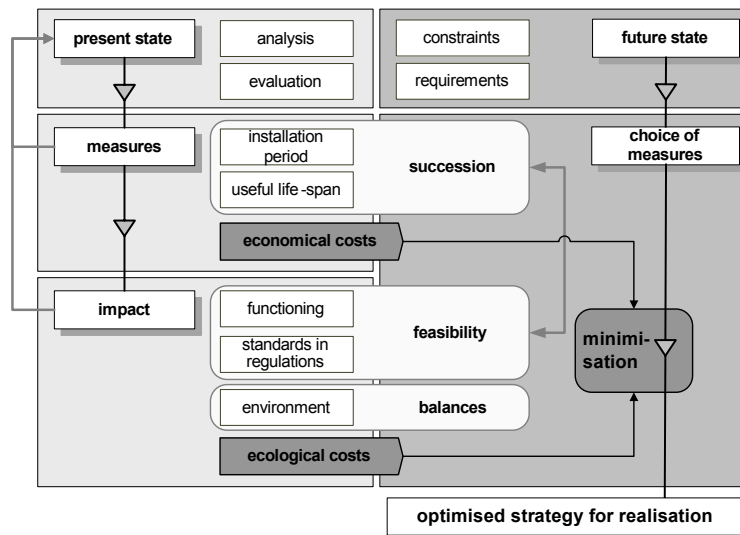


Figure 1. Scheme of mathematical optimisation model.

Based upon the boundary conditions of the present state and the favoured future state potentially realisable measures are provided for each subcatchment by using an own decision support tool (Schildwächter, 2006). At this, devices for SUDS, DESAR as well as drainage elements (sewers and surficial drains) are suggested depending on numerous parameters, e.g. topography, subsurface conditions, land use, space requirements or population density. For all measures investment costs and operating costs as well as installation periods were calculated in all subcatchments. Furthermore information about impacts on 'flows', discharge and pollution are linked to each measure allowing a simplified balancing of volumes and loads in different flow types and discharge paths (e.g. waterbodies, WWTP effluent, soil). The environmental impact is estimated with ecological costs expressing negative as well as positive ecological impacts. Within the optimising process the feasibility of the systems is verified in each time step, which decisively affects the succession of conversation measures.

Due to the requirements of the favoured future state now expedient measures are in such a way chosen, that on the one hand the hydraulic and legally allowed functioning of the systems is ensured at any time. On the other hand the succession of measures should cause the minimal economical and ecological costs.

## 2.2 Mathematical Modelling

The mathematical model itself is formulated as a bi-criteria mixed-integer program (MIP). The structure was built as a complex network of 'nodes' and 'arcs'. All transfer points of drainage systems (manholes, overflow structures, outfalls) as well as surface types (area of roofs, streets, yards, unpaved area) and foul water components were represented as nodes. Further nodes are the measures and devices and balancing nodes such as infiltration or evaporation. Arcs represent all possible connections between nodes and are characterised by capacities, costs and installation periods. All possible flow paths of dry and wet weather flow and their pollution are described by that way. That means e.g. that the surface type roof is linked to possible nodes as green roof, rainwater utilisation, infiltration, open drainage or stormwater sewer. But starting from one surface type many more arcs are necessary to describe the complex interdependencies (see Figure 2). The nodes-arc-network can easily reach dimensions of about 100 nodes and 300 arcs for each subcatchment.

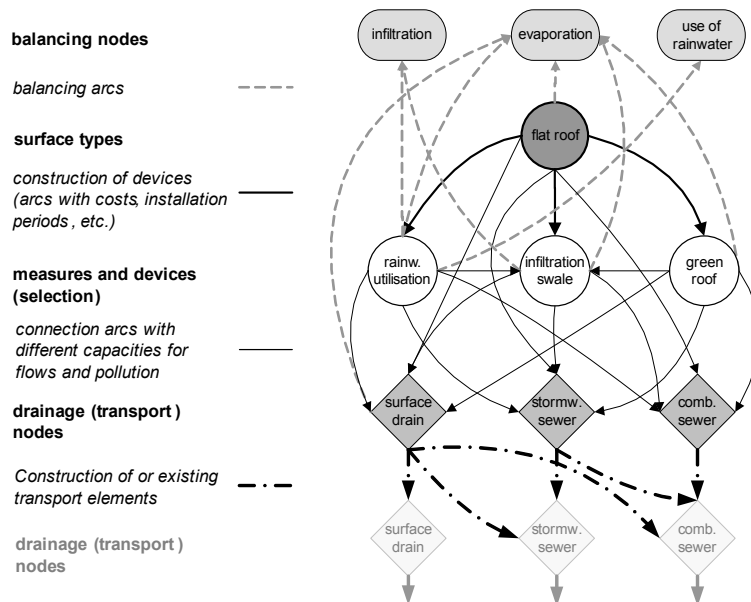


Figure 2: Model structure of nodes and arcs starting from one surface type.

Based on this structure a simultaneous project scheduling and network flow problem is defined. The challenge and specific is on the one hand, that not all specified expedient measures have to be chosen but just those measures should be selected, which lead to a Pareto-optimal strategy. On the other hand the network for the scheduling and flow problem is time dependent, as with the construction of different devices arcs are opened due to installation periods and closed when elements are replaced by new devices. By the implementation of different variables and adequate constraints within the mathematical modelling procedure all paths of the network are scanned in order to find a feasible optimal solution under the consideration of economical and ecological costs, the objective functions of the model.

The economical costs at every time step are the result of investment costs (€) of devices with beginning of construction in the regarded time step and operating costs (€/year) of all installed measures. Furthermore rehabilitation and reinvestment costs respectively (€) as well as costs for repairing damage caused by flooding (€/year) are

considered. Also a WWTP effluent charge (€/year) is balanced in a simplified way. The economical costs for the whole period under consideration are calculated as total project costs. The total project costs are the sum of money which is necessary for financing all measures based on today's cost level with a real interest rate of 3 percent. The ecological costs are not accounted monetarily but by a number resulting from a point system. Positive costs represent an environmental 'damage' whereas negative numbers express a benefit. The costs are calculated 'on-line' by simplified methods. The different criterions can be weighted individually. Table 1 lists the implemented ecological parameters, but as a matter of course others are possible as well. A more detailed description of the mathematical model can be seen in Kaufman et al. (2006).

impacts on	criteria	calculation of points per year
<b>water cycle</b>	distance from natural water cycle for infiltration rate, evaporation rate, use of rainwater	obliged value (%) - present value (%)
<b>resources</b>	distance from favoured resources protection for rate of reducing use of potable water, rate of water reuse, rate of fertiliser production	obliged value (%) - present value (%)
<b>emissions</b>	emitted pollution loads in water bodies	pollution load (kg/year) / 100
	discharge rate for overflows (ATV, 1992)	present rate - admissible rate
	dilution rate in overflow discharge	(required rate - present rate) · 2

Table 1: Evaluation of ecological costs.

### 3 IMPLEMENTATION OF THE MODEL

#### 3.1 Catchment and boundary conditions

The model has been implemented for a suburb of Kaiserslautern in Germany, a rural catchment of about 3,000 inhabitants. A section of the locality is shown in Figure 3.



Figure 3: Catchment for implementation of the model (section).

In a detailed preprocessing 32 subcatchments were determined. For a first model implementation they were merged to 6 subcatchments. The entire catchment has a drainage area of about 90 ha and implies 35 ha of paved area. About 30 % are drained by separate systems whereas the rest consists of combined sewer systems. Two combined sewer overflow devices (CSOs, see Figure 3) and one final sewer overflow tank (not included in section of Figure 3) are installed in the sewer system. A business park in the south of the suburb has an area of about 20 ha and its effluent shows the characteristics of domestic waste water. Dry weather flow amounts to 11.5 L/s and consist of 6.0 L/s foul sewage and 5.5 L/s infiltration water. The pollution of dry weather flow is 560 mg COD/L. Within the optimisation model (so far) only the parameter COD is implemented.

In this paper as an example of numerous potential future states it is determined that stormwater runoff and wastewater should not be mixed any more and a natural stormwater management should be achieved. The implementation of DESAR techniques should lead to a decentralised treatment of black-water (faeces and urine) whereas grey-water (all the wastewater produced in a house except by the WC) should be treated centrally in the WWTP. Additionally obliged values for water balance and resources protection for the future state are chosen as follows (Table 2).

impacts on	criteria	obliged value in future state
water cycle	infiltration rate	30 %
	evaporation rate	55 %
	use of rainwater	10 %
resources	rate of reducing water consumption	45 %
	rate of grey-water reuse	0 % (should be treated at WWTP)
	rate of fertiliser production	25 %

Table 2: Obligated values for future state.

For this example of implementation two different specifications are made for the optimisation process. In a first specification (S 1) both the ecological and the economical costs are equally weighted. In a second specification (S 2) the ecological costs are secondary weighted (half of S 1) and the future state should primarily be reached with minimal financial efforts.

### 3.2 Results and discussion

Figure 4 illustrates the *succession of measures* for the optimal strategy of transition for a subcatchment. The beginning of construction as well as installation periods (grey bars) and phases of rehabilitation (white bars) are shown for the two specifications.

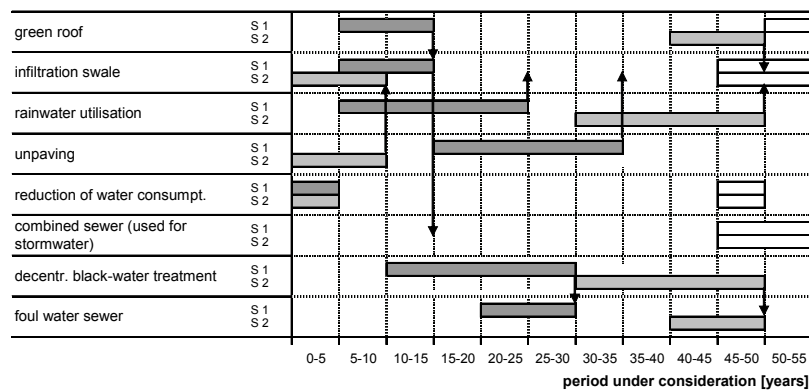


Figure 4: succession of measures for a subcatchment (No. 1) as result of mathematical modelling

At S 1 some devices are built earlier since negative ecological impacts are reduced more efficiently right from the start of the consideration period. In this subcatchment in both specifications the same devices are chosen to be implemented by the optimisation model, but the measure 'unpaving' is realised for a larger paved area in S 1.

The *economical costs* are demonstrated as annually costs (columns) and as summation curve for the period of consideration (lines) in Figure 5. Due to the real interest rate in S 2 many devices are implemented rather late being 'cheaper'. As ecological costs here are weighted lower there is no reason to build them earlier. All in all, in S 1 32.5 million € and in S 2 25.0 million € result as total costs. The higher weight on ecological costs in S 1 than in S 2.

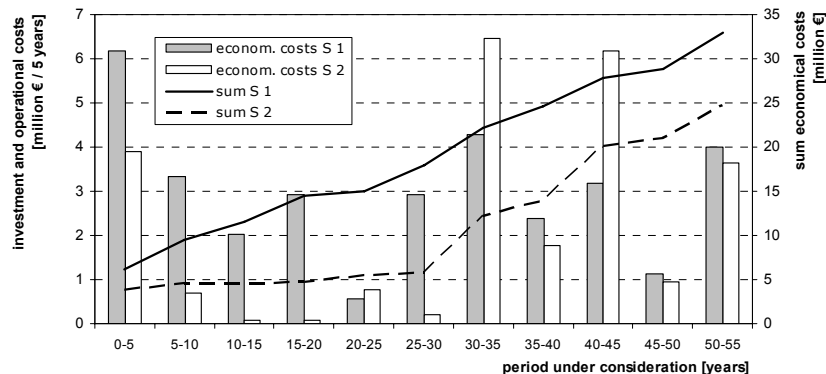


Figure 5: Economical costs for S 1 and S 2.

Annually *ecological costs* are reduced more rapidly in S 1 and accumulate to 2,720 'points' whereas in S 2 3,390 'points' result for the whole time. That means that the strategy in S 2 causes nearly 25 % more negative impacts. Taking e.g. the emitted COD loads at the stormwater tank into consideration in S 1 242 tons are emitted in the 55 years under consideration and in S 2 286 tons. Similarly the earlier adjustment to obliged values for water balance and resources protection is more distinctive.

As mentioned in chapter 2.1.1 you can see that more than one optimal solution exists. The results of both scenarios are each an optimal strategy for the implementation of more decentralised devices in regard to the favoured future state. The weighting of both kinds of costs (economical and ecological) as well as the weight of the ecological criteria among each other and of deficits influences the optimal solution. Only local deciders can make the final decision for a strategy.

#### 4 CONCLUSION AND OUTLOOK

Numerous alternatives for stormwater drainage are established in recent years whereas the use of components of domestic wastewater as resources is under consideration. This present change in exposure to wastewater causes intensive reconstruction work for existing centralised drainage systems. To ensure that every step of reconstruction ecologically and economically benefits the future an optimised strategy for the transition of systems should be investigated. A first tool to find such strategies was developed as a bi-criteria optimisation model and implemented for a rural area in Germany. The mathematical approach necessitates many simplifications due to the high complexity of interdependencies in the urban water and nutrient cycle. Nevertheless, the results are plausible and optimal strategies for the sequence of measures to more sustainable systems under ecological and economical aspects can be found. At the moment the enhancement of the solving process is in progress to achieve a more detailed consideration.

More reliable strategies could be developed if many more constraints, for instance the detailed consideration of wastewater treatment processes and receiving water or population development, are taken into account. The hitherto investigations have also shown, that current guidelines and regulations could increase the price of or even inhibit favoured restructuring of drainage systems. In the meantime there will be states where not all regulations could be fulfilled. Therefore standards should be adapted to changing systems. Furthermore it is essential to define the requirements and conditions of favoured future states, such as an obliged water balance or admissible emissions. They also have an important influence on costs and impacts of reconstruction measures.

The mathematical optimisation has been turned out to be an adequate instrument to find strategies for the realisation of sustainable urban water management. The developed tool possibly will be a support for decision-making processes. The potential of the approach will rise with the complexity of the specific application. For complex systems an optimal solution for transition to a favoured future state cannot be found manually.

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