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**Comment on “Chaotic-Integrable Transition in the Sachdev-Ye-Kitaev Model”**

In their Letter [1], the authors studied a variant of the Sachdev-Ye-Kitaev model with quadratic and quartic interactions (also known as a mass-deformed SYK). They claimed that the quantum Lyapunov exponent  $\lambda_L$  would vanish below some critical temperature. In this Comment, we show that this is not the case. We calculate  $\lambda_L$  exactly in the perturbative regime where the temperature  $T$  and the quartic coupling  $J$  are much smaller than the quadratic coupling  $\kappa$ . At leading order in  $T/\kappa$  and  $J/\kappa$ , we find

$$\frac{\lambda_L}{\kappa} = \frac{3T^2J^2}{\kappa^4}. \quad (1)$$

Therefore, this model has no  $\lambda_L = 0$  phase at small but nonzero  $T, J$ .

To derive Eq. (1), we improve upon the standard approach followed by the Letter’s authors and analyze the leading eigenvalue problem of the ladder kernel, Eq. (9) of the Letter:

$$K = K_2K_1, \quad (K_1f)(t) = [\kappa^2 + 3J^2G_{lr}^2(t)]f(t), \quad (2a)$$

$$(K_2f)(\omega) = \left| G_R \left( \omega + i \frac{\lambda_L}{2} \right) \right|^2 f(\omega), \quad (2b)$$

where  $G_R$  and  $G_{lr}$  are retarded and Wightman Green functions of a fermion, respectively. Recall also that  $\lambda_L$  is found by imposing that the largest eigenvalue of  $K$  is 1. We shall work perturbatively in  $J/\kappa$  and  $T/\kappa$  up to the leading order. Such a perturbative expansion is controlled since the quartic term is irrelevant in the low- $T$  limit where the model is a Fermi liquid. Our analysis thus differs from the Letter, which considered a perturbative expansion with respect to the relevant term.

To start, we compute  $G_{lr}$  and  $G_R$  to an adequate order. For  $G_{lr}$ , the conformal solution of the  $J = 0$  limit [2] suffices:

$$G_{lr}(t)^2 = \frac{T^2}{\kappa^2} \operatorname{sech}(t\pi T)^2. \quad (3)$$

For  $|G_R|^2$ , the  $J = 0$  conformal solution  $G_R(\omega) = -i/\kappa$  [2] is not enough, and we shall incorporate the leading correction. That comes from quasiparticle decay encoded in the imaginary part of the self-energy  $\Sigma_4(\tau) := J^2G(\tau)^3$  in

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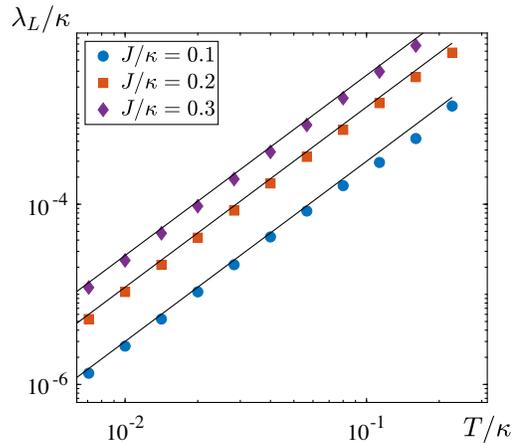


FIG. 1. Temperature dependence of the Lyapunov exponent  $\lambda_L$  for various interaction strengths (markers) compared to the prediction, Eq. (1) (solid lines).

Euclidean time. Continuing to real time, we find that the retarded propagator  $\Sigma_4^R$  satisfies

$$-\operatorname{Im}\Sigma_4^R(\omega) = \frac{J^2T^2}{2\kappa^3} + \frac{J^2\omega^2}{2\pi^2\kappa^3} \quad (4)$$

at leading order in  $J/\kappa$ . Together with the Schwinger-Dyson equation  $G_R(\omega)^{-1} = \omega - \kappa^2G_R(\omega) - \Sigma_{4,R}(\omega)$ , Eq. (4) implies that

$$\kappa^2 \left| G_R \left( \omega + i \frac{\lambda_L}{2} \right) \right|^2 = 1 - \frac{\lambda_L}{2\kappa} - \frac{J^2T^2}{2\kappa^4} - \frac{J^2}{2\pi^2\kappa^4} \omega^2, \quad (5)$$

where we also omitted higher orders of  $\lambda_L$  as we anticipated it to be small.

With Eqs. (3) and (5), the kernel, Eq. (2), reduces to

$$K = 1 - \frac{\lambda_L}{2\kappa} - \frac{J^2T^2}{2\kappa^4} - \frac{J^2T^2}{\kappa^4} \left[ -\frac{1}{2\pi^2T^2} \partial_t^2 - 3\operatorname{sech}(t\pi T)^2 \right]. \quad (6)$$

Upon rescaling  $s = t\pi T$  in the parenthesis, we recognize the 1D Schrödinger Hamiltonian with a Pöschl-Teller potential:  $-\frac{1}{2}\partial_s^2 - 3\operatorname{sech}(s)^2$ . Its ground state energy is  $E_0 = -2$  [3]. Thus, the largest eigenvalue of  $K$  equals  $1 - \lambda_L/(2\kappa) + 3J^2T^2/(2\kappa^4)$ . Imposing it to be 1 leads to Eq. (1).

We verified our prediction numerically in the large- $N$  limit by directly computing the leading eigenvalue of the kernel, Eq. (2), where the Green functions are (nonperturbative) numerical solutions of the real-time Schwinger-Dyson equations [2]. We find a nice agreement in the perturbative regime (see Fig. 1). Observe, however, that  $\lambda_L/\kappa$  can become very small and could be mistaken for 0 due to numerical artifacts. Finally, we note that Refs. [4–6] observed  $\lambda_L \sim T^2 > 0$  in other large- $N$  models of disordered Fermi liquid.

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