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Bibliographical anachronism: *The Art of Fugue* by Johann Sebastian Bach.

Konstantinos Alevizos
alevizos@prism.cnrs.fr
Aix-Marseille Univ, CNRS, PRISM, Marseille, France

A. Criteria to Establish the Progression[1].

The Art of Fugue is a collection of compositions in an austere contrapuntal style, all of which are based on the same subject. The latest research on the score reveals that Bach started this copious work around 1742[2]. However, it is known that the first edition was published in 1751[3]. Today, many different sources of the score exist, primarily including the autograph manuscript P200 and a limited series of copies of the first and second original editions of 1751 and 1752, respectively[4]. Over time, many unsettled issues have continued to vex scholars and musicians. These issues can be summarised as follows:

a. The explanation of the differences in the music score between manuscript P200 and the original edition.
b. The ordering of the composition.
c. The instrumentation of the work.

Unfortunately, none of these questions has been clarified based on historic or analytical evidence, thus it is impossible to provide a clear qualification of the information concerning *The Art of Fugue*. Regarding the ordering issue, it is of paramount importance to distinguish the material known today in relation to Bach's last position. Even if it were possible to establish an order that appears probable and is supported by evidence, it is unlikely we will be able to confirm this order was the one desired by Bach. Perhaps new information will come to light, and the situation will change in the future. For the moment, it appears that the answer is missing.

I believe that only an analysis of the musical score can provide the necessary indications and enable us to establish an order. It is therefore my conviction that family or professional anecdotes have led us away from the original order and the composer's intentions in this regard. In addition, it is imperative to determine the importance of manuscript P200, that is which was its intention. The justification of the ordering of the Fugues lies in the score, and certainly more attention must be given to the manuscript and its relation to the original edition's score than to the edition itself.

It also seems reasonable to presume that C. P. E. Bach (who is considered one of those responsible for the edition and perhaps the most important) would have used any part with

[1] In this text, all fugue numbers follow the Schmieder catalogue, BWV 1080.
[2] Christoph Wolff, *Johann Sebastian Bach: the learned musician*, Norton & Company, (New York, 2001), 433. Zoltán Göncz, op. cit., 118.
[3] The first edition was entitled *Die Kunst der Fuge durch Herrn Johann Sebastian Bach ehemahligen Capellmeister und Musikdirector zu Leipzig [sic]*.
[4] Today, five examples of the first edition of 1751 and eighteen of the second edition of 1752 remain in existence. Neue Ausgabe Samtlich Werke, *Die Neue Bach Ausgabe*, (Kassel, 1996).

relevance (to him) for the publication. We can be certain that apart from the known pieces, he did not find other manuscripts or related material. However, if so, why aren't the missing manuscripts included in P200?[5] If Wolff, as others, affirms that manuscript P200 is a fair (clear) copy[6], for what purpose was it written in the early forties and edited almost ten years later?

Based on these considerations and by using the modifications observed in the first edition, this article proposes an ordering for *The Art of Fugue* in relation to the manuscript. Through a detailed analysis of the entire work, it is possible to propose a detailed comparison of all the Fugues while trying to place them in the most credible, logical and probable order. Before starting on this thorough examination, it is necessary to qualify (having accepted the concept of the progression of complexity within the work)[7] the element (or elements) that characterises this progression. If, however, the concept of progression is not to be accepted, all these elements can be mixed.

- The subject and its rhythmic developments, rectus/inversus (the morphology of construction of the Fugues).
- The level of technicality of the counterpoint and its relation to the Fugues.
- The order of the Fugues in manuscript P200 compared with that of the first edition[8].
- Anecdotal (and not) and/or another extra-musical information.

Figure. A.1. Possible criteria, elements or techniques that support the establishment of a progression.

As I have mentioned, Bach's design was not understood nor followed by his successors. The most common problems that are encountered are the following: the differences between the manuscript and the edition of 1751; the inexplicable composition of the first edition; the existence of the organ choral; the anecdotes on the inversion of the unfinished Fugue; the use of the name BACH in this Fugue according to Forkel, including its relevance in the collection and its composition on two staves; the technique of the augmentation of the final voices in several Fugues (in the second section); the instrumentation of the work; the Fugues for two keyboards; Marpurg's introduction to the second edition of 1752; and the contents of the necrology. It is essential to define the period of composition in which the manuscript is included: undoubtedly intermediate between a first design and the edition (particularly if we consider the manuscript a fair copy). However, as it is known the manuscript P200 was written

[5] This question is obvious since much of the material included in the first edition is not included in the manuscript. Therefore, it is clear that at least one other source existed (and was used by the editor) that has not been preserved.
[6] Christoph Wolff, Johann *Sebastian Bach: the learned musician*, (New York, Norton & Company 2001), 433. Christoph Wolff, *Bach Essays on his life and music, (*U. S. Harvard University Press, 1995), 259-281.
[7] Adel Heinrich, Bach's *Die Kunst Der Fuge, A living compendium of fugal procedures, with a motivic analysis of al fugues,* (Washington D. C., University press of America, 1983), 2-3. Anne Bagnall, "The Simple Fugues" in "Bach's Art of Fugue: An Examination of the Sources, Seminar Report", directed by Christoph Wolff, *Current Musicology,* nº 19, (1975), 53. Laurence Dreyfus, Bach *and the Patterns of Invention,* (Cambridge, Mass., 1996), 160.
[8] George Stauffer, "Bach's Art of Fugue", in "An examination of the sources, Seminar report", Current *Musicology* XIX, (1975), 48-49.

a long time before being sent for publication. If this is the case, how could this order be explained?

Until nowadays, new suggestions have been made, more or less similar to those of the past. I list the most important (although I will inevitably forget some): beside the P200, the first original edition, Naegel, Czerny (Kalmus ED. for orchestra), Rust, Greaser (BG and NBG), Hauptmann, Riemann, Tovey, Schirmerand, Norton, Husmann, David, Gall, Vuataz, Schurich, Chailley, Bitsch, NBA, Walcha, Moroney, Butler, and the classification of Schmieder[9]. Several of these editions appear exaggerated and highly personalised with respect to the order of the Fugues. All of them consider different criteria for establishing an order. The main difference between the proposal advanced here and that of Butler is based on the ranking of the third section[10] and shall be explained in the following paragraphs.

This suggestion and the position of other authors accounts for the subject in *rectus* to progress with the subject in *inversus*. This position takes for granted that the presence of material in rectus or inversus influences the position of a Fugue inside the work and is regarded as a basic element. (If not, it is necessary to separately examine form phenomena and structures in each section). Another crucial factor that must be considered to proceed with the progression's criteria is the dotted rhythm of the subject. The dotted rhythm can already be found it in the first section[11] of the work in a more or less sporadic way. However, it makes its appearance as a primary element in the second section (fugues BWV 1080, 5-7)[12]. In addition, also in the second section, the element that characterises the progression becomes (more than the inversion of the subject) the *stretto* technique (considering that all of the Fugues use dotted rhythm) but always in relation to the inversion.

It is therefore possible to arrive at the following diagram, which takes into account the classification of the Fugues as presented in the edition (without the copy of Fugue 10)[13]. Regarding the last fugue (BWV 1080, 19), because of fortuitous evidence, it is possible to imagine that the unfinished Fugue (fuga a tre soggetti or die lezte fuge) was conceived as a conclusion to the entire collection, at least of the main part of the Fugues (Contrapunctus), as Butler was first to suppose[14].

Contrapunctus (BWV 1080, 1, 2, 3, 4)

Contrapunctus (BWV 1080, 5, 6, 7)

[9] Hans Georg Nageli, *Die Kunst der Fuge von Johann Sebastian Bach* (…) Zurich, 1802. Carl Czerny, *L'ART DE LA FUGUE* (. . .), Peters, Leipzig, 1838.

[10] Gregory Butler, "Ordering Problems in J. S. Bach's Art of Fugue Resolved", The *Musical Quarterly* LXIX n. 1, (1983), 56-57.

[11] *The Art of Fugue* can be divided into sections. Each section groups a different number of fugues (termed Contrapunctus), which are primarily assembled according to various parameters. These parameters include contrapuntal techniques, the use of the same subject and other compositional aspects. Konstantinos Alevizos, *L'Art de la Fugue de Johann Sebastian Bach*, Editions (EUE, Saarbrücken, 2016), 255-257.

[12] According to the deduction of the "consecutive entries", Chailley refers to a lost Contrapunctus. I could not adopt this assumption, although it corresponds to a solution of analytical nature and for this reason is close to the spirit of this essay. Jacques Chailley, *L'Art de la Fugue de J. S. Bach. Étude critique des sources. Remise en ordre du plan. Analyse de l'oeuvre*, (A. Leduc, Paris, 1971), 42.

[13] Contrapunctus BWV 1080, 10 is presented two times in the edition. The second is identical to the manuscript, and the first one has an addition of twenty-two measures at its opening.

[14] Christoph Wolff, Bach's "Art of Fugue", 76. Philipp Spitta, *Johann Sebastian Bach. His work and influence on the music of Germany*, Clara Bell, Fuller Maitland. vol. 3, (Novello & Company, London, 1899), 198-199.

Contrapunctus (BWV 1080, 9, 10)

Contrapunctus (BWV 1080, 11, 8 or 8, 11)

Contrapunctus (BWV 1080, 12, 13 a and b)

Contrapunctus (BWV 1080, 19)

Figure A.2. Suggested ordering of Contrapuncti.

In my opinion, the insertion of the canons[15] poses a balance problem within the work, a fact also noted in the past by others. For Dickinson, this imbalance is palpable already after the first eleven Contrapuncti and in particular with the Contrapuncti in inversion. However, if one accepts Butler's proposal of the possible presence of a section of four canons at the end, why not accept Chailley's suggestion, which proposes a general outline of the work in sections of four Fugues? It seems improbable that Bach would enclose the entire collection within less complex material. However, if the canons are to be considered part of the entire order, they should be either interposed in each section, inserted at the beginning, or appended at the end like an Appendix (though I believe not inside the sequence of the Fugues). Additionally, it seems unlikely that the two Fugues for keyboard belong to the series of Fugues. In contrast with the rest of the Fugues, the transcription of Fugue 13 appears to be the most likely candidate (as the title announces) as a Fugue especially conceived for the keyboard.

Thus, we arrive at a disposition of the Fugues of the collection in two parts, as mentioned in the form of an assumption by Vartolo and proposed by Dickinson years earlier in 1950.[16] Paradoxically, in the study by Houten and Kasbergen[17] (at least whose precision should be acknowledged), certain tables on *The Art of Fugue* propose the classification of the Fugues according to what is termed "the set of subjects according to their form". If the subjects according to this set are considered, it is possible to divide the entire collection of fugues into two parts: one that uses S1i (Subject 1 of Contrapunctus I) and its direct variations and another that uses new subjects (in combination with S1i or similar subjects).

A. Subject S1i[18] and similar subjects.

Contrapunctus (BWV 1080, 1, 2, 3, 4)

Contrapunctus (BWV 1080, 5, 6, 7)

Contrapunctus (BWV 1080, 12 a and b)

B. Other (new) subjects.

Contrapunctus (BWV 1080, 9, 10, 11, 8)

Contrapunctus (BWV 1080, 13 a and b)

[15] Gregory Butler, G. Stauffer, M. Greer, *About Bach*, (University of Illinois, Illinois, 2008), 119-120.
[16] Alan Dickinson, *The Art of J. S. Bach*, (Hinrischen ed. London, 1950), 63.
[17] Kees van Houten et Marinus Kasbergen, *Bach et le nombre*, (Mardaga, Sprimont, 2003), 236.
[18] Subject 1 of Contrapunctus 1.

Contrapunctus (BWV 1080, 19)

Figure A.3. Division of Contrapuncti according to subject.

The main issue in the above table lies the separation of the Fugues in complete inversion, (12 & 13) taking into account their subject and not their inversion. Additionally, although the possibility of inserting the two Fugues for two keyboards and the canons should be considered, it seems reasonable to create a type of Appendix with the canons at the end of the presentation of the Fugues. This consideration is not only to be accepted for reasons common in Bach's musical production (as Butler also seems to affirm) but because it appears impossible to find a meaning for their presence inside the section of the Fugues. On the other hand, as Hughes appears to suggest in his thesis on the unfinished Fugue, I agree that the term "resolved" is unsuitable: "(…) about Bach's intended order for the Canons and Fugues, a matter which Gregory Butler claims to have resolved (. . .)". Similarly, I could not confirm with certainty if the two Fugues for two keyboards were created for separate use or, which seems less probable, if there was an intention for them to be included in the collection. It seems logical that Bach composed these variations (in 4v.) to make the original compositions in 3v. "playable", considering that, as Schweitzer affirmed, "[Bach] must have been very proud of these two compositions".

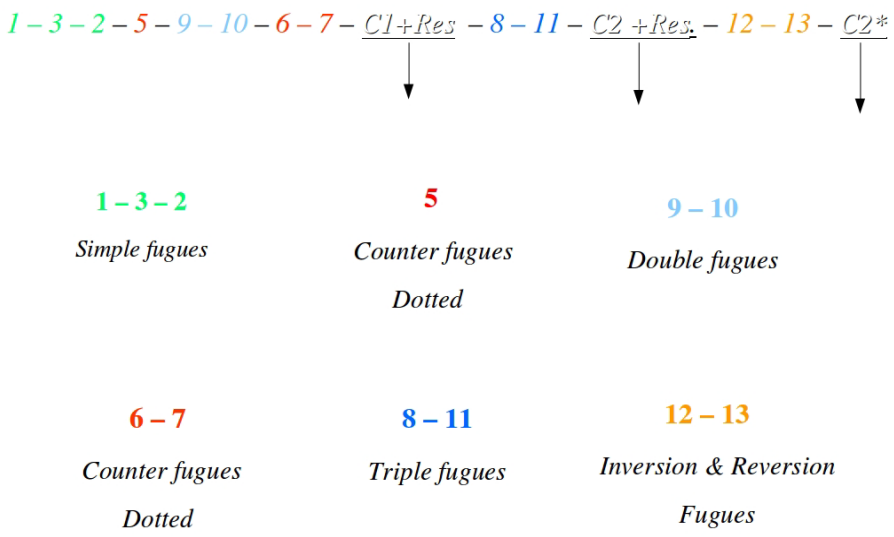


Figure A.4. The P200 manuscript. The numbers indicate the ordering according to the Schmieder catalogue BWV 1080. (Canons and *Resolutio canonis* have been omitted).

B. Sections and their ordering principle.

1. First section: Contrapuncti 1-4.

According to an order that appears logical and considers inversion to be the fundamental technique of progression (also according to Bagnall's conclusion), it is logical to assume that Bach first thought of starting with Contrapunctus 1 (which is considered for several reasons to be the matrix). Consequently, Contrapunctus 3 followed Contrapunctus 1, without introducing any other element than that of the inversion of the subject. The modification of Contrapunctus 2 (which became dotted, as Wiemer noted)[19] most likely depended on the modification of the entire section, which led to the composition of Contrapunctus 4. It is highly logical to believe that the modification of Contrapunctus 2 and that of the first three Fugues (including the modification of the cadences) is linked to the composition of Contrapunctus 4. Consequently, the final transformation of the section was to be two Fugues in rectus (in a different style; for this reason, Contrapunctus 2 becomes dotted) and two Fugues in inversus. To end the section, Bach composed Contrapunctus 4 (that is not to be found inside the P200), which is a simple monothematic fugue but much more technical and sophisticated (having all the characteristics of the first section) and therefore placed at the section's end. Thus, we arrive at the order 1-2-3-4. The same reasoning with respect to the progression can be found in other canonical collections of Bach, such as the Musical Offering and the Goldberg Variations, in which after the exposition of the basic subject we note a series of various techniques of development on the same topic – matrix.

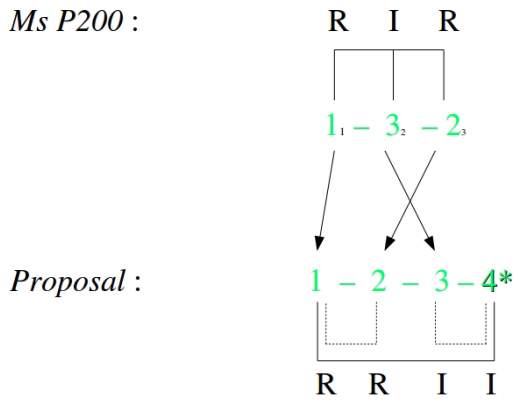


Figure B. 1. Transformation of the first section: Contrapunctus 1, 2, 3 and 4.[20]

In this way, the initial ordering RIR was transformed in RRII with the composition of the fourth Fugue. At the same time and in order to present a new element in regard to the section, Bach transformed the Contrapunctus 2 in dotted.

2. Second section: Contrapuncti 5-7.

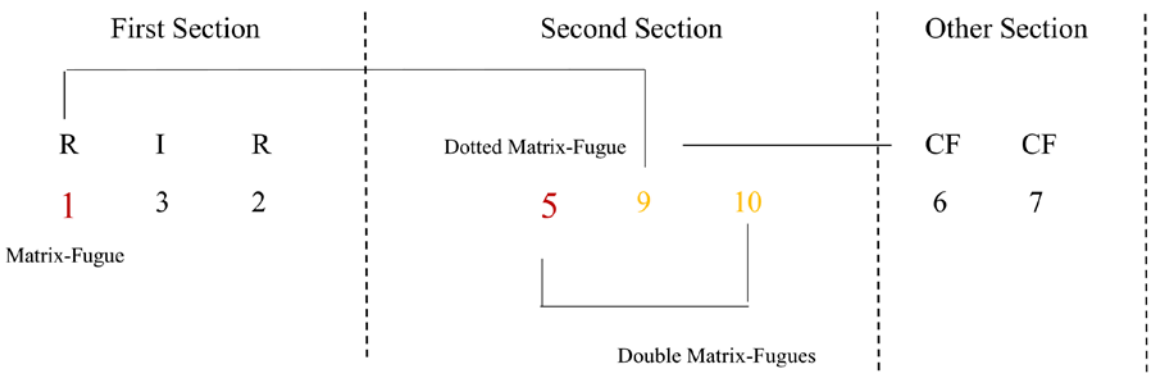
[19] Peter Shleuning, *Johann Sebastian Bachs "Kunst der Fuge" Ideologien, Entstehung, Analyse*, (Barenreiter-Verlag, Kassel, 1993), 125. Sergio Vartolo, *Johann Sebastian Bach, Homo Universalis, Die Kunst Der Fugue*, (SPES editore, Firenze, 2008), 50.
[20] The small numbers indicate the ordering of ms. P200.

Fugues 5, 6 and 7 could morphologically create a unit. However, not all researchers are keen on accepting this unit[21]. It is clear (considering its use in Contrapunctus 10[22]), that S1v[23] was conceived as a matrix of the *dotted rhythm subjects* (which is also consistent with the late addition of the dots in Contrapunctus 2). However, credence should be awarded to the isolation of Fugue 5 in the edition, as suggested by Vartolo and Butler[24].

Each Fugue among the three in this second section uses the stretto technique in a different way. It is most likely that the interposition of the double Fugue (BWV 1080, 9) in the manuscript was promptly adapted to fit its final destination in the (next) section of the double (and triple) Fugues. Similarly, Fugue 10 (BWV 1080. 10) was modified with the addition of 22 bars at the beginning to transform it into a natural and indisputable part of the third section, as a double fugue. That is because without these 22 bars it would have had an opening similar to the fugues of the second section directly with the dotted subject. In addition, Bach incorporated at least two subjects with separate exposition phases (according to the problematic mentioned by Bitsch) to create a common element for the section,[25] because all of the fugues in the third section use separate thematic phases for their different subjects' thematic phases.

It can therefore be supposed that the initial plan envisaged already a later development of the two basic subjects, S1i and S1v, to conclude the first part of the work. Thus, an initial plan could have included three simple Fugues: rectus-inversus-rectus (1-3-2). Fugue 5 (BWV 1080. 5) became the dotted Matrix-Fugue (presented separately to be elaborated later), and the two following Fugues (that later became doubles BWV 1080. 9 and 10) were conceived as a direct evolution of the two principal subjects, S1i and S1v. That is why Contrapunctus 9 elaborates the S1i together with a new subject and Contrapunctus 10 elaborates the S1v together with a new subject as well. To finish the cycle, Bach would have thought of inserting both stretto Fugues (Counter-Fugues) as elaborations of the dotted matrix, Fugue 5.

Possible initial planning of the P200.



[21] Jacques Chailley, *L'Art de la Fugue de J. S. Bach*, 41.
[22] BWV 1080, 10.
[23] Subject 1 of Contrapunctus 5.
[24] Sergio Vartolo, *Johann Sebastian Bach*, 52, 57.
[25] It should be noted that P200 presents Contrapunctus BWV 1080, 10 with the addition of S1v. In contrast, of the two versions, one can be found that directly introduces S1x (in the edition).

Or accordingly, another planning of the first fugues (that suits to the edition outline) could have been the presentation of each Matrix-Fugues followed by its straight elaborations, 1 to 3, 2 and 5 to 6, 7, by leaving the combined double fugues for the end of the section (1 to 9 and 5 to 10).

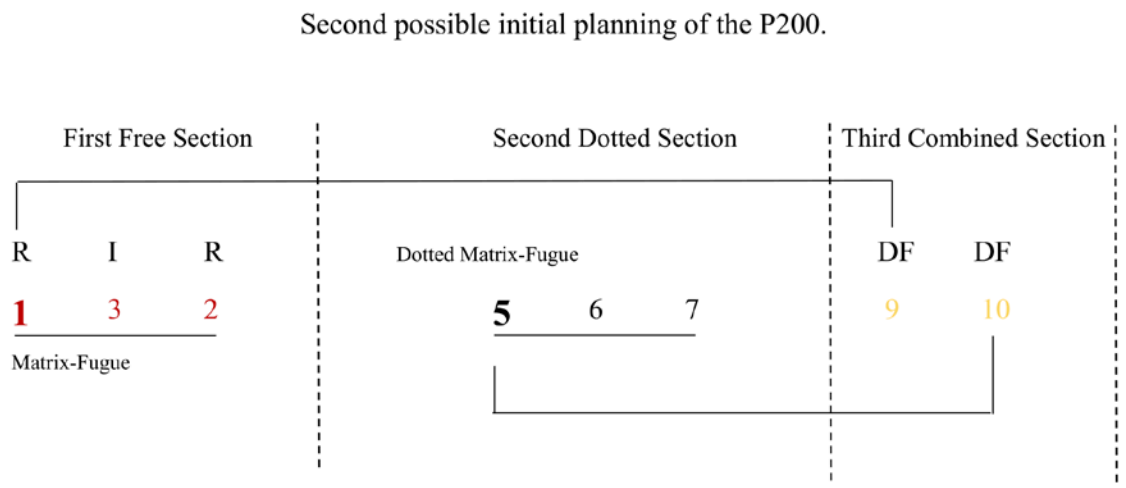


Figure B. 2. Examination of the initial planning of the P200.

The insertion of Contrapunctus 5 (BWV 1080, 5) would only have been used as a matrix-element to be developed inside BWV 1080, 10. Consequently and according to the first initial planning, the order 1-3-2-5-9-10-6-7 of the manuscript could have had a purpose.

Ms P200 (Before Cp. 4) 1* – 3 – 2 – 5** – 9* – 10** – 6 – 7

Division in sections

Section 1	Section 2	Section 3
1* – 2 – 3 – 4	5** – 6 – 7	9* – 10**

B. 3. Ordering of the eight first Fugues as presented in manuscript P200 before the composition of Contrapunctus 4 and proposal. All numbers correspond to the Schmieder catalogue.[26]

The composition of Fugue 4 (BWV 1080, 4) and the transformation of Fugues 9 and 10 into double Fugues (by using one of the two matrix subjects, S1i and S1v) results in the distribution of the Fugues into three sections. Fugue 5 becomes the first of the second section (as a dotted matrix subject) and as a stretto Fugue. This choice did not pose concerns regarding Fugues 9 and 10, which became double (with less allusion to the development of S1i and S1v) and were moved into the third section, as I discuss below. Therefore, the order was modified into 1-2-3-4 and 5-6-7-(9-10). However, it is a fact (not a problem) that we cannot explain the position of Fugue 6 (BWV 1080. 6) in the middle of the collection as an opening, knowing that Bach was keen on introducing an "Ouverture à la française" in the middle of other, greater compositions[27]. In my opinion, the collection cannot be divided into two parts to facilitate the introduction of an opening overture at the beginning of the second part (a technique frequently used by Bach).

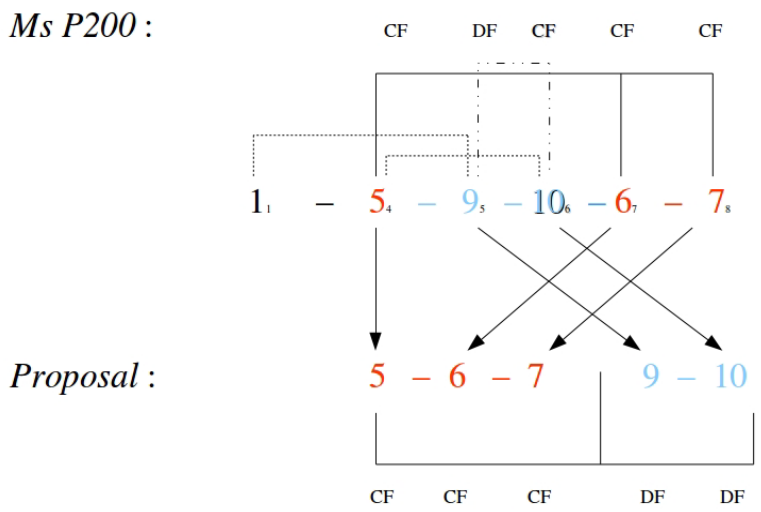


Figure B. 4. Second and third Sections. BWV 1080, 5 – 9 – 10 – 6 – 7.

Counter Fugues with dotted subject and double Fugues.

[26] Asterisks mark the Fugues that use the same subject: *=S1i and ** =S1v.

[27] Bach was keen on introducing "Ouvertures à la française" in the middle of greater compositions structured in series. Several scholars have unsuccessfully tried to apply this example of Bach's personal technique to *The Art of Fugue*. I can therefore mention the organ trio-sonata (BWV 528), the "overture" at the middle of the organ partita *Sei gegrüsset, Jesu gütig* (BWV 768), the fughetta super *Wir glauben all an einem Gott* (BWV 681) at the middle of the third part of the *Clavier Übung*, and the 13th prelude of the second book of the well-tempered clavier (BWV 882).

3. Third section: Contrapuncti 8-11.

All Fugues of this section pose a true challenge with respect to their unit and their morphological relation to one another. In this group (8-11), Fugue 10 is the most interesting, as there are two different versions in the edition. Regarding the remaining three Fugues, the differences between the edition and the manuscript are of a musical nature and less important[28] compared to the overall structure of the work.[29]

Regarding Contrapunctus 10, since the edition is posterior to the manuscript, I accept that Bach determined to add the first part of 22 bars, which is absent in P200[30], with the intention to separate it from the second section. Bach's choice to use S1i and S1v in double counterpoint (matrix subjects) together with new subjects drastically changed the order of the collection. Regarding the succession of Fugues 9 and 10, it is necessary to mention that Fugue 10 contains two clear subjects and two secondary subjects, for which Bach composed separate thematic phases. In contrast, Fugue 9[31] (Fugue 5 of the ms.) clearly contains only two. I note the presence of S1i in Fugue 9 and S1v in Fugue 10. In addition, Fugue 9 presents its second subject directly in stretto with the first one by transforming the Contrapunctus into a simple Fugue and not into a double, according to Bitsch's conclusion. However, Contrapunctus 10 (according to analytical facts) has separate thematic phases for each subject before their stretto expositions. I could therefore safely consider it a double Fugue. This evidence demonstrates that Fugue 9 should be placed before Fugue 10 according to the criteria of progression based on the increasing complexity of the Fugues and because Contrapunctus 9 is presented as the least elaborate of the section.

The existence of two versions of Cp. 10 forces us to acknowledge that Bach's intentions changed. After examining the morphology of the Fugue (with only one separate thematic phase set of S1ix: the 22 bars), we discover the parallelism between this Fugue and the previous one. The problem remains, that is which of the two subjects to present in each Fugue, a fact that Dickinson clearly noted[32]. I can assume that Bach modified the introduction of the Fugue by adding the first 22 bars (of the edition) to avoid similarities between Fugue 10 and the previous section, i.e., by opening with the dotted subject instead of the new one. Similarly, I can accept that Fugue 9 could have had a missing introductive part, as Chailley stated[33]. However, if this is the case, why do we only find the addition of Fugue 10 in the edition? Consequently, Fugues 9 and 10 acquire a structure of the following type:

9: (Hypothetical missing part) + 130 bars.

[28] Sergio Vartolo, *Johann Sebastian Bach*, 63-65.
[29] That is, if we do not consider the hypothetical - but not impossible - missing part suggested by Chailley for Fugue 9.
[30] Joseph Kerman, *The Art of fugue Bach fugues for keyboard 1715-1750*, (University of California Press, Los Angeles, 2005), 40-41.
[31] Sergio Vartolo, *Johann Sebastian Bach*, 57.
[32] Alan Dickinson, *Bach's fugal works, with an account of fugue before and after Bach*, I. Pitman, London, 1956, 133.
[33] Jacques Chailley, *L'Art de la Fugue*, 67-68.

10: (Addition of 22 bars) = 120 bars.

Or

9: 130 bars

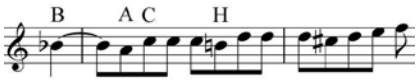
10: 98 bars.

Figure B. 5. Hypothetical ordering of Contrapuncti 9 and 10.

All Fugues in this section start with an altered (new) subject, with the exception of Fugue 11, which starts with an elaboration of S1i (probably because it may matche as an "answer" to Fugue 8). The position of the Fugues in the third section poses an unsolved problem. The assumption of Vartolo regarding the possibility of an edition in two parts, which is supported by Butler's theory involving the interpretation of the drawings of the edition at the end of Contrapunctus 5 (in relation to *Clavier Übung III*), does not seem concrete or demonstrable. The question remains regarding the explanation of the form of Fugue 9 (BWV 1080. 9)[34].

Regarding Fugues 8 and 11, if it is accepted that Bach would have introduced his name first in rectus (BACH) and not in inversus (for example, directly mirrored as HCAB), it could also be assumed that Contrapunctus 11 (BWV 1080. 11) is the rectus version of the two.[35] Another fact that results in the same conclusion is the direct development of S1i in the first thematic phase of Contrapunctus 11, i.e., S1xi (Subject 1 of Cp. 11) with the interposition of silences. It is therefore obvious that the rectus version of the pair is Fugue 11, not Fugue 8.

BACH Subject



S1xi

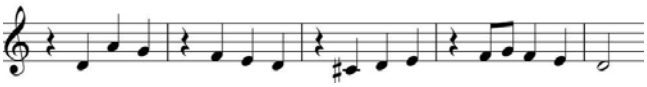


Figure B. 6. Two subjects (rectus) of Contrapunctus 11.

However, I must accept that the opening subject of Fugue 8 (S1viii, the second subject in inversion of Fugue 11) gives the impression of being in rectus. Though when put in stretto with the Sixi, the latter is presented always in its inversus form. It is thus difficult to believe

[34] Sergio Vartolo, *Johann Sebastian Bach*, 57. Gregory Butler, "Ordering Problems", 53.
[35] Sergio Vartolo, *Johann Sebastian Bach*, 63-65.

that the opposite could be true: that the second subject of Fugue 11 (S2xi) could be the rectus form[36]. I consequently believe that Bach could have composed the two Fugues almost simultaneously, and therefore, the rectus and inversus form of the subjects are mixed. A similar procedure can also be found inside Contrapunctus 13 with the "double mirror effect" where the subject is presented in rectus and inversus form inside the Rectus version, creating the opposite in the inversus one[37]. Dickinson's assumption (and that of those who support the work's keyboard destination) that Fugue 8 was composed with three voices to facilitate its execution on the keyboard cannot apply.

If in fact Bach composed the rectus form in the beginning to then continue with the inversus form (it is not about a mirror) and if the idea that *The Art of Fugue* follows a progression of complexity based on the technique of inversion can apply, the following statement is possible. If Contrapunctus 11 was to be ordered in the place of Contrapunctus 8 (or in any case inverted with 8), we should then expect a Fugue of enormous complexity and density in the ordering process, even more complex than Contrapunctus 11! In addition, if we assume that the concept of inversion of a Fugue constitutes an indication of a higher level of artifice of counterpoint (an idea also presented by Fux in the chapter on the Fugues in 3 voices), the succession 11-8 (BWV 1080, 11-8) can have a logical meaning. Unfortunately, the situation is different in the manuscript. It is therefore impossible to make assumptions on hypothetical lost material. We must work with the material we have[38].

In my opinion, the introduction of Contrapunctus 8 as an opening for the entire section of poly-thematic fugues cannot be accepted, first because Fugue 8 is constructed on material from Cp. 11 (as established by its inversus form) but also because Fugue 8 contains information and counterpoint techniques that do not appear in the Fugue 9 and 10 pair (including the inversion of the triple counterpoint). Therefore, Fugue 8 must be placed after Fugues 9 and 10. In addition, this logic may explain why Bach chose "to answer" the Titan Fugue 11 with a Fugue in three voices. It also helps us understand a similar assumption, which is valid for the Fugues with complete inversion (12 & 13), where the second pair is written in three voices and in a more skilful- way. It must be for the same reasons that Marpurg mentions the Fugue in three voices as a first example and the Fugue in four voices[39] as a second when examining Fugues in inversion.

The entire third section is characterized by the use of Sli (in dotted rhythm and with the interposition of silences) in stretto with new subjects. It is possible to distinguish six new subjects (and countersubjects) in addition to the development of S1i. I thus reach the conclusion of a possible ordering according to criteria that correspond to the technical progression 9-10-8-11[40] or 9-10-11-8[41] (as others have noted). The ordering of the Fugues of the first three sections must therefore be as follows:

[36] Fugue 11 can be defined as a triple fugue, which uses three subjects and a counter subject in quadruple counterpoint.
[37] Konstantinos Alevizos, The First Mirror inside the Contrapunctus 6 of the Art of Fugue by Johann Sebastian Bach, *Revue musicale OICRM* (*RMO*), No 7, (2020), 116-120.
[38] Jacques Chailley, *L'Art de la Fugue de J. S. Bach*, 50.
[39] Alfred Mann, The *Study of Fugue*, (Dover publications, New York, 1987), 187-190.
[40] André Verchaly, "L'Art de la Fugue by J. - S. Bach", Marcel Bitsch, *Revue de Musicologie*, T. 53, No. 2 (1967), 193.
[41] Jacques Chailley, L'Art *de la Fugue*, 73

Section 1	Section 2	Section 3	Section 4/5
1 – 2 – 3 – 4	5 – 6 – 7	9 – 10 – 11 – 8	12 – 13 – 19

B. 7. Suggested ordering.

In addition, that Fugues 8 and 11 are placed together in the manuscript cannot represent proof of order but only serves as an indication. It is important to consider and award significance to the use of the technique of the three voices of Contrapunctus 8. Understandably, we could consider placing Fugue 8 after Fugue 11 according to the progression concept because the second uses the inversion of the matrix-subject (and the BACH subject). Bach likely did not want to compose an inversion from Contrapunctus 11 to four voices considering the total climax of the section. However, as I explained, he sought to create a "perfect form" in the three voices with material from the previous titanic Fugue (11). It would be sufficient to place Contrapunctus 8 at the end of the section rather than at the beginning, and I assume that the person in charge of the edition considered the three voices as the key factor in regard erroneously to simplicity, for this reason he placed it, incorrectly, as first in the section.

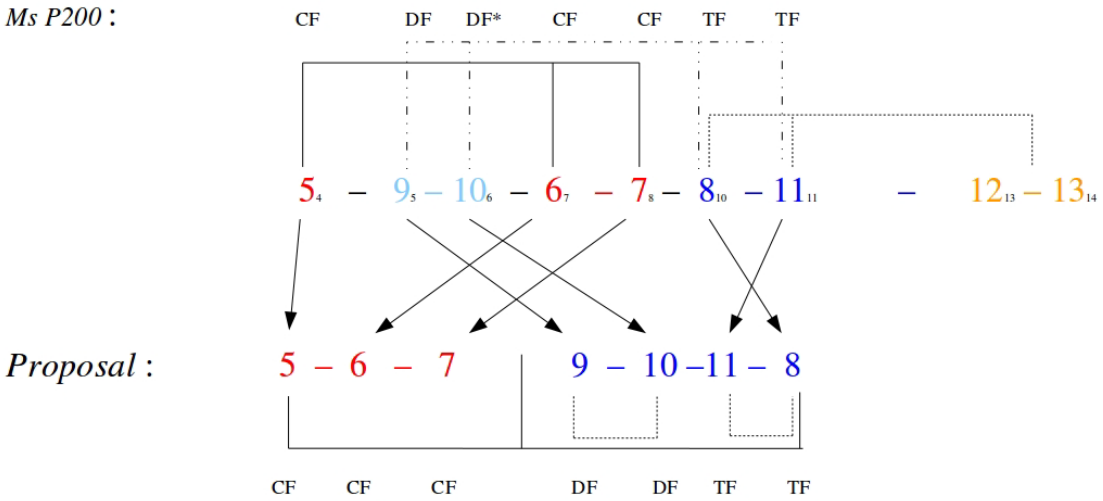


Figure B. 8. Completion of the third Section. BWV 1080, 9 – 10 – 11 – 8.

Double Fugues and triple Fugues.

4. Fourth section and the last Fugue: Contrapuncti 12, 13 and 19.

As in the previous section, the existence of two Fugues composed with three voices does not mean that they must be placed at the beginning according to the concept of progression. In contrast, the section begins with the two mirror Fugues that use the altered S1i: 1080.12 a, b[42]. It is astonishing that the first Fugue of the pair is in inversus and the second (which is the rectus) is marked as inversus in the first edition[43]. Regarding the second pair (1080, 13 a, b)[44], the new subject is presented in rectus and inversus in the same thematic phase to avoid making their distinction obvious, as stressed out by many. By establishing the order in relation to the presentation of the subject in rectus/inversus and by accepting the position of Contrapunctus 13 as the thirteenth Fugue, we indirectly affirm that the selection criterion is the development of the subject (S1xiii).[45]

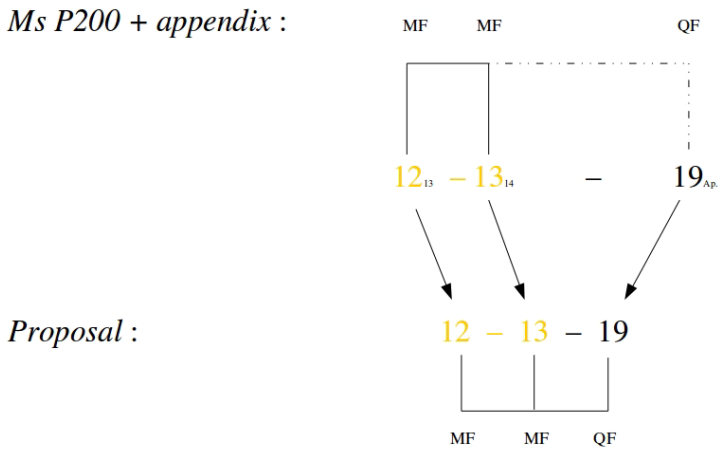


Figure B. 9. Fourth Section and Final Fugue. BWV 1080, 12 – 13 – 19.

Mirror Fugue, Double inverted Fugue and Quadruple Fugue.

The "last Fugue" (BWV 1080, 19) has at least the quality to finish the sections of the Fugues. There is nothing romantic in that statement. Much has been written on this Fugue, and nothing changes the fact that Bach could have intended to create its complete inversion but did not begin to compose it. In my opinion, the position of Wolff on the existence of a fragment "x" is possible (but without its realization, as Wolff suggested). However, a casual break at bar 239 of the fugue appears improbable, in order to be finished by another composer (as others

[42] Philipp Spitta, *Johann Sebastian Bach*, 202.
[43] Sergio Vartolo, *Johann Sebastian Bach*, 65.
[44] James Higgs, "Bach's 'Art of Fugue", *Proceedings of the Musical Association*, 3rd Sess. (1876 - 1877), 61-62.
[45] All of the preceding arguments should be considered to agree with what Heinrich terms the cumulative work and what led Bagnall to restore the order of the first section. Adel Heinrich, *Bach's Die Kunst Der Fuge*, 2-3.

have proposed)[46]. As I have observed on other occasions, it is highly probable that Bach never composed the complement for this Fugue despite any intentions he might have had to do so (Carl Philip's writing on the manuscript is proof of this statement)[47]. Goncz and Butler thoroughly clarified the situation concerning the fourth subject and the dimension of the part that is missing[48]. However, and if I am not mistaken, Butler's last proposal regarding the impossibility of playing a fourth subject on the keyboard in combination with the three already present does not facilitate an ordering of the pieces or the comprehension of the structure[49]. *The Art of Fugue* was not written for the keyboard.

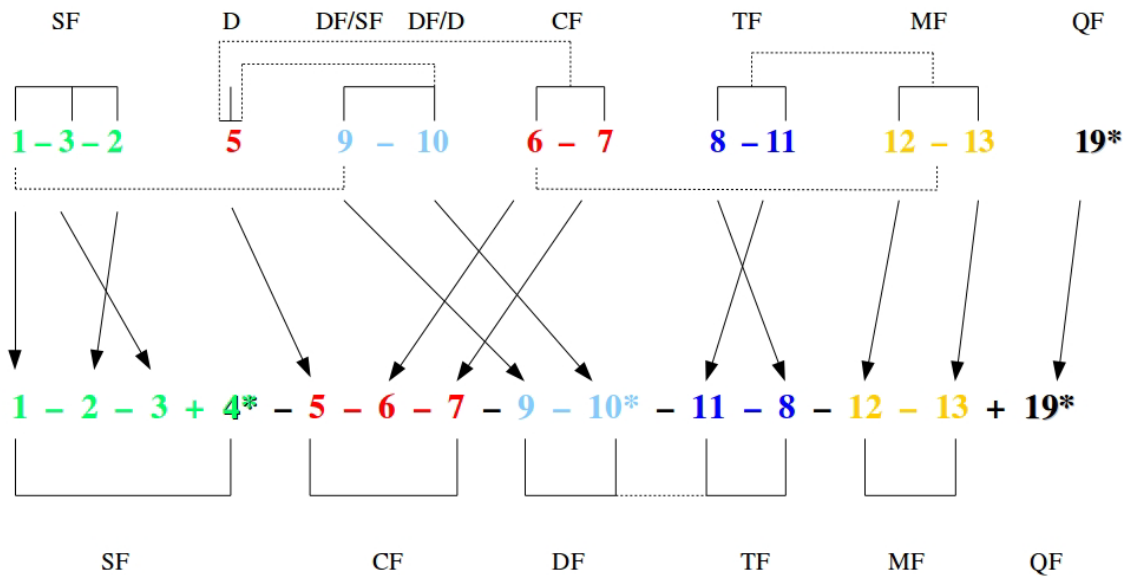


Figure B. 10. Ordering proposal according to the typology of each Fugue (numbers in conformity with the Schmieder catalogue, BWV 1080). Upper series: Ms. P200 ordering and appendix material; lower series: proposal with first edition's material. The continuous lines indicate coexistence inside the same section created by the use of similar technical criteria. The dotted lines indicate the use of common material. The arrows indicate the final placements of the fugues (Simple Fugues, Dotted, Double Fugues, Counter Fugues, Triple Fugues, Mirror Fugues, Quadruple Fugue).

[46] Laurence Dreyfus, Bach *and the Patterns of Invention*, Cambridge, Mass., 1996, 165. James Higgs, "Bach's 'Art of Fugue", 69.

[47] Roy Harris and M. D. Herter Norton, "The Art of the Fugue", *The Musical Quarterly*, Vol. 21, No. 2 (Apr. 1935), 169.

[48] Zoltan Goncz, *Bach's testament On the Philosophical and Theological Background of The Art of Fugue*, Contextual Bach Studies 4, (The Scarecrow press, Plymouth, 2013), 129-130.

[49] Gregory Butler, *About Bach*, 117, note 24.