

## Appendix

### A Symmetry-Breaking and Valid Inequalities: Proofs

In what follows, we present the proofs for each proposition in this paper.

#### A.1 Symmetry-Breaking Constraints

*Proof.* Given any NF  $n \in N$ , having the left-hand side of inequality (20) equals to zero (i.e.,  $x_{nu}^{sf} = 0$  for any  $s \in S, f \in F, u \in V_p$ ) implies that  $n$  was not assigned to any  $f \in F$  and hence it is available to host any NFS from any slice request. Since we check this feature in a paired, ordered way (i.e.,  $(n-1, n)$ ) and there is no utilization cost related to the uncapacitated functions from  $N$ , the right-hand side of the same inequality can hold any value equal or greater than zero; the related upper-bound will be imposed by constraints (4)-(12). Hence, (20) cut off part of the feasible region, while guaranteeing that at least one optimal solution from the original problem remains feasible.

#### A.2 Lower-Bound based Inequality

*Proof.* The left-hand side of inequality (21) is the sum of constraints (2) when isolation (4)-(5) constraints and capacity inequalities (12)-(13) are relaxed. Hence, this is the minimum value for the right-hand side of inequality (21), which is the sum of constraints-(3) and the optimized parameter in the objective function (14) disregarding the required capacities and their related costs.

#### A.3 Clique-based Isolation Inequalities: Virtual Isolation

*Proof.* Let  $\mathcal{C}$  be a clique in  $\mathcal{C}(\mathcal{H}_n)$ . If two tuples  $(s, f, u)$  and  $(t, g, v)$  from  $\mathcal{C}$  are mapped to a same network function  $n \in N$ , the related isolation constraints (4) will be violated. In other words, each pair of tuples from clique  $\mathcal{C}$  represents an infeasible packing of NFSs from different slices requests in an NF  $n \in N$ . Consequently, a clique in the graph  $\mathcal{H}_n$  corresponds to a set of NFS that cannot be simultaneously packed in any network function  $n \in N$ .

#### A.4 Clique-based Capacity Inequalities: Physical Links

*Proof.* Let  $\mathcal{C}$  be a clique in  $\mathcal{C}(\mathcal{H}_a)$ . If the flow of two pairs  $(k, f, g)$  and  $(k', f', g')$  from  $\mathcal{C}$  are carried through a same physical link  $a \in A_p$ , the related capacity constraint (13) will be violated. In other words, each pair of nodes from clique  $\mathcal{C}$  represents an infeasible routing from different traffic demands in a physical link  $a \in A_p$ . Consequently, a clique in the graph  $\mathcal{H}_a$  corresponds to a set of traffic demands that cannot be carried simultaneously through physical link  $a$ .

## B Node-Related Valid Inequalities for the NSDP

We now present new clique-based valid inequalities for the NSDP. They are related to both isolation and capacity constraints on physical nodes.

### B.1 Clique-based Isolation Inequalities: Physical Isolation

Let  $\mathcal{H}_u = (V_u, E_u)$  be the associated undirected graph obtained from an instance of NSDP as follows. A node  $v$  in  $V_u$  is associated with every tuple  $(s, f, n) : s \in S, f \in F, n \in N$  that  $\bar{x}_{nu}^{sf} > 0$  holds in the current solution. In addition, there exists an edge in  $E$  between any two nodes  $(s, f, n)$  and  $(t, g, n)$  from  $V_u$  if  $(q^{st} * q^{ts} = 0)$  holds. In other words, an edge in  $\mathcal{H}_u$  exists if two functions are packed together in any node  $u \in V_p$  while violating the isolation constraints imposed by the related slices.

**Proposition 5.** *Let  $\mathcal{C}(\mathcal{H}_u)$  be the set of all cliques on  $\mathcal{H}_u$ . Then, the following inequalities*

$$\sum_{(s,f,n) \in \mathcal{C}} x_{nu}^{sf} \leq 1 \quad \forall u \in V_p^{na}, \forall \mathcal{C} \in \mathcal{C}(\mathcal{H}_u) \quad (24)$$

are valid for the NSDP.

*Proof.* Let  $\mathcal{C}$  be a clique in  $\mathcal{C}(\mathcal{H}_u)$ . If two tuples  $(s, f, n)$  and  $(t, g, n)$  from  $\mathcal{C}$  are assigned to a same physical node  $u \in V_p$ , the related isolation constraints (5) will be violated. In other words, each pair of tuples from clique  $\mathcal{C}$  represents an infeasible packing of NFSs from different slices requests in a node  $u \in V_p$ . Consequently, a clique in the graph  $\mathcal{H}_u$  corresponds to a set of NFS that cannot be simultaneously installed in any physical node  $u \in V_p$ .

### B.2 Clique-based Capacity Inequalities: Physical Node

This class of valid inequalities depends directly on the values of  $x$  and  $w$  variables and therefore can be generated only regarding a solution for the presented formulation (1)-(14). Let  $\bar{w}_{nu}^{sf}$  and  $\bar{x}_{nu}^{sf}$  be the values of the related variables in the current solution. For each  $u \in V_p$ , let  $\mathcal{H}_u = (V_u, E_u)$  be the associated undirected graph obtained from an instance of NSDP as follows. A node  $v$  in  $V_u$  is associated with every tuple  $(s, f, n) : s \in S, f \in F, n \in N$  that  $\bar{x}_{nu}^{sf} > 0$  holds in the current solution. There exists an edge in  $E_u$  between any two nodes  $u = (s, f, n)$  and  $v = (t, g, m)$  from  $V_u$  if  $\frac{\bar{w}_{nu}^{sf}}{\bar{x}_{nu}^{sf}} c_f^c + \frac{\bar{w}_{mu}^{tg}}{\bar{x}_{mu}^{tg}} c_g^c > c_u^c$  holds for any physical resource  $c \in C$ .

**Proposition 6.** *Let  $\mathcal{C}(\mathcal{H}_u)$  be the set of all cliques on  $\mathcal{H}_u$ . Then, the following inequalities*

$$\sum_{(s,f,n) \in \mathcal{C}} x_{nu}^{sf} \leq 1 \quad , \forall u \in V_p, \forall \mathcal{C} \in \mathcal{C}(\mathcal{H}_u) \quad (25)$$

are valid for the NSDP.

### On the Network Slice Design problem

*Proof.* Let  $\mathcal{C}$  be a clique in  $\mathcal{C}(\mathcal{H}_u)$ . If the flow of two pairs  $(s, f, n)$  and  $(t, g, m)$  from  $\mathcal{C}$  are carried through a same physical node  $u \in V_p$ , the related capacity constraint (12) will be violated. In other words, each pair of nodes from clique  $\mathcal{C}$  represents an infeasible packing from different NFSs in a physical node  $u \in V_p$ . Consequently, a clique in the graph  $\mathcal{H}_u$  corresponds to a set of functions that cannot be packed simultaneously in the physical node  $u$ .