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SEPARATION OF RAIL AND WHEEL ROUGHNESS FROM ON-BOARD VIBROACOUSTIC MEASUREMENTS

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ABSTRACT

Railway noise contributes to environmental noise and may be a nuisance for people living near urban or interurban railway lines. Rolling noise is the main noise source over a wide speed range, resulting from the roughness of the railhead and wheel treads. The combined roughness at the wheel/rail contact zone produces vibrations that excite the track and the wheels, which then radiate noise. Maintaining a correct surface quality is a concern of rail network managers, requiring a knowledge of their network condition on large scales to optimise costly corrective actions. Indirect measurement methods, based on on-board vibration or acoustic sensors on a train set and combined with an inverse approach, allow the combined roughness to be estimated from the vibro-acoustic quantities. Then, removing the wheel roughness contribution is essential for properly assessing the rail condition. This can be achieved either by using smooth wheels, which is a stringent solution, or by implementing appropriate signal processing approaches taking advantage of the respective periodic or random properties of wheel and rail roughness. This paper reviews separation approaches available in the scientific literature. It also emphasizes the effect of the lateral shift of the contact point on the wheel tread, interfering with the deterministic nature of the wheel roughness contribution. This work is part of the French project MEEQUAI developing an on-board measurement system for the assessment of rail roughness.

1. INTRODUCTION

Railway transportation offers a major advantage by its low environmental impacts. In order to protect local residents by considering noise exposure, railway noise receives special attention, in particular through specific regulation and assessment like the European Directive 2002/49/CE [1]. Rolling noise is well known as being a main contribution to railway noise on a wide range of operational speeds, originating from the roughness of the wheel and rail running surfaces. Rail roughness quality is of the responsibility of infrastructure managers and their knowledge of the rail condition on the national scale is a challenging issue. Roughness investigation and monitoring by direct measurement using dedicated rulers or trolleys is unfeasible

on the whole railway network. Indirect solutions perform on-board acoustical or vibrational measurements to recover the roughness information behind the sensor signals through the inversion of the vibro-acoustic transformation [2]. The MEEQUAI project (On-board measurement of the acoustic quality of the railway infrastructure) aims at developing an on-board system combining model and measures to estimate the transfer function between the effective roughness and the on-board sensor signals [3]. The effective roughness combines the rail and the wheel roughness contributions, the challenge being then to separate them.

The paper reviews roughness separation approaches already used in existing systems. It presents an argumentative analysis of separation methods that could be considered in the MEEQUAI strategy, by leveraging the respective nature of the rail and wheel roughness, the former being a spatially infinite random variable while the latter is theoretically a periodic signal. However, this periodicity can be impaired by the lateral shift of the contact point position on the wheel tread during the bogie motion. The performance of a separation approach and its robustness to lateral shift will be illustrated by some test cases involving different rail roughness conditions.

2. ROUGHNESS AND ROLLING NOISE

Roughness characterizes the surface condition of rail and wheel treads respectively [4]. It is a 3-dimensional spatial quantity describing the magnitude of the vertical – generally microscopic – irregularities, as a function of the position (x, y) on the running surfaces. In practice, these surfaces are not transversally plane, but have a specific transverse profile influencing the stability position y of the contact area on the rolling treads [4]. While the train is in motion, and supposing a point contact between the rail and the wheel, the surface irregularities are "read" by the contact point along the longitudinal line of lateral position y on the rail (resp. on the wheel), defining the roughness $r_{rail}(x)$ (resp. $r_{wheel}(x)$) producing noise, where x is the longitudinal direction along the rail.

On the rail, where the contact point moves on an infinite running surface, roughness is a random variable. If the rail head condition is homogeneous along the studied line, rail roughness may be considered as spatially stationary. Its

spectrum $L_{r,rail}(\lambda)$ is expressed by the level (in dB ref. $1\mu m$) as a function of the wavelength λ , either in narrow band but more often in third-octave band.

Concerning the wheel, the 3D-roughness on the whole tread is deterministic for a given wheel. In stable conditions, the wheel roughness read by the contact point is periodic, with a spatial period L given by the wheel perimeter for the actual rolling radius. Its power spectrum may be described by a Fourier series. However, if the lateral position of the contact point drifts on the wheel tread and changes with the rotation, a part of variability and randomness impairs the periodic signal, due to transverse irregularities and according to the correlation degree of roughness in the lateral direction.

In the contact area, rail and wheel roughness combine to form the *combined roughness*. As a very first approximation, the combined roughness may be written as:

$$r_{comb}(x) = r_{rail}(x) - r_{wheel}(x) \quad (1)$$

if each roughness respectively is oriented in the outward direction of the steel surfaces. In general wheel and rail surface irregularities are uncorrelated and the spectrum of the combined roughness is the sum of their respective spectra [4]:

$$L_{r,comb}(\lambda) = L_{r,rail}(\lambda) \oplus L_{r,wheel}(\lambda) \quad (2)$$

where \oplus stands for the energetic summation. The contact area is actually not punctual but elliptic, with dimensions depending on various parameters, typically in the order of cm. This surface tends to dampen roughness components whose wavelengths are close to or shorter than the size of the contact area. Its effect, linear for small roughness amplitudes, behaves like a spatial filter of transfer function H_c to give the *combined effective roughness* $r_{eff}(x)$ [4]:

$$L_{r,eff}(\lambda) = 10 \log \left(|H_c(\lambda)|^2 \right) + L_{r,comb}(\lambda) \quad (3)$$

The combined effective roughness excites vibrations and is measured as a time signal on an on-board accelerometer (resp. an on-board microphone) through a roughness-acceleration (resp. roughness-acoustic pressure) transfer function, here generically named $H(f)$ at the frequency $f = \frac{V}{\lambda}$ where V is the train speed [2]. Thus, the measured quantity is related to the combined roughness at a constant speed with an overall transfer function $H_c(\lambda) \cdot H(\lambda)|_{\lambda=\frac{V}{f}}$. Since the inversion from the measured quantity to the roughness is generally implemented on quadratic data (power spectra), roughness separation may be more appropriate either before or after inversion depending on the approach.

3. STATE-OF-THE-ART ON ROUGHNESS SEPARATION BY INDIRECT SYSTEMS

For the most part, indirect roughness measurement systems aim at extracting rail roughness. Three main approaches may be identified.

3.1 Use of smooth wheels

Systems considering wheel roughness as insignificant in comparison with rail roughness ("smooth wheel" assumption) constitute an important part of the existing systems. They concern both global (no frequency information provided) and frequency solutions for rail roughness qualification. However, the assumption introduces a stronger constraint in the latter case since it must be fulfilled in each useful frequency band. In any case, it defines stringent operational requirements:

- on the type of equipped wheel or wheelset, in particular its braking mode in order to prevent wheel tread damage in operational conditions,
- on scheduled maintenance through wheel(s) grinding.

3.1.1 Global qualification systems

The global indirect methods are often used for monitoring rail head degradation and initiating a maintenance operation when the global quantity exceeds a threshold value. The assumption of insignificant wheel roughness results in:

$$\int_{(X)} r_{comb}^2(x) dx \approx \int_{(X)} r_{rail}^2(x) dx \quad (4)$$

where (X) refers to the rail section analysed.

Depending on the system description in the literature, requirements for low wheel roughness do not specify maintenance imperatives (VIMTO [5], NoiseMon [6]) or they prescribe a periodical wheel control [7]. One solution of the MONA project, working on the wide band 1-5 kHz, includes an acceptability criterion of the wheel tread condition regarding smooth wheel assumption vs. the threshold value [8].

3.1.2 Frequency qualification systems

Systems in this category are quite numerous and most of them do not specifically perform roughness separation but require using "smooth" or low roughness wheels, such that:

$$\int_{(X)} r_{\Delta\lambda,comb}^2(x; \lambda_c) dx \approx \int_{(X)} r_{\Delta\lambda,rail}^2(x; \lambda_c) dx \quad (5)$$

where $r_{\Delta\lambda}(x; \lambda_c)$ is the roughness filtered in the bandwidth $\Delta\lambda$ (generally third-octave, except octave on the Milan metro) around central wavelength λ_c .

According to their wording for supporting the hypothesis, some systems specify that wheels must be grinded (Milan's metro [9]) or reprofiled so that their roughness level be lower than those of the rails (HSRCA [10]), that the wheels should be unbraked and often grinded (TRC Strix [11]), reprofiled or replaced in case of occurrence of defects (SWECO), installed on coaches equipped with a suitable braking mode (Infrabel). The Austrian system ÖBB also falls into the low wheel roughness category.

For these systems, there is generally no indication on the monitoring strategy or of an acceptability criterion for

wheel roughness to correctly fulfill the hypothesis. Also, a periodical monitoring of the wheels conditions is achieved for the Infrabel system.

3.2 Roughness separation in the frequency domain

Another approach consists, from the combined roughness spectrum estimated by the indirect method, to remove the wheel contribution by energetical subtraction of its roughness spectrum:

$$L_{r,rail}(\lambda) = L_{r,comb}(\lambda) \ominus L_{r,wheel}(\lambda) \quad (6)$$

The essential condition is then to know the wheel roughness spectrum either by direct measurement or by indirect estimation through the on-board sensors.

3.2.1 Direct measurement of wheel roughness

Two systems, using on-board microphones, fall in this category.

The ARROW system uses a direct roughness measure of each wheel of the bogie equipped with sensors, and subtraction (6) is performed for each wheel separately [12]. It is essential that the wheels have a low roughness, in any case lower than those of the rails. Indeed, the wheel roughness affects the sensitivity limit of the indirect system. There is no information on the selection of lines to be preferably used on the wheel tread.

In its first phase, the Quiet Track project has used direct wheel roughness measured one month after reprofiling [13]. Several parallel lines were measured and the wheels turned out to have low roughness levels, inferior to the ISO 3095 curve. No information mentioned a possible line combination to apply prior to subtraction. This approach was a transitional phase towards the final one in the project, described later.

Thus, the direct wheel roughness approach raises two issues:

- the line selection on the wheel tread and the processing to get the wheel roughness spectrum characteristic of the actual contact condition involved at the time of on-board sensor signal recording,
- the evolution and wear of the wheel surface between occasional direct controls,

both impacting the accuracy of the rail roughness estimate.

3.2.2 Indirect wheel roughness estimation

In this approach, wheel roughness spectra are indirectly estimated from the on-board sensors. They require the use of a tachometer to allow the synchronisation of signal sampling to the wheel rotation and to take advantage of the wheel periodicity. Indeed, the combined spectrum calculated on signal sections of several wheel rotations concentrates the wheel power contribution on the discrete spatial frequencies harmonic to the basic wheel rotation, whereas the rail power spectrum extends on the whole frequency range.

By following this observation, the MONA project has tested the extraction of the wheel component by spectral analysis and by cepstral analysis, the later one proving to be underperforming [8].

As well, the final approach of the system developed in the Quiet Track project, although not much detailed, seems to follow the same principle [14]. With signal sampling controlled by the tachometer, the wheel roughness spectrum is extracted at the rotation harmonic frequencies from a specific measurement performed once a day. Then, it would be subtracted from the combined roughness spectrum estimated when the trainset is running on the railway network.

3.3 Separation by synchronous spatial averaging

The approach by synchronous averaging takes advantage of the periodic vs. random contributions of wheel and rail respectively, by operating in the spatial domain. Requiring a tachometer to follow wheel rotation and provide a spatial sampling that gets rid of possible speed variations, it consists in slicing the sensor signal in sections of the duration of one wheel rotation and in averaging them to bring out the deterministic wheel contribution while removing the random rail input. To make it possible, it is mandatory to implement this averaging process on the signal before the inversion that operates on quadratic quantities. The averaging result is the estimate of the wheel roughness contribution in the sensor signal. As proposed in the continuation of the VIMTO project [15], two options may be considered:

- either the averaged result is used to estimate the wheel roughness spectrum and subtract it from the combined roughness spectrum in a similar vein as in Section 3.2. However, if the indirect method involves multiple sensors and leverages intercorrelation between sensors, two inversions (one for each term of subtraction) could be necessary. But the wheel component estimation is not necessarily carried out concomitantly with the rail investigation and can be anticipated. The ratio of wheel to rail roughness plays an important role in the accuracy of the rail roughness estimate [15].
- or the averaged wheel contribution section is periodised and directly subtracted from the sensor signal(s) to extract the contribution of the rail roughness in the sensor signals and next apply inversion. This approach provides very good rail roughness estimates, even with a wheel in poor condition, as long as the periodicity is perfect.

The same synchronous averaging approach was sooner considered in the project MONA [8] as an interesting alternative solution to the acceptability criterion presented in section 3.1.1. None of these projects have tested the synchronous averaging method on real-world data, but only on controlled simulations.

3.4 Discussion on roughness separation approaches

The existing roughness separation methods for rail roughness determination may be grouped in four classes, implying decreasing constraints on the wheel roughness characteristics:

- The "smooth wheel" assumption brings strong constraints on the coach type, wheel tread quality, direct monitoring procedure and acceptability criterion. It provides no guarantee in case of damage occurrence between control operations. In return, no processing for roughness separation is needed.
- Spectra subtraction with direct wheel roughness measurement loosens the constraint on wheel tread quality while maintaining wheel inspection and the limits due to changes occurring between controls.

These two approaches are best suited to a system set up on a dedicated coach or trainset due to the need of regular wheel inspection.

- Spectra subtraction with on-board wheel roughness measurement reduces requirements on the rolling stock and the periodical monitoring of the wheels by providing updated information on the wheel tread condition. However for a higher accuracy, the use of low roughness wheels is recommended, favouring unbraked or disc braked wheelsets.
- The synchronous spatial averaging method sounds attractive in theory. The separation carried out on the measured raw signal does not require wheel roughness spectrum calculation and is tolerant to a wide range of rail/wheel roughness ratios.

The two last separation approaches relying on the on-board sensors are based on the periodicity of the wheel roughness read by the contact area. They require the use of a tachometer for synchronising the signals to the wheel rotation. A variation of the lateral position of the contact point on the wheel with the rotations might undermine the periodicity assumption by changing the actual wheel roughness line over time, thus transferring part of wheel roughness information to rail roughness estimate. This issue is considered in the next sections.

4. SYNCHRONOUS SPATIAL AVERAGING IN A PERIODICAL CONTEXT

The performance of the synchronous spatial averaging approach for estimating the rail roughness spectrum is illustrated for two rail roughness conditions: one rough rail and one smooth rail. Direct roughness measurements of the rails and of a wheel are used to build a combined roughness using Equation (1), after prior post-processing of the respective roughness measures by the spike removal technique and curvature processing according to ISO 3095 standard.

The spatial averaging method is directly tested on the simulated combined roughness instead of vibro(-acoustic)

signals as would be the case on real data. Indeed, subject to the linear transformation by the contact filter and the vibro(-acoustic) transfer function with the overall transfer function $H_c(\lambda) \cdot H(\lambda)$ (see section 2), behaviour and conclusions on the roughness separation remain similar.

In the perfectly periodic case, the wheel roughness measured on one line along one perimeter L is repeated on M periods. The same length ML of rail roughness is used. In the present illustration, $L = 2.686$ m and $M = 35$, corresponding to a total length of about 100 m. With these parameters used in operational conditions of an indirect measurement system, the infrastructure sampling rate for issuing rail roughness spectra is 100 m.

In the test of the separation method, the combined roughness is sliced in 35 one-rotation long sections and averaged in order to get an estimate of the wheel contribution. It is then periodised and subtracted from the original combined signal. The power spectrum of the residual signal is calculated as an estimate of the rail roughness power spectrum. The difference of this estimated spectrum with the actual rail roughness spectrum assesses the estimation error. Figure 1 shows that the error is quite low on the whole wavelength range, even in the case of the smooth rail where errors do not exceed 0.32 dB. A shorter average length with $M = 19$ (rail section length 51 m) gives a spectrum underestimate, by less than a maximum of 0.8 dB whatever the wavelength (not shown), which is perfectly acceptable.

5. INFLUENCE OF A LATERAL SHIFT OF THE CONTACT POINT

As mentioned sooner, various reasons may cause the wheelset to laterally shift while the train is running. Tests carried out during the project MEEQUAI using a video camera showed wheelset shifts exceeding 10 mm peak-to-peak on a straight track in only several wheel rotations, as shown by clear trends in Figure 2 despite some local image inaccuracies. This variation may make the contact position shift on large lateral distances on the wheel tread [16].

As a simple illustration for testing the effect of a lateral shift of the contact point on the rail roughness estimate with the synchronous spatial averaging method, it is supposed here that the contact point shifts over the small width of 5 mm, sinusoidally oscillating between two parallel lines distant of 5 mm on the wheel tread, so that the roughness read by the contact point is the mixture:

$$r_{w,shift}(x) = \sqrt{\alpha} r_{w,1}(x) + \sqrt{1-\alpha} r_{w,2}(x) \quad (7)$$

$$\alpha = \left[1 + \cos \left(2\pi \frac{x}{X_s} \right) \right] / 2 \quad (8)$$

where $r_{w,1}(x)$ and $r_{w,2}(x)$ are two roughness lines distant of 5 mm recorded on a wheel tread by a direct measurement instrument and X_s is the longitudinal period of the lateral shift in the mixture. The mixture roughness spectrum is similar – with intermediate levels – to both original roughness spectra.

Synchronous spatial averaging is applied similarly to the procedure depicted in section 4, with the same rails and

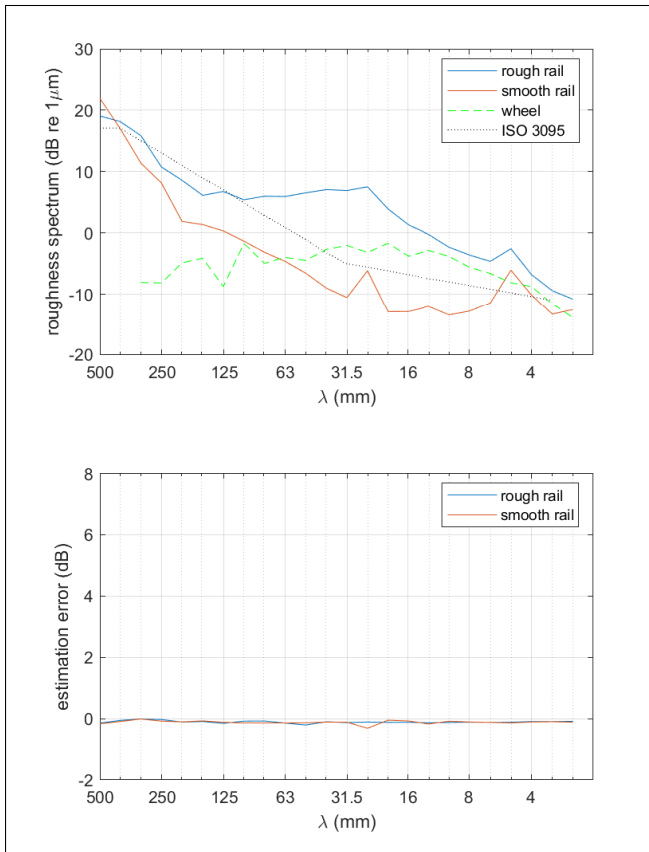


Figure 1. Estimation of the rail roughness spectrum by the synchronous spatial averaging method, for a rough and a smooth rail. Top: Actual rail and wheel roughness. Bottom: Rail roughness estimation error.

parameters ($L = 2.686$ m and $M = 35$) and with the wheel roughness given by the mixture roughness. It appears that the results of the rail roughness estimates, still fair with the rough rail, are much worse in case of a rather smooth rail, even for slow contact point oscillations. Figure 3 corresponds to an oscillation period of $X_s = 100$ m, to be compared to Figure 1. This highlights the high sensitivity of the method to disturbance in the wheel roughness waveform periodicity. The rail to wheel roughness ratio turns out to be a crucial parameter to the method effectiveness in that case. Comparatively, roughness separation relying on spectra difference methods shows a stronger robustness against such distortion (not illustrated here).

6. CONCLUSIONS

Several approaches of roughness separation have been developed in existing indirect systems for the estimation of rail roughness. Systems requiring the use of smooth wheels involve an easy analysis procedure but imply stringent equipment specificities for remaining really efficient on a wide range of rail conditions. Those implementing the subtraction of the wheel roughness spectrum partly relax the constraint on wheel roughness, whether known by direct or indirect measurements, but still require the use of wheels in rather good condition for a better result ac-

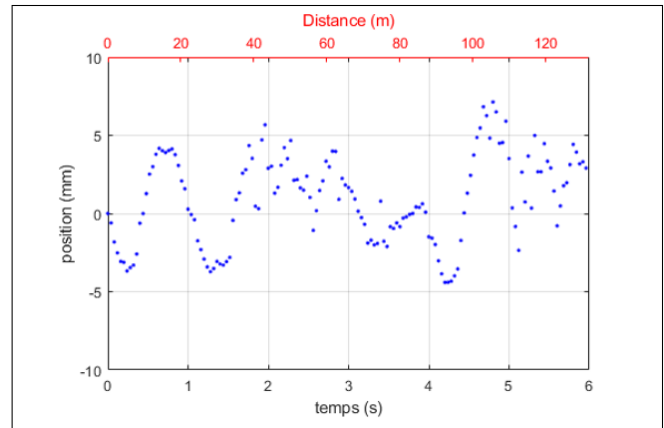


Figure 2. Relative shift of the wheelset lateral position as a function of time (bottom horizontal scale) or distance (upper horizontal scale) during train motion.

curacy. Indirect wheel roughness estimates before spectra subtraction, taking advantage of the periodic character of wheel signals, exempt the operator from relying on regular wheel control and constantly update the wheel condition. Synchronous spatial averaging removes wheel contribution from the measured signal in the spatial domain with a high efficiency whatever the wheel/rail roughness ratio in perfect periodic conditions. However, it is strongly sensitive to disturbances on the periodic signal like those induced by a lateral shift of the contact point on the wheel tread during train motion, clearing its theoretical strength if used in real operational conditions.

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8. REFERENCES

- [1] "Directive 2002/49/EC of the European Parliament and of the Council of 25 June 2002 relating to the assessment and management of environmental noise," 2002.
- [2] A.-R. Tufano, O. Chiello, M.-A. Pallas, B. Faure, C. Chaufour, E. Reynaud, and N. Vincent, "On-board indirect measurements of the acoustic quality of railway track: state-of-the art and simulations," in *Proc. of Internoise 2019*, (Madrid, Spain), 2019.

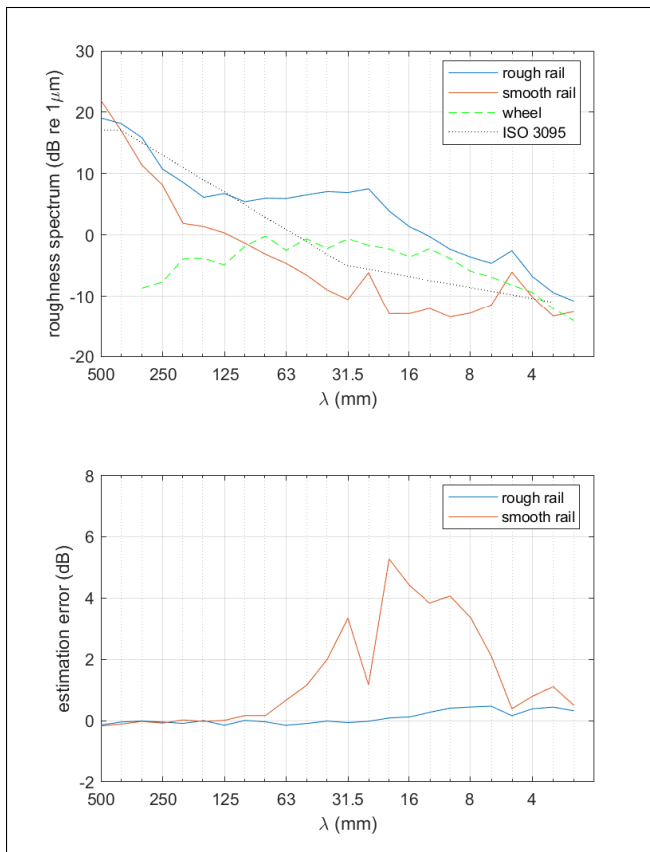


Figure 3. Estimation of the rail roughness spectrum by the synchronous spatial averaging method, for a rough and a smooth rail, with a lateral oscillation of the contact point ($X_s = 100$ m). Top: Actual rail and wheel roughness. Bottom: Rail roughness estimation error.

- [3] A.-R. Tufano, O. Chiello, M.-A. Pallas, B. Faure, C. Chaufour, E. Reynaud, and N. Vincent, “Numerical and experimental analysis of transfer functions for on-board indirect measurements of rail acoustic roughness,” in *Proc. of IWRN13*, (Leuven, Belgium), 2019.
- [4] D. Thompson, *Railway Noise and Vibration: Mechanisms, Modelling and Means of Control*. Oxford, UK: Elsevier, 2009.
- [5] P. Fidler, S. Hartley, and J. Talbot, “Project VIMTO: A new system for the vibration and impact monitoring of tram operations,” in *SHMII 2017 - 8th International Conference on Structural Health Monitoring of Intelligent Infrastructure, Proceedings*, (Brisbane, Australia), 2017.
- [6] R. R. K. Jones and A. J. Packham, “Acoustic Monitoring of Rail-Head Roughness for Targeted Grinding and Noise Modelling,” in *2006 IET International Conference On Railway Condition Monitoring*, pp. 61–62, 2006.
- [7] B. H. Asmussen, H. Onnich, R. Stube, L. M. Greven, S. Schröder, K. Jäger, and K. G. Degen, “Status and perspectives of the ”Specially Monitored Track”,” *Journal of Sound and Vibration*, vol. 293, no. 3, pp. 1070–1077, 2006.
- [8] N. Vincent, P. Bouvet, and P. Balthazard, “Aide à la conception d’un cahier des charges pour une chaîne de mesure acoustique du bruit de roulement - Projet MONA,” 1995.
- [9] M. Bocciolone, A. Capriolo, A. Cigada, and A. Colina, “A measurement system for quick rail inspection and effective track maintenance strategy,” *Mechanical Systems and Signal Processing*, vol. 21, no. 3, pp. 1242–1254, 2007.
- [10] P. E. Chartrain, *Lecture acoustique de la voie ferrée*. PhD thesis, Université d’Aix-Marseille, 2013.
- [11] J. Spännar, “A new approach of assessing rail roughness,” in *4th IET International Conference on Railway Condition Monitoring*, (Derby, UK), 2008.
- [12] A. Kuijpers, W. Schwanen, and E. Bongini, “Indirect Rail Roughness Measurement: The ARROW System within the LECAV Project,” in *Noise and Vibration Mitigation for Rail Transportation Systems*, pp. 563–570, Springer, 2012.
- [13] M. Höjer and M. Almgren, “Monitoring system for track roughness,” in *Euronoise 2015*, (Maastricht, the Netherlands), 2015.
- [14] M. Höjer, “QUIET-TRACK - WP 1, Monitoring system for noise related track parameters and severe wear.” QUIET TRACK Final Dissemination Conference & Workshop, 2016.
- [15] T. Carrigan and J. Talbot, “Extracting information from axle-box accelerometers: on the derivation of rail roughness spectra in the presence of wheel roughness,” in *IWRN13*, (Ghent, Belgium), 2019.
- [16] D. J. Thompson, “The influence of the contact zone on the excitation of wheel/rail noise,” *Journal of Sound and Vibration*, vol. 267, no. 3, pp. 523–535, 2003.