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A MATHEMATICAL MODEL FOR TREATMENT OF PAPILLARY THYROID CANCER USING THE ALLEE EFFECT

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The incidence of thyroid cancer is rising all over the world, and the papillary subtype (PTC) is the primary factor for this increase. The presence of thyroid tumors is commonly associated with increased levels of cytokines, such as interleukin 6 (IL-6). Considering PTC patients treated with thyroidectomy and radioactive iodine ^{131}I (RAI), we propose an ordinary differential system using four variables: the RAI activity, the number of cancer cells and the serum concentrations of IL-6 and thyroglobulin (Tg). Our objective is to study the efficacy of different therapeutic doses of RAI in the treatment of thyroid cancer. The Allee effect is taken into account when modeling cancer cells growth under the influence of IL-6. From the results obtained, the main factors and conditions correlated with successful treatment, such as the RAI activity used and the tumor response are addressed. The detection of minimal doses of RAI that can cause tumor extinction is performed, though this has also meant longer periods for tumor cell elimination. The critical number of tumor cells due to the Allee effect is analyzed and linked to the immune system or biological factors that can slow the progression of the tumor but are

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insufficient after thyroid resection surgery.

Keywords: Thyroid tumor; Interleukin 6; Radioiodine; Ordinary differential equations.

1. Introduction

The number of thyroid cancer diagnoses in the United States is predicted to exceed that of colorectal cancer by 2030,¹ making it the fourth most prevalent cancer. Despite this, a simultaneous increase in the number of thyroid cancer-related deaths is not predicted.² When weighing the evidence of the higher number of small tumors diagnosed and the improved or unchanged survival rate, most studies attribute the increased incidence to an overdiagnosis of papillary thyroid cancer (PTC),³ whereas some studies consider that it's more than a case of overdiagnosis but an actual increase in the numbers of tumors.^{4,5} The PTC is by far the most common histological subtype, representing 90% of differentiated thyroid carcinomas (DTC) and 85-90% of all thyroid cancers.^{6,7}

The American Thyroid Association (ATA) guidelines⁸ recommend thyroid lobectomy or thyroidectomy surgery, if the cytology results indicate a primary thyroid malignancy. Though, in some cases, only an active surveillance management approach may be endorsed. After surgery, the RAI remnant ablation depends on many factors, e.g., the presence of local or distant metastases, locoregional tumor invasion, aggressive histology, among others. However, this treatment is not routinely recommended after thyroidectomy for PTC patients classified with low risk of recurrence, as seen in Ref. 8. In PTC, this therapy is frequently used for the following purposes: residual thyroid tissue elimination; adjuvant therapy, due to its potential tumoricidal effect; resource for a post-treatment full-body scan to find unknown metastases; and therapy in the case of persistent disease.

RAI therapy has been used, since the 1940s, for both malignant and benign thyroid conditions, such as hyperthyroidism due to Graves' disease, toxic multinodular goiter, and hyperfunctioning thyroid nodules.⁹ According to Wishart *et al.*,¹⁰ the iodine-131 causes mutation and death in cells due to its mode of beta decay, and its therapeutic effects are a result of the ionizing radiation absorbed by the thyroidal tissue. Although the thyroid gland naturally uptakes iodine from the body, the success of RAI treatment for PTCs depends on how much iodine is taken up by iodide transporter of the thyroid. I-131 uptake is almost entirely done by the thyroid gland, and so only an insignificant amount of radiation exposure occurs outside the gland.¹⁰ Among the ways to evaluate response to RAI therapy in PTC, analysis of thyroglobulin (Tg) levels are indicated.^{11,12} For instance, an excellent response to the treatment with total thyroidectomy and RAI ablation in PTC patients is if the thyrotropin-stimulated Tg is lower than 1 ng/mL in the absence of anti-Tg antibodies.⁸

The immune system is a significant agent in cancer regulation, which can both prevent or facilitate tumor progression. According to Dunn *et al.*¹³ and Yu *et al.*,¹⁴ the manner in which tumor cells escape immune system surveillance must be under-

stood primarily from an immunosuppressive dysfunction perspective, for instance, a dysregulation of Tregs cells, immunomodulatory cytokines or both. Interleukin 6 (IL-6) is a cytokine that plays a central role in the regulation of inflammatory, immune and hematopoietic processes,¹⁵ and has been the target of several studies regarding its influence on various types of tumor malignancy, including thyroid cancer.^{16–21} Provatopoulou *et al.*²² observed IL-6 levels significantly higher than baseline levels (2.19 ± 3.30 vs 5.42 ± 9.76 , $p = 0.006$) in thyroid cancer patients and proposed its use among other cytokines to understand the mechanisms underlying the disease. Working on the significance of IL-6 in papillary thyroid carcinomas, Kobawala *et al.*¹⁹ noted that, in addition to increased levels, this cytokine was generally associated with a lower survival rate of PTC patients, playing an essential role in disease progression.

To study tumor growth, in addition to the mathematical models employing exponential, logistic or Gompertz equations,²³ some studies adopted a law of population ecology known as the Allee effect.^{24–28} According to Stephens *et al.*,²⁹ this concept established by W. C. Allee in the 1930s, refers to a fact observed in some populations in which there is extinction below a certain threshold. In general, this phenomenon is marked by the existence of an unstable equilibrium, where below it the growth is negative.³⁰ Among the recent studies using the Allee effect for cancer research, Korolev *et al.*²⁴ considered tumor cells as an endangered species to find potential vulnerabilities related to extinction processes. In their paper, they used concepts from conservation biology, ecology, and evolution to identify mechanisms that can lead cancer to extinction or inhibit its adaptation. Moreover, the use of the Allee effect in processes such as cancer cell cooperative behavior until tumor recurrence in glioblastoma,²⁸ population growth kinetics in small tumors,²⁵ tumor growth and necrotic core formation²⁶ have been acknowledged.

Mathematical modeling has long been used in cancer biology research.^{31–34} Until 2018, most models of thyroid cancer used ordinary differential equations.³⁵ They have been used to investigate thyroid cancer in terms of diagnosis, treatment or follow-up of patients with neoplasms (see also Refs. 36–40). In another study,⁴¹ a linear-quadratic model was employed to optimize the amount of RAI (¹³¹I) to be applied in cases of remaining thyroid cells after surgery or in the presence of metastasis. The evaluation of the effective half-life of radioactive iodine in different compartments of the human body, and a criteria proposal to classify patients with thyroid cancer as responders or not to RAI treatment also used ordinary differential models.^{40, 42} Considering RAI treatment after thyroidectomy in PTC patients and metastasis, Barbolosi *et al.*⁴² constructed a mathematical model to evaluate the effectiveness of RAI treatment. From the results obtained, a classification criterion was created based on the parameter associated with the tumor doubling-time under treatment to recognize patients as responders or non-responders to RAI in the early stages of therapy. The proposed model is a system composed of three ordinary differential equations using the variables: the RAI activity, the number of cancer cells and the thyroglobulin concentration. The serum Tg concentration at time in

the blood was used as a means to assess treatment efficacy.

Taking into account the tumor cell proliferation driven by the Allee effect and IL-6 influence, we propose a mathematical model to evaluate the efficacy of RAI treatment in PTC. The amount of interleukin 6 considered excessive is produced by the tumor mass and also stimulated by the RAI administration. The success of the treatment is analyzed based on the number of cancer cells and serum Tg concentration over time. From linear stability analysis and numerical simulations, we discuss scenarios in which parameters representing the applied RAI dose, IL-6 levels, and critical tumor cell number are recognized as key factors for success or not of the treatment.

This paper is structured as follows: in Section 2, we present the proposed mathematical model and its parameters. In Section 3, we discuss the positivity and the existence of equilibrium points of the model and the influence of parameter Q in the absence of RAI treatment. In Section 4, we consider the introduction of the RAI treatment in simulations. In Section 5, we present our results and discussions. Finally, Section 6 closes the article with the main conclusions to the area of study.

2. Mathematical Model

In this model, the study of RAI effectiveness in the PTC treatment is based on four variables depending on time t . Denoting by A the RAI activity, by N the number of the cancer cells, by I the IL-6 and by T_g the thyroglobulin, both with serum concentrations over time, the model proposed is as follows:

$$\begin{cases} \frac{dA}{dt} = -a \log(2)A, \\ \frac{dN}{dt} = \alpha(1 - k \exp(-\beta I)) N \left(\frac{N}{Q} - 1 \right) \left(1 - \frac{N}{K} \right) - \rho AN, \\ \frac{dI}{dt} = \sigma + \frac{cN}{\gamma + N} + bA - mI, \\ \frac{dT_g}{dt} = pN - dT_g. \end{cases} \quad (2.1)$$

The equations for A and T_g are based on Barbolosi *et al.*⁴² The first equation comes from the exponential decay law $A(t) = A_0 \exp(-\lambda t)$, in that $t_{1/2} = 1/a$ and $\lambda = a \log(2)$ is the decay constant. The equation of A does not describe the physical half-life of RAI, but rather its continuous therapeutic effect on cancer cells (see Ref. 42). The treatment consists of only one administration of RAI to ablate remaining thyroid tissue after the total thyroidectomy. The expression $\alpha(1 - k \exp(-\beta I))$ represents the proliferation rate of cancer cells under the influence of IL-6, in that α is its maximum value; the parameter k , with $0 < k < 1$, defines its minimum value $\alpha(1 - k)$; and $\beta > 0$ refers to the rate at which the proliferation rate varies as the IL-6 concentration increases. The parameter Q , with $0 < Q < K$, is the Allee threshold. When $N < Q$, the term $(N/Q - 1)$ is negative. Therefore, a natural decrease occurs in the tumor cell population. The parameter K is the carrying

capacity of the cancer cells, and ρ is the efficiency rate of iodine on tumor cells. The constant σ refers to the natural IL-6 production, while c is the production rate of this interleukin by tumor mass. Since IL-6 production may occur in multiple ways near the tumor, including the tumor cells,¹⁸ the term $cN/(\gamma + N)$ represents the IL-6 production by the tumor mass, where γ is the number of cancer cells by which the IL-6 production rate is half of its maximum. The parameter b refers to the increase of IL-6 due to RAI treatment and m is its natural elimination rate. The parameter p is the T_g production rate by the tumor cells, and d represents the thyroglobulin elimination rate (For more details, see Tables 1, 2 and Fig. 1).

Table 1. Variables used in the model.

Variable	Meaning	Units
A	Radioiodine therapy activity	GBq
N	Number of the cancer cells	cells
I	Serum IL-6 concentration	$\mu\text{g/L}$
T_g	Serum Tg concentration	pg/mL

Note: According to the International System of Units (SI)⁴⁶ the symbol Bq (Becquerel) is the activity referred to a radionuclide and $1 \text{ GBq} = 10^3 \text{ Bq}$.

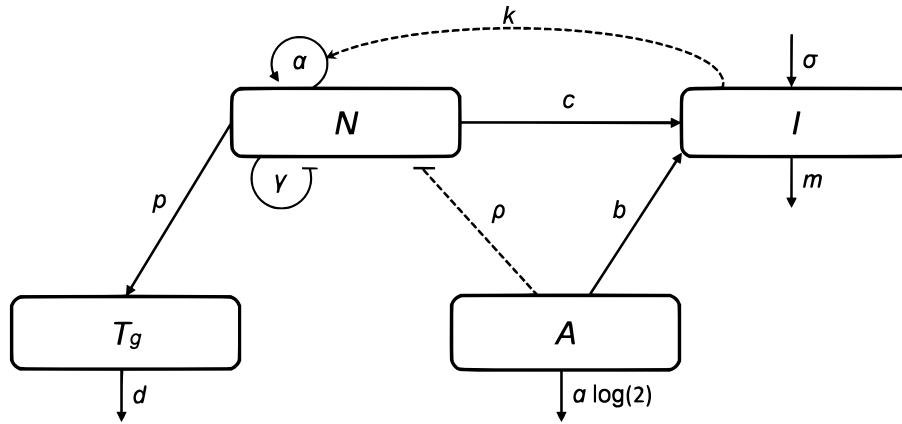


Fig. 1. Schematic model diagram with the dynamics proposed between the activity of RAI, A ; the number of cancer cells, N ; and the serum concentrations of IL-6 and thyroglobulin, I and T_g , respectively. Solid lines represent actions without interaction between compartments and dashed lines, the opposite.

3. Analysis of the model

First of all, it is not difficult to see that the solution of the system (2.1) is non-negative provided that the initial condition is non-negative. To study the equilibrium points of the model (2.1) and their stability, the last equation will not be considered. The variable T_g is used in numerical simulations due to its importance as a tumor marker. The equilibrium points are given by:

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- $P_1 = \left(0, 0, \frac{\sigma}{m}\right)$: extinction of cancer cells,
- $P_2 = \left(0, Q, \frac{\sigma}{m} + \frac{Qc}{(Q + \gamma)m}\right)$: number unstable cancer cells,
- $P_3 = \left(0, K, \frac{\sigma}{m} + \frac{Kc}{(K + \gamma)m}\right)$: scenario of metastasis.

The Jacobian matrices and eigenvalues associated to points P_1 , P_2 and P_3 , are given below:

$$\begin{aligned}
 J(P_1) &= \begin{pmatrix} -a \ln(2) & 0 & 0 \\ 0 & -H_1 & 0 \\ b & \frac{c}{\gamma} & -m \end{pmatrix}, & \text{with } \begin{cases} \lambda_1 = -a \ln(2), \\ \lambda_2 = -m, \\ \lambda_3 = -H_1, \end{cases} \\
 J(P_2) &= \begin{pmatrix} -a \ln(2) & 0 & 0 \\ -rQ & H_2 & 0 \\ b & \frac{c\gamma}{(Q+\gamma)^2} & -m \end{pmatrix}, & \text{with } \begin{cases} \lambda_1 = -a \ln(2), \\ \lambda_2 = -m, \\ \lambda_3 = H_2, \end{cases} \\
 J(P_3) &= \begin{pmatrix} -a \ln(2) & 0 & 0 \\ -rK & -H_3 & 0 \\ b & \frac{c\gamma}{(K+\gamma)^2} & -m \end{pmatrix}, & \text{with } \begin{cases} \lambda_1 = -a \ln(2), \\ \lambda_2 = -m, \\ \lambda_3 = -H_3, \end{cases}
 \end{aligned}$$

where

$$H_1 = \alpha \left(1 - k \exp\left(-\frac{\sigma\beta}{m}\right)\right), \quad (3.1)$$

$$H_2 = (K - Q) \alpha \left(1 - k \exp\left(-\beta \left(\frac{\sigma}{m} + \frac{Qc}{(Q + \gamma)m}\right)\right)\right) K^{-1}, \quad (3.2)$$

$$H_3 = (K - Q) \alpha \left(1 - k \exp\left(-\beta \left(\frac{\sigma}{m} + \frac{Kc}{(K + \gamma)m}\right)\right)\right) Q^{-1}. \quad (3.3)$$

Since $0 < Q < K$ and $0 < k < 1$, then $H_i > 0$, $i = 1, 2, 3$. Consequently, P_1 and P_3 are locally asymptotically stable, while P_2 is a saddle point. In the first part of linear stability analysis, we consider $A = 0$ and obtain scenarios in which complementary treatment with RAI is required or not. Fig. 2 shows the overall system behavior around equilibrium points and the dependence of solutions on their initial conditions and values considered for the parameter Q . The dashed curve that contains the squares representing the equilibrium points can be obtained from the variables N and I coordinates at these points. In $N = Q$ there is a vertical line containing point P_2 , separating the two attraction regions which include the points P_1 and P_3 .

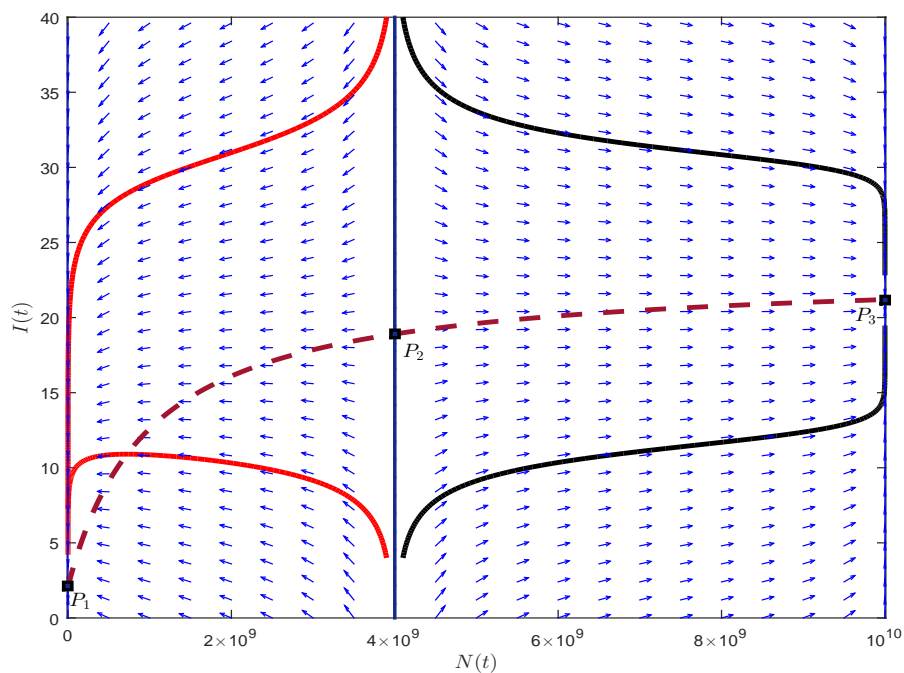


Fig. 2. Phase portrait $N \times I$ using the parameters as indicated in Table 2, except for parameter Q , in which is $Q = 4 \times 10^9$ cells. Solutions obtained from initial conditions $(N_0, I_0) = (3.9 \times 10^9, 2.1)$, $(4.1 \times 10^9, 2.1)$, $(3.9 \times 10^9, 38)$ and $(4.1 \times 10^9, 38)$ are represented by solid curves. Equilibrium points are given by $P_1 = (0, 2.1)$, $P_2 = (4 \times 10^9, 18.9)$ and $P_3 = (10^{10}, 21.2)$, and denoted by solid circles. The curve dashed connecting the circles is given by the equation $I = \frac{\sigma}{m} + \frac{Nc}{(N+\gamma)m}$.

The critic value Q plays a decisive role in terms of which equilibrium point works as an attractor to the solutions. In this context, we relate this parameter to biological factors that prevent malignant cell growth after surgery. If the number of cancer cells becomes greater than the threshold Q , then genetic and epigenetic changes must have occurred to allow them to grow continuously without being impaired by the defense mechanisms, also known as immune evasion.¹³

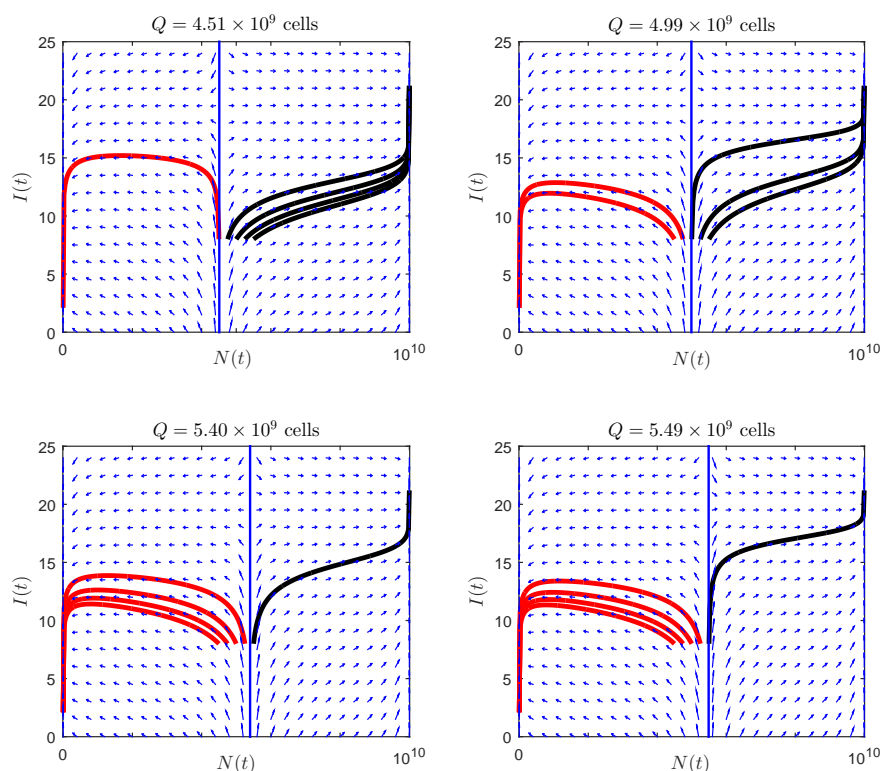


Fig. 3. Influence of parameter Q on solutions representing malignant cell number and IL-6 concentrations over time, $t \geq 0$. Solutions obtained using parameters as indicated in Table 2. The parameter Q ranges from 4.0×10^9 to 5.5×10^9 cells. The initial conditions are $N_0 = 4.5 \times 10^9$, 4.75×10^9 , 5×10^9 , 5.25×10^9 and 5.5×10^9 cells, while $I(0) = 8$ pg/mL. The behavior of the solutions varies depending on the value assumed by Q , and as this parameter becomes greater than some N_0 the corresponding solution leaves the equilibrium point P_3 and becomes attracted to P_1 , which represents the extinction of the tumor.

Furthermore, when $N \leq Q$, the rate of change of N becomes negative, and, consequently, cancer extinction occurs, as predicted by the Allee effect. The point P_2 is a saddle and the values chosen for the initial condition $N(0)$ near this point are decisive for the behavior of the solutions, as shown in Fig. 3. In the equilibrium point P_1 we have $N = 0$ which denotes the extinction of the tumor cells. The equilibrium point P_3 indicates situations where the disease shows resistance over time, overcoming antitumor action of the immune system.

4. Introduction of the treatment A

Considering that there is an initial number of cancer cells N_0 greater than Q and the introduction of treatment A , if the radioiodine reduces the tumor cells population to some threshold, denoted by N^- , then the treatment success is verified, as can be seen in Fig. 4. This results, however, raises an important question: is there an optimal RAI activity according to each patient, or the more RAI used, the higher the chances of success and the faster cancer extinction occurs regardless of the particularities of the patient? According to the literature, many factors are involved in this process, as seen in the next section.

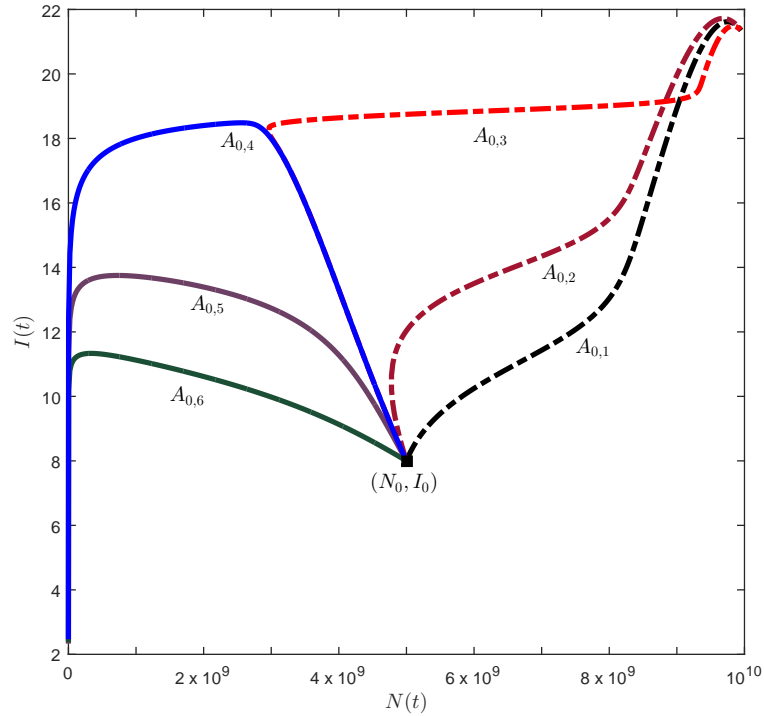


Fig. 4. The number of malignant cells and IL-6 concentrations over time, $t \geq 0$, at different RAI doses. The model (2.1) presents scenarios in which cancer is driven to extinction or not, depending on the RAI doses used. The parameters used are indicated in Table 2, except for the parameter Q , in which is $Q = 2 \times 10^9$ cells. Initial conditions are given by $N_0 = 5 \times 10^9$ cells, $I_0 = 8$ pg/mL and $A_{0,1} = 3.7$, $A_{0,2} = 4.3$, $A_{0,3} = 4.4132407$, $A_{0,4} = 4.4132409$, $A_{0,5} = 4.5$, and $A_{0,6} = 5.55$ GBq.

The bifurcation observed in Fig. 4 applying the parameters $A_{0,3}$ and $A_{0,4}$, could also be verified using less specific values. However, from the values used, we can see the exact instant when the solution alters the equilibrium point attractor. Moreover, we can also note the entry of treatment as a new variable that strangles the regions

previously considered as the basin of attraction, where the vertical line $N = Q$ is the separatrix. With $A_{0,4}$ the solution previously attracted to point P_3 migrates to the other region of asymptotic stability. The bifurcation then occurs when the number of cancer cells is, approximately, $N^- = 3 \times 10^9$ cells, a number different from Q with the scales considered. The value of N^- where the stability change occurs is not fixed, but depends on the parameters, especially those involved in cancer cell proliferation and elimination.

5. Results and discussion

In order to perform some of the numerical simulations, we collect data on interleukin-6 (IL-6) serum concentrations presented in Morais *et al.*,⁴³ consisting of the average IL-6 concentration in individuals under diverse conditions. The data obtained from this study were: concentration of 2.10 pg/mL for IL-6 in healthy subjects; 7.98 pg/mL for IL-6 concentration in papillary thyroid cancer (PTC) patients, measured after the thyroidectomy but before RAI treatment; and 22.99 pg/mL after RAI treatment. The initial condition $I_0 = 7.98$ pg/mL is considered in the simulations because it refers to the target stage of this PTC treatment study.

According to Weinberg,⁴⁴ angiogenesis begins when the tumor has about 10^7 cells and is palpable when it reaches 10^9 cells. Wilkie and Hahnfeldt⁴⁵ used a cell number of 10^{10} for the carrying capacity of cancer in the mathematical models employed to study acute myeloid leukemia. After thyroidectomy, some residual thyroid tissue remains in the patient. Nodules with a reservoir of cancer cells sometimes persist, with the possibility of metastasis occurring.⁸ Barbolosi *et al.*⁴² considered the average number of $N_0 = 1.12 \times 10^9$ cancer cells in patients after the thyroidectomy and with the presence of metastasis. We assume in our study three values as the initial number of cancer cells after the thyroidectomy and before RAI treatment: $N_{0,1} = 6 \times 10^8$, $N_{0,2} = 9 \times 10^8$, and $N_{0,3} = 1.12 \times 10^9$ cells. Thus, three stages of PTC under RAI treatment are assumed.

Regarding the RAI activity assumed in the simulations, the doses of $A_{0,1} = 3.7$, $A_{0,2} = 5.55$, and $A_{0,3} = 7.4$ GBq are considered. These values presented in Morais *et al.*⁴³ as the most used doses during the treatment of 31 PTC patients. In Barbolosi *et al.* study,⁴² RAI activities used in a group of 50 PTC patients ranged from 3.7 to 5.5 GBq.

According to Haugen *et al.*,⁸ specific levels of thyroglobulin that distinguish normal residual tissue from those with cancer are unknown, albeit increasing Tg values over time may indicate cancer persistence. In patients undergoing thyroidectomy and RAI treatment, the criteria for disease-free status include low serum Tg levels, $Tg < 1$ $\mu\text{g/L}$, in the absence of interfering antibodies. Therefore, the value of 10 $\mu\text{g/L}$ is considered for $T_{g,0}$ as the initial condition in numerical simulations.

The parameters of the model (2.1) received values as found in the literature or assume biologically relevant values to allow the system to illustrate some expected results during the treatment (Table 2). The model's numerical simulations

are performed Runge-Kutta fourth-order method, using step-size $h = 0.01$.

Table 2. Parameters and used values in the model's numerical simulations.

Parameter	Value	Unity	Reference/comment
a	1.69×10^{-2}	t^{-1}	Ref. 42
α	7.5×10^{-1}	$\text{cells} \times t^{-1}$	—
k	7.2×10^{-1}	—	—
β	2×10^{-3}	$(\text{pg/mL})^{-1}$	—
Q	5×10^8	cells	based on Ref. 45
K	1×10^{10}	cells	Ref. 45
ρ	4.07×10^{-2}	$(\text{GBq} \times t)^{-1}$	based on Ref. 42
σ	5×10^{-2}	$\text{pg} \times (\text{mL} \times t)^{-1}$	—
c	5×10^{-1}	$\text{pg} \times (\text{mL} \times t)^{-1}$	—
γ	1×10^9	cells	—
b	2×10^{-2}	$\text{pg} \times (\text{mL} \times t \times \text{GBq})^{-1}$	—
m	2.38×10^{-2}	t^{-1}	—
p	3.86×10^{-9}	$\mu\text{g} \times (\text{L} \times t)^{-1}$	Ref. 42
d	3.19×10^{-1}	t^{-1}	Ref. 42

Note: According to the International System of Units (SI)⁴⁶ the following conversions are worth: $1 \text{ pg} = 10^{-15} \text{ kg}$, $1 \mu\text{g} = 10^{-9} \text{ kg}$ and $1 \text{ mL} = 10^{-3} \text{ L}$. The dash used in the reference column means that no values were found in the literature for this parameter.

The analysis began by considering a scenario with the absence of treatment to verify a possible evolution of remnant cancer cells at different risk stages after total thyroidectomy. Due to the Allee effect, two situations are considered. In the first, the values for the initial number of cancer cells, represented by $N_{0,i}$, $i=1,2,3$, indicate situations in which the tumor mass is quite advanced, with possible foci in other parts of the body, especially when considered $N_{0,3}$. In the second situation, we considered a tumor with well-localized malignant cells, in which the thyroidectomy eliminated most cancer cells in the thyroid gland, with only 10^6 cells remaining.

Furthermore, the $Q \gg 10^6$ in the second situation entails an active and effective defense system as well as the absence of factors necessary for the growth of the tumor population, such as angiogenesis.⁴⁷ Moreover, because the tumor mass is not spread, it is likely that this cancer is in its early stages or is a low-risk PTC,⁸ and therefore RAI treatment is not required to eliminate remnants of cancer, as seen in Fig. 5. However, when the initial number for cancer cells is $N_0 > Q$, these populations continue to grow toward the carrying capacity, meaning the development of metastasis.

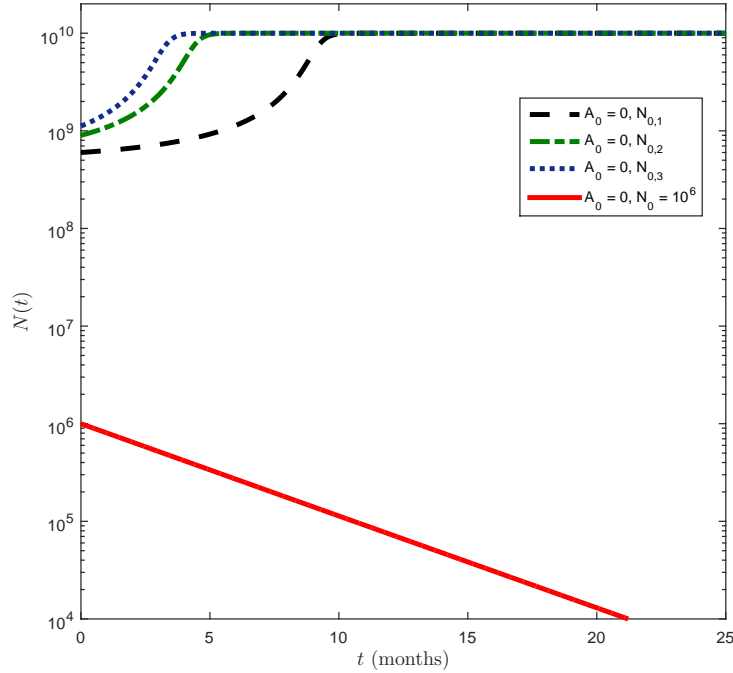


Fig. 5. Evolution of cancer cells in the absence of treatment over time $t \geq 0$. The parameters used are indicated in Table 2. Initial conditions are given by $N_{0,1} = 6 \times 10^8$, $N_{0,2} = 9 \times 10^8$, $N_{0,3} = 1.12 \times 10^9$ and $N_0 = 10^6$ cells, $I_0 = 7.98$ pg/mL and $A_0 = 0$ GBq. The time difference that the populations $N_{0,1}$ and $N_{0,3}$ take to reach carrying capacity is approximately 6 months. In the situation where thyroidectomy has eliminated most cancer cells, the body is in a position to lead the remaining cells, as used in this example, $N(0) = 10^6$, to extinction.

Next, we simulate the use of RAI treatment (see Fig. 6). With the initial dose $A_{0,1}$, only the solution corresponding to the initial condition $N_{0,1}$ approaches the equilibrium point P_1 , while the other solutions evolve towards point P_3 . When we use the dose $A_{0,2}$, the solution corresponding to $N_{0,2}$ also approaches point P_1 and the same happens to solution corresponding to $N_{0,3}$ when $A_{0,3}$ is used (see Fig. 6a). Hence, it is clear that part of a successful treatment is linked to the dose used in the therapy. However, following this dynamic, if we consider the value for N_0 closer to K , the dose required to eliminate the disease could be physically impractical due to aggressive side effects, and, consequently, the treatment should be performed through various spaced RAI applications, as considered in Barbolosi *et al.*⁴² From numerical simulations, we can also verify scenarios with a possible association between the dose used in RAI treatment and its effectiveness in destroying the cancer cells (see Fig. 6b).

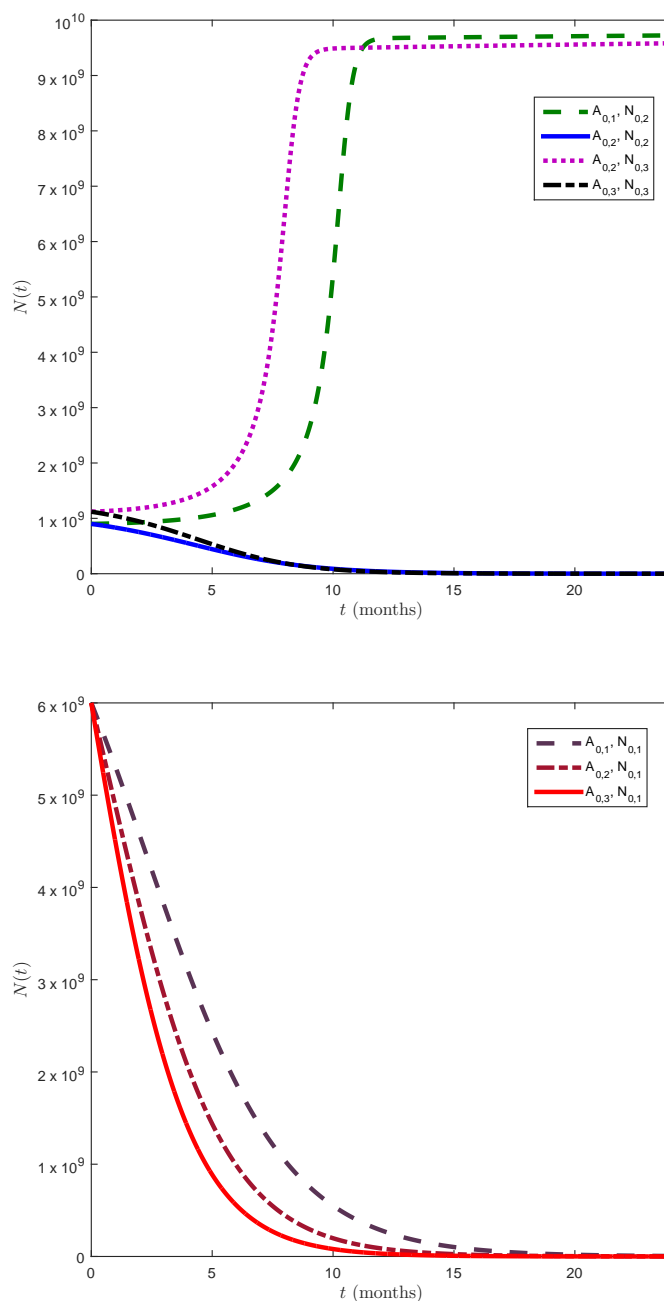


Fig. 6. Growth or decrease in the number of malignant cells according to RAI activity used over time, $t \geq 0$. The parameters used are indicated in Table 2. The values used in the initial conditions are given by $N_{0,1} = 6 \times 10^8$, $N_{0,2} = 9 \times 10^8$, and $N_{0,3} = 1.12 \times 10^9$ cells; $I_0 = 7.98$ pg/mL and $A_{0,1} = 3.7$, $A_{0,2} = 5.55$, and $A_{0,3} = 7.4$ GBq. Despite the same initial conditions for $N(0)$, in some situations successful treatment depends on the RAI activity administered to patients.

The use of solutions only formed from initial condition $N_{0,1}$, shown in Fig. 6b, aims to enable us to monitor RAI destructive force when its doses are more substantial. From the database obtained from the numerical simulations resulting files, we observed that for a reduction of N below 10^4 cells the time spent in months for $A_{0,1}$, $A_{0,2}$ and $A_{0,3}$, were 36, 29, and 24 months, respectively. Thus, the model presents scenarios with a positive association between the amount of RAI used and the tumor extinction time. Notwithstanding, it must be noted that the system (2.1) does not consider aspects such as side effects of each RAI amount used, genetic mutations and the patient's physical condition to support such doses of the drug.

Fig. 7 describes a few scenarios with the association between cancer cell number and thyroglobulin level in the patients. These results agree with the use of Tg as a means of evaluating treatment outcomes.⁸ The value reached by variable T_g when the tumor population approaches the carrying capacity depends on values used for parameters p , d , and K . When considered $A_{0,3}$ and $N_{0,2}$, the level of T_g falls below $2 \mu\text{g/L}$ after 9 months, and continue to decrease over time.

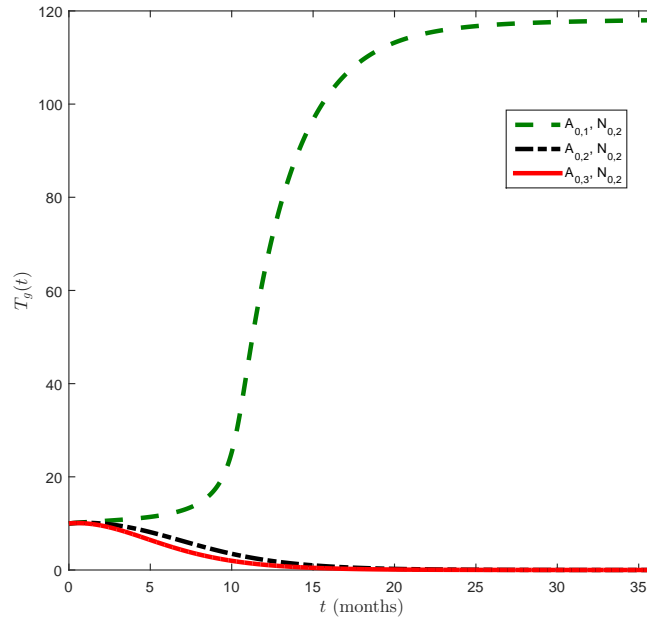


Fig. 7. Thyroglobulin concentration in patients under treatment with RAI over time $t \geq 0$. The parameters used are indicated in Table 2. The values used in the initial conditions are given by $N_{0,2} = 9 \times 10^8$ cells, $I_0 = 7.98 \text{ pg/mL}$, $A_{0,1} = 3.7$, $A_{0,2} = 5.55$, and $A_{0,3} = 7.4 \text{ GBq}$, and $T_{g,0} = 10 \text{ µg/L}$. When RAI dose is sufficient to eliminate cancer cells, the level of Tg decreases as the cancer cell population is eliminated.

Next, Fig. 8 presents different scenarios with IL-6 levels over time. From the

solution curves, we note that, although tumor mass triggers a higher production of this interleukin compared to the presence of iodine in the body, when used $A_{0,3}$ with $N_{0,2}$ the production of IL-6 tends to approach the solution obtained from $N_{0,3}$. This fact suggests two observations: firstly, that altered IL-6 levels may remain for a long time and, secondly, that side effects triggered by increased inflammation due to the higher dose of RAI also contribute to higher levels of interleukin.

Another fact to note is the slower rate in which IL-6 levels increases as cancer progresses to its carrying capacity. Regarding this slow increase, the IL-6 limit given by the value obtained from the model (2.1) is $\frac{\sigma+c}{m} \approx 23.1$ pg/mL, and of all simulations performed, the highest value obtained for the variable I was, approximately, 20 pg/mL, with this value being verified as a plateau level over time.

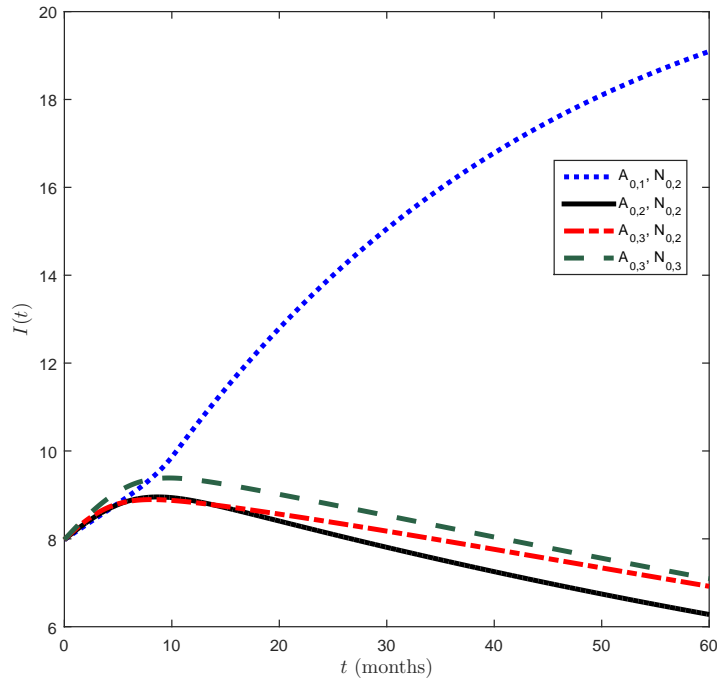


Fig. 8. IL-6 concentration in PTC patients undergoing RAI treatment over time, $t \geq 0$. The parameters used are indicated in Table 2. The values used in the initial conditions are given by: $N_{0,2} = 9 \times 10^8$ and $N_{0,3} = 1.12 \times 10^9$ cells; $I_0 = 7.98$ pg/mL; and $A_{0,1} = 3.7$, $A_{0,2} = 5.55$, and $A_{0,3} = 7.4$ GBq. From the solutions, we observed that the tumor mass is primarily responsible for the increase of this interleukin levels.

The results obtained through numerical simulations from the model (2.1) present

a wide range of scenarios; however, solutions as those obtained in Fig. 6 do not intend to end the discussion of treatment success, perhaps only suggest larger doses to eliminate the disease without taking into account the side effects of the higher dosage. There are other variables not considered in the model as the natural concentration of sodium iodide symporter (NIS), which is crucial for iodine uptake by thyroid cells.⁴⁸ Morais *et al.*⁴³ studied the levels of NIS glycoprotein as a first step in deciding whether to use RAI in the PTC treatment. Furthermore, an assessment of tumor cells' NIS expression by molecular imaging, as suggested by Barbolosi *et al.*,⁴² is presented as a defining factor on the effectiveness of radioiodine therapy. Therefore, considering factors like these in the interpretation of results, it is necessary as a way to complement for some variables not included in the proposed model. The results shown in Figs. 7 and 8 aim to show general behavior over Tg and IL-6 concentrations over time, however, it does not necessarily reflect the results found in patients. In general, the time spent for cancer elimination ranged from 12 to 40 months; therefore, it is understandable that Tg and IL-6 levels also take time slowly decrease. Finally, relapses situations can be understood as presented via solution from $A_{0,1}$ and $N_{0,2}$ in Figs. 6 and 8, where just seven months after starting treatment, the number of cancer cells and IL-6 concentrations shows growth at an advanced rate.

6. Conclusions

Using a mathematical model based on the RAI treatment applied to PTC, we evaluated the parameters of RAI activity, IL-6 levels and the number of remnant cancer cells in determining the effectiveness of treatment. The simulations performed associated the time taken to eliminate tumor cells with the RAI activity used. Notwithstanding, we have also obtained scenarios with the lowest possible activity for A_0 resulting in a successful treatment. These results corroborated new treatment proposals with the least possible side effects. Besides the treatment and, the application of the Allee effect, other factors that promote the reduction of cancer cells were considered in the model from the parameter Q . In this sense, simulations presented underscore the importance of natural processes or defenses in controlling or eliminating malignant cells. Finally, from the parameters k and β , we observed how components of tumor mass may contribute to the development of tumors, increasing the aggressiveness and proliferation rate of cancer cells via IL-6 levels. Thus, the proposed model presented new elements in the discussion about the optimal amount of RAI used in PTC patients; the importance of natural defense mechanisms for the elimination of post-surgery remnant cells, with and without the use of RAI; and presented scenarios in that IL-6 is considered a favorable agent for tumor progression.

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