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Comment on “Chaotic-Integrable Transition in the Sachdev-Ye-Kitaev Model”

In their Letter [1], the authors studied a variant of the Sachdev-Ye-Kitaev model with quadratic and quartic interactions (also known as a mass-deformed SYK). They claimed that the quantum Lyapunov exponent λ_L would vanish below some critical temperature. In this Comment, we show that this is not the case. We calculate λ_L exactly in the perturbative regime where the temperature T and the quartic coupling J are much smaller than the quadratic coupling κ . At leading order in T/κ and J/κ , we find

$$\frac{\lambda_L}{\kappa} = \frac{3T^2J^2}{\kappa^4}. \quad (1)$$

Therefore, this model has no $\lambda_L = 0$ phase at small but nonzero T, J .

To derive Eq. (1), we improve upon the standard approach followed by the Letter’s authors and analyze the leading eigenvalue problem of the ladder kernel, Eq. (9) of the Letter:

$$K = K_2 K_1, \quad (K_1 f)(t) = [\kappa^2 + 3J^2 G_{lr}^2(t)]f(t), \quad (2a)$$

$$(K_2 f)(\omega) = \left| G_R \left(\omega + i \frac{\lambda_L}{2} \right) \right|^2 f(\omega), \quad (2b)$$

where G_R and G_{lr} are retarded and Wightman Green functions of a fermion, respectively. Recall also that λ_L is found by imposing that the largest eigenvalue of K is 1. We shall work perturbatively in J/κ and T/κ up to the leading order. Such a perturbative expansion is controlled since the quartic term is irrelevant in the low- T limit where the model is a Fermi liquid. Our analysis thus differs from the Letter, which considered a perturbative expansion with respect to the relevant term.

To start, we compute G_{lr} and G_R to an adequate order. For G_{lr} , the conformal solution of the $J = 0$ limit [2] suffices:

$$G_{lr}(t)^2 = \frac{T^2}{\kappa^2} \text{sech}(t\pi T)^2. \quad (3)$$

For $|G_R|^2$, the $J = 0$ conformal solution $G_R(\omega) = -i/\kappa$ [2] is not enough, and we shall incorporate the leading correction. That comes from quasiparticle decay encoded in the imaginary part of the self-energy $\Sigma_4(\tau) := J^2 G(\tau)^3$ in

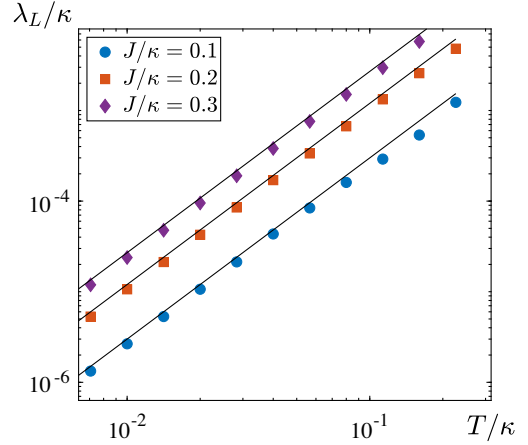


FIG. 1. Temperature dependence of the Lyapunov exponent λ_L for various interaction strengths (markers) compared to the prediction, Eq. (1) (solid lines).

Euclidean time. Continuing to real time, we find that the retarded propagator Σ_4^R satisfies

$$-\text{Im}\Sigma_4^R(\omega) = \frac{J^2 T^2}{2\kappa^3} + \frac{J^2 \omega^2}{2\pi^2 \kappa^3} \quad (4)$$

at leading order in J/κ . Together with the Schwinger-Dyson equation $G_R(\omega)^{-1} = \omega - \kappa^2 G_R(\omega) - \Sigma_{4,R}(\omega)$, Eq. (4) implies that

$$\kappa^2 \left| G_R \left(\omega + i \frac{\lambda_L}{2} \right) \right|^2 = 1 - \frac{\lambda_L}{2\kappa} - \frac{J^2 T^2}{2\kappa^4} - \frac{J^2}{2\pi^2 \kappa^4} \omega^2, \quad (5)$$

where we also omitted higher orders of λ_L as we anticipated it to be small.

With Eqs. (3) and (5), the kernel, Eq. (2), reduces to

$$K = 1 - \frac{\lambda_L}{2\kappa} - \frac{J^2 T^2}{2\kappa^4} - \frac{J^2 T^2}{\kappa^4} \left[-\frac{1}{2\pi^2 T^2} \partial_t^2 - 3 \text{sech}(t\pi T)^2 \right]. \quad (6)$$

Upon rescaling $s = t\pi T$ in the parenthesis, we recognize the 1D Schrödinger Hamiltonian with a Pöschl-Teller potential: $-\frac{1}{2}\partial_s^2 - 3\text{sech}(s)^2$. Its ground state energy is $E_0 = -2$ [3]. Thus, the largest eigenvalue of K equals $1 - \lambda_L/(2\kappa) + 3J^2 T^2/(2\kappa^4)$. Imposing it to be 1 leads to Eq. (1).

We verified our prediction numerically in the large- N limit by directly computing the leading eigenvalue of the kernel, Eq. (2), where the Green functions are (nonperturbative) numerical solutions of the real-time Schwinger-Dyson equations [2]. We find a nice agreement in the perturbative regime (see Fig. 1). Observe, however, that λ_L/κ can become very small and could be mistaken for 0 due to numerical artifacts. Finally, we note that Refs. [4–6] observed $\lambda_L \sim T^2 > 0$ in other large- N models of disordered Fermi liquid.


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Jaewon Kim¹ and Xiangyu Cao²

¹Department of Physics, University of California
Berkeley, California 94720, USA

²Laboratoire de Physique de l'Ecole Normale Supérieure
ENS, Université PSL, CNRS
Sorbonne Université
Université de Paris, 75005 Paris, France

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