



**HAL**  
open science

## **A Computational and Spectroscopic Study of the Electronic Structure of V<sub>2</sub>O<sub>5</sub>-Based Cathode Materials**

Evgenii Roginskii, Mikhail Smirnov, Konstantin Smirnov, Rita Baddour-Hadjean, Jean Pierre Pereira-Ramos, Alexander Smirnov, Valery Yu. Davydov

► **To cite this version:**

Evgenii Roginskii, Mikhail Smirnov, Konstantin Smirnov, Rita Baddour-Hadjean, Jean Pierre Pereira-Ramos, et al.. A Computational and Spectroscopic Study of the Electronic Structure of V<sub>2</sub>O<sub>5</sub>-Based Cathode Materials. *Journal of Physical Chemistry C*, 2021, <10.1021/acs.jpcc.0c11285>. <hal-03163013>

**HAL Id: hal-03163013**

**<https://hal.science/hal-03163013v1>**

Submitted on 9 Mar 2021

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

# A Computational and Spectroscopic Study of Electronic Structure of V<sub>2</sub>O<sub>5</sub>-based Cathode Materials

Evgenii M. Roginskii,<sup>\*,†</sup> Mikhail B. Smirnov,<sup>‡</sup> Konstantin S. Smirnov,<sup>\*,¶</sup>  
Rita Baddour-Hadjean,<sup>§</sup> Jean-Pierre Pereira-Ramos,<sup>§</sup> Alexander N. Smirnov,<sup>†</sup> and  
Valery Yu. Davydov<sup>†</sup>

1

<sup>†</sup>*Ioffe Institute, Polytekhnicheskaya 26, 194021 St. Petersburg, Russia*

<sup>‡</sup>*Faculty of Physics, St. Petersburg State University, Petrodvoretz, 194508 St. Petersburg,  
Russia*

<sup>¶</sup>*Univ. Lille, CNRS, UMR 8516 – LASIRE – Laboratoire Avancé de Spectroscopie pour les  
Interactions la Réactivité et l'Environnement, F-59000 Lille, France*

<sup>§</sup>*Institut de Chimie et des Matériaux Paris-Est (ICMPE), UMR 7182 CNRS et Université  
Paris-Est Créteil, 2 rue Henri Dunant, 94320 Thiais, France*

E-mail: [e.roginskii@mail.ioffe.ru](mailto:e.roginskii@mail.ioffe.ru); [konstantin.smirnov@univ-lille.fr](mailto:konstantin.smirnov@univ-lille.fr)

## Abstract

The electronic structure of  $\alpha$ -V<sub>2</sub>O<sub>5</sub>,  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> and  $\gamma$ -MeV<sub>2</sub>O<sub>5</sub> (Me = Li, Na) bronzes is studied by quantum-chemical calculations completed by spectroscopic experiments. The calculations are performed using  $G_0W_0$  method with DFT+ $U$  self-consistent wavefunction as an initial approximation. The electronic band gap  $E_g = 2.89$  eV calculated for  $\alpha$ -V<sub>2</sub>O<sub>5</sub> is found to be in a fair agreement with available experimental data. The strategy was then applied to studying the electronic structure of the  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> phase and  $\gamma$ -MeV<sub>2</sub>O<sub>5</sub> bronzes for which no experimental band gap data exist in the literature. Computed  $E_g$  values are equal to 3.17 eV, 1.21 eV and 1.18 eV for  $\gamma'$ -V<sub>2</sub>O<sub>5</sub>,  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> and  $\gamma$ -NaV<sub>2</sub>O<sub>5</sub>, respectively. The nature of alkali metal atom is obtained to have little influence on the structure and electronic states of the bronzes. Raman spectra recorded with different wavelengths of exciting radiation have allowed the determination of energy threshold corresponding to the transition from off-resonance to resonance Raman scattering process. In this way, a band gap value in the range 2.54 – 2.71 eV for  $\alpha$ -V<sub>2</sub>O<sub>5</sub> and  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> is obtained in a good agreement with the experimental values for the  $\alpha$ -phase. Raman spectra of  $\gamma$ -MeV<sub>2</sub>O<sub>5</sub> suggest the band gap smaller than 1.58 eV in these materials, whereas the photoluminescence measurements yield  $E_g \approx 0.95$  eV for the  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> bronze. Remarkably, the result of the  $G_0W_0$  calculations lies in between the experimental estimates. Strong similarity of structures and electronic states of  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> and  $\gamma$ -NaV<sub>2</sub>O<sub>5</sub> accounts for their the same operating voltage when used as cathodes in Li(Na)-ion batteries.

# 1 Introduction

2 The rich chemistry and polymorphism of vanadium pentoxide ( $V_2O_5$ ) have made this ma-  
3 terial very interesting for fundamental research and for many applications in heterogeneous  
4 catalysis,<sup>1</sup> electrochromism<sup>2</sup> and electrochemical energy storage.<sup>3</sup> Vanadium pentoxide is a  
5 typical intercalation compound because of its layered structure. This structural peculiarity  
6 is illustrated in Figure 1 for the  $\alpha$ - $V_2O_5$  and  $\gamma'$ - $V_2O_5$  phases which are the main objects of  
7 the present work. These structures exhibit characteristics typical of all  $V_2O_5$  polymorphs,  
8 *i.e.*  $V_2O_5$  chains running in the  $b$  crystallographic direction and connected in layers *via*  
9 V-O2 "ladder" steps; the layers are then stacked in the third dimension and held together by  
10 weak interlayer interactions. Puckering layers in the  $\gamma'$ - $V_2O_5$  lattice compared to flat layers  
11 in the  $\alpha$ - $V_2O_5$  structure results in the  $c$  lattice parameter of the  $\gamma'$ - $V_2O_5$  unit cell being  
12 approximately twice of the parameter of  $\alpha$ - $V_2O_5$  one.

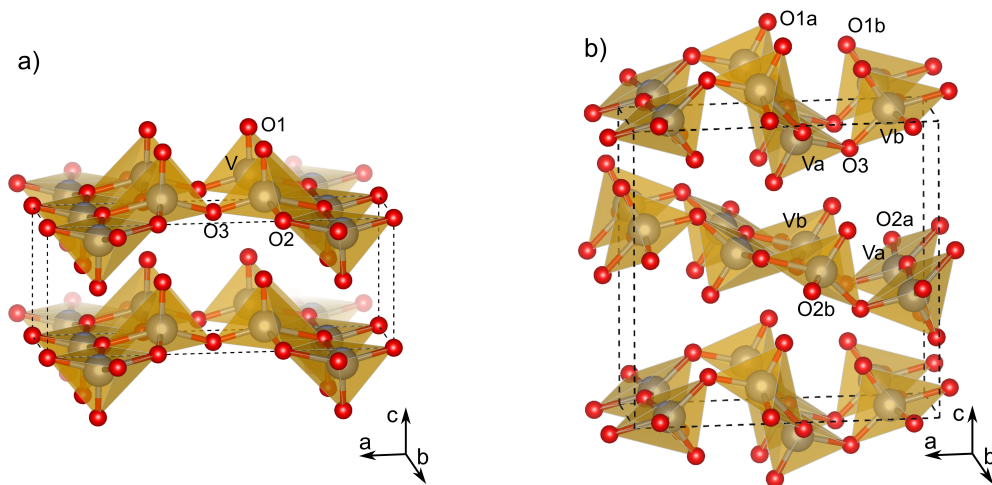


Figure 1: Structures of  $\alpha$ - $V_2O_5$  (a) and  $\gamma'$ - $V_2O_5$  (b). The crystallographic unit cells are shown by dashed lines; atom labels correspond to those used throughout the paper.

13 Very early, the  $\alpha$ -form (Figure 1a) has been identified as a promising cathode material  
14 for secondary lithium batteries.<sup>3-5</sup> Functioning of the batteries relies on the electron transfer  
15 reaction accompanied by the electrochemical intercalation/de-intercalation of alkali metal  
16 atoms into the oxide structure. The insertion of alkali metal (Me) in the interlayer space

1 of  $V_2O_5$  lattice leads to vanadium oxide bronzes with general formula  $Me_xV_2O_5$  showing  
2 remarkable versatility as a function of the nature of intercalated alkali atom.<sup>6</sup> The presence  
3 of two  $V^{5+}$  ions per  $V_2O_5$  unit allows to reach a high storage capacity that can amount to  
4 300 mAh  $g^{-1}$  corresponding to the reversible accommodation of two lithium ions/mole in the  
5 2.3–3.5 V potential window.<sup>3,5</sup> The interest of the  $\gamma'$ - $V_2O_5$  polymorph (Figure 1b) for the  
6 electrochemical applications has been recently demonstrated in the case of lithium<sup>7,8</sup> and  
7 sodium insertion.<sup>9,10</sup> A higher open circuit voltage combined with lower diffusion barriers  
8 afford superior electrochemical properties to this material compared to the  $\alpha$ - $V_2O_5$  form.<sup>8,10</sup>  
9 A readily accessible  $V^{5+}/V^{4+}$  redox couple and the stability of different local coordination  
10 environments of the vanadium atoms (octahedral, distorted octahedral, square pyramid, . . . ),  
11 result in the flexibility of the layered  $V_2O_5$  structures and in their ability to accommodate  
12 point defects through a crystallographic shear and/or charge localization/delocalization.<sup>11</sup>  
13 The lattice of  $\gamma$ - $MeV_2O_5$  bronzes ( $Me = Li, Na$ ) is very close to that of  $\gamma'$ - $V_2O_5$  shown in  
14 Figure 1b and in what follows, we therefore use the same notation for the atoms V and O  
15 atoms in both the pure and intercalated structures.

16 The electronic structure and the energy difference between the top of the valence band  
17 and the bottom of the conduction band, the band gap  $E_g$ , are fundamental characteristics  
18 of material that determine its electric conductivity and optical properties. The conductiv-  
19 ity is directly related to the concentration of charge carriers that, in its turn, depends on  
20 the band gap, and impacts high rate capabilities of electrode materials.<sup>12</sup> The band gap is  
21 also an important factor in electrochemical applications, in particular for designing an elec-  
22 trode–electrolyte pair.<sup>13,14</sup> Among different structural modifications of vanadium pentoxide,  
23 a value of the band gap has been measured only for the  $\alpha$ - $V_2O_5$  polymorph,<sup>15–17</sup> while no such  
24 data exist for  $\gamma$ - $Me_xV_2O_5$  bronzes which, as it was mentioned above, have more promising  
25 properties for the usage as cathode material for Me-ion batteries.<sup>9,10,18</sup>

26 Nowadays, the lack of experimental data can in part be compensated by the use of  
27 atomistic simulations that have become a valuable and reliable tool in materials research.

1 Most of electronic-structure calculations of materials employs methods based on the density  
2 functional theory (DFT). However, a correct description of electronic structure with DFT  
3 encounters problems due to the energy derivative discontinuity and the local character of  
4 exchange-correlation (XC) functionals of commonly used local density approximation (LDA)  
5 and generalized gradient approximation (GGA) to DFT. The derivative discontinuity makes  
6 the band gap to be known up to a constant,<sup>19</sup> while the local character of LDA and GGA XC  
7 functionals causes artifacts in the calculations for materials with occupied  $d$  and  $f$  states,<sup>20</sup>  
8 such as transition metal oxides. To overcome the shortcomings, computational investigations  
9 of the electronic structure of solids often rely on the DFT+ $U$  approach, that introduces an  
10 on-site repulsion for localized electrons.<sup>21</sup> Alternatively, the local character of the LDA and  
11 GGA XC functionals is remedied in hybrid functionals, such as PBE0<sup>22</sup> and HSE06<sup>23,24</sup>,  
12 that bring a non-local exchange energy of Hartee-Fock (HF) theory into DFT. While these  
13 approaches generally improve results for the electronic structure of materials, they introduce  
14 empirical parameters (Hubbard parameter  $U$  in DFT+ $U$  and the portion of HF exchange in  
15 hybrid functionals) that may not work properly outside the range of systems (properties) to  
16 which they are fitted.

17 In order to sidestep the deficiencies inherent to DFT, one can employ computations  
18 that use a  $GW$  approximation based on the many-body perturbation theory. The  $GW$   
19 calculations permit a more accurate description of the electronic states in a systems of  
20 interacting electrons and nuclei,<sup>25,26</sup> but at the expense of being much more computationally  
21 expensive than their DFT counterparts, especially if the self-consistent solution is sought.  
22 Although the  $GW$  method provides results in a good agreement with experimental data, it  
23 was found that the computations overestimate the band gap if the Green's function  $G$  and  
24 the screened Coulomb interaction matrix  $W$  are updated until a self-consistency. On the  
25 other hand, a "single-shot"  $G_0W_0$  calculations including of one iteration step underestimate  
26 experimental values. A compromise was found in the case when only the Green's function is  
27 updated until the self-consistency, while the screened Coulomb potential is kept unchanged

1 ( $GW_0$  approximation).<sup>25</sup> Nevertheless, even  $GW_0$  remains computationally demanding for  
2 systems with dozens of atoms in the unit cell. Bruneval and Marques<sup>27</sup> reported on results  
3 of  $G_0W_0$  calculations starting from electronic states obtained by different methods. The  
4 authors found that the accuracy of the calculations could be improved by using a solution  
5 obtained with a hybrid XC functional as an initial approximation. Alternatively, Scheffler  
6 and co-workers have shown that the use of LDA+ $U$  solution in  $G_0W_0$  method provides  
7 an accurate description of electronic structure in systems with occupied  $d$ - or  $f$ -electron  
8 shells.<sup>28,29</sup>

9 The structure, lattice dynamics and electronic states of the  $\alpha$ -phase of vanadium pentox-  
10 ide have been studied with DFT methods in a number of works.<sup>30-37</sup> In order the calculations  
11 to correctly reproduce the layered structure of the material, the use of semi-empirical disper-  
12 sion correction was found to be indispensable.<sup>35,36</sup> Band gap values computed in ref 30-32,34  
13 were obtained to be smaller than the experimental data<sup>15-17</sup> and the agreement with the ex-  
14 periment could be improved by the inclusion of on-site Hubbard term penalizing the electron  
15 delocalization into the Hamiltonian.<sup>34</sup> Like the experimental research, computational DFT  
16 studies of other  $V_2O_5$  structures paid little attention to their electronic states.<sup>32,37-39</sup> Inves-  
17 tigation of  $V_2O_5$  materials with the  $GW$  methods are scarce. The electronic structure of  
18  $\alpha$ - $V_2O_5$  was investigated in self-consistent  $GW$  calculations by Bhandari and Lambrecht.<sup>40</sup>  
19 In line with aforementioned results, the authors obtained the band gap significantly larger  
20 than the experimental values.<sup>15-17</sup> The discrepancy was explained as a result of enhanced  
21 screening of the Coulomb potential due to an increase of the static dielectric constant because  
22 of large LO-TO splitting in the system. To the best of our knowledge, no other attempts of  
23 exploring  $V_2O_5$ -based materials with  $GW$  methods were undertaken.

24 The present work reports on a combined computational and experimental study of the  
25 electronic structure, particularly the band gap, of pure  $\alpha$ - $V_2O_5$ ,  $\gamma'$ - $V_2O_5$  and of alkali-atom  
26 intercalated  $\gamma$ -Me $V_2O_5$  (Me = Li, Na) materials. The computations have adopted a strategy  
27 in which the electronic structure of  $\alpha$ - $V_2O_5$  was investigated by  $G_0W_0$  method with the self-

1 consistent LDA+ $U$  solution as an initial approximation to the many-particle wavefunction.<sup>29</sup>  
2 To avoid the arbitrariness in the choice of Hubbard parameter, value of  $U$  was obtained using  
3 density functional perturbation theory.<sup>41</sup> A good agreement of the calculated band gap value  
4 with the available experimental data for this structure enabled us to extend the strategy to  
5 the computation of the electronic structure of  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> and of the MeV<sub>2</sub>O<sub>5</sub> bronzes. In  
6 the absence of any experimental information on the band gap in these materials, Raman  
7 and photoluminescence experiments performed with different wavelengths of the exciting  
8 radiation provided experimental estimates for  $E_g$  and validated results of the electronic-  
9 structure calculations.

## 10 Methods

### 11 Experimental procedure and materials characterisation

12  $\alpha$ -V<sub>2</sub>O<sub>5</sub> was prepared through the polyol method described in ref 42. Chemical lithiation of  
13  $\alpha$ -V<sub>2</sub>O<sub>5</sub> was performed under mild conditions, using lithium iodide in excess (molar ratio 2:1)  
14 to obtain the  $\delta$ -LiV<sub>2</sub>O<sub>5</sub> precursor that transforms to  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> when heat-treated at 300°C.<sup>43</sup>  
15  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> is obtained by chemical oxidation of  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> using NO<sub>2</sub>BF<sub>4</sub> in molar excess (2:1),  
16 as described in ref 18.  $\gamma$ -NaV<sub>2</sub>O<sub>5</sub> is come by chemical sodiation of  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> using anhydrous  
17 sodium iodide (molar excess 2:1).<sup>11</sup> The synthesis procedure results in a change of samples  
18 color from orange for the pure V<sub>2</sub>O<sub>5</sub> phases to dark green (nearly black) for the alkali-atom  
19 intercalated compounds. The structural parameters of  $\alpha$ -V<sub>2</sub>O<sub>5</sub>,  $\gamma'$ -V<sub>2</sub>O<sub>5</sub>,  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub>,  $\gamma$ -NaV<sub>2</sub>O<sub>5</sub>  
20 are summarized in Table S1 of Supporting Information (SI). The experimental Raman spectra  
21 of the materials are plotted in Figure S1 of SI, while the assignment of spectral features in  
22 the spectra is discussed in detail elsewhere.<sup>18,39,44–46</sup>

23 The Raman measurements of  $\alpha$ -V<sub>2</sub>O<sub>5</sub> and  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> were performed at room temperature  
24 using LaBRAM HR 800 (Jobin-Yvon-Horiba) micro-Raman spectrometer equipped with a  
25 back illuminated charge coupled device detector (Spex CCD) cooled by Peltier effect to

1 200 K. In the measurements, the exciting light at four wavelengths was used, namely: He-Ne  
2 laser ( $\lambda = 632.8$  nm), and tunable Ar<sup>+</sup> laser ( $\lambda = 514, 488$  and  $458$  nm). A 100 $\times$  LWD  
3 objective was used to focus the laser light to a spot size of  $1 \mu\text{m}^2$  on the sample surface. To  
4 avoid a local heating of the sample, the power of the laser beam was adjusted to 0.2–0.5 mW  
5 with neutral filters of various optical densities. The Raman and photoluminescence mea-  
6 surements of  $\gamma\text{-LiV}_2\text{O}_5$  were performed with a T64000 (Horiba Jobin-Yvon) spectrometer  
7 equipped with a confocal microscope and a silicon CCD detector cooled by liquid nitrogen.  
8 Room temperature Raman spectra were excited by a 442 nm HeCd laser line, 532 nm line  
9 of a Nd:YAG laser (Torus, Laser Quantum, Inc.), and  $\lambda_0 = 785$  nm line of nearinfrared  
10 laser diode (TEC, Thorlabs Inc.). Photoluminescence spectra were excited by  $\lambda_0 = 785$  nm  
11 (TEC, Thorlabs Inc.) and  $\lambda_0 = 244$  nm (Coherent Innova 300C MotoFreD Ion Laser). The  
12 measurements in the temperature range from 78 to 300 K were carried out in a temperature  
13 controlled microscope stage Linkam THMS600. All spectra were measured in a backscatter-  
14 ing geometry.

## 15 **Computational procedure**

16 The *ab initio* calculations presented in the paper were carried out with ABINIT software  
17 package.<sup>47,48</sup> The LDA calculations employed the pseudopotential method and the plane wave  
18 basis set for valence electronic states, and used the PW exchange-correlation functional.<sup>49</sup>  
19 The interactions between the core and valence electrons were described with the projector-  
20 augmented wave (PAW) method.<sup>50,51</sup> The  $3p3d4s$  states of V atom,  $2s2p$  states of O atom  
21 and  $2s2p3s$  states of sodium were considered as valence states; the Li atom was treated at  
22 the all-electron level. In order to take into account the reduction of the transition metal upon  
23 the intercalation of alkali atoms, spin-polarized calculations were performed. The localized  
24 character of the  $3d$  states of V atoms was taken into consideration via the Hubbard term  
25 in the Hamiltonian (DFT+ $U$  method).<sup>21</sup> Choice of the Hubbard parameter  $U$  is discussed  
26 below in detail.

1 For each system studied the equilibrium structure was obtained in the following way.  
2 First, the geometry optimization of the unit cell was performed for a series of five volumes.  
3 In these calculations, both atomic positions and lattice parameters were allowed to vary,  
4 whereas the volume and the symmetry of the crystalline lattice were kept fixed. The Kohn-  
5 Sham equations were solved iteratively with a stopping criterion  $10^{-12}$  Ha for the energy  
6 convergence and the geometry optimization was considered to be completed if the maximum  
7 force on ions was less than  $5 \times 10^{-5}$  Ha/Bohr. The obtained energy *vs.* volume dependence  
8 was fitted by the Murnaghan equation of state<sup>52</sup> that provided an equilibrium cell volume.  
9 Then, the volume was used in a final optimization run that produced the equilibrium lattice  
10 parameters and atomic coordinates. This approach permitted to avoid problems related  
11 to the Pulay stress and to the variation of size of plane wave basis set that accompany  
12 calculations with simultaneous optimization of both the volume and atomic positions. The  
13 total energy was tested for the convergence with respect to the  $k$ -point grid and number  
14 of plane waves, the latter is controlled by a plane wave kinetic energy cutoff. It was found  
15 that the convergence of the total energy within  $10^{-4}$  Ha/atom was achieved with the energy  
16 cutoff of 40 Ha and  $2 \times 6 \times 6$  grids and  $2 \times 6 \times 2$  grids of  $k$ -points chosen according to the  
17 Monkhorst-Pack scheme<sup>53</sup> in the irreducible part of the Brillouin zone (BZ) of the  $\alpha$ - $V_2O_5$ ,  
18 and  $\gamma'$ - $V_2O_5$  and  $\gamma$ - $MeV_2O_5$  structures, respectively. The so-obtained equilibrium structures  
19 were employed in subsequent calculations that used the above calculation parameters and  
20 the convergence criterion. The LDA+ $U$  calculations were performed considering a partial  
21 occupation of electronic states with a gaussian smearing of 1 mHa width.

22 The quasi-particle  $GW$  calculations<sup>54</sup> used the frequency dependence of the screened  
23 Coulomb interaction obtained within the random-phase approximation by the contour de-  
24 formation method.<sup>55</sup> The exchange and correlation components of the self-energy were com-  
25 puted using the plane wave basis set with the energy cutoffs of 40 Ha and 10 Ha, respectively.  
26 The calculation of the polarizability and Green's function employed 268 unoccupied states in  
27 the conduction band. The BZ integration was performed over the  $2 \times 4 \times 4$  and  $2 \times 4 \times 2$  grid

1 for  $\alpha$ -V<sub>2</sub>O<sub>5</sub> and for  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> and  $\gamma$ -MeV<sub>2</sub>O<sub>5</sub>, respectively. The initial single-particle Green's  
 2 function  $G_0$  and the screened Coulomb potential  $W_0$  were constructed using the LDA+ $U$   
 3 eigenstates and eigenvalues, that is known as  $G_0W_0@LDA+U$  approach.<sup>28,29,56</sup> In order to  
 4 exclude double-counting term contained in the LDA+ $U$  eigenvalues,<sup>57</sup> the quasi-particle en-  
 5 ergies  $\mathcal{E}_{n\mathbf{k}}^\sigma$  were calculated using following expression:<sup>29</sup>

$$\mathcal{E}_{n\mathbf{k}}^\sigma = \bar{\epsilon}_{n\mathbf{k}}^\sigma + Z_{n\mathbf{k}}(\bar{\epsilon}_{n\mathbf{k}}^\sigma) [\Sigma_{n\mathbf{k}}^\sigma(\bar{\epsilon}_{n\mathbf{k}}^\sigma) - (\nu_{xc})_{n\mathbf{k}}^\sigma], \quad (1)$$

6 where  $\sigma$  is the spin index (up/down),  $\mathbf{k}$  – wavevector,  $\bar{\epsilon}_{n\mathbf{k}}^\sigma$  – single-particle energies calculated  
 7 with LDA+ $U$  wave functions,  $Z_{n\mathbf{k}}$  – renormalization factor, and  $\nu_{xc}$  – Kohn-Sham (KS)  
 8 exchange-correlation operator. The band structure in the  $GW$  calculations was obtained  
 9 from the KS band structure by applying the energy-dependent scissors operator generated  
 10 by a fit of  $GW$ -KS energy differences over a dense grid of  $k$ -points as a function of the KS  
 11 eigenvalues.<sup>58,59</sup>

## 12 Results and discussion

13 **Choice of Hubbard parameter  $U$ .** The use of the LDA+ $U$  solution in the  $GW$  calcu-  
 14 lations requires the knowledge of the Hubbard parameter  $U$ . Previous DFT calculation of  
 15 V<sub>2</sub>O<sub>5</sub> bronzes<sup>34,39</sup> made use of  $U = 4$  eV that was obtained from comparison of computed  
 16 band gap to  $E_g$  values measured by the X-ray and UV photoelectron spectroscopies.<sup>60,61</sup>  
 17 However, this value of  $U$  could not be simply transferred to the present calculations. In  
 18 fact, the used DFT+ $U$  approach relies on two Hubbard parameters  $U$  and  $J$  which being  
 19 combined give an effective parameter  $U_{eff} = U - J$  and it is  $U_{eff}$  that corresponds to  $U$  in  
 20 ref <sup>34,39</sup>. Making use of the linear response method,<sup>41,62,63</sup> we have computed  $U = 4.5$  eV  
 21 and  $U = 4.9$  eV for the  $\alpha$ -V<sub>2</sub>O<sub>5</sub> and  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> structures, respectively. For the alkali atom  
 22 intercalated  $\gamma$ -MeV<sub>2</sub>O<sub>5</sub> compound the method has yielded an  $U$  value of 5.2 eV. Constrained  
 23 DFT calculations of  $U$  and  $J$  for several transition metal oxides have given  $U \approx 5$  eV and

1  $J \approx 1$  eV<sup>29</sup> that results in  $U_{eff} \approx 4$  eV. In our choice of  $J$  we have relied on an estimation  
2  $J \approx 0.1U$  that can be proposed considering the explicit formulas for these quantities.<sup>21,37</sup>  
3 Given the likeness of the results obtained by the linear response method and the outcome  
4 of the previous calculations,<sup>29</sup> we have chosen to use  $U = 4.5$  eV and  $J = 0.5$  eV for all the  
5 systems in the study without adjustment.

6 The comparison of electron density of states (DOS) obtained for  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> with these  
7 values of  $U$  and  $J$  to DOS computed in ref 39 shows their a good agreement (see Figure S2).  
8 Futhermore, auxiliary calculations have revealed a small difference between the target quan-  
9 tities computed with the system-specific (obtained with the linear response method) and the  
10 generic  $U$  and  $J$  parameters (Table S3). A weak dependence of the electronic structure on  
11 the parameter values indicate that their values are "physically" meaningful.<sup>29</sup>

12 **Electronic structure and Raman spectra of pure V<sub>2</sub>O<sub>5</sub> structures.** The geometry  
13 optimization of the pristine V<sub>2</sub>O<sub>5</sub> structures at the LDA+ $U$  level does not notably improve  
14 the agreement between the computed and experimental structural characteristics in compar-  
15 ison to the conventional LDA calculations (Table S1). The result is perfectly understandable  
16 because none of the  $3d$  states of vanadium atoms, which are subjected to the Hubbard term,  
17 is occupied in these systems. However, the use of  $U$  in the LDA calculations pushes the  
18 bottom of the conduction band up, thus increasing the electronic band gap  $E_g$ . The direct  
19 band gap  $E_g^d$  for the  $\alpha$ -V<sub>2</sub>O<sub>5</sub> structure computed at the LDA+ $U$  level is equal to 1.79 eV,  
20 1.84 eV and 1.87 eV for  $U_{eff} = 0$  eV,  $U_{eff} = 2.7$  eV and  $U_{eff} = 4$  eV, respectively. The opti-  
21 cal absorption measurements on  $\alpha$ -V<sub>2</sub>O<sub>5</sub> monocrystals have given  $E_g = 2.35 \pm 0.01$  eV<sup>15</sup> and  
22  $E_g = 2.31 \pm 0.01$  eV<sup>16</sup> and the band gap at  $T = 0$  K estimated by extrapolating these exper-  
23 imental data is 2.49 eV to 2.54 eV.<sup>16</sup> More recent photoemission spectroscopic experiments  
24 reported the band gap of 2.8 eV for this material.<sup>17</sup> Although the LDA+ $U$  slightly increases  
25  $E_g$  value, the band gap still remains markedly smaller than the experimental counterparts  
26 and the energy derivative discontinuity makes the quantity known up to a constant.<sup>19</sup> As it

1 was mentioned in Introduction, one may expect to solve the problem, at least in part, by  
 2 using the *GW* method while a high computational cost of such calculations can be reduced  
 3 by taking the self-consistent LDA+*U* solution as an initial approximation.<sup>56</sup>

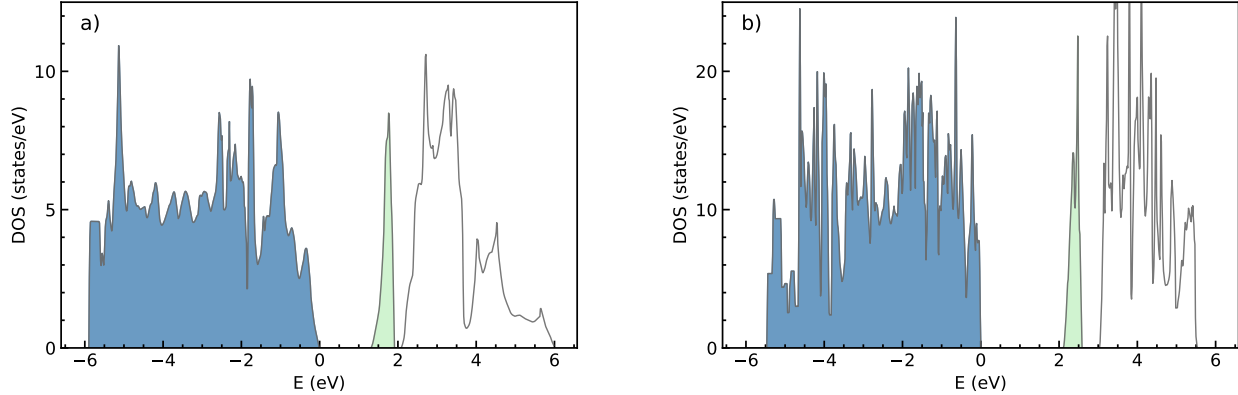


Figure 2: Densities of states (DOS) for  $\alpha$ -V<sub>2</sub>O<sub>5</sub> (a) and  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> obtained in the LDA+*U* calculations. The density of states in valence band and in the V<sup>3d</sup> conduction band discussed in the text are shaded by the blue and light green color, respectively. The origin of the energy axis is placed at the Fermi energy.

4 Density of electronic states (DOS) for  $\alpha$ -V<sub>2</sub>O<sub>5</sub> computed at the LDA+*U* level is presented  
 5 in Figure 2a while the partial DOS (PDOS) can be found in Figure S3. The first conduction  
 6 band with lowest energy appears as a sharp peak in DOS (shaded with light green color in  
 7 Figure 2). The nature of states in the band is important for understanding the change of  
 8 the electronic structure upon the intercalation of alkali metal atoms. As it has been found  
 9 previously, the conduction band is dominated by *d*-states of vanadium atoms<sup>32,40</sup> and this  
 10 conclusion is confirmed by the analysis of PDOS (Figure S3). Since these V<sup>3d</sup> states are  
 11 unoccupied, the vanadium atoms are in the 5+ oxidation state in the pure compound. Such  
 12 a band of localized V<sup>3d</sup> states is typical for vanadium oxides including  $\gamma$ -V<sub>2</sub>O<sub>5</sub> (Figure 2b)<sup>64</sup>.  
 13 Anticipating the following discussion, the intercalation of alkali metal atoms injects addi-  
 14 tional electrons into the system that fill the states thus making the band valence and radically  
 15 decreasing the band gap in the material.

16 Figure 3a presents the band structure of  $\alpha$ -V<sub>2</sub>O<sub>5</sub> obtained in an "one-shot" *G*<sub>0</sub>*W*<sub>0</sub>@LDA+*U*  
 17 calculation. The top of the valence band and the bottom of the conduction band are in the

1 vicinity of the T-point  $(0, \frac{1}{2}, \frac{1}{2})$  and in the  $\Gamma$ -point of the BZ, respectively. The direct gap  
 2  $E_g^d$  is equal to 2.89 eV and corresponds to the transition in the  $(0, \frac{1}{4}, \frac{1}{2})$  point of the BZ, the  
 3 indirect band gap  $E_g^i$  amounts to 2.42 eV. The computed  $E_g^d$  value is slightly greater than  
 4 the estimation of 2.49 – 2.54 eV at  $T = 0$  K obtained from the UV-vis adsorption data,<sup>15,16</sup>  
 5 while it is close to an estimate 2.98 – 3.01 eV at  $T = 0$  K found by extrapolating the band  
 6 gap measured in the photoemission experiments.<sup>17</sup> Our trial calculations have shown that  
 7 a small variation of  $U_{eff}$  does not significantly influence the values of the band gap and  
 8 therefore, the choice of  $U_{eff} = 4$  eV seems to be reasonable.

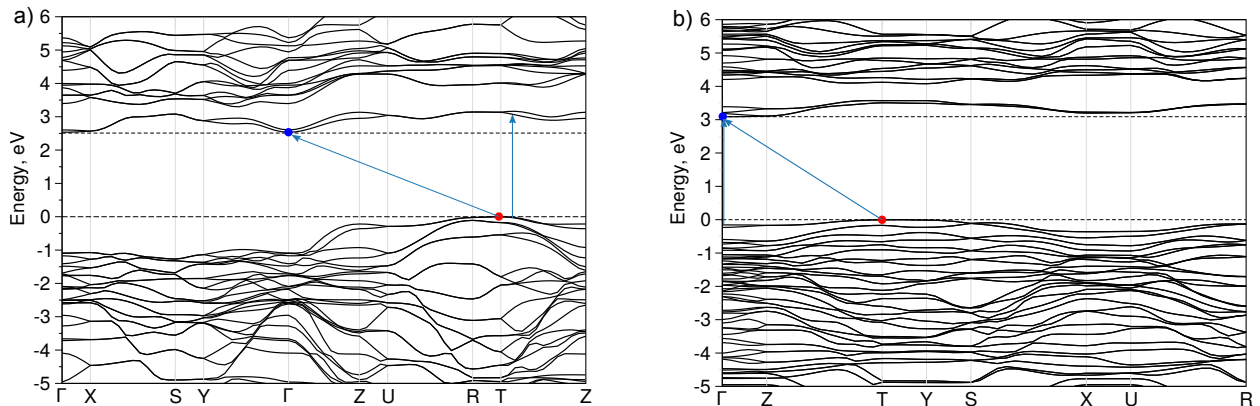


Figure 3: Electronic band structure of  $\alpha$ - $V_2O_5$  (a) and  $\gamma'$ - $V_2O_5$  (b) calculated in  $G_0W_0@LDA+U$  approximation; dashed lines indicate the positions of the Fermi level and the bottom of the conduction band. The bottom of conduction band and the top of the valence band pointed out by blue and red dots, respectively. Direct and indirect interband transitions are respectively shown by vertical and oblique arrows. The origin of energy axis is placed at  $E_F$  (Fermi energy).

9 Previous self-consistent  $GW$  calculations<sup>40</sup> of  $\alpha$ - $V_2O_5$  have given band gap values  $E_g^d =$   
 10 4.8 eV and  $E_g^i = 4$  eV that significantly exceed the experimental data.<sup>15–17</sup> The overestimation  
 11 has been explained as a consequence of large LO-TO splitting that reduces screened Coulomb  
 12 interaction in this polar material thus, increasing the gap. Due to the presence of the  
 13 integrable singularity of Coulomb potential at the  $\Gamma$ -point, which hinders the convergence  
 14 with respect to the number of  $k$ -points used to sample the BZ, the bare Coulomb potential in  
 15 the present calculations was replaced by an auxiliary function<sup>65</sup> within a sphere of radius 150  
 16 Bohr that has resulted in the convergence of exact exchange energy better than  $5 \times 10^{-4}$  Ha.

1 The results presented above show that the use of this approach and of a DFT solution as  
2 the input to the  $G_0W_0$  calculations, gives the band gap in the  $\alpha$ - $V_2O_5$  in a much better  
3 agreement with the experimental data than the  $E_g$  values obtained in the self-consistent  
4  $GW$  calculations.<sup>40</sup>

5 The electronic band structure obtained for  $\gamma'$ - $V_2O_5$  in a  $G_0W_0@LDA+U$  calculation using  
6 the above approach is presented in Figure 3b. The top of the valence band and the bottom  
7 of the conduction band are found in the vicinity of the T-point  $(0, \frac{1}{2}, \frac{1}{2})$  and in the  $\Gamma$ -point  
8  $(0, 0, 0)$  of the BZ, respectively. The calculation yields the direct and indirect band gaps of  
9 3.17 eV and 3.09 eV, respectively, and the direct gap corresponds to the transition in the  $\Gamma$ -  
10 point of the BZ.  $G_0W_0@LDA+U$  calculations using the  $U$  value obtained by linear response  
11 method has given slightly lower  $E_g$  values, namely 3.12 eV and 3.04 eV for the direct and  
12 indirect band gap, respectively (Table S3). To the best of our knowledge, no experimental  
13 data exist on the band gap in the material. Previously, Willinger *et al.*<sup>32</sup> computed  $E_g$  of  
14 1.70 eV in  $\gamma'$ - $V_2O_5$  by using the full potential linearized augmented plane waves method; the  
15 value  $E_g = 1.75$  eV was obtained for  $\alpha$ - $V_2O_5$  by the authors.

16 Table 1 collects experimental and theoretical band gap values for the  $\alpha$ - $V_2O_5$  and  $\gamma'$ - $V_2O_5$   
17 materials. Comparison of the  $E_g$  values computed at different levels of theory shows that  
18 the use of the DFT+ $U$  approach yields a better agreement with the experimental data, in  
19 line with the outcome of previous studies<sup>34</sup>. Computations with the hybrid PBE0 functional  
20 using both the LCAO basis set<sup>64</sup> and plane waves (this work) strongly overestimate the band  
21 gap in the  $\alpha$  phase. The HSE06 functional performs better, yet the band gap in the material  
22 is markedly greater than the experimental values. A better agreement with the experiment  
23 has been obtained at the GGA+ $U$  level in ref 34 that is not surprising as  $U$  in that study  
24 has been fitted to reproduce the experimental band gap. Despite a relatively high compu-  
25 tational cost, the  $G_0W_0$  calculation using LDA+ $U$  solution gives a very good estimation of  
26  $E_g$  obtained in a purely *ab initio* way with the  $U$  parameter found with the linear response  
27 method<sup>41</sup>.

Table 1: Experimental and calculated values of band gap  $E_g$  for the  $\alpha$ -V<sub>2</sub>O<sub>5</sub> and  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> polymorphs.

Polymorph	$E_g$ (eV)	Method	Reference
$\alpha$ -V <sub>2</sub> O <sub>5</sub>	$2.35 \pm 0.01$	Exp. <sup>a</sup> , OA	15
	$2.31 \pm 0.01$	Exp., OA	16
	2.8	Exp., PES	17
	1.74	Calc. <sup>b</sup> , ASW/LDA	31
	1.6 (2.1)	Calc., PAW/GGA (GGA+ $U$ )	66
	1.38 (2.06)	Calc., PAW/LDA (PBE)	67
	1.92	Calc., PAW/PW91	68
	1.74	Calc., PAW/PBE	69
	2.26	Calc., PAW/PBE+ $U$	34
	1.75	Calc., FPLAPW	32
	4.04	Calc., TZVP/PBE0	64
	4.83	Calc., SC GW	40
	1.79 (1.87)	Calc., PAW/LDA (LDA+ $U$ )	This work
	4.17	Calc., PW-NCPP/PBE0	This work
	3.43	Calc., PW-NCPP/HSE06	This work
2.89	Calc., G <sub>0</sub> W <sub>0</sub> @LDA+ $U$	This work	
$\gamma'$ -V <sub>2</sub> O <sub>5</sub>	1.70	Calc., FPLAPW	32
	4.23	Calc., TZVP/PBE0 <sup>e</sup>	64
	1.93 (2.27)	Calc., PAW/LDA (LDA+ $U$ )	This work
	4.41	Calc., PW-NCPP/PBE0	This work
	3.64	Calc., PW-NCPP/HSE06	This work
	3.17	Calc., G <sub>0</sub> W <sub>0</sub> @LDA+ $U$	This work

<sup>a</sup>Experiment: OA – optical absorption, PES – photoemission spectroscopy;

<sup>b</sup>Calculations: LDA, GGA - local density approximation and generalized gradient approximation to DFT, respectively; + $U$  – inclusion of Hubbard term into Hamiltonian; PBE, PW91 – GGA exchange-correlation functionals; PBE0, HSE06 – hybrid HF-DFT functionals; ASW – augmented spherical wave; FPLAPW – full potential linearized augmented plane wave; SC GW – self-consistent GW approach; PAW – projector augmented wave; TZVP - polarization quality triple-zeta valence atomic basis set, PW-NCPP – plane wave basis set with norm-conserving pseudopotentials.

1 Besides the optical absorption or photoemission experiments, Raman spectroscopy can,  
 2 in principle, provide information on the band gap value. Indeed, one can expect a notable  
 3 change in the Raman spectrum of material as soon as the energy of exciting radiation exceeds  
 4  $E_g$  and thus, the scattering process occurs in the electron resonance conditions. Figure 4 dis-  
 5 plays Raman spectra of  $\alpha$ -V<sub>2</sub>O<sub>5</sub> and  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> samples measured with four different excitation  
 1 wavelengths. One sees that all the spectra obtained with  $\lambda_0 > 458$  nm are very similar to  
 2 each other, whereas the spectra obtained with  $\lambda_0 = 458$  nm strikingly differ from the spectra  
 3 measured with excitation wavelengths of smaller energy. Despite a variation of shape of the  
 4 most intense peak at *ca.* 500 cm<sup>-1</sup>, the 458 nm spectra obtained from different spots on the  
 5 sample's surface show a good reproducibility. Hence, such a dependence of the spectra on the  
 6 excitation wavelength cannot be viewed as an artifact and the behavior allows us to suppose  
 7 that the 458 nm spectra were measured in the electron resonance conditions. Then, it is  
 8 reasonable to assume that the band gap value for the  $\alpha$ -V<sub>2</sub>O<sub>5</sub> and  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> materials at room  
 9 temperature lies between 2.71 eV ( $\lambda_0 = 458$  nm) and 2.54 eV ( $\lambda_0 = 488$  nm). Making use of  
 10 the relative band gap values in these materials obtained in the  $G_0W_0@LDA+U$  calculations,  
 11 one can estimate the band gap in  $\alpha$ -V<sub>2</sub>O<sub>5</sub> to be slightly greater than 2.54 eV that well agrees  
 12 with the previous experimental results.<sup>16,17</sup> The gap in  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> can then be evaluated to be  
 13 slightly lesser than 2.71 eV that provides a first experimental estimation of this quantity for  
 14 the material. It should be noted that the  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> sample was obtained by the chemical de-  
 15 intercalation of Li atoms from the  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> bronze and, therefore its structure can contain  
 16 a number of defects. Consequently, the above value likely underestimates the band gap in a  
 17 defect-less structure of  $\gamma'$ -V<sub>2</sub>O<sub>5</sub>.

18 Figure 4 shows that the resonance Raman spectra of both the pure V<sub>2</sub>O<sub>5</sub> structures  
 1 are dominated by a peak at 500 cm<sup>-1</sup>. According to our previous calculations,<sup>39,44,46</sup> the  
 2 Raman features in this spectral region originate from vibrational modes involving stretching  
 3 vibrations of V-O2 contacts in the ladder steps and of the bonds in the V-O3-V bridges<sup>46</sup>  
 4 (see Figure 1b). The electronic-structure calculations indicate that the main contribution to

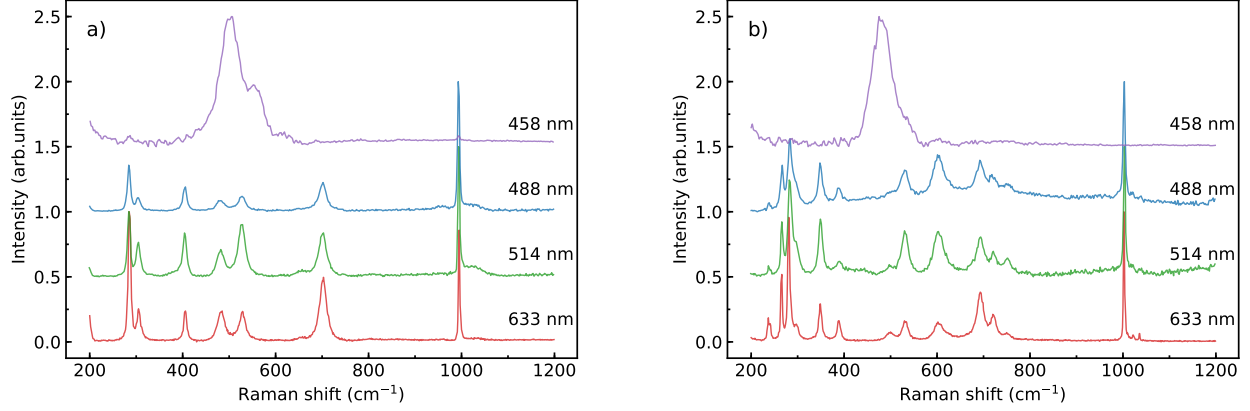


Figure 4: Raman spectra of  $\alpha$ - $V_2O_5$  (a) and  $\gamma'$ - $V_2O_5$  (b) obtained with different wavelengths  $\lambda_0$  of the incident radiation:  $\lambda_0 = 633$  nm (red),  $\lambda_0 = 514$  nm (green),  $\lambda_0 = 488$  nm (blue), and  $\lambda_0 = 458$  nm (violet).

5 the top of the valence band comes from the O2 and O3 oxygen atoms, whereas the bottom of  
6 the conduction band is formed by the  $d$  states of vanadiums. It is, therefore, not surprising  
7 that the Raman peaks due to vibrational modes involving displacements of these atoms are  
8 the most concerned by the electron resonance effect.

9  **$\gamma$ -MeV<sub>2</sub>O<sub>5</sub> bronzes.** In contrast to the pure  $V_2O_5$  materials, a computational investiga-  
10 tion of alkali atom intercalated MeV<sub>2</sub>O<sub>5</sub> structures necessitates the use of theoretical ap-  
11 proach that takes into account the localized character of partially occupied  $3d$  vanadium  
12 states.<sup>34,39</sup> Figure 6 shows the  $G_0W_0@LDA+U$  electron band structure of  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub>, the  
13 band structure of the  $\gamma$ -NaV<sub>2</sub>O<sub>5</sub> bronze is very similar with a tiny difference in a value of  
14 the band gap (Figure S3). The electronic states at the Fermi level are formed by the  $d$   
15 orbitals of Vb atoms occupied by electrons ejected from the outer  $s$  shell of the alkali metals,  
16 while the bottom of the conduction band is formed by the  $d$  orbitals of Va atoms, Figure 1b.  
17 From Figure 6 it follows that the  $\gamma$ -MeV<sub>2</sub>O<sub>5</sub> bronzes are the direct transition semiconductors  
18 with the direct optical transition at the Y-point and the band gap values obtained in the  
19  $G_0W_0@LDA+U$  calculation are equal to 1.21 eV and 1.18 eV for the  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> and  $\gamma$ -NaV<sub>2</sub>O<sub>5</sub>  
20 structures, respectively. Calculations using  $U$  obtained by the linear response method results  
21 in  $E_g$  values slightly greater (by 0.1 eV) than those computed with the generic  $U$  (Table S3).

22 In the absence of any experimental data concerning the band gap in these materials, we at-  
 23 tempt to estimate this quantity by using the Raman and photoluminescence spectroscopies.  
 24 Note that the above theoretical  $E_g$  values imply that the Raman spectra measured using the  
 25 light sources in the visible region are all obtained in the resonance conditions.

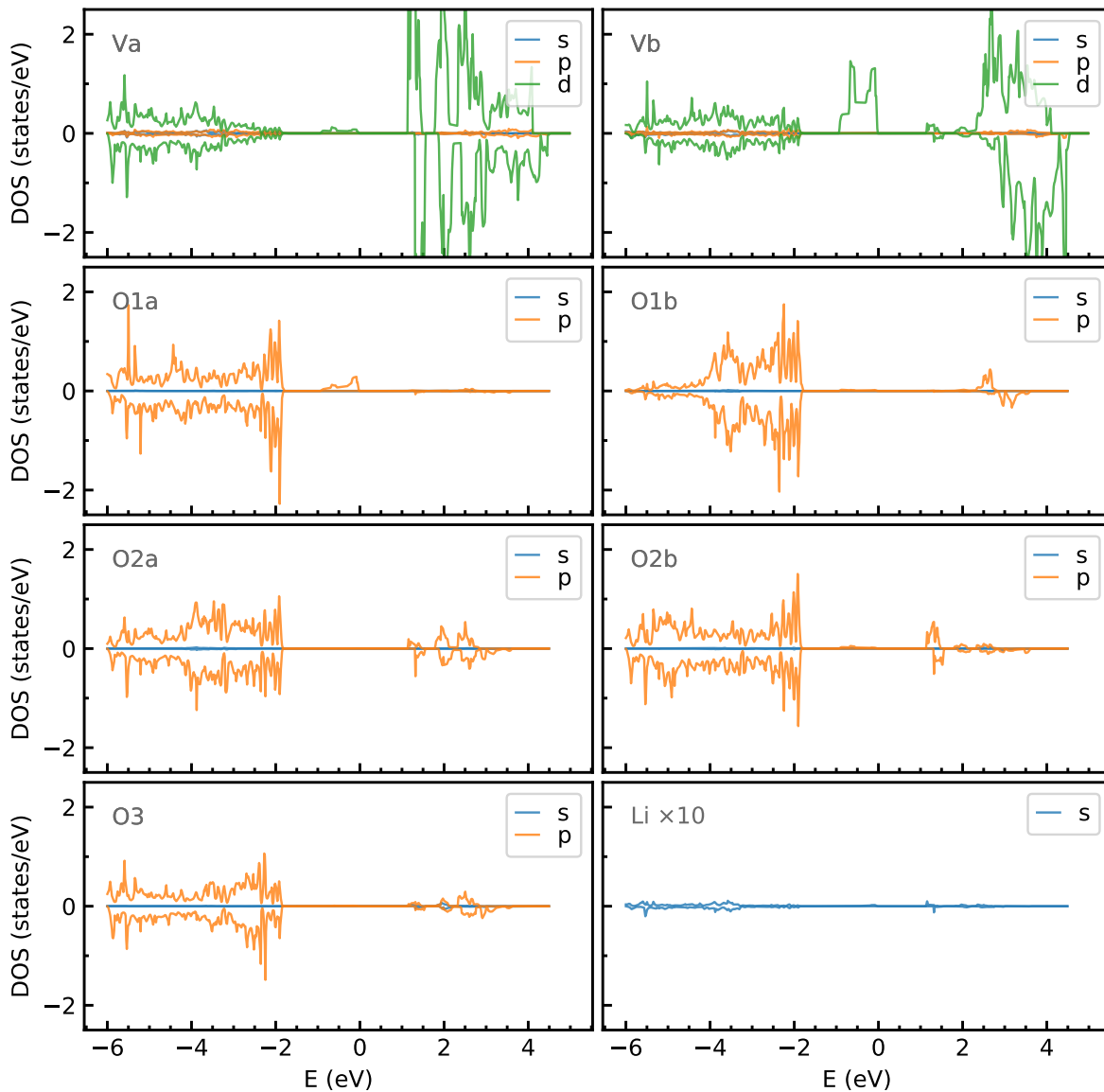


Figure 5: Partial densities of states (PDOS) for  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> obtained in the spin-polarized LDA+ $U$  calculation. For each atom type, the positive and negative parts of DOS correspond to the spin up and spin down states. The origin of the energy axis is placed at the Fermi energy.

26 Figure 5 shows PDOS for the  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> bronze obtained in the spin-polarized LDA+*U*  
 1 calculation (see Figure S2 for the total DOS). One readily sees the difference between "spin  
 2 up" and "spin down" states in the range -0.7 to 0 eV corresponding to the band of localized  
 3 3*d* states of Vb atoms. The difference is explained by the fact that with four intercalated  
 4 Li atoms per unit cell, four electrons ejected from the 2*s* orbitals of lithiums occupy the 3*d*  
 5 states of Vb atoms in the "spin up" configuration. This leads to the splitting of conduction  
 1 V<sup>3*d*</sup> band to the valence one formed by filled *d*-states of Vb atom and to the conducting  
 2 band formed by empty *d*-states of Va atom (Figure 6). The cation sublattice now consists  
 3 of vanadium atoms in the 4+ (Vb) and 5+ (Va) oxidation states. This conclusion is in line  
 4 with the results reported in ref<sup>34</sup> and complete the data in ref<sup>39</sup>.

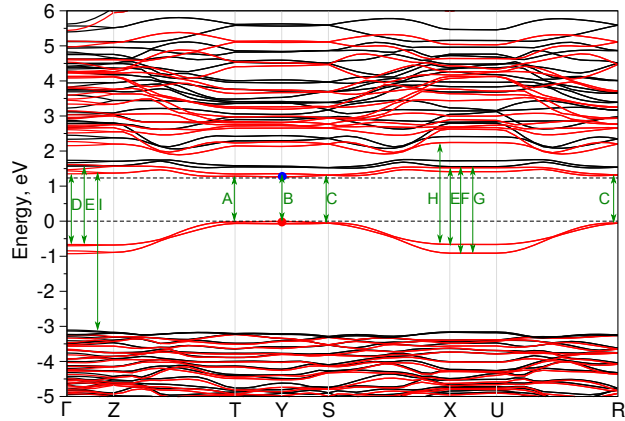


Figure 6: Electron band structure of  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> obtained in  $G_0W_0@LDA+U$  calculation. Red and black lines stand for the spin up and spin down states, respectively; dashed lines indicate the positions of the Fermi level and bottom of the conduction band. The bottom of conduction band and the top of the valence band pointed out by blue and red dots respectively. Arrows indicate optical interband transitions, see text for details. The origin of the energy axis is placed at  $E_F$  (Fermi energy)

5 Figure 7 presents the Raman spectra of the  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> bronze measured with the exciting  
 6 radiation of different wavelengths and the spectrum of  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> obtained in off-resonance  
 7 conditions (corresponding spectra of  $\gamma$ -NaV<sub>2</sub>O<sub>5</sub> are given in Figure S4). One sees that peaks  
 8 in the spectra of  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> are characterized by larger widths and higher intensities compared  
 9 to the counterpart peaks in the spectrum of the pure compound. It could be supposed that

10 these peculiarities are due to a more disordered structure of the bronze compared to that  
 11 of  $\gamma'$ - $V_2O_5$ , however, it is the contrary that should be true. Indeed,  $\gamma'$ - $V_2O_5$  is obtained by  
 12 chemical de-intercalation of alkali-metal atoms from  $\gamma$ - $LiV_2O_5$  and therefore, its resultant  
 13 structure is expected to contain more defects than that of the parent material. Nevertheless,  
 14 the peaks in the Raman spectrum of  $\gamma'$ - $V_2O_5$  shown in Figure 7 are characterized by a  
 15 notably smaller width. Such a difference between the spectra of the pure and lithiated  $\gamma$ -  
 1 phases indicates, in our opinion, that the spectra of  $\gamma$ - $LiV_2O_5$  (and of its sodium analogue)  
 2 were measured in the electron resonance conditions. If true, this gives an estimation of  
 3 the band gap in the material  $E_g < 1.58$  eV (785 nm) that is in line with the results of  
 4 the calculations. Unfortunately, all our attempts to obtain the Raman spectra of  $MeV_2O_5$   
 5 bronzes with an excitation source of energy smaller than 1.58 eV ( $\lambda_0 > 785$  nm) did not  
 6 succeed due to technical problems.

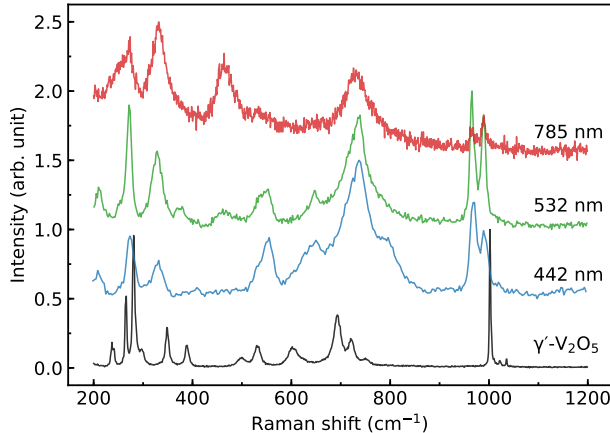


Figure 7: Experimental Raman spectra of  $\gamma$ - $LiV_2O_5$  measured with the excitation wave-  
 lengths  $\lambda_0 = 785$  nm (red),  $\lambda_0 = 532$  nm (green) and  $\lambda_0 = 442$  nm (blue); black line labeled  
 $\gamma'$ - $V_2O_5$  presents the experimental Raman spectrum of  $\gamma'$ - $V_2O_5$  obtained with  $\lambda_0 = 633$  nm.

7 Then, we have attempted to elucidate this issue by using the photoluminescence spec-  
 8 troscopy. Photoluminescence spectra of  $\gamma$ - $LiV_2O_5$  obtained with different excitation sources  
 9 are shown in Figure 8 and bands in the spectra are related to specific interband transitions  
 10 depicted with arrows in Figure 6. The spectrum excited by a low energy radiation, Fig-

11 ure 8a, has three clear bands labeled *A* to *C* that can be related to transitions from the top  
 12 of the valence band to the bottom of the conduction band in the vicinity of the T, Y, S and  
 13 R points of the BZ. In the high-energy region, Figure 8b, the bands *D* and *E* (Figure 8b)  
 14 can be associated with transitions in the  $\Gamma$ -point of the BZ and the *E*, *F*, *G* and *H* bands  
 15 corresponds to electron transitions at zone boundary points. Finally, the *I* band may be  
 16 related to transitions from lower-lying states of the valence zone in the *Z*-point of the BZ.  
 1 The calculated electronic structure suggests that the transitions *A*–*D* and *H* occur between  
 2 states of the same spin, while the transitions *E* – *G* and *I* involve states of different spin.

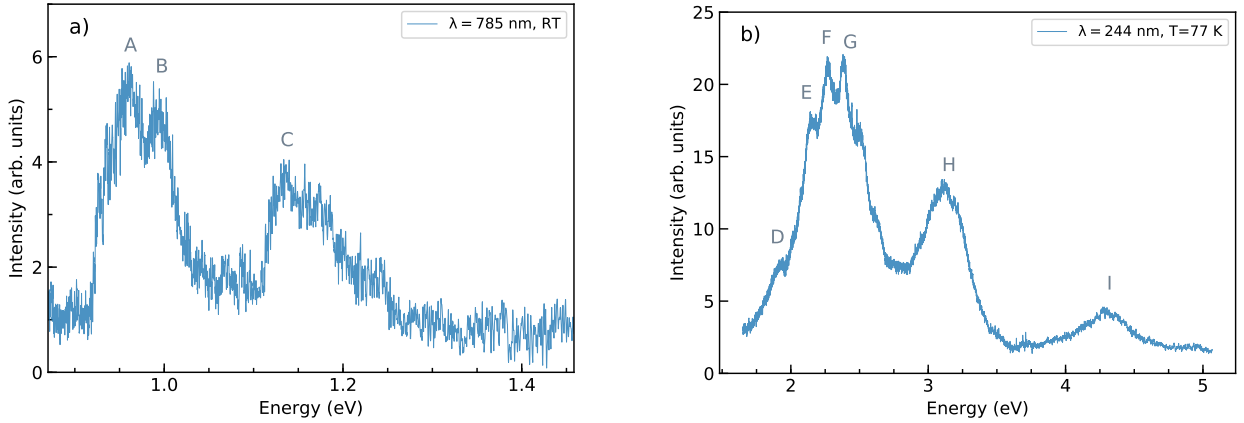


Figure 8: Photoluminescence spectra of  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> sample: a) room temperature (RT), excitation wavelength  $\lambda = 785$  nm; b) temperature  $T = 77$  K, excitation wavelength  $\lambda = 244$  nm. Letters labeling bands in the spectra refer to transitions depicted in Figure 6.

3 The energy of the band *A*, Figure 8a, can be used to estimate the direct band gap value  
 4 in  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> that gives  $E_g$  of *ca.* 0.95 eV at room temperature. Assuming the validity  
 5 of temperature dependence of  $E_g$  in  $\alpha$ -V<sub>2</sub>O<sub>5</sub> for the bronzes, an approximate band gap at  
 6  $T = 0$  K in these materials is  $E_g \approx 1.15$  eV that reasonably agrees with the value of 1.21 eV  
 7 obtained in the calculations.

8 It is noteworthy that all the spectra in Figure 7a-c were measured with the energy of  
 9 incident radiation smaller than 2.8 eV, *i.e.* the resonance electronic excitation corresponds  
 10 to the transitions *A* – *G*, see Figure 6 and Figure 8. These transitions occur between the  
 11 localized states near the Fermi level formed by the *d*-orbitals of Vb atoms and the bottom of

12 the conduction band. This can account for the fact that the most marked difference between  
13 the spectra of the  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> and  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> samples, Figure 7, is seen for Raman peaks which,  
14 according to our calculations, correspond to vibrational modes involving displacements of  
15 Vb atoms (see Figure 1).

## 16 Conclusions

1 This work reports results of a combined computational and experimental study of the elec-  
2 tronic structure of  $\alpha$ -V<sub>2</sub>O<sub>5</sub>,  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> and  $\gamma$ -MeV<sub>2</sub>O<sub>5</sub> (Me = Li, Na) materials. The electronic  
3 states in these compounds were investigated with the quasi-particle *GW* method which used  
4 a self-consistent LDA+*U* solution for constructing the initial Green's function and screened  
5 Coulomb potential. Such a "single-shot" *G*<sub>0</sub>*W*<sub>0</sub>@LDA+*U* calculations have given the direct  
6 electronic band gap value  $E_g = 2.89$  eV for the  $\alpha$ -V<sub>2</sub>O<sub>5</sub> structure that well agrees with  
7 the available experimental data.<sup>15-17</sup> The strategy was subsequently applied to compute the  
8 electronic structure of the  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> phase and of the  $\gamma$ -MeV<sub>2</sub>O<sub>5</sub> bronzes for which no ex-  
9 perimental information on band gap value exists. The calculations have given  $E_g = 3.17$   
10 eV,  $E_g = 1.21$  eV and  $E_g = 1.18$  eV for the  $\gamma'$ -V<sub>2</sub>O<sub>5</sub>,  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> and  $\gamma$ -NaV<sub>2</sub>O<sub>5</sub> structures,  
11 respectively. The calculations have shown a close similarity between the electronic structures  
12 of the two alkali-atom intercalated materials that accounts for their peculiar and attractive  
13 electrochemical characteristics.<sup>18</sup> Indeed, while Na insertion systematically occurs at much  
14 lower voltage than Li one ( $\Delta E \approx 1.5$  V in the case of  $\alpha$ -V<sub>2</sub>O<sub>5</sub><sup>70</sup>), the same high voltage of  
15 3.3 V *vs.* Na<sup>+</sup>/Na is remarkably achieved for Li and Na reactions in  $\gamma$ -MeV<sub>2</sub>O<sub>5</sub> bronzes.<sup>18</sup>

16 The results of electronic-structure calculations were accompanied with data of spectro-  
17 scopic experiments. Thus, measuring Raman spectra of the materials with different wave-  
18 lengths of exciting radiation has allowed to estimate a threshold energy corresponding to  
19 the transition from off-resonance to resonance Raman scattering process. The band gap  
20 value obtained in this way for  $\alpha$ -V<sub>2</sub>O<sub>5</sub> and  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> was found to lie between 2.54 eV and

21 2.71 eV at room temperature that is in a good agreement with the experimental values for  
22 the  $\alpha$ -phase and with the result of the  $G_0W_0@LDA+U$  calculation. The Raman spectra of  
23  $\gamma$ -MeV<sub>2</sub>O<sub>5</sub> bronzes revealed a band gap smaller than 1.58 eV, whereas the comparison of the  
24 calculated band structure of  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> with the photoluminescence spectra yielded the band  
25 gap  $E_g \approx 0.95$  eV for this material. The result of the quasi-particle  $G_0W_0$  calculations is just  
26 between the two experimental estimates and is very close to a 0 K value extrapolated from  
27 the photoluminescence data. The present work shows that the calculations do not necessari-  
1 tate using adjusted parameters, *i.e.* they can be performed in a fully nonempirical way. The  
2 successful approach combining the *ab initio* calculations and spectroscopy will be applied to  
3 the investigation of other  $\gamma$ -Me<sub>x</sub>V<sub>2</sub>O<sub>5</sub> (Me = K<sup>+</sup>, Mg<sup>2+</sup>, Ca<sup>2+</sup>, ...) bronzes having a crucial  
4 interest for next generation Me-ion batteries.

## 5 **Supporting Information**

6 Supporting Information is available free of charge on the ACS Publications website at DOI:  
7 10.1021/xxxxyyzzz.

8 Table with lattice parameters of  $\alpha$ -V<sub>2</sub>O<sub>5</sub>,  $\gamma'$ -V<sub>2</sub>O<sub>5</sub>,  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> and  $\gamma$ -NaV<sub>2</sub>O<sub>5</sub>; table with  
9 experimental and calculated structural characteristics of  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> and  $\gamma$ -NaV<sub>2</sub>O<sub>5</sub>; Raman  
10 spectra of the  $\alpha$ -V<sub>2</sub>O<sub>5</sub>,  $\gamma'$ -V<sub>2</sub>O<sub>5</sub>,  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> and  $\gamma$ -NaV<sub>2</sub>O<sub>5</sub> materials; electronic band struc-  
11 ture and Raman spectra of  $\gamma$ -NaV<sub>2</sub>O<sub>5</sub> (PDF).

## 12 **Author Information**

### 13 **ORCID**

14 Evgenii M. Roginskii: 0000-0002-5627-5877

15 Mikhail B. Smirnov: 0000-0002-4292-1989

16 Konstantin S. Smirnov: 0000-0002-8370-8797

17 Rita Baddour-Hadjean: 0000-0002-3158-1851  
18 Jean-Pierre Pereira-Ramos: 0000-0001-5381-900X

## 19 Notes

20 Authors declare no competing financial interest.

## 21 Acknowledgements

1 E.M.R. acknowledges the financial support of Université Paris Est-Créteil in the frame of  
2 the invited professor program UPE 2020. A part of the calculations was performed using the  
3 facilities of the Computational Center of Research Park of St. Petersburg State University.  
4 The authors also thank the anonymous reviewers for valuable comments.

## 5 References

- 6 (1) Weckhuysen, B. M.; Keller, D. E. Chemistry, Spectroscopy and the Role of Supported  
7 Vanadium Oxides in Heterogeneous Catalysis. *Catal. Today* **2003**, *78*, 25–46, DOI:  
8 [https://doi.org/10.1016/S0920-5861\(02\)00323-1](https://doi.org/10.1016/S0920-5861(02)00323-1).
- 9 (2) Chernova, N. A.; Roppolo, M.; Dillon, A. C.; Whittingham, M. S. Layered Vanadium  
10 and Molybdenum Oxides: Batteries and Electrochromics. *J. Mater. Chem.* **2009**, *19*,  
11 2526–2552, DOI: <https://doi.org/10.1039/B819629J>.
- 12 (3) Whittingham, M. S. Lithium Batteries and Cathode Materials. *Chem. Rev.* **2004**, *104*,  
13 4271–4301, DOI: [10.1021/cr020731c](https://doi.org/10.1021/cr020731c).
- 14 (4) Whittingham, M. S. The Role of Ternary Phases in Cathode Reactions. *J. Electrochem.*  
15 *Soc.* **1976**, *123*, 315–320, DOI: [10.1149/1.2132817](https://doi.org/10.1149/1.2132817).

- 16 (5) Delmas, C.; Cognac-Auradou, H.; Cocciantelli, J. M.; Menetrier, M.; Doumerc, J. P.  
17 The  $\text{Li}_x\text{V}_2\text{O}_5$  System: An Overview of the Structure Modifications Induced  
18 by the Lithium Intercalation. *Solid State Ion.* **1994**, *69*, 257–264, DOI:  
19 10.1016/0167-2738(94)90414-6.
- 20 (6) Galy, J. Vanadium Pentoxide and Vanadium Oxide Bronzes - Structural Chemistry of  
21 Single (S) and Double (D) Layer  $\text{M}_x\text{V}_2\text{O}_5$  phases. *J. Solid State Chem.* **1992**, *100*,  
22 229–45, DOI: 10.1016/0022-4596(92)90097-F.
- 23 (7) Baddour-Hadjean, R.; Safrany Renard, M.; Pereira-Ramos, J. P. Unraveling the Struc-  
1 tural Mechanism of Li Insertion in  $\gamma'$ - $\text{V}_2\text{O}_5$  and Its Effect on Cycling Properties. *Acta*  
2 *Mater.* **2019**, *165*, 183–191, DOI: 10.1016/j.actamat.2018.11.043.
- 3 (8) Baddour-Hadjean, R.; Safrany Renard, M.; Pereira-Ramos, J. P. Kinetic Insight into the  
4 Electrochemical Lithium Insertion Process in the Puckered-Layer  $\gamma'$ - $\text{V}_2\text{O}_5$  Polymorph.  
5 *J. Electrochem. Soc.* **2019**, *166*, A3474–A3479, DOI: 10.1149/2.1211914jes.
- 6 (9) Safrany Renard, M.; Emery, N.; Baddour-Hadjean, R.; Pereira-Ramos, J.-P.  $\gamma'$ - $\text{V}_2\text{O}_5$ : A  
7 New High Voltage Cathode Material for Sodium-Ion Battery. *Electrochim. Acta* **2017**,  
8 *252*, 4–11, DOI: 10.1016/j.electacta.2017.08.175.
- 9 (10) Baddour-Hadjean, R.; Safrany Renard, M.; Emery, N.; Huynh, L. T. N.; Le, M.  
10 L. P.; Pereira-Ramos, J. P. The Richness of  $\text{V}_2\text{O}_5$  Polymorphs as Superior Cath-  
11 ode Materials for Sodium Insertion. *Electrochim. Acta* **2018**, *270*, 129–137, DOI:  
12 10.1016/j.electacta.2018.03.062.
- 13 (11) Zavalij, P. Y.; Whittingham, M. S. Structural Chemistry of Vanadium Ox-  
14 ides with Open Frameworks. *Acta Cryst. B* **1999**, *55*, 627–663, DOI:  
15 10.1107/S0108768199004000.
- 16 (12) Eftekhari, A. Lithium-Ion Batteries with High Rate Capabilities. *ACS Sustainable*  
17 *Chem. Eng.* **2017**, *5*, 2799–2816, DOI: 10.1021/acssuschemeng.7b00046.

- 18 (13) Bisquert, J.; Cendula, P.; Bertoluzzi, L.; Gimenez, S. Energy Diagram of Semi-  
19 conductor/Electrolyte Junctions. *J. Phys. Chem. Lett.* **2014**, *5*, 205–207, DOI:  
20 10.1021/jz402703d.
- 21 (14) Liu, C.; Neale, Z. G.; Cao, G. Understanding Electrochemical Potentials of Cathode  
22 Materials in Rechargeable Batteries. *Mater. Today* **2016**, *19*, 109–123.
- 23 (15) Kenny, N.; Kannewurf, C.; Whitmore, D. Optical Absorption Coefficients of Vana-  
1 dium Pentoxide Single Crystals. *J. Phys. Chem. Solids* **1966**, *27*, 1237 – 1246, DOI:  
2 [https://doi.org/10.1016/0022-3697\(66\)90007-2](https://doi.org/10.1016/0022-3697(66)90007-2).
- 3 (16) Bodó, Z.; Hevesi, I. Optical Absorption near the Absorption Edge in V<sub>2</sub>O<sub>5</sub> Single  
4 Crystals. *Phys. Status Solidi* **1967**, *20*, K45–K49, DOI: 10.1002/pssb.19670200153.
- 5 (17) Meyer, J.; Zilberberg, K.; Riedl, T.; Kahn, A. Electronic Structure of Vanadium Pen-  
6 toxide: An Efficient Hole Injector for Organic Electronic Materials. *J. Appl. Phys.*  
7 **2011**, *110*, 033710, DOI: 10.1063/1.3611392.
- 8 (18) Emery, N.; Baddour-Hadjean, R.; Batyrbekuly, D.; Laik, B.; Bakenov, Z.; Pereira-  
9 Ramos, J.-P.  $\gamma$ -Na<sub>0.96</sub>V<sub>2</sub>O<sub>5</sub>: A New Competitive Cathode Material for Sodium-Ion  
10 Batteries Synthesized by a Soft Chemistry Route. *Chem. Mater.* **2018**, *30*, 5305–5314,  
11 DOI: 10.1021/acs.chemmater.8b02066.
- 12 (19) Perdew, J. P. Density Functional Theory and The Band Gap Problem. *Int. J. Quantum*  
13 *Chem.* **2009**, *28*, 497–523, DOI: 10.1002/qua.560280846.
- 14 (20) Mori-Sánchez, P.; Cohen, A. J.; Yang, W. Localization and Delocalization Errors in  
15 Density Functional Theory and Implications for Band-Gap Prediction. *Phys. Rev. Lett.*  
16 **2008**, *100*, DOI: 10.1103/physrevlett.100.146401.
- 17 (21) Anisimov, V. I.; Solovyev, I. V.; Korotin, M. A.; Czyżyk, M. T.; Sawatzky, G. A.

- 18 Density Functional Theory and NiO Photoemission Spectra. *Phys. Rev. B* **1993**, *48*,  
19 16929–16934, DOI: 10.1103/PhysRevB.48.16929.
- 20 (22) Perdew, J. P.; Ernzerhof, M.; Burke, K. Rationale for Mixing Exact Exchange with  
21 Density Functional Approximations. *J. Chem. Phys.* **1996**, *105*, 9982–9985, DOI:  
22 10.1063/1.472933.
- 23 (23) Heyd, J.; Scuseria, G. E.; Ernzerhof, M. Hybrid Functionals Based on a Screened  
1 Coulomb Potential. *J. Chem. Phys.* **2003**, *118*, 8207–8215, DOI: 10.1063/1.1564060.
- 2 (24) Heyd, J.; Scuseria, G. E.; Ernzerhof, M. Erratum: "Hybrid Functionals Based on a  
3 Screened Coulomb Potential" [J. Chem. Phys. 118, 8207 (2003)]. *J. Chem. Phys.* **2006**,  
4 *124*, 219906, DOI: 10.1063/1.2204597.
- 5 (25) Shishkin, M.; Kresse, G. Self-Consistent GW Calculations for Semiconductors and In-  
6 sulators. *Phys. Rev. B* **2007**, *75*, 235102, DOI: 10.1103/physrevb.75.235102.
- 7 (26) Hedin, L. New Method for Calculating the One-Particle Green's Function with  
8 Application to the Electron-Gas Problem. *Phys. Rev. A* **1965**, *139*, 796, DOI:  
9 <https://doi.org/10.1103/PhysRev.139.A796>.
- 10 (27) Bruneval, F.; Marques, M. A. L. Benchmarking the Starting Points of the GW  
11 Approximation for Molecules. *J. Chem. Theory Comput.* **2012**, *9*, 324–329, DOI:  
12 10.1021/ct300835h.
- 13 (28) Jiang, H.; Gomez-Abal, R. I.; Rinke, P.; Scheffler, M. Localized and Itinerant States  
14 in Lanthanide Oxides United by  $GW@LDA+U$ . *Phys. Rev. Lett.* **2009**, *102*, DOI:  
15 10.1103/physrevlett.102.126403.
- 16 (29) Jiang, H.; Gomez-Abal, R. I.; Rinke, P.; Scheffler, M. First-Principles Modeling of  
17 Localized  $d$  States with the  $GW@LDA+U$  Approach. *Phys. Rev. B* **2010**, *82*, DOI:  
18 10.1103/physrevb.82.045108.

- 19 (30) Kempf, J. Y.; Silvi, B.; Dietrich, A.; Catlow, C. R. A.; Maignet, B. Theoretical Investi-  
20 gations of the Electronic Properties of Vanadium Oxides. 1. Pseudopotential Periodic  
21 Hartree-Fock Study of Vanadium Pentoxide Crystal Lattice. *Chem. Mater.* **1993**, *5*,  
22 641–647, DOI: 10.1021/cm00029a011.
- 23 (31) Eyert, V.; Höck, K.-H. Electronic Structure of  $V_2O_5$ : Role of Octahedral Deformations.  
1 *Phys. Rev. B* **1998**, *57*, 12727–12737, DOI: 10.1103/PhysRevB.57.12727.
- 2 (32) Willinger, M.; Pinna, N.; Su, D. S.; Schlögl, R. Geometric and Electronic Structure of  
3  $\gamma$ - $V_2O_5$ : Comparison Between  $\alpha$ - $V_2O_5$  and  $\gamma$ - $V_2O_5$ . *Phys. Rev. B* **2004**, *69*, 155114,  
4 DOI: <https://doi.org/10.1103/PhysRevB.69.155114>.
- 5 (33) Brázdová, V.; Ganduglia-Pirovano, M. V.; Sauer, J. Periodic Density Functional Study  
6 on Structural and Vibrational Properties of Vanadium Oxide Aggregates. *Phys. Rev.*  
7 *B* **2004**, *69*, 165420, DOI: 10.1103/PhysRevB.69.165420.
- 8 (34) Scanlon, D. O.; Walsh, A.; Morgan, B. J.; Watson, G. W. An ab initio Study of Re-  
9 duction of  $V_2O_5$  through the Formation of Oxygen Vacancies and Li Intercalation. *J.*  
10 *Phys. Chem. C* **2008**, *112*, 9903–9911, DOI: <https://doi.org/10.1021/jp711334f>.
- 11 (35) Londero, E.; Schröder, E. Role of van der Waals Bonding in the Layered Oxide  $V_2O_5$ :  
12 First-principles Density-functional Calculations. *Phys. Rev. B* **2010**, *82*, 054116, DOI:  
13 10.1103/PhysRevB.82.054116.
- 14 (36) Londero, E.; Schröder, E. Vanadium Pentoxide ( $V_2O_5$ ): A van der Waals Den-  
15 sity Functional Study. *Comput. Phys. Commun.* **2011**, *182*, 1805 – 1809, DOI:  
16 <https://doi.org/10.1016/j.cpc.2010.12.036>.
- 17 (37) Carrasco, J. Role of van der Waals Forces in Thermodynamics and Kinet-  
18 ics of Layered Transition Metal Oxide Electrodes: Alkali and Alkaline-Earth  
19 Ion Insertion into  $V_2O_5$ . *J. Phys. Chem. C* **2014**, *118*, 19599–19607, DOI:  
20 <https://doi.org/10.1021/jp505821w>.

- 21 (38) Zhou, B.; He, D. Raman Spectrum of Vanadium Pentoxide from Density-  
22 Functional Perturbation Theory. *J. Raman Spectrosc.* **2008**, *39*, 1475–1481, DOI:  
23 10.1002/jrs.2025.
- 24 (39) Smirnov, M. B.; Roginskii, E.; Kazimirov, V. Y.; Smirnov, K. S.; Baddour-Hadjean, R.;  
1 Pereira-Ramos, J.-P.; Zhandun, V. Spectroscopic and Computational Study of Struc-  
2 tural Changes in  $\gamma$ -LiV<sub>2</sub>O<sub>5</sub> Cathodic Material Induced by Lithium Intercalation. *J.*  
3 *Phys. Chem. C* **2015**, *119*, 20801–20809, DOI: 10.1021/acs.jpcc.5b05540.
- 4 (40) Bhandari, C.; Lambrecht, W. R. L.; van Schilfgaarde, M. Quasiparticle Self-Consistent  
5 GW Calculations of the Electronic Band Structure of Bulk and Monolayer V<sub>2</sub>O<sub>5</sub>. *Phys.*  
6 *Rev. B* **2015**, *91*, 125116, DOI: 10.1103/physrevb.91.125116.
- 7 (41) Cococcioni, M.; de Gironcoli, S. Linear Response Approach to the Calculation of the  
8 Effective Interaction Parameters in the LDA + U Method. *Phys. Rev. B* **2005**, *71*,  
9 035105, DOI: 10.1103/PhysRevB.71.035105.
- 10 (42) Mjejri, I.; Rougier, A.; Gaudon, M. Low-Cost and Facile Synthesis of the  
11 Vanadium Oxides V<sub>2</sub>O<sub>3</sub>, VO<sub>2</sub>, and V<sub>2</sub>O<sub>5</sub> and Their Magnetic, Thermo-chromic  
12 and Electrochromic Properties. *Inorg. Chem.* **2017**, *56*, 1734–1741, DOI:  
13 10.1021/acs.inorgchem.6b02880.
- 14 (43) Murphy, D. W.; Christian, P. A.; DiSalvo, F. J.; Waszczak, J. V. Lithium In-  
15 corporation by Vanadium Pentoxide. *Inorg. Chem.* **1979**, *18*, 2800–2803, DOI:  
16 10.1021/ic50200a034.
- 17 (44) Baddour-Hadjean, R.; Smirnov, M. B.; Kazimirov, V. Y.; Smirnov, K. S.; Pereira-  
18 Ramos, J.-P. The Raman Spectrum of the  $\gamma'$ -V<sub>2</sub>O<sub>5</sub> Polymorph: A Combined  
19 Experimental and DFT Study. *J. Raman Spectrosc.* **2015**, *46*, 406–412, DOI:  
20 10.1002/jrs.4660.

- 21 (45) Safrany Renard, M.; Emery, N.; Roginskii, E. M.; Baddour-Hadjean, R.; Pereira-  
22 Ramos, J. P. Crystal Structure Determination of a New Sodium Vanadium  
23 Bronze Electrochemically Formed. *J. Solid State Chem.* **2017**, *254*, 62–68, DOI:  
24 10.1016/j.jssc.2017.07.012.
- 25 (46) Smirnov, M. B.; Roginskii, E. M.; Smirnov, K. S.; Baddour-Hadjean, R.; Pereira-  
1 Ramos, J.-P. Unraveling the Structure-Raman Spectra Relationships in V<sub>2</sub>O<sub>5</sub> Poly-  
2 morphs via a Comprehensive Experimental and DFT Study. *Inorg. Chem.* **2018**, *57*,  
3 9190–9204, DOI: 10.1021/acs.inorgchem.8b01212.
- 4 (47) Gonze, X. et al. ABINIT: First-Principles Approach to Material and Nanosys-  
5 tem Properties. *Comput. Phys. Commun.* **2009**, *180*, 2582–2615, DOI:  
6 <https://doi.org/10.1016/j.cpc.2009.07.007>.
- 7 (48) Gonze, X. et al. A Brief Introduction to the ABINIT Software Package. *Z. Kristallogr.*  
8 *Cryst. Mater.* **2005**, *220*, 558–562, DOI: 10.1524/zkri.220.5.558.65066.
- 9 (49) Perdew, J. P.; Wang, Y. Accurate and Simple Analytic Representation of the  
10 Electron-Gas Correlation Energy. *Phys. Rev. B* **1992**, *45*, 13244–13249, DOI:  
11 10.1103/physrevb.45.13244.
- 12 (50) Blöchl, P. E. Projector Augmented-Wave Method. *Phys. Rev. B* **1994**, *50*, 17953–17979,  
13 DOI: <https://doi.org/10.1103/PhysRevB.50.17953>.
- 14 (51) Kresse, G.; Joubert, D. From Ultrasoft Pseudopotentials to the Projec-  
15 tor Augmented-Wave Method. *Phys. Rev. B* **1999**, *59*, 1758–1775, DOI:  
16 <https://doi.org/10.1103/PhysRevB.59.1758>.
- 17 (52) Murnaghan, F. D. The Compressibility of Media under Extreme Pressures. *PNAS*  
18 **1944**, *30*, 244–247, DOI: <https://doi.org/10.1073/pnas.30.9.244>.

- 19 (53) Monkhorst, H. J.; Pack, J. D. Special Points for Brillouin-Zone Integrations. *Phys. Rev.*  
20 *B* **1976**, *13*, 5188–5192, DOI: <https://doi.org/10.1103/PhysRevB.13.5188>.
- 21 (54) Aulbur, W. G.; Jönsson, L.; Wilkins, J. W. *Solid State Phys.*; Academic Press, 2000;  
22 Vol. 54; pp 1–218.
- 23 (55) Lebègue, S.; Arnaud, B.; Alouani, M.; Bloechl, P. E. Implementation of an All-Electron  
1 GW Approximation Based on the Projector Augmented Wave Method Without Plas-  
2 mon Pole Approximation: Application to Si, SiC, AlAs, InAs, NaH, and KH. *Phys.*  
3 *Rev. B* **2003**, *67*, DOI: [10.1103/physrevb.67.155208](https://doi.org/10.1103/physrevb.67.155208).
- 4 (56) Patrick, C. E.; Giustino, F. GW Quasiparticle Bandgaps of Anatase TiO<sub>2</sub>  
5 Starting from DFT + U. *J. Phys. Condens. Matter* **2012**, *24*, 202201, DOI:  
6 [10.1088/0953-8984/24/20/202201](https://doi.org/10.1088/0953-8984/24/20/202201).
- 7 (57) Himmetoglu, B.; Floris, A.; de Gironcoli, S.; Cococcioni, M. Hubbard-Corrected DFT  
8 Energy Functionals: the LDA+*U* Description of Correlated Systems. *Int. J. Quantum*  
9 *Chem.* **2013**, *114*, 14–49, DOI: [DOI: 10.1002/qua.24521](https://doi.org/10.1002/qua.24521).
- 10 (58) Levine, Z. H.; Allan, D. C. Linear Optical Response in Silicon and Germa-  
11 nium Including Self-Energy Effects. *Phys. Rev. Lett.* **1989**, *63*, 1719–1722, DOI:  
12 [10.1103/physrevlett.63.1719](https://doi.org/10.1103/physrevlett.63.1719).
- 13 (59) The official github mirror of the ABINIT repository.
- 14 (60) Wu Q. H., J. W., Thissen A. Photoelectron Spectroscopic Study of Li Intercalation into  
15 V<sub>2</sub>O<sub>5</sub> Thin Films. *Surf. Sci.* **2005**, *578*, 203–212, DOI: [10.1016/j.susc.2005.01.042](https://doi.org/10.1016/j.susc.2005.01.042).
- 16 (61) Wu Q. H., J. W., Thissen A. Photoelectron Spectroscopic Study of Na In-  
17 tercalation into V<sub>2</sub>O<sub>5</sub> Thin Films. *Solid State Ion.* **2004**, *167*, 155–163, DOI:  
18 [10.1016/j.ssi.2003.12.016](https://doi.org/10.1016/j.ssi.2003.12.016).

- 19 (62) Shick, A. B.; Liechtenstein, A. I.; Pickett, W. E. Implementation of the LDA+ $U$  Method  
20 Using the Full-Potential Linearized Augmented Plane-Wave Basis. *Phys. Rev. B* **1999**,  
21 *60*, 10763–10769, DOI: 10.1103/physrevb.60.10763.
- 22 (63) Bennett, J. W.; Hudson, B. G.; Metz, I. K.; Liang, D.; Spurgeon, S.; Cui, Q.;  
23 Mason, S. E. A Systematic Determination of Hubbard  $U$  Using the GBRV  
24 Ultrasoft Pseudopotential Set. *Comput. Mater. Sci.* **2019**, *170*, 109137, DOI:  
1 10.1016/j.commatsci.2019.109137.
- 2 (64) Porsev, V. V.; Bandura, A. V.; Evarestov, R. A. Hybrid Hartree-Fock-Density Func-  
3 tional Theory Study of  $V_2O_5$  Three Phases: Comparison of Bulk and Layer Sta-  
4 bility, Electron and Phonon Properties. *Acta Materialia* **2014**, *75*, 246 – 258, DOI:  
5 <https://doi.org/10.1016/j.actamat.2014.04.068>.
- 6 (65) Carrier, P.; Rohra, S.; Görling, A. General Treatment of the Singularities in Hartree-  
7 Fock and Exact-Exchange Kohn-Sham Methods for Solids. *Phys. Rev. B* **2007**, *75*,  
8 205126, DOI: 10.1103/PhysRevB.75.205126.
- 9 (66) Wang, L.; Maxisch, T.; Ceder, G. Oxidation Energies of Transition Metal Ox-  
10 ides within the GGA +  $U$  Framework. *Phys. Rev. B* **2006**, *73*, 195107, DOI:  
11 10.1103/PhysRevB.73.195107.
- 12 (67) Da Silva, J. L. F.; Ganduglia-Pirovano, M. V.; Sauer, J. Formation of the Cerium  
13 Orthovanadate  $CeVO_4$ : DFT +  $U$  Study. *Phys. Rev. B* **2007**, *76*, 125117, DOI:  
14 10.1103/PhysRevB.76.125117.
- 15 (68) Goclon, J.; Grybos, R.; Witko, M.; Hafner, J. Relative Stability of Low-Index  
16  $V_2O_5$  surfaces: A Density Dunctional Investigation. *J. Phys.: Condens. Matter* **2009**,  
17 *21*, 095008, DOI: 10.1088/0953-8984/21/9/095008.
- 18 (69) Gallardo-Amores, J. M.; Biskup, N.; Amador, U.; Persson, K.; Ceder, G.; Morán, E.;  
19 Arroyo y de Dompablo, M. E. Computational and Experimental Investigation of the

- 20 Transformation of  $V_2O_5$  Under Pressure. *Chem. Mater.* **2007**, *19*, 5262–5271, DOI:  
21 10.1021/cm071360p.
- 22 (70) Muller-Bouvet, D.; Baddour-Hadjean, R.; Tanabe, M.; Huynh, L.; Le, M.;  
23 Pereira-Ramos, J. Electrochemically Formed  $\alpha'$ - $NaV_2O_5$ : A New Sodium  
24 Intercalation Compound. *Electrochim. Acta* **2015**, *176*, 586 – 593, DOI:  
1 <https://doi.org/10.1016/j.electacta.2015.07.030>.