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The Pyramid of Complexity Interpreted as an Energy Lattice With the Energy Operators

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Abstract

The Pyramid of Complexity (PoC) is a multi-levels structure classifying our knowledge of the matter from particles to more complex organisms [1]. We are here introducing the PoC using the family of energy operators defined in our previous works [3, 5, 6]. To do so, we formulate three postulates to define the application of the energy operator theory to the PoC. We also associate a PoC with a specific space-time metric which is the basis to define the energy operators in a specific referential. The theory is then applied using two examples related to the electromagnetic and gravitational coupling defined in [3]. The first example is related to an electromagnetic cavity, whereas the second one is based on the human heart.

1. Introduction

1.1. General Concept of the Pyramid of Complexity

The *Pyramid of Complexity* (PoC) is a multi-levels structure defined in [1] and classify our knowledge of the matter from the basic elements and particles (quarks, nucleons, ..) at the bottom of the pyramid, to the multi-level complexity defining advanced organisms in the highest level of the PoC. We call advanced organisms, those ones constituted with various levels of the PoC, i.e. cells, biomolecules, atoms, ... etc. With our expanding knowledge and understanding of our environment, the PoC is always subject to changes from both the top and the bottom of the pyramid. Therefore, we can wonder what is the structure smaller than the quark (or the precedent level in the PoC). At the opposite what are the most complex structures (solar system, cluster of galaxies, ...) at the top of the pyramid?

The PoC has been introduced in relationship with the entropy of the universe [1]. Structures or elements in the PoC arise, because the process cannot occur fast enough in the expanding universe to maintain equilibrium. The second assumption is that each type of fundamental forces would always form the objects which are the most stable from its own perspective. For example, strong forces would turn all nuclei into iron; electric force would turn all atoms into noble gases [1]. Beyond the definition of the PoC, some works have argued in favor of a multiverse where the PoC should be defined for various subsets constituting it. This work does not include the multiverse versus universe debate.

Note that fundamental forces are here defined as the four forces, which are the interactions that do not appear to be reducible to more basic interactions. Namely, the gravitational and electromagnetic interactions, which produce significant long-range forces whose effects can be seen directly at the macroscale; and the strong and weak interactions, which produce forces at microscale or subatomic distances governing

nuclear interactions. Furthermore, the PoC associates each element with a scale magnitude, i.e. a quark is in the magnitude of 10^{-20} cm, the human being is around 10^2 cm, whereas our planetary system orbiting the sun is in the 10^{15} cm [1]. Each structure or object within a level of the PoC can interact with other structures in the other levels of the PoC. In the following, the structures or objects can be called observers. This work is the development of these three postulates.

Postulate 1. *Each level of the PoC classifying a type of structure can be associated with an energy level in relationship with its interaction between one and several fundamental forces.*

Postulate 2. *The PoC is an energy lattice associated with a nominated space-time referential and a class of energy operators.*

Postulate 3. *Each structure within a PoC is intrinsically defined by a variation of energy resulting from its interaction with one or several fundamental forces.*

In the following, we discuss and define in details the formulation of these postulates.

1.2. Work Overview and Relationship with the Energy Operators

The energy operators are a generalization of the Teager-Kaiser energy operators, first introduced in the measure of instant energy of a periodic signal and found various signal processing applications [18]. The generalized energy operators with their conjugates were then redefined on the mathematical basis of the Schwartz space [5, 6] subspace of the L2-space [2, 7, 9]. With this definition, the generalized energy operators found some applications in the solutions of the wave equation [5, 6], linear and non-linear partial differential equations (NL/LPDEs) [2]) and more recently in the definition of an electromagnetic and gravitational coupling [3].

This work starts with the definition of the generalized energy operators associated with a particular space-time metric extending the previous definition given in [6]. Subsequently, [2] defined the concept of *multiplicity of solutions* where additional solutions of a given PDE are generated using the properties of these subspaces. Here, the different class of energy operators are used to define the PoC with different energy levels, hence following the general definition of an energy lattice. Note that the energy operators are introduced in the equation formalism by assuming that there is a variation of energy quantity associated with a given structure as described with *Postulate 3*.

1.3. Notation and Symbols

In this work, several symbols are used. The set of integer numbers \mathbb{Z} is sometimes called only for the positive integer such as \mathbb{Z}^+ or \mathbb{Z}_+^m (for a space with dimension m). When the integer 0 is not included, it is explicitly mentioned such as $\mathbb{Z}^+ - \{0\}$. The set of natural numbers is \mathbb{N} , with only the positive numbers defined as \mathbb{N}^+ . \mathbb{R} is the set of real numbers. Also, the Schwartz space is here called $\mathbf{S}^-(\mathbb{R}^m)$ which is the notation used in previous works such as [2] and [3]. Several notations describe the relationship between spaces such as intersection (\cap), union (\cup), inclusion (\subset , inclusion without the equality \subsetneq , inclusion with equality \subseteq). Reader can refer to [9] or advanced mathematical textbooks for more explanations.

2. Mathematical formulation

2.1. L-2 norm and Schwartz space

Here, we are recalling some definitions and results developed in [3] where the Schwartz space was defined using the L-2 norm (p equal to 2 for the L^p norm). It allows to state the Plancherel identity $\forall f \in L^2(\mathbb{R}^m)$:

$$\int_{\mathbb{R}^m} |f|^2 dt = \int_{\mathbb{R}^m} |\mathcal{F}(f)(\xi)|^2 d\xi \quad (1)$$

We restrict our work to the functions belonging to the Schwartz space $f \in \mathbf{S}^-(\mathbb{R}^m) \subset L^2(\mathbb{R}^m)$. In other words, these functions are finite energy function described in (see *Remark 1* of [3]). The Schwartz space $\mathbf{S}^-(\mathbb{R}^m)$ is defined as (for $m \in [1, 2]$ [2, 6], for $m \in \mathbb{Z}^+$ [3]:

$$\mathbf{S}^-(\mathbb{R}^m) = \{f \in \mathbf{C}^\infty(\mathbb{R}^m) \mid \|f\|_{\alpha, \beta} < \infty, \quad \forall \alpha, \beta \in \mathbb{Z}_+^m\} \quad (2)$$

where α, β are multi-indices and

$$\|f\|_{\alpha, \beta} = \sup_{t \in \mathbb{R}^m} |t^\beta D^\alpha f(t)| \quad (3)$$

Note that one can define $\mathbf{S}^-(\mathbb{R}^m)$ with $\forall \alpha, \beta \in \mathbb{Z}^m$, but we decide to use \mathbb{Z}_+^m following the development in the next sections. In [3] (see *Properties 1*), the properties of $\mathbf{S}^-(\mathbb{R}^m)$ are developed and discussed, in particular the stability with Fourier transform. It then leads to recall the equation:

for $f \in \mathbf{S}^-(\mathbb{R}^m)$, $k \in \mathbb{N}$

$$\begin{aligned} \sup_{\xi \in \mathbb{R}^m} \{|(1 + |\xi|^2)^{k/2} \mathcal{F}(f)(\xi)|\} &< \infty \\ \Leftrightarrow \exists a \in \mathbb{R}, \quad |(1 + |\xi|^2)^{k/2} \mathcal{F}(f)(\xi)| &\leq \frac{a}{1 + |\xi|^2} \end{aligned} \quad (4)$$

Now, let us recall the definition of the Hilbert spaces $H^{k,p}(\mathbb{R}^m)$ (Sobolev spaces $W^{k,2}(\mathbb{R}^m) = W^k(\mathbb{R}^m)$) (see [3] **Definition I.1**) :

$$W^k(\mathbb{R}^m) = H^k(\mathbb{R}^m) := \{f \in \mathbf{S}^{*, -}(\mathbb{R}^m) \mid (1 + |\xi|^2)^{k/2} \mathcal{F}(f) \in L^2(\mathbb{R}^m)\} \quad (5)$$

Note that $\mathbf{S}^{*, -}(\mathbb{R}^m)$ is the space of tempered distributions, dual of $\mathbf{S}^-(\mathbb{R}^m)$ via the Fourier transform. A function belongs to $L^2(\mathbb{R}^m)$ if and only if its Fourier transform belongs to $L^2(\mathbb{R}^m)$ and the Fourier transform preserves the L^2 -norm. As a result, the Fourier transform provides a simple way to define L^2 -Sobolev spaces on \mathbb{R}^m (including ones of fractional and negative order m [8]). Finally, the stability via Fourier transform is the key for $\mathbf{S}^-(\mathbb{R}^m) \subsetneq \mathbf{H}^k(\mathbb{R}^m)$.

Remark (3) Following the remark (*Remark 3.4* in [10]) and the general properties of the Fourier transform, one can state the equivalence relationship in $L^2(\mathbb{R}^m)$

$$\begin{aligned} f \in \mathbf{H}^k(\mathbb{R}^m) &\Leftrightarrow D^\alpha f \in L^2(\mathbb{R}^m) \forall |\alpha| \leq k \quad \Leftrightarrow \quad \mathcal{F}(D^\alpha f) \in L^2(\mathbb{R}^m) \forall |\alpha| \leq k \\ &\Leftrightarrow \xi^\alpha \mathcal{F}(f) \in L^2(\mathbb{R}^m) \forall |\alpha| \leq k \quad \Leftrightarrow \quad (1 + |\xi|^2)^{|\alpha|/2} \mathcal{F}(f) \in L^2(\mathbb{R}^m) \forall |\alpha| \leq k \end{aligned}$$

Using the definition of $\mathbf{H}^k(\mathbb{R}^m)$ and the properties of the Fourier transform, it is also possible to show that for $k > k'$, $\mathbf{H}^k(\mathbb{R}^m) \subset \mathbf{H}^{k'}(\mathbb{R}^m)$ [21], and the relationship $\mathbf{H}^0(\mathbb{R}^m) = L^2(\mathbb{R}^m)$. It is also possible to define $\mathbf{H}^\infty(\mathbb{R}^m) = \bigcap_{k \in \mathbb{N}} \mathbf{H}^k(\mathbb{R}^m)$ with $\mathbf{S}^-(\mathbb{R}^m) \subset \mathbf{H}^\infty(\mathbb{R}^m)$, and to extend this equality to $k \in \mathbb{R}$ following [10].

2.2. Energy Operators and a Flat Space-Time Metric

2.2.1 Definition and properties of the Energy operators in $\mathbf{S}^-(\mathbb{R}^m)$

Let us call the set $\mathcal{F}(\mathbf{S}^-(\mathbb{R}^m), \mathbf{S}^-(\mathbb{R}^m))$ all Schwartz functions (or operators) defined such as $\gamma : \mathbf{S}^-(\mathbb{R}^m) \rightarrow \mathbf{S}^-(\mathbb{R}^m)$. For $f \in \mathbf{S}^-(\mathbb{R}^m)$, let us define $\partial_i^k f$ ($k \in \mathbb{Z}$, $i \in [1, \dots, m]$), with f defined with the vector parameter $\mathbf{T} = [t_1, t_2, \dots, t_m] \in \mathbb{R}^m$ such as

$$\begin{cases} \partial_i^k f = & \frac{\partial^k f}{\partial t_i^k}, \quad \forall i \in [1, \dots, m], \quad \forall k \in \mathbb{Z}^+ - \{0\} \\ \partial_i^k f = & \int_{-\infty}^{t_i} (\dots (\int_{-\infty}^{\tau_2} f(t_1, t_2, \dots, \tau_1, t_{i+1}, \dots, t_m) d\tau_1) \dots) d\tau_k, \\ & \forall i \in [1, \dots, m], \quad \forall k \in \mathbb{Z}^- - \{0\} \\ \partial_i^0 f = & f, \quad \forall i \in [1, \dots, m] \end{cases} \quad (6)$$

Combining multiple integrals and derivatives justify the use of the Schwartz space $\mathbf{S}^-(\mathbb{R}^m)$ and echoes the choice made previously in [5] (see equation (10)). The definitions and results given in [5] and [6] in the case $\mathbf{S}^-(\mathbb{R})$ are now formulated for $\mathbf{S}^-(\mathbb{R}^m)$. Section 2 in [5] and Section 4 in [6] defined the energy operators

Ψ_k^+ , Ψ_k^- (k in \mathbb{Z}) and the generalized energy operators $[[\cdot]^p]_k^+$ and $[[\cdot]^p]_k^-$ (p in \mathbb{Z}^+). Following [6], let us define the energy operators with multi-index derivative:

$$\begin{aligned}\Psi_k^+(\cdot) &= \sum_{i=1}^m \partial_i^1 \cdot \partial_i^{k-1} \cdot + \partial_i^0 \cdot \partial_i^k \cdot \\ \Psi_k^+(\cdot) &= \sum_{i=1}^m \psi_{k,i}^+(\cdot) \\ [\cdot, \cdot]_k^+ &= \Psi_k^+(\cdot) \\ [\cdot, \cdot]_{k,i}^+ &= \psi_{k,i}^+(\cdot)\end{aligned}\tag{7}$$

Further more, we also use the short notation $[\cdot, \cdot]_k^+ = [\cdot]_k^+$ in the remainder of this work. Note that Ψ_k^- is the conjugate operator of Ψ_k^+ and $\psi_{k,i}^-$ respectively to $\psi_{k,i}^+$.

Remark (4) The families of (generalized) energy operators $([[\cdot]^p]_k^+)_{k \in \mathbb{Z}}$ and $([[\cdot]^p]_k^-)_{k \in \mathbb{Z}}$ (p in \mathbb{Z}^+) are also called families of differential energy operator (DEO) [5] [6].

Furthermore, [6] defined the generalized energy operators $[[\cdot]^1]_k^+$ and $[[\cdot]^1]_k^-$ ($k \in \mathbb{Z}$):

$$\begin{aligned}[[\cdot, \cdot]_{k,i}^+, [\cdot, \cdot]_{k,i}^+]^+ &= \partial_i^1 \psi_{k,i}^+(\cdot) \partial_i^{k-1} \psi_{k,i}^+(\cdot) + \partial_i^0 \psi_{k,i}^+(\cdot) \partial_i^k \psi_{k,i}^+(\cdot) \\ [[\cdot, \cdot]_{k,i}^+, [\cdot, \cdot]_{k,i}^+]^+ &= \partial_i^1 [[\cdot]^0]_{k,i}^+ \partial_i^{k-1} [[\cdot]^0]_{k,i}^+ + \partial_i^0 [[\cdot]^0]_{k,i}^+ \partial_i^k [[\cdot]^0]_{k,i}^+ \\ [[\cdot, \cdot]_k^+, [\cdot, \cdot]_k^+]^+ &= \sum_{i=1}^m [[\cdot, \cdot]_{k,i}^+, [\cdot, \cdot]_{k,i}^+]^+ \\ [[\cdot, \cdot]_k^+, [\cdot, \cdot]_k^+]^+ &= \sum_{i=1}^m [[\cdot]^1]_{k,i}^+ \\ &= [[\cdot]^1]_k^+\end{aligned}\tag{8}$$

By *iterating* the bracket $[\cdot]$, [6] defined the generalized operator $[[\cdot]^p]_{k,i}^-$ and the conjugate $[[\cdot]^p]_{k,i}^+$ with p in \mathbb{Z}^+ . Note that $[[f]^p]_{1,i}^- = 0 \forall p$ in \mathbb{Z}^+ and i in \mathbb{Z} .

Now, the derivative chain rule property and bilinearity of the energy operators and generalized operators (for i in $[1, 2]$) are shown respectively in [5], Section 2 and [6], Proposition 3. The generalisation of this property to i in $[1, \dots, m]$ for the operators $\psi_{k,i}^+(\cdot)$, $\psi_{k,i}^-(\cdot)$, $[[\cdot]^p]_{k,i}^-$ and $[[\cdot]^p]_{k,i}^+$ ($k \in \mathbb{Z}$, $p \in \mathbb{Z}^+$) is trivial due to the linearity of the derivatives and integrals when defining ∂_i^k in (6). Due to the linearity of the sum, the bilinearity property is also generalized to $\Psi_k^+(\cdot)$, $\Psi_k^-(\cdot)$, $[[\cdot]^1]_k^+$ and $[[\cdot]^1]_k^-$ ($k \in \mathbb{Z}$, $p \in \mathbb{Z}^+$).

2.2.2 Association with a Space Time Metric

For the following sections related to the physical meaning of the energy operators, we introduce a space-time metric. In other words, we need to associate the energy operators definition with a metric. Therefore, we restrict \mathbb{R}^m , with specific value of m in \mathbb{Z}^+ . The spacetime metric captures all the geometric and causal structure of spacetime in the theory of general relativity [23], being used to define notions such as time, distance, volume, curvature, angle, and separating the future and the past. As a first example, We are interested in the four dimension associating 3D space and time with a signature $(-, +, +, +)$ also called a flat space time defined as:

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2\tag{9}$$

where interval ds^2 imparts information about the causal structure of spacetime. When $ds^2 < 0$, the interval is time like and the square root of the absolute value of ds^2 is an incremental proper time. Only time like

intervals can be physically traversed by a massive object. When $ds^2 = 0$, the interval is lightlike, and can only be traversed by light. When $ds^2 > 0$, the interval is spacelike and the square root of ds^2 acts as an incremental proper length [23]. Thus, one can write the partial differential equation:

$$ds = \frac{\partial s}{\partial t} dt + \frac{\partial s}{\partial x} dx + \frac{\partial s}{\partial y} dy + \frac{\partial s}{\partial z} dz \quad (10)$$

In [5, 3], the generalized energy operators are defined omitting the cross-derivatives. Thus, one can define the energy operator with the space time metric s following the previous section:

$$\begin{aligned} \psi_{k,x}^+(\cdot) &= \partial_x^1 \cdot \partial_x^{k-1} \cdot + \partial_x^0 \cdot \partial_x^k \\ \Psi_{k,s}^+(\cdot) &= \psi_{k,x}^+(\cdot) + \psi_{k,y}^+(\cdot) + \psi_{k,z}^+(\cdot) + \psi_{k,t}^+(\cdot) \\ [\cdot, \cdot]_{k,x}^+ &= \psi_{k,x}^+(\cdot) \\ [\cdot, \cdot]_{k,s}^+ &= \Psi_{k,s}^+(\cdot) \end{aligned} \quad (11)$$

Now, let us take another example, the Friedmann–Lemaître–Robertson–Walker (FLRW) metric. The FLRW metric starts with the assumption of homogeneity and isotropy of space. It also assumes that the spatial component of the metric can be time-dependent, defined as (in Cartesian coordinates):

$$\begin{aligned} -c^2 ds^2 &= -c^2 dt^2 + a(t)^2 d\Sigma^2 \\ ds^2 &= dt^2 - \frac{a(t)^2}{c^2} d\Sigma^2 \\ d\Sigma^2 &= dx^2 + dy^2 + dz^2 \end{aligned} \quad (12)$$

where $d\Sigma$ ranges over a 3D space of uniform curvature, that is, elliptical space, Euclidean space, or hyperbolic space. It is normally written as a function of three spatial coordinates as shown above in Cartesian coordinates. Note that all of the time dependence is in the function $a(t)$, known as the scale factor. For more information, a reader can refer to [23]. Now, we can define the energy operators as above using the same space time basis (i.e. $\Psi_{k,s}^+(\cdot)$). Thus, we can conclude that with the choice of a given metric associated with a specific space-time referential, we then define the energy operators.

2.2.3 Energy Spaces, Subspaces of the Hilbert space

The energy spaces and their properties were defined in [2] and [3]. Here are the main definitions based only on the formalism in [3]. Properties and theorems are recalled in the appendices.

Definition 3 [2], *Definition 3*: The energy space $\mathbf{E}_p \subsetneq \mathbf{S}^-(\mathbb{R}^m)$, with p in \mathbb{Z}^+ , is equal to $\mathbf{E}_p = \bigcup_{v \in \mathbb{Z}^+ \cup \{0\}} \mathbf{M}_p^v$.

with $\mathbf{M}_p^v \subsetneq \mathbf{S}^-(\mathbb{R}^m)$ for v in \mathbb{Z}^+ defined as

$$\mathbf{M}_p^v = \{g \in \mathbf{S}^-(\mathbb{R}^m) \mid g = \partial_i^k ([f]^p)_1^+)^n, [f]^p_1^+ \in \mathbf{S}^-(\mathbb{R}^m), k \in \mathbb{Z}^+, \forall k \leq v, n \in \mathbb{Z}^+ - \{0\}, i \in [1, \dots, m]\} \quad (13)$$

The energy spaces, $\mathbf{S}_p^-(\mathbb{R}^m)$ and $\mathbf{s}_p^-(\mathbb{R}^m)$ ($p \in \mathbb{Z}^+$), cited in **Lemma 2** (in [3] and recall in the appendices) and **Theorem 2** (in [3] and recall in the appendices) are defined:

$$\begin{aligned} \mathbf{S}_p^-(\mathbb{R}^m) &= \{\mathbf{E}_p = \bigcup_{i \in \mathbb{Z}^+ \cup \{0\}} \mathbf{M}_p^i\} \\ \mathbf{s}_p^-(\mathbb{R}^m) &= \{f \in \mathbf{S}_p^-(\mathbb{R}^m) \mid f \notin \bigcup_{i \in [1, \dots, m]} (\bigcup_{k \in \mathbb{Z}} \text{Ker}([f]^p)_{k,i}^+) \cup (\bigcup_{k \in \mathbb{Z} - \{1\}} \text{Ker}([f]^p)_{k,i}^-)\} \end{aligned} \quad (14)$$

Now, let us recall Properties 2 in [3] where it was shown in this previous work the relationship with the Hilbert spaces $\mathbf{H}^k(\mathbb{R}^m)$.

Properties 2: $\forall v$ in \mathbb{Z}^+ , and in particular v_1, v_2 in \mathbb{Z}^+ (with $v_1 < v_2$), p in \mathbb{Z}^+ , we have the following inclusions:

- 1- $\mathbf{M}_p^v \subsetneq \mathbf{H}^v(\mathbb{R}^m)$
- 2- $\mathbf{M}_p^{v_2} \subsetneq \mathbf{M}_p^{v_1}$
- 3- $\mathbf{E}_p = \bigcup_{v \in \mathbb{Z}^+ \cup \{0\}} \mathbf{M}_p^v \subsetneq \mathbf{H}^0(\mathbb{R}^m)$

Remark From [3] (**Definition I.1**), let us recall the definition of the Hilbert space on \mathbb{R}^m .

$$\mathbf{H}^v(\mathbb{R}^m) = \{f \in L^2(\mathbb{R}^m) | D^\alpha f \in L^2(\mathbb{R}^m), \forall |\alpha| \leq v\} \quad (15)$$

Looking at the definition of the energy space \mathbf{M}_p^v and $\mathbf{H}^v(\mathbb{R}^m)$, one can notice the similitude. However, the multi-index derivative D^α ([22]) contains also the cross-derivatives (e.g., $\frac{\partial^2}{\partial t_1 \partial t_2}$), whereas there are no cross-derivatives in the definition of ∂_i^v in [3] (**Appendix I**). Thus, the energy spaces \mathbf{M}_p^v ($p \in \mathbb{Z}^+$, $v \in \mathbb{Z}^+ - \{0\}$) is defined without the cross-derivatives.

Now, we call a *class of energy operators*, all energy operators (or the family of energy operators) associated with the energy subspace \mathbf{E}_p with p in \mathbb{Z}^+ defined above. In other words, p defines the class of energy operators. To recall *Postulate 2*, the definition of the energy operators with a space-time metric and what we call *class of energy operators*, we then implicitly associate an energy space (\mathbf{E}_p) to a given PoC.

Remark X1 Furthermore, *Postulate 1* and *Postulate 2* also imply that for each level of the PoC defining specific structures at a nominated scale, the class of energy operators is associated with each structure. This definition is based on *Postulate 3*, where we assume that each structure is associated with an intrinsic energy variation (due to one or several interactions with the fundamental forces) and thus directly related to the energy operators. The relationship with an energy variation is formulated in the general case by *Proposition 1* in [3] and recalled in the appendices as *Proposition I.1*. Therefore there is a direct relationship between the structures and the energy spaces via the energy operators.

3. Discussions Using Examples with the Electromagnetic and Gravitational Coupling

In this section, we discuss two examples based on the EM and Gravitational (EM & G) coupling which was defined in [3] based on the Woodward effect. The Woodward effect, also referred to as a Mach effect, is part of a hypothesis proposed by James F. Woodward in 1990 [4]. The hypothesis states that transient mass fluctuations arise in any object that absorbs internal energy while undergoing a proper acceleration. Note that the Woodward effect is derived in [4] using the FLRW metric (see Section 2).

Recently, the Woodward effect was applied to asymmetric EM cavities (i.e. frustum) due to EM waves reflected on the cavity's wall, and creating a momentum [3, 11]. The assumption is that the EM energy density variation results from the evanescent waves taking place in the skin depth of the asymmetric EM cavity's walls.

The second example is the human heart where we also apply the hypothesis of the EM & G coupling.

3.1. Example of EM Cavity

The Woodward effect is based on a formula which the author implicitly assumed that the rest mass of the piezoelectric material via the famous Einstein's relation in special relativity $\mathcal{E} = mc^2$ (\mathcal{E} the rest energy associated with the rest mass m) and its variation via electrostrictive effect. In order to apply this formula to an asymmetric EM cavity, the author in [3] formulated the hypothesis that the EM excitation on the walls creates electric charges (i.e. electrons) which makes the rest mass varying with time. The Woodward

effect can be mathematically derived in various ways [13, 14, 17]. Note that in the appendices, we also show a derivation based on the model of a point mass particle moving in an electric field.

If we define the mass density such as $\rho = m/V$, then from [17], one can write the elementary mass variation per unit of volume

$$\begin{aligned}\delta\rho &= \frac{\delta m}{V} \\ &\sim d\rho \text{ (infinitesimally small variation)} \\ d\rho &= \frac{1}{4\pi G} \left[\frac{1}{\rho} \partial_t^2 \rho - \frac{1}{\rho^2} (\partial_t \rho)^2 \right]\end{aligned}\tag{16}$$

Let us define the the rest energy $\mathcal{E} = \rho c^2$, then

$$\begin{aligned}d\rho &= \frac{1}{4\pi G} \left[\frac{1}{\rho c^2} \partial_t^2 \mathcal{E} - \frac{1}{(\rho c^2)^2} (\partial_t \mathcal{E})^2 \right] \\ d\rho &= \frac{1}{4\pi G} \left[\frac{1}{\mathcal{E}} \partial_t^2 \mathcal{E} - \frac{1}{(\mathcal{E})^2} (\partial_t \mathcal{E})^2 \right]\end{aligned}\tag{17}$$

In some particular cases such as an EM cavity, we assume that the variation in time of the rest energy is equal to the variation of EM energy density u (i.e. $\partial_t \mathcal{E} \simeq \partial_t u$), but the rest energy is much bigger than the EM energy density $\mathcal{E} \gg u$. It allows then to state the relationship between the Woodward effect and the EM energy density

$$d\rho = \frac{1}{4\pi G} \left[\frac{1}{\mathcal{E}} \partial_t^2 u - \frac{1}{(\mathcal{E})^2} (\partial_t u)^2 \right]\tag{18}$$

The EM energy density u follows the general definition of the sum of energy density from the electric (u_E) and magnetic (u_B) fields [12]. Note that in [3], the author defines the Electro magnetic and gravitational coupling using equation (18).

Discussion (2) The above equation shows that the variation of mass density is a linear relationship with the first and second derivative of the EM energy density. To recall [3], we underline the relationship between the order of the derivatives of the EM energy density and the energy spaces. As we are dealing with evanescent waves (functions in $\mathbf{S}^-(\mathbb{R}^m) \subset L^2(\mathbb{R}^m)$ with $m = 4$, with $3D$ space and time) in the skin layer of the EM cavity, we can apply the results of Example 2 in [3] and [11], with the multiplicity of the solutions (i.e. *Theorem 1*). The interpretation of the Woodward effect using the energy parallax is that the solutions are in \mathbf{M}_0^k (k in $\{0, 1, 2\}$) using the same definitions as in Section 2. In other words, we need to take into account the evanescent waves associated with the electric and magnetic fields and their first and second derivative in time.

3.2. The Heart Modelled as an EM and Gravitational Coupling

Electric fields due to all muscle cells produce voltage variations in the body that are sensed by Electro Cardiogram (ECG). The voltages vary with time indicating the depolarization (contraction) of the right and left ventricles during systole (*QRS* complex), the repolarization of the atria masked by this, and the repolarization of the ventricles (*T* wave). The difference in potential across the cell membrane of the cardiac muscle cells changes during the depolarization and subsequent repolarization of artial and ventricular heart muscles during each cycle, and this changes the electric potential near the heart.

Some works have investigated to model this depolarization and subsequent polarization sequences as a capacitor at the cell level and inside the heart tissues. Here, we base our assumptions on a multipole heart model following [16]. The capacitor model is generally formulated at the single cell level (ventricular) and

the heart tissue following:

$$\begin{aligned}\frac{dV}{dt} &= -\frac{I_{ion} + I_{stim}}{C_m} \quad (single \text{ cell}) \\ C_m \frac{dV}{dt} &= -(I_{ion} + I_{stim}) + \nabla \cdot (D \nabla V) \quad (tissu \text{ model})\end{aligned}\tag{19}$$

$$\frac{dV}{dt} = \frac{I}{C}\tag{20}$$

where V is the voltage across the membrane, t is time, I_{ion} is the total transmembrane current, I_{stim} is the stimulus current applied externally, and C_m is the capacitance of the cell. D is the diffusion tensor describing the conductivities of the tissue along different directions. Thus, the equation for the single cell (first equation in Eq. (19)) is a direct analogy at a capacitance when comparing with the general equation of the capacitor Eq. (20), whereas the tissue model is nonlinear with the diffusion term $\nabla \cdot (D \nabla V)$. For the general purpose of our comparison of the heart cycle with a charge/discharge capacitance, we will not go into the modelling of this diffusion phenomenon. In addition, we consider the heart as a multipole due to each cell charging and discharging following the heart cycle. Now, one can consider each cell with a capacitance C_i and a charge q_i . The total voltage is then equal to:

$$\begin{aligned}V(t) &= \sum_{i=1}^N \frac{q_i(t)}{C_i} \\ V(t) &= \frac{q(t)}{C} + \frac{dq(t)}{C \cdot dt}\end{aligned}\tag{21}$$

Therefore, the heart is now considered as a relativistic capacitor as defined in [11]. Here, the variations of charge is assumed to be over each ECG cycle due to possible inhomogeneity of the cells [15], hence creating a slight variations in to the average voltage over a cycle.

Before discussing the application of the Woodward effect to heart modeled as a capacitance using the EM & G coupling, one need to carefully recall some important hypothesis how the Woodward effect was derived in the first place. From [17], it is stated that the lowest order of *mass-energy density moment* possible to trigger any dynamics in general relativity is the quadrupole. Nothing can then be triggered with a monopole or a dipole. The reason for the non-existence of monopole and dipole gravitational waves is that the quanta of gravitation (assuming the quantization of general relativity), the "graviton", would be a massless particle with spin 2, and therefore it is impossible to put a graviton in a state consistent with monopole or dipole radiation. A graviton can have only two helicity states (the projection of the graviton's spin onto its direction of momentum). Some works, i.e. [13], have discussed an alternative way using the Hoyle-Narlikar theory, where an additional scalar field is introduced, the so-called mass field. An equivalent term of the original Woodward effect is then derived. This theory supports the excitation of monopole waves, positing the existence of a (unverified) particle called the "dilaton".

Various models of the human heart have been developed such as the dipole, quadrupole [19] and multipole [20]. Our example does not investigate these different models. In our above description of the heart, we justify the use of a multipole in our assumption due to the cells charging/discharging along the cycle such as capacitors, hence postulating that the existence of the Woodward effect.

Based on the previous example, the Woodward effect can only take place if there is a variation of energy quantity. We then need to make the assumption that the charge/discharge of electricity in each cell along the heart cycle, can be formulated in terms of variation of EM energy density and then induce a variation of mass via an EM & G coupling with Eq. (17). This assumption is formulated such as a variation of mass of a capacitance similar to the relative capacitance model in [11] described with Eq. (21). Now, we would like to give an order of magnitude of this effect for this specific case. However, modeling each cell with the relative capacitance model is not straightforward. Instead, we will consider the whole heart cycle modeled

as a relative capacitance. This gross approximation allows us to give an order of magnitude to the terms in the original Woodward equation [17]. Therefore, we quantify the Woodward effect following [11] such as:

$$\begin{aligned}\delta\rho_0(t) &= \frac{1}{4\pi G} \left[\frac{1}{\rho_0 c^2} \partial_t^2 \mathcal{E} - \left(\frac{1}{\rho_0 c^2} \right)^2 (\partial_t \mathcal{E})^2 \right] \\ \delta\rho_0(t) &\simeq \frac{1}{4\pi G} \left[\frac{1}{\rho_0 c^2} \partial_t^2 \mathcal{E} \right]\end{aligned}\quad (22)$$

ρ_0 is the transient mass source and c speed of light. Considering a rest energy \mathcal{E} , roughly the energy of the capacitor at rest, one can state the famous Einstein's relationship in special relativity between \mathcal{E} and the rest mass m , $\mathcal{E} = mc^2$. In [3] and [11], we justify the assumption that the variation with time of \mathcal{E} equal the variation of EM energy density with the capacitor model and replacing the transient mass source ρ_0 with the mass density $\delta\rho = \frac{\delta m}{V}$. Therefore, if we give an order of magnitude of $\partial_t^2 \mathcal{E} \sim 1$ [mV], $c^2 \sim 9.10^{16}$ [m/s], $\rho_0 \sim 10^1$ to 10^5 [Kg/L] (adult), and $\frac{1}{4\pi G} \sim 1/(6.10^{-10})$, we obtain $\delta\rho_0 \sim [2.10^{-9} - 2.10^{-13}]$. This is a very small mass variation for an adult (70 Kg, with a heart volume of 1 L).

Finally, [24] concluded in his study of the Woodward effect that to see any fluctuations due to a Machian effect one needs to take into account the ratio of the mass of the device divided by a general approximation of the total mass of the matter in our observable universe (10^{54} Kg). Knowing that the mass of the human heart is around 0.3 to 0.4 Kg, we can quickly see that the above ratio is infinitesimally small, hence practically insignificant and therefore down-scaling the possibility of measuring such effect at the human scale. Note that [25] and [26] have argued against this development quoting *[24] claims that such rest mass fluctuations with significant magnitude, save in the vicinity of black holes, are not to be found in general relativity notwithstanding that they appear in the relativistic Newtonian approximation. [24] is mistaken in this claim, as I have already explained at some length in [25]*.

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Appendix I : Some Definitions

Definition I.1 [5]: $\forall f$ in $\mathbf{S}^-(\mathbb{R}^m)$, $\forall v \in \mathbb{Z}^+ - \{0\}$, $\forall n \in \mathbb{Z}^+$ and $n > 1$, the family of operators $(G_k)_{k \in \mathbb{Z}}$ (with $(G_k)_{k \in \mathbb{Z}} \subseteq \mathcal{F}(\mathbf{S}^-(\mathbb{R}^m), \mathbf{S}^-(\mathbb{R}^m))$) decomposes $\partial_i^v f^n$ in \mathbb{R}^m ($i \in [1, \dots, m]$), if it exists $(N_j)_{j \in \mathbb{Z}^+ \cup \{0\}} \subseteq \mathbb{Z}^+$, $(C_l)_{l=-N_j}^{N_j} \subseteq \mathbb{R}$, and it exists (α_j) and r in $\mathbb{Z}^+ \cup \{0\}$ (with $r < v$) such as $\partial_i^v f^n = \sum_{j=0}^{v-1} \binom{v-1}{j} \partial_i^{v-1-j} f^{n-r} \sum_{u=-N_j}^{N_j} C_u G_u(\partial_i^{\alpha_u} f)$.

In addition, one has to define $\mathbf{s}^-(\mathbb{R}^m)$ as:

$$\mathbf{s}^-(\mathbb{R}^m) = \{f \in \mathbf{S}^-(\mathbb{R}^m) | f \notin (\cup_{k \in \mathbb{Z}} \text{Ker}(\Psi_k^+)) \cup (\cup_{k \in \mathbb{Z} - \{1\}} \text{Ker}(\Psi_k^-))\} \quad (23)$$

or with the energy operators $\psi_{k,i}^+$ and $\psi_{k,i}^-$ defined in (7)

$$\mathbf{s}^-(\mathbb{R}^m) = \{f \in \mathbf{S}^-(\mathbb{R}^m) | f \notin \cup_{i \in [1, \dots, m]} (\cup_{k \in \mathbb{Z}} \text{Ker}(\psi_{k,i}^+(f)) \cup (\cup_{k \in \mathbb{Z} - \{1\}} \text{Ker}(\psi_{k,i}^-(f)))\} \quad (24)$$

$\text{Ker}(\cdot)$ is the notation for the kernel associated here with the operators Ψ_k^+ , Ψ_k^- , $\psi_{k,i}^+$ and $\psi_{k,i}^-$ (k in \mathbb{Z}) (see [5], *Properties* 1 and 2). By definition, one can state that $\mathbf{s}^-(\mathbb{R}^m) \subsetneq \mathbf{S}^-(\mathbb{R}^m)$. Following **Definition** 1, the *uniqueness* of the decomposition in $\mathbf{s}^-(\mathbb{R}^m)$ with the families of differential operators can be stated as:

Proposition I.1 (*Proposition 1 in [3]*): *If for $n \in \mathbb{Z}^+$, $f^n \in \mathbf{S}^-(\mathbb{R}^m)$ and analytic; for any $(p_i, q_i) \in \mathbb{R}^2$ and $\tau_i \in [q_i, p_i]$ ($\forall i \in [1, \dots, m]$), and $\mathcal{E}(f^n)$ is analytic, where*

$$\mathcal{E}(f^n(\tau_i)) = \int_{q_i}^{\tau_i} |f^n(t_i)|^2 dt_i < \infty \quad (25)$$

then

$$\mathcal{E}(f^n(p_i)) = \mathcal{E}(f^n(q_i)) + \sum_{k=0}^{\infty} \partial_{t_i}^k (f^n(q_i))^2 \frac{(p_i - q_i)^k}{k!} < \infty \quad (26)$$

is a convergent series.

Note that the proof is given in P.4 of [2] and discuss in P.9 of [3].

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