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Limiting amplitude principle and resonances in plasmonic structures with corners: numerical investigation

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Abstract

The limiting amplitude principle states that the response of a scatterer to a harmonic light excitation is asymptotically harmonic with the same pulsation. Depending on the geometry and nature of the scatterer, there might or might not be an established theoretical proof validating this principle. In this paper, we investigate a case where the theory is missing: we consider a two-dimensional dispersive Drude structure with corners. In the non lossy case, it is well known that looking for harmonic solutions leads to an ill-posed problem for a specific range of critical pulsations, characterized by the metal's properties and the aperture of the corners. Ill-posedness is then due to highly oscillatory resonances at the corners called black-hole waves. However, a time-domain formulation with a harmonic excitation is always mathematically valid. Based on this observation, we conjecture that the limiting amplitude principle might not hold for all pulsations. Using a time-domain setting, we propose a systematic numerical approach that allows to give numerical evidences of the latter conjecture, and find clear signature of the critical pulsations. Furthermore, we connect our results to the underlying physical plasmonic resonances that occur in the lossy physical metallic case.

Keywords: Limiting amplitude principle, Plasmonics, Black-hole waves

1. Introduction

Plasmonic structures are commonly made of noble metals (silver, gold, etc.) and dielectrics (air, vacuum, glass). At optical frequencies, metals can be dispersive, allowing the propagation of localized surface waves at the metal-dielectric interface called surface plasmons [1]. The field of plasmonics is very active as surface plasmons offer strong light enhancement, with applications to next-generation sensors, antennas, high-resolution imaging, cloaking and other [2, 3, 4, 5, 6, 7]. Several models are available in the literature to model dispersive materials. In particular, Drude model [8] is relevant for classical noble materials: in this approximation, the metal is considered as a free electrons gas (with a static lattice of positive ions). Then interactions of these electrons with the ion lattice manifest through a collision frequency parameter, representing dissipation

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11 in the equations. Over the past decades, new models have been developed, including the so-
12 called negative-index metamaterial, and interesting ideal cases (negligible dissipation) have been
13 uncovered.

14 If the source of incident illumination is monochromatic, one would naturally expect the time
15 dependent electromagnetic field to evolve asymptotically (in time) to a harmonic state with the
16 corresponding incident frequency. This asymptotic harmonic behavior is called *Limiting ampli-*
17 *tude principle* and allows to work with the associated frequency-domain boundary value problem.
18 The limiting amplitude principle has been investigated for a long time, and is well understood
19 for the wave equation and related classical scattering problems [9, 10, 11, 12, 13]. Recently there
20 has been a new interest in exploring this principle in the context of emerging plasmonic struc-
21 tures [14, 15]. In particular, the specific case of a planar interface with a non lossy Lorentz model
22 has been fully investigated in [15]. However for other configurations, the landscape is different:
23 this is especially not clear for (non lossy) plasmonic structures with corners.

24 The limiting amplitude principle is closely related to well-posedness of the corresponding
25 harmonic equation. Although the time-dependent equations system is mathematically well-posed
26 (in the usual function spaces), the frequency-domain counterpart has proven to be more challeng-
27 ing [16, 17, 18, 19, 20, 21, 22]. A key point lies in the fact that the Fourier transform of a non
28 lossy metal's constitutive law can correspond to a real negative permittivity¹. The induced possi-
29 ble change of sign of the permittivity at the interface affects the optical response. If the structure
30 has corners, the frequency-domain equations system may be mathematically ill-posed for a range
31 of critical frequencies (corresponding to a critical range of permittivities). In this range of fre-
32 quencies, hypersingular behaviors arise at the interface (especially at corners), requiring specific
33 numerical treatments to avoid spurious reflections and inaccurate predictions. Ill-posedness in
34 frequency-domain corresponds to an unphysical *infinite* electromagnetic energy, indicating that
35 *the limiting amplitude principle should not hold in that case*. This conjecture motivates our
36 exploration.

37 In this paper we provide a systematic approach to numerically assess the latter conjecture
38 in non lossy subwavelength plasmonic structures with corners. We base our strategy on a time-
39 domain framework. From typical quantities of interest (fields, energy, cross sections, Poynt-
40 ing flux, etc.), we manage to identify a signature of the underlying critical interval from the
41 frequency-domain, by using time-domain simulations. Our results show a clear change of be-
42 havior at critical frequencies. Additionally, we find this signature also when considering physical
43 structures (incorporating losses): in other words the limit non lossy case is useful to highlight
44 intrinsic resonances in physical plasmonic structures.

45 The paper is organized as follows. Section 2 presents the general context, the model problem
46 along with relevant quantities of interest. In Section 3, we specify the two-dimensional (or 2D),
47 geometrical, physical and numerical framework that we precisely consider to explore the limiting
48 amplitude principle. The numerical evidences that assess our conjecture are detailed in Section 4.
49 Then, in Section 5, we continue our efforts towards a more physical discussion. Finally Section
50 6 presents our concluding remarks.

51 **2. General context: plasmonics and limiting amplitude principle**

52 *2.1. Drude Model in plasmonics*

53 As mentioned in the introduction, plasmonic structures are commonly made of noble metals

¹It commonly provides some imaginary part for lossy materials.

54 and dielectrics, where surface plasmons arise at the interface at optical frequencies. We present
 55 below the well-known Drude model and related equations to model the electromagnetic field in
 56 those structures.

57 Metals at optical frequencies are known to be dispersive: each monochromatic wave trav-
 58 els with different speeds through the metallic material. To accurately model optical properties of
 59 metallic structures, one has thus to rely on models that take into account the frequency-dependent
 60 velocity of the wave. This dispersion phenomenon is equivalently explained as a delay effect in
 61 the reaction of the electrons of the metal to light excitation. In this work, we will use the well-
 62 known Drude model to account for this dispersion phenomenon. It is based on the kinetic theory
 63 of gases [8], considering the metal as a static lattice of positive ions immersed in a free electrons
 64 gas. In the case of scattering by a metallic obstacle, the set of (linearized) equations can be even-
 65 tually summarized as follows.

66 The time-dependent electromagnetic field is computed using time-domain Maxwell's equations
 67 with variables $(\mathbf{D}, \mathbf{E}, \mathbf{B}, \mathbf{H})^2$ where dispersive effects are incorporated through the electric consti-
 68 tutive law. The latter relates the electric displacement \mathbf{D} and the electric field \mathbf{E} and incorporates
 69 the possible time history (when dispersive effects are taken into account) via a time convolution
 70 (denoted $*_t$):

$$\mathbf{D} = \varepsilon *_t \mathbf{E}, \quad (1)$$

71 where

$$\varepsilon(t, \cdot) := \delta_0(t) \varepsilon_0 \varepsilon_r(\cdot) + \chi(t, \cdot), \quad (2)$$

72 is the space-time dielectric permittivity, ε_0 the vacuum permittivity, ε_r the relative permittivity
 73 and χ is the electric sensitivity. These quantities are defined in \mathbb{R}^3 and such that causality property
 74 holds (see *e.g.* [23] for a nice review). Since we do not take any dispersive effects into account in
 75 the dielectric, one sets $\chi = 0$ there. However, in the metallic obstacle, χ is non vanishing. If one
 76 defines the polarization current \mathbf{J} as $\mathbf{J} := -\partial_t(\chi *_t \mathbf{E})$, one can rewrite the whole set of Maxwell's
 77 equations in terms of $(\mathbf{E}, \mathbf{H}, \mathbf{J})$ variables only. In particular, \mathbf{J} verifies a linear differential equation
 78 that is linearly coupled to (\mathbf{E}, \mathbf{H}) through classical Maxwell's equations. With this approach, we
 79 do not need the expression of χ explicitly. The reason is that Drude model is entirely determined
 80 via the variable \mathbf{J} (see below). We will see later that χ plays an important role in frequency-
 81 domain.

82 We fix an *end time* $T > 0$, and a *domain* Ω , that is an open and connected subset of \mathbb{R}^3 with
 83 Lipschitz boundary. In our model, the domain Ω is the metallic obstacle, and it is immersed in
 84 a homogeneous dielectric background. For the practical choice of the end time T in numerical
 85 simulations, we refer to subsection 3.4. In what follows, μ_0 denotes the permeability of vacuum,
 86 ε_d denotes the dielectric relative permittivity of the dielectric and ε_∞ the relative permittivity (at
 87 infinite frequency) of the metallic obstacle Ω . We now set

$$\varepsilon_r(\mathbf{x}) := \begin{cases} \varepsilon_d, & \text{for } \mathbf{x} \in \mathbb{R}^3 \setminus \bar{\Omega}, \\ \varepsilon_\infty, & \text{for } \mathbf{x} \in \Omega, \end{cases} \quad (3)$$

²respectively electric displacement, electric field, magnetic induction, magnetic field.

88 and we will denote $\varepsilon := \varepsilon_0 \varepsilon_r$. Thereafter, Drude model in the time-domain writes on $[0, T]$ as:

$$\mu_0 \frac{\partial \mathbf{H}}{\partial t} = -\mathbf{curl} \mathbf{E} \text{ in } \mathbb{R}^3, \quad (4a)$$

$$\varepsilon_0 \varepsilon_d \frac{\partial \mathbf{E}}{\partial t} = \mathbf{curl} \mathbf{H} + \mathbf{J}_{ext} \text{ in } \mathbb{R}^3 \setminus \bar{\Omega}, \quad (4b)$$

$$\varepsilon_0 \varepsilon_\infty \frac{\partial \mathbf{E}}{\partial t} = \mathbf{curl} \mathbf{H} - \mathbf{J} + \mathbf{J}_{ext} \text{ in } \Omega, \quad (4c)$$

$$\frac{\partial \mathbf{J}}{\partial t} = \omega_p^2 \varepsilon_0 \mathbf{E} - \gamma \mathbf{J} \text{ in } \Omega, \quad (4d)$$

$$\mathbf{J} = 0, \text{ in } \mathbb{R}^3 \setminus \bar{\Omega}, \quad (4e)$$

89 where ω_p is the plasma angular frequency, and γ the collision frequency (coming from Drude
90 model). Here \mathbf{J}_{ext} denotes a possible external current that we will use to model volumic source
91 excitation in the following.

92 **Remark 1.** *Note that the plasma angular frequency characterizes the angular frequency above*
93 *which an incident wave can completely penetrate the metal. On the other hand, the strong*
94 *plasmonic effects induced by surface plasmons are obtained by an illumination, below the plasma*
95 *angular frequency, of subwavelength metallic structures.*

96 We will call this system *time-dependent Maxwell-Drude equations in plasmonic structures*.

97 *Well-posedness.* As commonly done, in order to compute the solution, we will artificially truncate
98 the *exterior domain* $\mathbb{R}^3 \setminus \bar{\Omega}$ and close the system (4) by adding approximate transparent
99 boundary conditions (for \mathbf{E} and \mathbf{H}), transmission conditions at $\partial\Omega$ (for \mathbf{E} and \mathbf{H}) and initial
100 conditions (for \mathbf{E} , \mathbf{H} and \mathbf{J}). At the artificial boundary, to approximate transparent boundary
101 conditions, we will use classical first order Silver-Müller boundary conditions. In this setting,
102 using classical semi-group theory, one can prove that system (4) is well posed³ (see e.g. [24] for
103 details).

104 *Excitation.* Several excitations of the scatterer are possible. A physically compliant one consists
105 of using an incident illumination that we denote $(\mathbf{E}_{inc}, \mathbf{H}_{inc})$. To take this illumination into account
106 in the set of equations, we use the non homogeneous Silver-Müller boundary conditions
107 as:

$$\mathbf{n} \times \mathbf{E} - \mathbf{n} \times \left(\sqrt{\frac{\mu_0}{\varepsilon_0}} \mathbf{H} \times \mathbf{n} \right) = \mathbf{n} \times \mathbf{g}_{inc}, \quad (5)$$

108 with $\mathbf{g}_{inc} = \mathbf{E}_{inc} - \left(\sqrt{\frac{\mu_0}{\varepsilon_0}} \mathbf{H}_{inc} \times \mathbf{n} \right)$ and \mathbf{n} the outward normal to the exterior artificial boundary.

109 **Remark 2.** *As a result, the total electromagnetic field (\mathbf{E}, \mathbf{H}) can be decomposed into an incident*
110 *contribution $(\mathbf{E}_{inc}, \mathbf{H}_{inc})$ and a scattered one $(\mathbf{E}_{sca}, \mathbf{H}_{sca})$. The scattered field $(\mathbf{E}_{sca}, \mathbf{H}_{sca})$ verifies*
111 *Maxwell's equations with homogeneous radiation condition and a source term \mathbf{J}_{ext} .*

³this result is obtained in the natural space $C^0([0, T], H(\mathbf{curl})) \times C^0([0, T], H(\mathbf{curl})) \times C^0([0, T], L^2)$ with L^2 tangential traces for \mathbf{E} and \mathbf{H} .

112 *Electromagnetic energy, Poynting vector.* We define the time-dependent total energy of system
 113 (4) by

$$\mathcal{E}(t) = \frac{1}{2} \|\sqrt{\varepsilon_0 \varepsilon_r} \mathbf{E}(\cdot, t)\|_{L^2(\mathbb{R}^3)}^2 + \frac{1}{2} \|\sqrt{\mu_0} \mathbf{H}(\cdot, t)\|_{L^2(\mathbb{R}^3)}^2 + \frac{1}{2\varepsilon_0 \omega_p^2} \|\mathbf{J}(\cdot, t)\|_{L^2(\mathbb{R}^3)}^2. \quad (6)$$

114 The space-time dependent Poynting vector also plays a central role in the study of the energy's
 115 variations, classically defined as

$$\mathbf{\Pi} = \mathbf{E} \times \mathbf{H}. \quad (7)$$

116 Recalling that we have $\operatorname{div}(\mathbf{E} \times \mathbf{H}) = \mathbf{H} \cdot \operatorname{curl} \mathbf{E} - \mathbf{E} \cdot \operatorname{curl} \mathbf{H}$, formally we get, using equations (4)

$$\begin{aligned} \frac{\partial \mathcal{E}}{\partial t}(t) &= \int_{\mathbb{R}^3} (\operatorname{div}(\mathbf{E}(\mathbf{x}, t) \times \mathbf{H}(\mathbf{x}, t)) + \mathbf{J}_{ext}(\mathbf{x}, t) \cdot \mathbf{E}(\mathbf{x}, t)) d\mathbf{x} \\ &\quad + \int_{\Omega} \mathbf{J}(\mathbf{x}, t) \cdot \mathbf{E}(\mathbf{x}, t) - \mathbf{E}(\mathbf{x}, t) \cdot \mathbf{J}(\mathbf{x}, t) d\mathbf{x} - \frac{\gamma}{\varepsilon_0 \omega_p^2} \int_{\Omega} \mathbf{J}(\mathbf{x}, t) \cdot \mathbf{J}(\mathbf{x}, t) d\mathbf{x}, \\ &= \int_{\mathbb{R}^3} \operatorname{div}(\mathbf{\Pi}(\mathbf{x}, t)) d\mathbf{x} + \int_{\mathbb{R}^3} \mathbf{J}_{ext}(\mathbf{x}, t) \cdot \mathbf{E}(\mathbf{x}, t) d\mathbf{x} - \frac{\gamma}{\varepsilon_0 \omega_p^2} \int_{\Omega} \mathbf{J}(\mathbf{x}, t) \cdot \mathbf{J}(\mathbf{x}, t) d\mathbf{x}. \end{aligned} \quad (8)$$

117 The pointwise version of the equality is the Poynting theorem. From (8), we deduce that if
 118 $\mathbf{J}_{ext} \equiv 0$, $\operatorname{div}(\mathbf{\Pi}(\mathbf{x}, t)) \equiv 0$ and $\gamma = 0$, then the energy is preserved. If $\mathbf{J}_{ext} \equiv 0$ and the quantity
 119 $\operatorname{div}(\mathbf{\Pi}(\mathbf{x}, t)) \leq 0$, then the energy is dissipated. In the rest of the paper, we focus on the limit case
 120 where there is no physical dissipation, i.e. $\gamma = 0$.

121 **Remark 3.** *When using first order Silver-Müller boundary conditions, we introduce artificial*
 122 *dissipation in the system and as a result $\operatorname{div}(\mathbf{\Pi}(\mathbf{x}, t)) \leq 0$ if the condition is homogeneous.*

123 *Long time asymptotics.* If the source is monochromatic, one would naturally expect the solution
 124 to evolve asymptotically (in time) to a harmonic state with the corresponding incident frequency.
 125 This asymptotic harmonic behavior is called *Limiting amplitude principle*. This principle holds
 126 for standard settings and is closely related to well-posedness of the corresponding harmonic
 127 equation. This principle is well-understood in classic dielectric materials. However in the non
 128 lossy case and for objects with corners, the landscape is different and less trodden.

129 2.2. Limiting amplitude principle

130 The limiting amplitude principle has been studied for a long time (e.g. [9, 10, 11, 12, 13]) and
 131 states the following. Given a source $t \mapsto e^{-i\omega t} \mathbf{F}(\cdot)$, with $\mathbf{F} \in L^2(\mathbb{R}^3)$ (and support $\operatorname{supp} \mathbf{F} \Subset \mathbb{R}^3$), a
 132 given pulsation $\omega > 0$, and a problem of the form $\partial_t^2 \mathbf{U} + \mathcal{L} \mathbf{U} = e^{-i\omega t} \mathbf{F}$, with \mathcal{L} a linear differential
 133 operator, then after a long time the solution asymptotically behaves as $\mathbf{U} = e^{-i\omega t} \mathbf{W}$ with \mathbf{W}
 134 satisfying a problem of the form $-\omega^2 \mathbf{W} + \mathcal{L} \mathbf{W} = \mathbf{F}$.

135 This statement indicates that a periodic regime is asymptotically established and therefore it is
 136 natural to consider the problem in the time-harmonic regime (stationary problem).

137 Assume for now we can write the external current $\mathbf{J}_{ext}(\mathbf{x}, t) = \Re(\underline{\mathbf{J}}_{ext}(\mathbf{x}) e^{-i\omega t})$, and $(\mathbf{E}, \mathbf{H}, \mathbf{J})(\mathbf{x}, t) =$
 138 $\Re(\underline{\mathbf{E}}(\mathbf{x}) e^{-i\omega t}, \underline{\mathbf{H}}(\mathbf{x}) e^{-i\omega t}, \underline{\mathbf{J}}(\mathbf{x}) e^{-i\omega t})$, with $\underline{\mathbf{J}}_{ext}, \underline{\mathbf{E}}, \underline{\mathbf{H}}, \underline{\mathbf{J}}$ denoting complex-valued fields. Then sys-
 139 tem (4) (with $\gamma = 0$) simplifies to

$$-i\omega \mu_0 \underline{\mathbf{H}} = -\operatorname{curl} \underline{\mathbf{E}} \quad \text{in } \mathbb{R}^3, \quad (9a)$$

$$-i\omega \varepsilon_0 \hat{\varepsilon}_r \underline{\mathbf{E}} = \operatorname{curl} \underline{\mathbf{H}} + \underline{\mathbf{J}}_{ext} \quad \text{in } \mathbb{R}^3, \quad (9b)$$

140 with

$$\hat{\varepsilon}_r(\mathbf{x}, \omega) := \begin{cases} \varepsilon_d > 0, & \text{for } \mathbf{x} \in \mathbb{R}^3 \setminus \bar{\Omega}, \\ \varepsilon_m(\omega) = \left(\varepsilon_\infty - \frac{\omega_p^2}{\omega^2} \right), & \text{for } \mathbf{x} \in \Omega \end{cases}, \quad (10)$$

141 and transmission conditions, plus some radiation condition at infinity. Indeed, $\underline{\mathbf{J}}$ is known, and
 142 equal to $i \frac{\omega_p^2 \varepsilon_0}{\omega} \underline{\mathbf{E}}$ in Ω , respectively 0 in $\mathbb{R}^3 \setminus \bar{\Omega}$. We will also denote $\hat{\varepsilon} := \varepsilon_0 \hat{\varepsilon}_r$. Above $\varepsilon_0 \varepsilon_m(\omega)$
 143 represents the non lossy Drude model permittivity. Let us point out that if $0 < \omega < \frac{\omega_p}{\sqrt{\varepsilon_\infty}}$ (optical
 144 frequency range), then $\varepsilon_m(\omega) < 0$. System (9) will be called the *frequency-dependent Maxwell-Drude*
 145 *equations in plasmonic structures*.

147 **Remark 4.** We make the abuse of terminology to denote ω by the terms *pulsation, frequency, or*
 148 *angular frequency*. However in numerical experiments, ω will be always given in rad.s^{-1} .

149 *Well-posedness.* Classical theory considers $\underline{\mathbf{E}}, \underline{\mathbf{H}} \in \mathbf{H}_{\text{loc}}(\mathbf{curl}) := \{\mathbf{X} \in L^2_{\text{loc}}(\mathbb{R}^3) \mid \forall \xi \in C_c^\infty(\mathbb{R}^3), \xi \mathbf{X} \in$
 150 $\mathbf{H}(\mathbf{curl})\}$, and (9) is equivalent to solve:

$$\mathbf{curl} \hat{\varepsilon}_r^{-1} \mathbf{curl} \underline{\mathbf{H}} - k^2 \underline{\mathbf{H}} = -\mathbf{curl} \hat{\varepsilon}_r^{-1} \underline{\mathbf{J}}_{\text{ext}} \text{ in } \mathbb{R}^3, \quad (11a)$$

$$-i\omega \varepsilon_0 \hat{\varepsilon}_r \underline{\mathbf{E}} = \mathbf{curl} \underline{\mathbf{H}} + \underline{\mathbf{J}}_{\text{ext}} \text{ in } \mathbb{R}^3, \quad (11b)$$

151 with $k = \omega \sqrt{\varepsilon_0 \mu_0}$. One can also consider the system

$$-i\omega \mu_0 \underline{\mathbf{H}} = -\mathbf{curl} \underline{\mathbf{E}} \text{ in } \mathbb{R}^3, \quad (12a)$$

$$\mathbf{curl} \mathbf{curl} \underline{\mathbf{E}} - k^2 \hat{\varepsilon}_r \underline{\mathbf{E}} = -i\omega \mu_0 \mathbf{curl} \underline{\mathbf{J}}_{\text{ext}} \text{ in } \mathbb{R}^3. \quad (12b)$$

152 Note that, if one chooses $\underline{\mathbf{J}}_{\text{ext}}$ so that $\text{div}(\underline{\mathbf{J}}_{\text{ext}}) = 0$, then $(\underline{\mathbf{E}}, \underline{\mathbf{H}}) \in \mathbf{H}_{\text{loc}}(\mathbf{curl})^2$ solution of (12) or
 153 (11) also belongs to $\mathbf{V}_{\text{loc}}(\hat{\varepsilon}; \mathbf{curl}) \times \mathbf{V}_{\text{loc}}(\mu_0; \mathbf{curl})$, with $\mathbf{V}_{\text{loc}}(\zeta; \mathbf{curl}) := \{\mathbf{X} \in \mathbf{H}_{\text{loc}}(\mathbf{curl}) \mid \text{div}(\zeta \mathbf{X}) =$
 154 $0\}$.

155 Contrary to the time-domain case, due to the change of sign of $\hat{\varepsilon}_r$ at optical frequencies,
 156 the problems (11)-(12) can be ill-posed in $\mathbf{V}_{\text{loc}}(\hat{\varepsilon}; \mathbf{curl}) \times \mathbf{V}_{\text{loc}}(\mu_0; \mathbf{curl})$. With the T-coercivity
 157 approach it has been shown (e.g. [25, 16, 26, 17, 18, 20, 21]) that there exists two cases depending
 158 on the contrast $\kappa_\varepsilon := \frac{\varepsilon_m}{\varepsilon_d}$:

159 • for contrasts κ_ε far enough from -1 , then the problem is well-posed in $\mathbf{V}_{\text{loc}}(\hat{\varepsilon}; \mathbf{curl}) \times$
 160 $\mathbf{V}_{\text{loc}}(\mu_0; \mathbf{curl})$.

161 • for contrasts κ_ε close to -1 , plasmonic hypersingularities arise at the corners of the inter-
 162 face (if any), and the problems is ill-posed in $\mathbf{V}_{\text{loc}}(\hat{\varepsilon}; \mathbf{curl}) \times \mathbf{V}_{\text{loc}}(\mu_0; \mathbf{curl})$.

163 Those guidelines can be refined for the specific case of Maxwell 2D. In that case the interval
 164 of contrasts (acceptable or not) is explicitly known. For now, let us denote I_c this interval. We
 165 will provide explicit bounds if needed for numerical purposes. Let us note that this interval I_c
 166 corresponds to a *critical interval* of angular frequencies I_ω , and that it holds that

$$\kappa_\varepsilon = -1 \quad \text{if, and only if,} \quad \omega := \omega_{sp} := \frac{\omega_p}{\sqrt{\varepsilon_d + \varepsilon_\infty}} \quad (13)$$

167 with ω_{sp} denoting the surface plasmon angular frequency. The specific case $\omega = \omega_{sp}$ is very
 168 peculiar and the problem is strongly-ill posed. In what follows we will exclude this case.

169 To sum up, in the frequency-domain, there is a critical range of angular frequencies for which
 170 the problem is then ill-posed, whereas in the time-domain the problem is always mathematically
 171 well-posed. This interesting result questions the validity of the limiting amplitude principle at
 172 critical angular frequencies, indicating that

- 173 • If $\omega \notin I_\omega$: the limiting amplitude principle holds.
- 174 • If $\omega \in I_\omega$: the limiting amplitude should not hold.

175 Using this conjecture, the rest of the paper is dedicated to provide several approaches and results
 176 to find signature of the critical interval I_ω in time-domain simulations. To that aim we will need
 177 to compute quantities of interest in frequency-domain.

178 **Remark 5.** *The limiting amplitude principle has been studied for Lorentz metamaterials (both*
 179 *permeability and permittivity can change sign in frequency-domain) for planar interfaces. It has*
 180 *been shown that this principle doesn't hold for $\kappa_\varepsilon = -1$, and that in this case the fields' amplitude*
 181 *increases linearly with respect to time [15].*

182 *Electromagnetic energy, Poynting vector and Cross sections.* Time-domain quantities such as
 183 the electromagnetic energy and the Poynting vector can be compared to frequency-domain ones
 184 if harmonic behavior is achieved. In the time-domain, we consider a real-valued harmonic ex-
 185 citation of the form $\mathbf{J}_{e,xt}(\mathbf{x}, t) = \Re(\underline{\mathbf{J}}_{e,xt}(\mathbf{x})e^{-i\omega t})$, with $\omega > 0$ and $\underline{\mathbf{J}}_{e,xt}$ a complex-valued field.
 186 If we denote $(\underline{\mathbf{E}}, \underline{\mathbf{H}})$ the solution of (9) with source term $\underline{\mathbf{J}}_{e,xt}$, then if the solution of (4) is har-
 187 monic, it should write as $(\mathbf{E}(\mathbf{x}, t), \mathbf{H}(\mathbf{x}, t), \mathbf{J}(\mathbf{x}, t)) = \Re(\underline{\mathbf{E}}(\mathbf{x})e^{-i\omega t}, \underline{\mathbf{H}}(\mathbf{x})e^{-i\omega t}, \underline{\mathbf{J}}(\mathbf{x})e^{-i\omega t})$. Then to
 188 relate frequency- and time-domain energy, the adequate quantity to start with is the time average
 189 energy

$$\underline{\mathcal{E}} = \frac{1}{T(\omega)} \int_{t_0}^{t_0+T(\omega)} \mathcal{E}(t) dt, \quad (14)$$

190 where $T(\omega)$ is equal to the time period, i.e. $T(\omega) = 2\pi\omega^{-1}$, and $t_0 \geq 0$. Using expression (6), the
 191 average energy becomes⁴

$$\begin{aligned} \underline{\mathcal{E}} &= \frac{1}{2T(\omega)} \int_{t_0}^{t_0+T(\omega)} \left\| \frac{\sqrt{\varepsilon}}{2} (\underline{\mathbf{E}}e^{-i\omega t} + \underline{\mathbf{E}}^*e^{i\omega t}) \right\|_{L^2(\mathbb{R}^3)}^2 + \left\| \frac{\sqrt{\mu_0}}{2} (\underline{\mathbf{H}}e^{-i\omega t} + \underline{\mathbf{H}}^*e^{i\omega t}) \right\|_{L^2(\mathbb{R}^3)}^2 \\ &\quad + \frac{1}{\varepsilon_0\omega^2} \left\| \frac{1}{2} (\underline{\mathbf{J}}e^{-i\omega t} + \underline{\mathbf{J}}^*e^{i\omega t}) \right\|_{L^2(\mathbb{R}^3)}^2 dt, \quad (15) \\ &= \frac{1}{4} \left(\left\| \sqrt{\varepsilon} \underline{\mathbf{E}} \right\|_{L^2(\mathbb{R}^3)}^2 + \left\| \sqrt{\mu_0} \underline{\mathbf{H}} \right\|_{L^2(\mathbb{R}^3)}^2 + \frac{1}{\varepsilon_0\omega^2} \left\| \underline{\mathbf{J}} \right\|_{L^2(\mathbb{R}^3)}^2 \right), \end{aligned}$$

192 with $\underline{\mathbf{V}}^*$ denoting the complex conjugate of $\underline{\mathbf{V}}$.

193 **Remark 6.** *We here point out a very straightforward fact that will be used later in the computa-*
 194 *tions. For the time-domain fields to have a harmonic behavior, the time average of the energy on*
 195 *an interval of length $T(\omega)$ must not depend on the chosen interval. This simple remark provides*
 196 *us with a necessary condition for a signal to be harmonic.*

⁴Recall that $\underline{\mathbf{J}} = 0$ in $\mathbb{R}^3 \setminus \tilde{\Omega}$.

197 Similarly, we can compute the time average Poynting vector over the time period $T(\omega)$ defined
 198 as follows:

$$\underline{\Pi}(\omega) = \frac{1}{T(\omega)} \int_{t_0}^{t_0+T(\omega)} \Pi(t) dt = \frac{1}{2} \Re(\underline{\mathbf{E}} \times \underline{\mathbf{H}}^*). \quad (16)$$

199 We will omit to write the space dependence using the abuse of notations $\underline{\Pi}(\omega) = \underline{\Pi}(\cdot, \omega)$,
 200 $\Pi(t) = \Pi(\cdot, t)$.

201 To further exploit information from the Poynting vector, it is natural to introduce physical
 202 quantities called cross sections. As introduced in Remark 2, we separate the contributions from
 203 the scattered fields $(\underline{\mathbf{E}}_{sca}, \underline{\mathbf{H}}_{sca})$ and the incident fields $(\underline{\mathbf{E}}_{inc}, \underline{\mathbf{H}}_{inc})$: we define $\Pi_{sca} = \underline{\mathbf{E}}_{sca} \times \underline{\mathbf{H}}_{sca}$,

$$\underline{\Pi}_{sca}(\omega) = \frac{1}{T(\omega)} \int_{t_0}^{t_0+T(\omega)} \Pi_{sca}(t) dt, \text{ and similarly } \underline{\Pi}_{inc} \text{ using the incident electromagnetic fields.}$$

205 Note that $|\underline{\Pi}_{inc}|$ is independent of the spatial variables.

206 To quantify the amount of absorbed energy \underline{P}_{abs} and scattered energy \underline{P}_{sca} at a given pulsation,
 207 we compute the fluxes of, respectively, the total Poynting vector $\underline{\Pi}$ and the scattered Poynting
 208 vector $\underline{\Pi}_{sca}$ on a closed surface S enclosing the scatterer:

$$\underline{P}_{abs}(\omega) =: - \int_S \underline{\Pi}(\omega) \cdot \mathbf{n} dS, \quad \underline{P}_{sca}(\omega) =: - \int_S \underline{\Pi}_{sca}(\omega) \cdot \mathbf{n} dS, \quad (17)$$

209 where \mathbf{n} is the outward normal vector to S . If one denotes by V the bounded volume such that
 210 $S = \partial V$, one has obviously $\underline{P}_{abs}(\omega) = - \int_V \text{div} \underline{\Pi}(\omega) d\mathbf{x}$. If there is a scatterer in the domain,
 211 not all the energy entering the volume delimited by S will leave it: some energy is absorbed
 212 ($\underline{P}_{abs}(\omega) > 0$). The cross sections are then defined relative to the power density (per unit area) of
 213 the incident field:

$$C_{abs} = \frac{\underline{P}_{abs}}{|\underline{\Pi}_{inc}|}, \quad C_{sca} = \frac{\underline{P}_{sca}}{|\underline{\Pi}_{inc}|}, \quad (18)$$

214 where C_{abs} denotes the absorption cross section, C_{sca} the scattering cross section⁵. These frequency-
 215 domain quantities are widely used to measure the absorption or the scattering features of a given
 216 scatterer. For some standard structures, it is also possible to have their analytical expression (see
 217 *e.g.* [27] and references therein).

218 3. The two-dimensional case: theoretical and numerical guidelines

219 We focus on the light scattering by a rod structure with transversal section \mathbb{D} . We seek
 220 solutions of system (4) that have an invariance with respect to the direction of the rod's axis.
 221 In this setting the tridimensional Maxwell's equations can be recast in two 2D sets of equations
 222 defining two transverse modes: TE (Transverse Electric) and TM (Transverse Magnetic).

223 In the rest of this paper, we consider that Ω is a metallic rod of bounded section \mathbb{D} , $\Omega := \mathbb{D} \times \mathbb{R}$
 224 and we concentrate on the 2D TM polarization. Then $(\vec{E}_\perp, H_z, \vec{J}_\perp)$, with $\vec{V}_\perp := (V_x, V_y)^t$, is
 225 solution of the corresponding two-dimensional version of Maxwell's equations.

⁵one can also define C_{ext} , the extinction cross section as $C_{ext} = C_{abs} + C_{sca}$. It will not be used in this work.

226 *3.1. An explicit theoretical critical interval*

227 As mentioned previously, there exists a critical interval I_ω , centered around the surface plas-
 228 mon frequency ω_{sp} , for which the problem is ill-posed in frequency-domain. In some cases, this
 229 interval is explicitly known, and hypersingular behaviors have been identified in the ill-posed
 230 configurations. We will use this framework to assert if the limiting amplitude principle holds.

231 According to (11a), in frequency-domain, the problem in \underline{H}_z becomes

$$\operatorname{curl} \hat{\varepsilon}_r^{-1} \operatorname{curl} \underline{H}_z - k^2 \underline{H}_z = -\operatorname{curl} \hat{\varepsilon}_r^{-1} \vec{J}_{ext,\perp} \text{ in } \mathbb{R}^2,$$

232 and similarly for the problem in \underline{E}_\perp (cf. (12b)). Classical theory considers $\vec{E}_\perp \in \mathbf{L}_{loc}^2(\mathbb{R}^2)$ so that
 233 $\underline{H}_z \in H_{loc}^1(\mathbb{R}^2)$, and the bounds of the interval I_c depends on the interface's geometry. Suppose
 234 that the interface $\Sigma := \partial\mathbb{D}$ is polygonal with $0 < \alpha < 2\pi$ the sharpest interior angle in \mathbb{D} . We
 235 define $I_\alpha := \max\left(\frac{\alpha}{2\pi-\alpha}; \frac{2\pi-\alpha}{\alpha}\right) > 1$, then $I_c := [-I_\alpha; -1/I_\alpha]$ (details about the derivation can be
 236 found in [16, Theorem 3.3], [28, Theorem 1]). This gives us

$$\begin{aligned} -I_\alpha \leq \kappa_\varepsilon \leq -\frac{1}{I_\alpha} &\iff \frac{\omega_p}{\sqrt{I_\alpha \varepsilon_d + \varepsilon_\infty}} \leq \omega \leq \frac{\omega_p}{\sqrt{\varepsilon_\infty + \frac{\varepsilon_d}{I_\alpha}}}, \\ I_\omega &:= \left[\frac{\omega_p}{\sqrt{I_\alpha \varepsilon_d + \varepsilon_\infty}}; \frac{\omega_p}{\sqrt{\varepsilon_\infty + \frac{\varepsilon_d}{I_\alpha}}} \right] \end{aligned} \quad (19)$$

237 Moreover, we have the following result:

- 238 • If $\omega \notin \left[\frac{\omega_p}{\sqrt{I_\alpha \varepsilon_d + \varepsilon_\infty}}; \frac{\omega_p}{\sqrt{\varepsilon_\infty + \frac{\varepsilon_d}{I_\alpha}}} \right]$: problem in \underline{H}_z is well-posed in $H_{loc}^1(\mathbb{R}^2)$. Mathematical
 239 well-posedness in this function space guarantees to have a bounded total electromagnetic
 240 energy.
- 241 • If $\omega \in I_\omega \setminus \{\omega_{sp}\}$: problem in \underline{H}_z is ill-posed in $H_{loc}^1(\mathbb{R}^2)$. There exist black-hole waves
 242 $s \notin H_{loc}^1(\mathbb{R}^2)$ that propagate towards the corners.

243 **Remark 7.** Given a polygonal interface Σ with N corners c_i , $i = 1, \dots, N$, and denoting α_i ,
 244 $i = 1, \dots, N$ all the interior angles in \mathbb{D} , one can define subintervals

$$I_{c_i} := [-I_{\alpha_i}; -1/I_{\alpha_i}], \text{ and } I_{c_i} \subseteq I_c, i = 1, \dots, N, \text{ or equivalently } I_{\omega_i} \subseteq I_\omega, i = 1, \dots, N.$$

245 This means that, depending on the contrast κ_ε (and therefore depending on the angular frequency
 246 ω), all black-hole waves, or only some of them, can be excited. This will play a certain role when
 247 interpreting numerical results.

248 **Remark 8.** Black-hole waves can be characterized as follows. Given a corner c , we denote
 249 (r, θ) the polar coordinates centered at c , the black-hole wave propagating towards the corner c
 250 is of the form $s(r, \theta) = r^{i\lambda} \Phi(\theta)$, with $\lambda \in \mathbb{R}^*$, and Φ a periodic function. Moreover it has been
 251 established that (see [20] for details):

- 252 • If $\omega \in \left[\frac{\omega_p}{\sqrt{I_\alpha \varepsilon_d + \varepsilon_\infty}}, \omega_{sp} \right)$, the black-hole wave is an odd coupled plasmon. This means
 253 that the black-hole wave exhibits two localized oscillating behaviors along the interface
 254 that are skew-symmetric with respect to the angle's bisector (Φ is an odd function).

- 255 • If $\omega \in \left(\omega_{sp}, \frac{\omega_p}{\sqrt{\varepsilon_\infty + \frac{\varepsilon_d}{I_\omega}}} \right]$, the black-hole wave is an even coupled plasmon. This means that
 256 the black-hole wave exhibits two localized oscillating behaviors along the interface that
 257 are symmetric with respect to the angle's bisector (Φ is an even function).

258 Figure 1 represents the two types of black-hole waves near a corner.

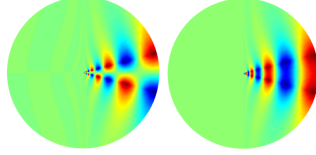


Figure 1: Representation of black-hole waves near a corner: odd (left), and even (right).

259 **Remark 9.** The specific case $\omega = \omega_{sp}$ is strongly ill-posed, the provided black-hole characteri-
 260 zation is valid for $\omega \in I_\omega \setminus \{\omega_{sp}\}$. We refer for example to [15, 19, 29] for more details.

261 The two-dimensional case is fully characterized in frequency-domain. It provides the ade-
 262 quate framework to investigate if the limiting amplitude principle holds in plasmonic structures.
 263 In particular, we will look for a signature of this critical interval I_ω in time-domain.

264 3.2. Physical problem

265 In order to investigate situations with corners, we choose an isosceles triangle of upper aper-
 266 ture $\frac{\pi}{6}$, with characteristic size (height of longest bisector) equal to 20nm for the transversal
 267 section \mathbb{D} (see Figure 2) and with area $a_T \approx 1.07 \times 10^{-16} \text{m}^2$. It is tilted so that the edge ab is
 268 vertical.

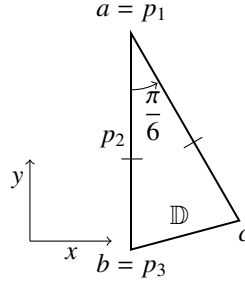


Figure 2: Physical domain and notations. 2D section in the (x, y) -plane of the metallic rod.

269 The exterior domain $\mathbb{R}^2 \setminus \bar{\mathbb{D}}$ is filled with vacuum ($\varepsilon_d = 1$). The section \mathbb{D} will either consist
 270 of

- 271 (i) Dielectric: $\varepsilon_\infty = 3.73$, $\omega_p = 0 \text{rad.s}^{-1}$.
 272 (ii) Gold: $\varepsilon_\infty = 1$, $\omega_p = 13.87 \times 10^{15} \text{rad.s}^{-1}$, with values taken from [30].
 273 (iii) Another Drude material: $\varepsilon_\infty = 3.7362$, $\omega_p = 13.87 \times 10^{15} \text{rad.s}^{-1}$.

274 We will illuminate the structure at a range of pulsations $[\omega_{min}, \omega_{max}]$ that includes the critical
 275 interval I_ω associated to both materials⁶ and that is such that $\omega_{max} \leq \omega_p$. Therefore the smallest
 276 wavelength is greater than $\frac{2\pi c_0}{\omega_p} \approx 135\text{nm}$, with $c_0 = \frac{1}{\sqrt{\epsilon_0 \epsilon_d \mu_0}}$. In this regard, the metallic structure
 277 is subwavelength for incident illuminations below the plasma angular frequency ω_p .

278 Some quantities will be visualized at three selected probe points: p_1 situated at the top vertex
 279 a , p_2 is the middle of segment $[ab]$ and p_3 situated at the left bottom vertex b . To investigate the
 280 limiting amplitude principle, we use an incident illumination $(\vec{E}_{\perp,inc}, \vec{H}_{z,inc})$ (added to radiation
 281 conditions). The latter will be

- 282 (a) a monochromatic plane wave (solution of Maxwell's in vacuum), or
- 283 (b) a polychromatic gaussian pulse (Gaussian modulated plane wave).

284 We choose the vertical direction of propagation $-y$ for the incident plane wave field. By tilting
 285 the triangle, we break the symmetry, allowing us to excite both odd and even coupled plasmons.

286 3.3. Limiting amplitude principle requirements

287 The monochromatic case (a) is readily covered by the limiting amplitude principle frame-
 288 work. Indeed, as already mentioned in Remark 2, the total electromagnetic field can then be
 289 decomposed into the incident contribution $(\vec{E}_{inc,\perp}, \vec{H}_{inc,z})$ and the scattered one $(\vec{E}_{sca,\perp}, \vec{H}_{sca,z})$.
 290 As a result, the scattered field $(\vec{E}_{sca,\perp}, \vec{H}_{sca,z})$ verifies Maxwell's equations with homogeneous
 291 radiation conditions and source term $\vec{J}_{ext,\perp}$ with support in \mathbb{D} . This source term expresses the fact
 292 that the incident plane wave $(\vec{E}_{inc,\perp}, \vec{H}_{inc,z})$ is solution of Maxwell's equation in vacuum, but is
 293 not solution in the scatterer. Since the incident field is monochromatic, so is the source term. In
 294 other words, our source term is monochromatic, with support $\subseteq \mathbb{R}^2$ and in $L^2(\mathbb{R}^2)$ which fits in
 295 the theoretical framework led by [11, 12] to investigate the limiting amplitude principle.

296 Same procedure can be applied with the Gaussian modulated plane wave (b). However, in
 297 this case, the resulting source term $\vec{J}_{ext,\perp}$ in the scattered field equation is not monochromatic
 298 anymore. The latter is in addition attenuated. This case does not readily fall into the limiting
 299 amplitude principle framework. However, such an incident field allows for the excitation of the
 300 scatterer by a whole range of pulsations using one single excitation. Moreover, using Fourier
 301 transform, the spectral response of the scatterer is easily attainable once the time-domain fields
 302 are known. Source (b) provides a practical (but empirical) approach to investigate the problem.

303 3.4. Numerical framework and strategy

304 In what follows, we will need to compute a numerical approximation of the solution of the
 305 time-domain equations. To do so, we consider a Discontinuous Galerkin Time Domain (DGTD)
 306 framework as developed in [31]. This numerical framework is particularly adapted to the chal-
 307 lenges encountered for scattering problems and has been assessed on several occasions especially
 308 for plasmonic problems (see *e.g.* [32, 33] and references therein). In the numerical tests, we use
 309 a non-dissipative DGTD scheme for the whole system with unknowns $(\mathbf{E}, \mathbf{H}, \mathbf{J})$. It relies on a
 310 discontinuous Galerkin finite element space discretization (with Lagrange nodal basis) with centered
 311 fluxes, and a leap-frog scheme in time. This scheme has the advantage to be explicit; the

⁶Here, if $\tilde{\omega} \in I_\omega$, then $\tilde{\omega} \leq \frac{\omega_p}{\sqrt{\epsilon_{\infty} + \frac{\epsilon_d}{\epsilon_a}}} \leq \omega_p$.

312 price to pay is that one should choose discretization parameters according to a CFL constraint.
 313 Computations are made on an adimensionalized version of the system, quantities plotted later in
 314 the paper have been re-dimensionalized.

315 We approximate the solution over a sufficiently long physical time T relative to the period
 316 of the incident signal: T represents 100 to 200 times the period of the monochromatic source
 317 (a), or the period of the smallest frequency in the pulse of the polychromatic source (b). This
 318 time has been empirically adjusted so that it does not affect our conclusions with regards to the
 319 convergence of the computed quantities. We are able to compute all the quantities mentioned
 320 in Section 2: time evolution of the energy, time evolution of the fields at probe points, and time
 321 averaged quantities. In particular, we compute the discrete time evolution of the total discrete
 322 energy (on the whole computational domain) and in a small domain surrounding each corner.
 323 When considering a polychromatic source (b), we compute cross sections and Poynting fluxes at
 324 the end of the simulation, using a Fourier transform that is computed "on the fly" (done in one
 325 simulation run). For illuminations considered in this work, the quantity $|\underline{\Pi}_{inc}|$ that appears in (18)
 can be computed analytically.

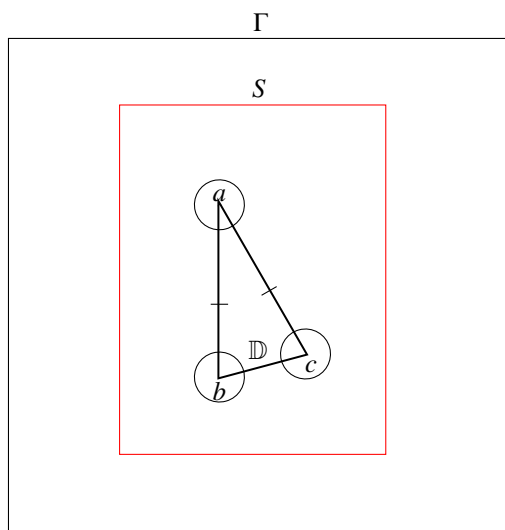


Figure 3: The computational domain is delimited by an artificial boundary Γ . A side of Γ has a length of 60nm. The cross sections are computed on a line S around the scatterer, which is approximately 20nm away from it. The black-hole fluxes and energy are computed in small disks centered at each corner.

326 As mentioned previously, the monochromatic source type (a) falls into the exact limiting
 327 amplitude principle setting, and therefore will be used to find a clear indication of a non-harmonic
 328 response to the harmonic incident field. The polychromatic source type (b) will allow to obtain
 329 a spectral response and investigate physical quantities over the whole spectral band of interest,
 330 and in one single run. The two approaches are thus complementary and are used to thoroughly
 331 test our approach.

333 The scheme has been implemented in a in house 2D Fortran code developed within the Inria
 334 Atlantis project team (Inria Sophia Antipolis, France)⁷. Previous versions of this code have

⁷<http://www-sop.inria.fr/atlantiss/>

335 been already exploited in the context of [34] and [35]. Discretization parameters have been
 336 fixed so that we use a discretization fine enough with respect to the incident wavelength and
 337 fulfill the CFL condition. If Δt denotes the physical time step, and h_{max} the space discretization
 338 parameter, we use $\Delta t \approx 10^{-19}$ s and $h_{max} \approx 1$ nm (the mesh is non uniform and is appropriately
 339 refined at the corners of the domain and close to the interface, where the size of the mesh is
 340 approximately $\frac{1}{5}h_{max}$). Unless specified, we use a \mathbb{P}_2 (polynomials of degree less than or equal to
 341 2) basis for our finite element space. Finally, in Figure 3 we detail the *computational domain* and
 342 geometrical entities that we use to compute the solution and quantities of interest. Numerically,
 343 one computes Poynting fluxes, called *black-hole fluxes* for short, around each corner, for ω in the
 344 range of pulsations of interest:

$$F_k(\omega) := \int_{D_k} \operatorname{div} \underline{\Pi}(\omega) d\mathbf{x}, \quad k = \{a, b, c\}, \quad (20)$$

345 where $(D_k)_{k=\{a,b,c\}}$ are (small) disks of radius 2nm around each corner a, b, c , respectively. Simi-
 346 larly, the energies at the vicinity of each corner are computed for $k = \{a, b, c\}$ and $t \in [0, T]$ using

$$\mathcal{E}_k(t) = \frac{1}{2} \|\sqrt{\varepsilon_0 \varepsilon_r} \vec{E}_\perp(t)\|_{L^2(D_k)}^2 + \frac{1}{2} \|\sqrt{\mu_0} H_z(t)\|_{L^2(D_k)}^2 + \frac{1}{2\varepsilon_0 \omega_p^2} \|\vec{J}_\perp(t)\|_{L^2(D_k)}^2. \quad (21)$$

349 4. Numerical results

350 First, we investigate the situation where the limiting amplitude principle (LAP) holds. This
 351 is the situation where one considers for example a dielectric inclusion (case (i) in Section 3.2).
 352 We use this simple case as a benchmark to validate our strategy. Then, we consider situations
 353 where the LAP might not hold (cases (ii) and (iii) in Section 3.2).

354 4.1. When the limiting amplitude principle holds

355 We consider here case (i), of a dielectric inclusion⁸.

356 4.1.1. Response to monochromatic illumination.

357 We consider a monochromatic incident field (a) of pulsation ω , with $\omega \in [2 \times 10^{15}, 13.8 \times 10^{15}]$
 358 rad.s⁻¹.

359 *Study of the energy.* Figure 4 represents the evolution of the electromagnetic energy \mathcal{E} over the
 360 last 10% of the total physical time i.e. $t \in [0.9T, T]$, for some incident pulsations ω . Results
 361 show that the electromagnetic energy stays clearly bounded over time and is periodic. Moreover,
 362 for each pulsation, we observe that the value of the energy mean $\underline{\mathcal{E}}$ (see Figure 5) varies in the
 363 range $[2.255 \times 10^{-15}, 2.285 \times 10^{-15}]$. Thus, it stays of the same order of magnitude over pulsations
 364 and varies fairly little (relative variation of $\approx 1\%$).

365 In the spirit of Remark 6, at each fixed pulsation ω , we compute the mean value of the energy
 366 over several time intervals of length $T(\omega)$ (these intervals are chosen around the end of the
 367 physical simulation time). We observe only relative variations of maximum 10^{-6} , that allows us
 368 to conclude that (for a fixed pulsation) the mean value of the energy is numerically independent
 369 of the chosen interval: the signal appears to be harmonic at the expected frequency.

⁸To be complete, and for a further validation of the benchmark, the very simple case of vacuum has also been tested. The results are conclusive and as expected. We choose not to reproduce them here, since the situation is completely straightforward. The results will be only used sometimes for comparison, to support our reasoning.

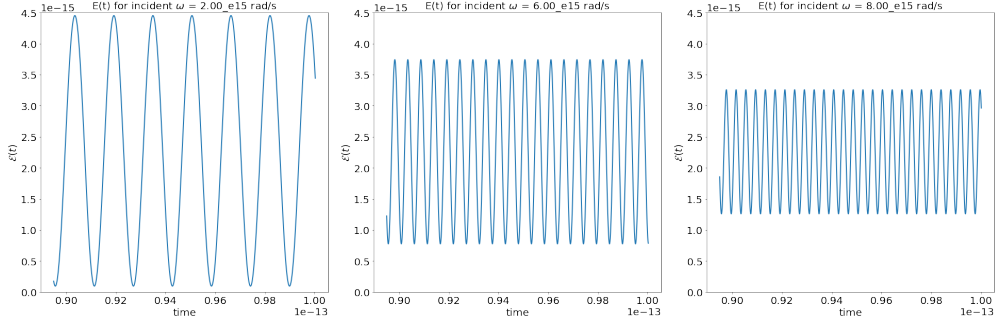


Figure 4: Representation of $\mathcal{E}(t)$ (computed via (6)) for different incident fields. The incident field is monochromatic, we vary the pulsation ω and represent the result for $\omega = 2 \times 10^{15} \text{rad.s}^{-1}$, $\omega = 6 \times 10^{15} \text{rad.s}^{-1}$, $\omega = 8 \times 10^{15} \text{rad.s}^{-1}$.

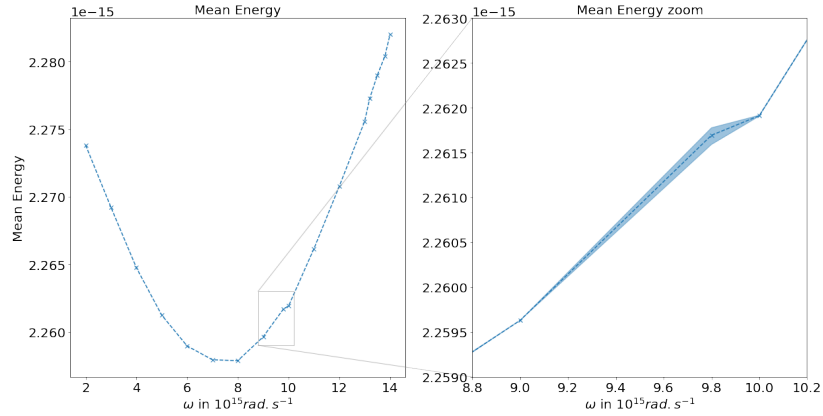


Figure 5: (Left) Mean energy $\bar{\mathcal{E}}$ (computed with (15)) with respect to the incident pulsation. For each value of the pulsation ω , we compute the mean of the energy on different time intervals of length $T(\omega)$ over the simulation time duration. (Right) Zoom of the energy mean where there is a maximum of variations, scaled by a factor 10. Computations show relative variations of order 10^{-6} .

370 *Fourier transform.* We now compute the Fourier transform (via FFT) of the magnetic field over
 371 the range of frequencies of interest at chosen probe points (see Section 3.2), and compute the
 372 relative error between the computed main pulsation and the chosen incident pulsation ω . Figure
 373 6 (Left) and Table 1 show that we recover harmonic signals centered within less than 0.4%
 374 of relative error from the incident pulsation. To observe whether these effects are also visible
 375 globally, we also plot in Figure 6 (right) the L^2 -norm in space of the Fourier transform (in time)
 376 of the total electromagnetic field. Here again, we recover a (numerical) harmonic behavior.
 377 The above observations can be viewed as strong numerical evidences that the limiting amplitude
 378 principle holds, as expected for dielectric materials.

379 4.1.2. Response to polychromatic illumination

380 We also investigate the FFT of the magnetic field for a polychromatic illumination. We
 381 choose here to represent the field H_z since this is the field that naturally compares to frequency-
 382 domain approach via equation (11a), but we could have also represented the two components of

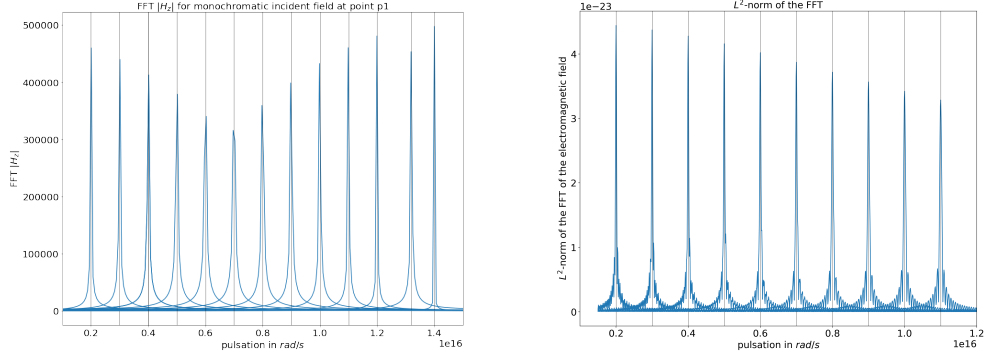


Figure 6: Left: FFT of H_z at first probe points p_1 . Similar plots are obtained at other probe points and we do not represent them here to ease the reading. Right: L^2 -norm of the FFT of the total electromagnetic field on the whole computational domain. Vertical lines represent the chosen incident ω . All obtained peaks match the incident pulsation.

ω (rad.s $^{-1}$)	Error p_1	Error p_2	Error p_3
2e15	4.61e-3	4.61e-3	4.61e-3
4e15	4.61e-3	4.61e-3	4.61e-3
6e15	4.61e-3	4.61e-3	4.61e-3
8e15	3.23e-3	3.23e-3	3.23e-3
10e15	1.66e-3	1.66e-3	1.66e-3
12e15	6.17e-4	6.17e-4	6.17e-4

Table 1: Relative errors of the computed main pulsations at the chosen probe points (via FFT) with the exact pulsation ω , with $\omega \in [2 \times 10^{15}, 12 \times 10^{15}] \text{rad.s}^{-1}$.

383 the electric fields (leading to similar conclusions). This allows to: (i) alleviate any discrepancy in
 384 the Fourier signal that may be sensitive to a single pulsation, (ii) test multiple incident pulsations
 385 in one single run. Figure 7 represents the FFT of the magnetic field at probe points in the case
 386 of propagation of a polychromatic pulse (b). Results show that a Gaussian Fourier signal is
 387 recovered without any discrepancy. Same conclusion holds for the global L^2 -norm of the Fourier
 transform, that we do not reproduce here.

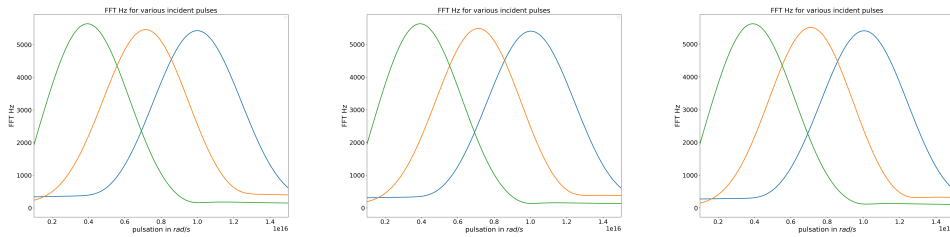


Figure 7: Modulus of the Fourier transform for various gaussian pulses at probe points p_1 (left), p_2 (middle), p_3 (right) for several Gaussian pulses. We use several central frequencies (4×10^{15} , 7×10^{15} and $10 \times 10^{15} \text{rad.s}^{-1}$).

389 *4.2. Breaking the limiting amplitude principle*

390 We now consider a metallic scatterer with parameters from case (ii) or (iii). We will follow
391 the same strategy as in Section 4.1, but first we make use of results from Section 3.1.

392 *4.2.1. Explicit critical interval of pulsations*

393 In this section we specify I_ω given in (19) for cases (ii) and (iii). Given the geometry, the
394 critical interval is associated to corner a with aperture $\frac{\pi}{6}$ (then $I_\alpha = 11$). Using Remark 7 we
395 compute the critical subintervals associated to the other corners b, c to identify when black-hole
396 waves may appear.

- 397 • For material (ii) (corresponding to gold) we obtain

$$\omega \in I_\omega \iff \frac{\omega_p}{\sqrt{12}} \leq \omega \leq \frac{\omega_p}{\sqrt{\frac{12}{11}}}$$

398 leading to $I_\omega = [4.0039 \times 10^{15}, 13.2795 \times 10^{15}] \text{rad.s}^{-1}$, and the surface plasmon angular
399 frequency (13) is equal to

$$\omega_{sp} := \frac{\omega_p}{\sqrt{2}} \simeq 9.8076 \times 10^{15} \text{ rad.s}^{-1}.$$

400 The other two corners b, c of angle $\frac{5\pi}{12}$, provide $I_{\omega_b} = I_{\omega_c} = [6.3307 \times 10^{15}, 12.3409 \times$
401 $10^{15}] \text{rad.s}^{-1}$.

- 402 • For material (iii) we obtain

$$\omega \in I_\omega \iff \frac{\omega_p}{\sqrt{11 + 3.7362}} \leq \omega \leq \frac{\omega_p}{\sqrt{3.7362 + \frac{1}{11}}}$$

403 leading to $I_\omega = [3.6131 \times 10^{15}, 7.0899 \times 10^{15}] \text{rad.s}^{-1}$, and the surface plasmon angular
404 frequency (13) is equal to

$$\omega_{sp} := \frac{\omega_p}{\sqrt{1 + 3.7362}} \simeq 6.3732 \times 10^{15} \text{ rad.s}^{-1}.$$

405 Further we obtain $I_{\omega_b} = I_{\omega_c} = [5.0524 \times 10^{15}, 6.9355 \times 10^{15}] \text{rad.s}^{-1}$.

406 **Remark 10.** *In what follows, we will indicate I_ω in light red, and the subinterval I_{ω_b} in dark red*
407 *in the plots.*

408 *4.2.2. Response to monochromatic illumination*

409 We consider a monochromatic incident field of pulsation ω , with $\omega \in [2 \times 10^{15}, 13.8 \times$
410 $10^{15}] \text{rad.s}^{-1}$. The covered pulsation range includes the critical interval I_ω associated to both
411 materials. Contrary to the previous case we expect changes for $\omega \in I_\omega$.

412 *Study of the energy.* Figure 8 represents the evolution of the energy for several incident pulsation
413 values for both cases. Contrary to the previous case, we observe a drastic change of behavior
414 of the energy when the pulsation ω of the monochromatic source belongs to I_ω : the energy
415 drastically increases by several orders of magnitude (10^{-13} compared to 10^{-15}), and doesn't
416 exhibit a clear periodic behavior. This change is clearly visible when ω "enters" the critical
417 interval. Moreover, at lower pulsations, the energy exhibits a periodic behavior. When ω "leaves"
418 the critical interval, the energy drastically decreases. For case (ii) it is not clear that we recover
419 a periodic signal at the chosen pulsation (located right outside of the critical interval), however
for case (iii) the periodic behavior for $\omega \notin I_\omega$ is more visible. Figure 9 represents the means of

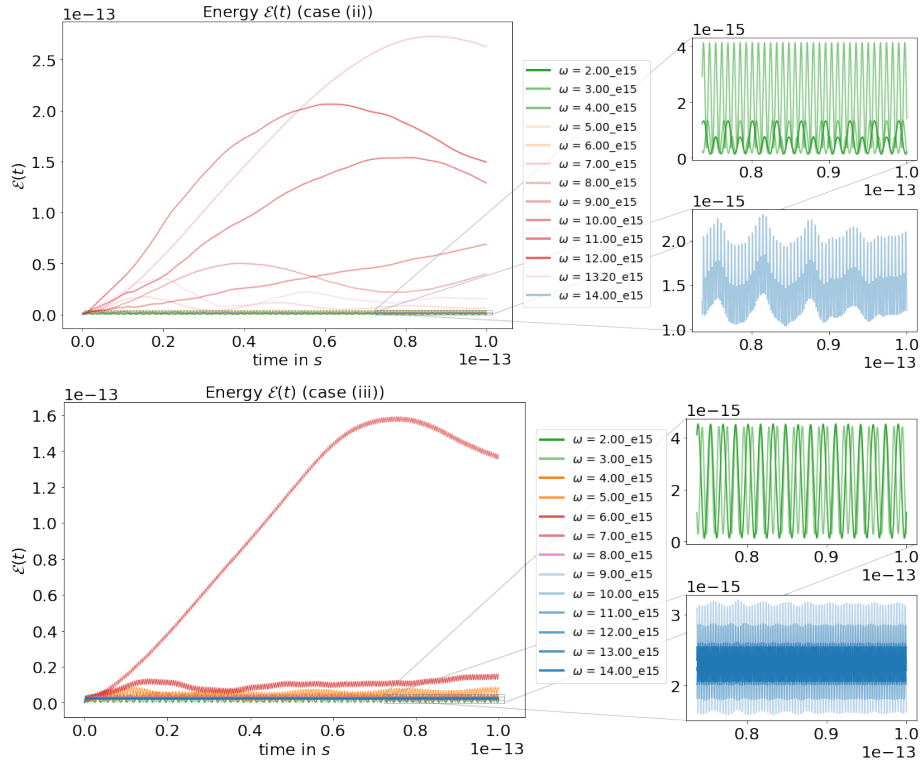


Figure 8: Representation of $\mathcal{E}(t)$ (computed via (6)) for different incident fields for case (ii) (top) and for case (iii) (bottom), with zooms at the long time simulation. The green and blue curves correspond to $\omega \notin I_\omega$, whereas the warm colored curves correspond to $\omega \in I_\omega$.

420 energy \mathcal{E} with respect to the monochromatic pulsation. For each incident source, we compute the
421 mean of the energy for different time intervals of length $T(\omega)$ over the final part of the simulation
422 time duration. The light blue shadow indicates the variations between those computations (we
423 report the minimal and maximal values), scaled by a factor 10. As observed before, the energy
424 is considerably more important at critical pulsations (indicated by the red zones). Additionally
425 the computation of the mean \mathcal{E} is highly sensitive to the time interval when we choose $\omega \in I_\omega$,
426 indicating that a periodic regime may not be established. Note that the energy mean is two orders
427 of magnitude stronger than what was observed in Section 4.1. Furthermore, one can observe that
428 the strongest variations within the means are obtained when all corners are excited ($\omega \in I_{\omega_b}$).
429

430 Based on the energy observations, one can conclude that there is definitely a change of behavior
 at critical pulsations, indicating that the limiting amplitude principle should not hold.

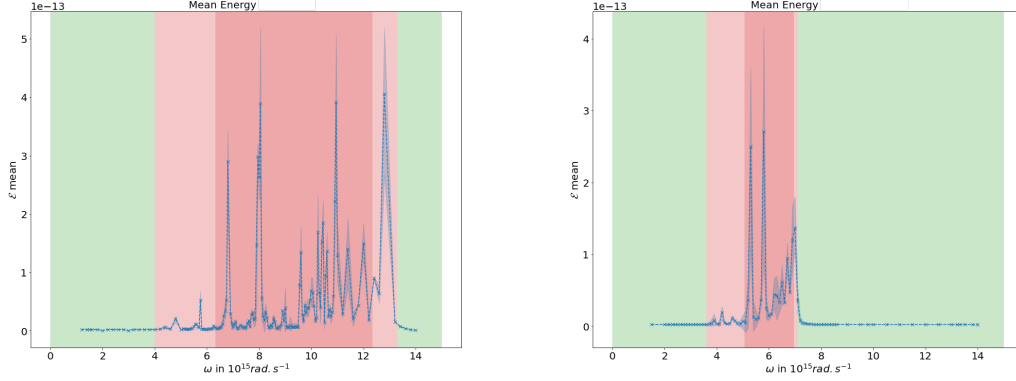


Figure 9: Mean of energy $\underline{\mathcal{E}}$ (computed with (15)) with respect to the monochromatic pulsation: for case (ii) (left), for case (iii) (right). The green zones indicate when $\omega \notin I_\omega$, the red zones indicate when $\omega \in I_\omega$. The darker red zone indicates the critical subinterval $\omega \in I_{\omega_b}$.

431

432 *Fourier transform at probe points.* Figure 10 represents the Fourier transform of the magnetic
 433 field over the range of frequencies of interest at probe point p_1 (see Section 3.2). Similar plots
 434 have been obtained for other probe points, we do not present them here. Figure 11 represents the
 435 L^2 -norm in space of the Fourier transform (in time) of the whole electromagnetic field (\vec{E}_\perp, H_z) .

436 Results show that we still recover harmonic-like signals centered at the incident pulsation,
 437 however the signal is perturbed for critical pulsations. We can make several observations:

- 438 • at each frequency, one main peak occurs at the pulsation of the incident field. The numer-
 439 ical relative error to the exact value does not exceed the one obtained in Section 4.1,
- 440 • for some pulsations inside the critical interval, the main peak is wider and/or stronger in
 441 intensity,
- 442 • for pulsations inside the critical subinterval, secondary peaks do appear.

443 The last two items above invalidate the limiting amplitude principle.

444 In the next section we compute the Fourier transform when considering a Gaussian pulse,
 445 where the break of the harmonic signal is significantly more striking.

446 4.2.3. Response to polychromatic illumination

447 We now investigate the response of the metallic scatterer to a pulse illumination (b). As
 448 before, we investigate the Fourier transform of the magnetic field.

449 *Fourier transform.* Figure 12 represents the Fourier transform of the magnetic field at the probe
 450 points p_1, p_2, p_3 for a Gaussian pulse centered at $4 \times 10^{15}, 7 \times 10^{15}$ and 10×10^{15} rad.s^{-1} . One
 451 clearly observes that the Gaussian signal is recovered for $\omega \notin I_\omega$ and completely perturbed when
 452 $\omega \in I_\omega$. These effects are also observable globally. In Figure 13, we plot the L^2 -norm (in space)
 453 of the Fourier transform of the whole electromagnetic field (\vec{E}_\perp, H_z) (we here choose to represent
 454 only one central frequency 7×10^{15} rad.s^{-1} , the others being similar).

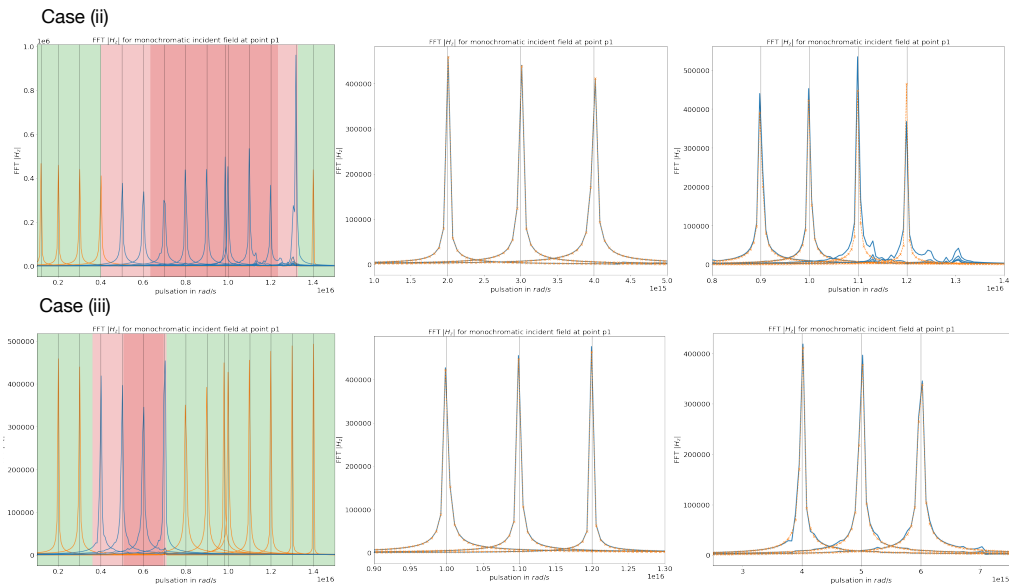


Figure 10: (Left) FFT of H_z at first probe point p_1 : for case (ii) (top row), for case (iii) (bottom row). Vertical lines represent the chosen ω . The green zones indicate when $\omega \notin I_\omega$, the red zones indicate when $\omega \in I_\omega$. The darker red zone indicates the critical subinterval $\omega \in I_{\omega_b}$. (Middle, Right): samples of FFT from the two cases: for $\omega \notin I_\omega$ (middle), and for $\omega \in I_\omega$ (right). The orange 'x' curves correspond to FFT peaks in vacuum (where the response is always harmonic).

4.2.4. Conclusion

To sum up, through various quantities of interests, we can clearly identify a change of behavior in the spectral response in the critical interval. This provides numerical evidences about the proposed limiting amplitude principle conjecture. Moreover, using polychromatic pulse illumination, one is directly able to find precisely traces of the critical interval. In what follows, we continue our investigation and examine the impact of underlying black-hole waves on the time-domain simulations.

5. Black-hole waves resonances

Results from previous sections clearly highlight the break of the limiting amplitude principle for critical pulsations. In this section we investigate its impact on more physical quantities and situations.

5.1. Cross sections and black-hole fluxes

The amount of light diffracted or absorbed by an illuminated tridimensional structure is measured by energy fluxes. The intrinsic capacity of an object to diffract or absorb light is then measured relative to the power of the incident light beam excitation. One way to quantify this is to measure the diffraction or absorption cross sections (defined in (18)). As a matter of fact, these provide the equivalent area of the incident beam that would have to be used to obtain the same energy than that provided by the illuminated object. Thus when a scatterer absorbs or scatters light on a much larger area compared to its physical size, it transpires in the absorption and

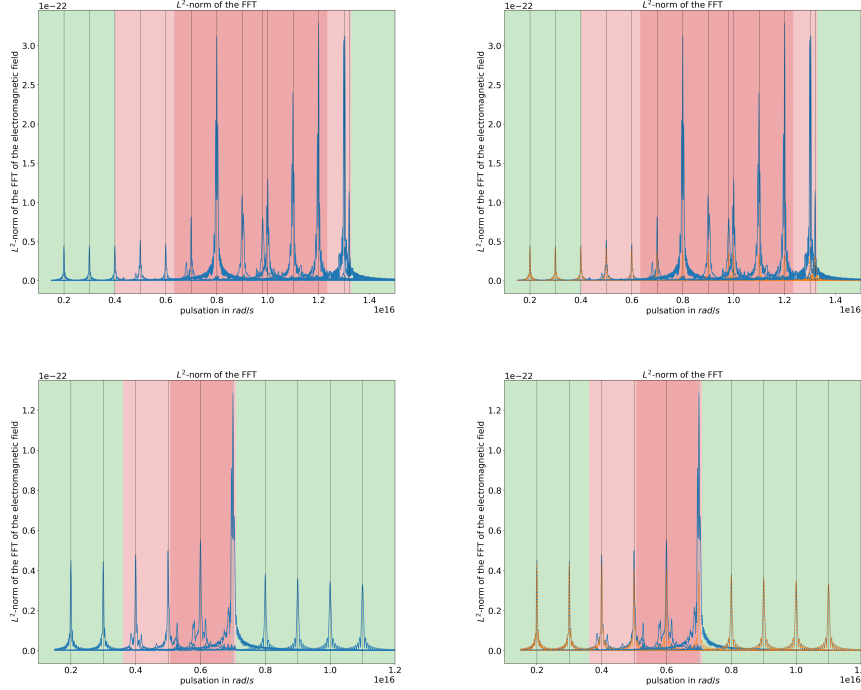


Figure 11: L^2 -norm of FFT of the whole electromagnetic field (left) comparison with vacuum results (right): for case (ii) (top row), for case (iii) (bottom row). The orange ‘ \times ’ curves correspond to FFT peaks in vacuum and the computations have been performed on same meshes for both cases. Vertical lines represent the chosen ω . The green zones indicate when $\omega \notin I_\omega$, the red zones indicate when $\omega \in I_\omega$. The darker red zone indicates the critical subinterval $\omega \in I_{\omega_b}$.

474 scattering cross sections as intense peaks, and their location indicates the associated resonance
 475 frequency. Cross sections are by nature positive and in the 2D setting that we consider, cross sec-
 476 tions have the dimension of a length and provide an equivalent perimeter. We now investigate
 477 how they vary for cases (ii) and (iii), in the context of a polychromatic illumination.

478 **Remark 11.** We choose a polychromatic source that illuminates the range of interest [$1 \times$
 479 10^{15} , 14×10^{15}] $rad.s^{-1}$. With these chosen parameters, the range of frequencies at which we
 480 illuminate the structure lies in the visible-near UV range. Furthermore, as mentioned in Section
 481 3.2, the structure used is subwavelength.

482 *Cross sections.* Figure 14 represents the scattering and absorption cross sections obtained with
 483 an incident Gaussian pulse for both Drude materials. It must be emphasized that our interest
 484 lies more in finding a clear trace of the critical interval than in extracting a precise position of
 485 resonances. Indeed, results show a clear trace of the critical interval: strong resonances do appear
 486 for $\omega \in I_\omega$. While C_{sca} remains positive, C_{abs} presents quite significant unphysical oscillations
 487 and negative values. We observe that the latter is also sensitive to mesh discretization and the
 488 chosen degree of interpolation (even for a refined mesh).

489 These observations can be explained. Scattering cross section C_{sca} tracks the far-field’s re-
 490 sponse whereas absorption cross section C_{abs} is linked to the near-field’s response of the scat-

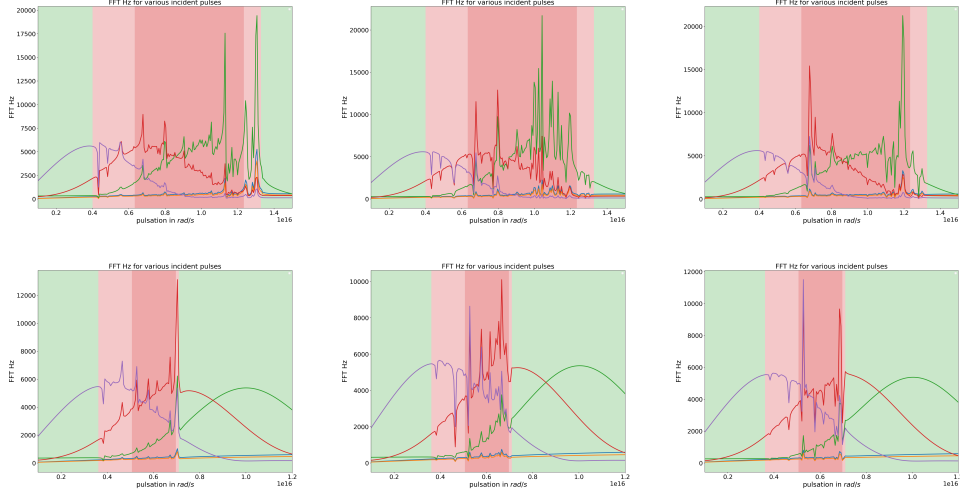


Figure 12: FFT of $|H_z|$ at probe points p_1 (left), p_2 (middle), p_3 (right) for several Gaussian pulses centered at 4×10^{15} , 7×10^{15} or 10×10^{15} $\text{rad}\cdot\text{s}^{-1}$ and two widths: for case (ii) (top row), for case (iii) (bottom row). The green zones indicate when $\omega \notin I_\omega$, the red zones indicate when $\omega \in I_\omega$. The darker red zone indicates the critical subinterval $\omega \in I_{\omega_b}$.

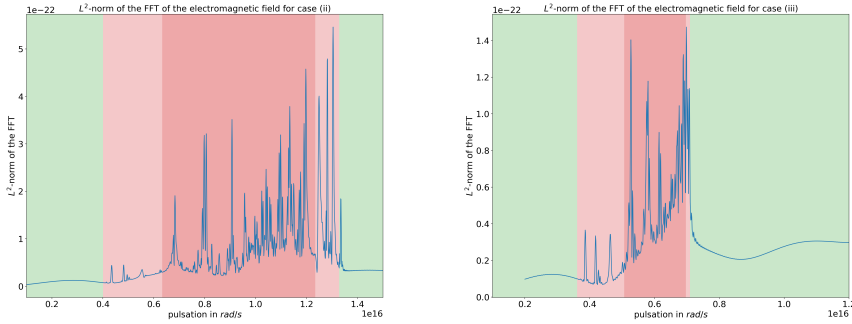


Figure 13: L^2 -norm in space of the time FFT of the whole electromagnetic field for a Gaussian pulse centered at 7×10^{15} $\text{rad}\cdot\text{s}^{-1}$: for case (ii) (left), for case (iii) (right). The green zones indicate when $\omega \notin I_\omega$, the red zones indicate when $\omega \in I_\omega$. The darker red zone indicates the critical subinterval $\omega \in I_{\omega_b}$.

491 terer. The more erratic behavior of C_{abs} can thus be explained by the difficulties to accurately
 492 capture black-hole waves close to the corners, where discretization has to be fine enough to avoid
 493 spurious reflections. This phenomenon has been well characterized in frequency-domain [20],
 494 where an efficient modified finite element method (FEM) approximation with corner treatments
 495 has been developed. Results may indicate that, even for time-domain formulations for which
 496 the problem is mathematically well-posed, the discretization fails to approximate those highly-
 497 oscillatory behaviors and would benefit from a similar specific corner treatment. This will be
 498 part of future investigations. As mentioned before, while the polychromatic illumination doesn't
 499 fit the theoretical LAP framework, it allows to highlight the predicted phenomena in a single

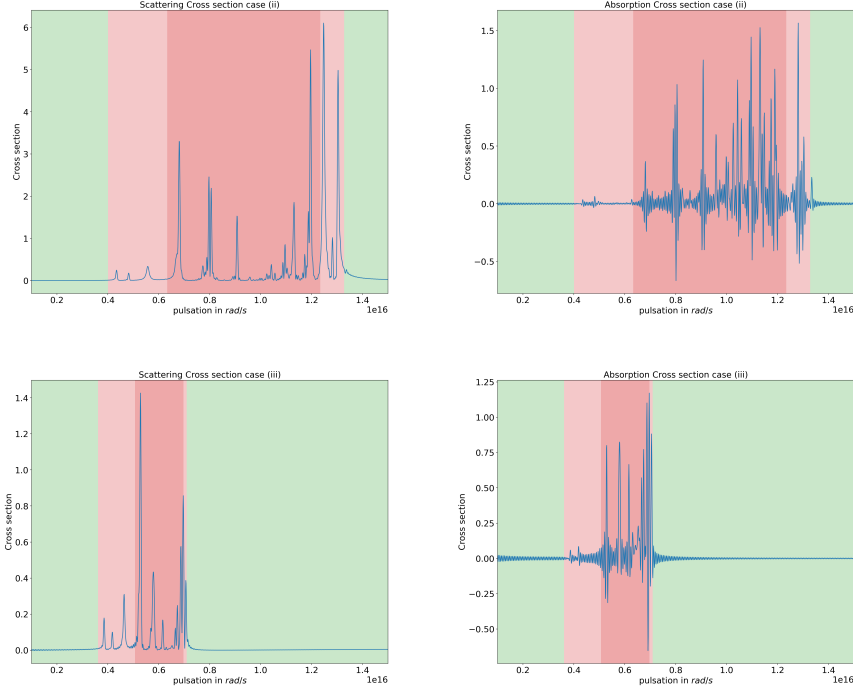


Figure 14: (Left): Scattering cross sections C_{sca} (computed with (18)) when considering a Gaussian pulse: for case (ii) (top row), for case (iii) (bottom row). (Right): Absorption cross sections C_{abs} when considering a Gaussian pulse: for case (ii) (top row), for case (iii) (bottom row).

500 run. This strongly suggests a systematic strategy to numerically identify signatures of a critical
 501 interval on a given structure, even when the theory is not known.

502 *Poynting fluxes.* Figure 15 compares the total Poynting flux to the black-hole fluxes around each
 503 corner of the triangle scatterer. The black-hole fluxes $(F_k)_{k=\{a,b,c\}}$ are computed in a disk centred
 504 at the corner and of radius 2nm, see (20) and Figure 3 for details. Results show that:

- 505 (i) all black-hole fluxes are (almost) equal to zero when $\omega \notin I_\omega$ (no black-hole waves are
 506 excited);
- 507 (ii) black-hole fluxes remain small when $\omega \in I_\omega \setminus I_{\omega_b}$, that is when only the black-hole singu-
 508 larities located at the corner a can be excited;
- 509 (iii) all black-hole fluxes are significant when $\omega \in I_{\omega_b}$ (corresponding to all black-hole singu-
 510 larities being excited); in this situation, we also observe that almost all the contributions
 511 to the Poynting flux are due to the corners.

512 All those observations are in accordance with theory from frequency-domain detailed in [20]:
 513 this is closely related to black-hole excitation.

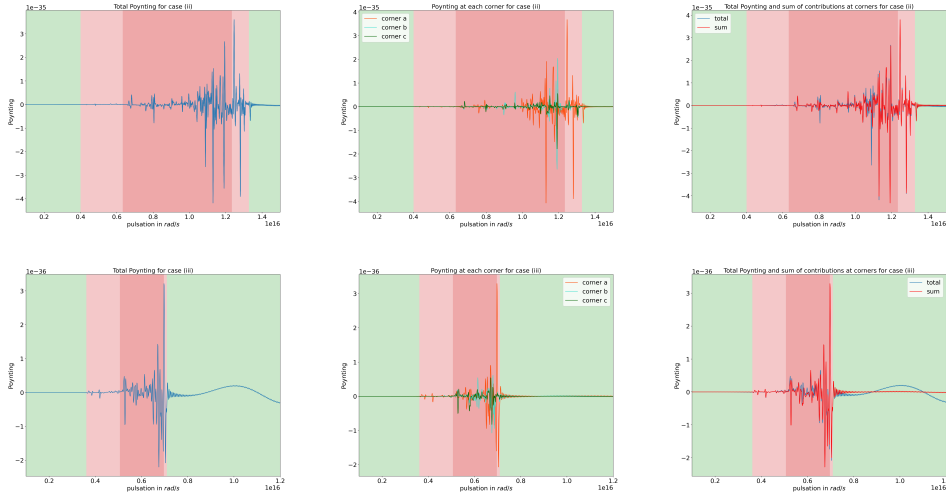


Figure 15: Poynting fluxes when considering a Gaussian pulse illumination, for case (ii) (top row) and for case (iii) (bottom row). We compute the total Poynting flux (left column), the black-hole fluxes (middle column), and compare the total Poynting flux to the sum of the black-hole fluxes (right column).

514 All results above illustrate that strong responses arise when illuminating a polygonal metallic
 515 obstacle with a source swiping critical pulsations ω , and those strong behaviors are directly
 516 connected to the black-hole waves that are known to exist in frequency-domain. Here we considered
 517 an *ideal* case without dissipation. In what follows we compare results with and without
 518 dissipation: this allows to identify whether the above observations are degenerate behaviors (i.e.
 519 they only occur in the absence of dissipation), or intrinsic behaviors (i.e. they are observable also
 520 with dissipation), of the physical structure.

521 5.2. Back to physics: the role of dissipation

522 Metals are always lossy, meaning that in practice one considers $\gamma \neq 0$ in equation (4d). In
 523 this section we study the impact of introducing dissipation ($\gamma \neq 0$) in our computations. Note
 524 that adding dissipation changes the asymptotics of the solution since the solution will be damped
 525 (up to vanishing). Moreover, problem (9) in frequency-domain is always mathematically well-
 526 posed in presence of dissipation. This implies that there are actually no critical pulsations to
 527 consider. We explore the question of finding a signature of the limit problem (and consequently
 528 limit behaviors) in lossy cases.

529 Figures 16 and 17 present comparisons between previous cross sections and Poynting fluxes,
 530 and the ones obtained when we add dissipation: we now consider models (ii) and (iii) with the
 531 physical value $\gamma = 4.515 \times 10^{13} \text{rad.s}^{-1}$. Obtained cross sections for lossy cases remain positive
 532 (which is more physically relevant) and less sensitive to the mesh discretization. However
 533 in both configurations (non lossy, lossy), cross sections present similar behaviors: strong reso-
 534 nances arise at "critical" pulsations. Those resonances have less intensity with dissipation, and
 535 dissipation prevents strong spurious resonances mentioned above in the non lossy case (assuming
 536 the mesh is sufficiently refined at the corners). The fact that intense resonance peaks remain can

537 be explained via the frequency-domain framework [36, 20]. By adding dissipation, the frequency
 538 problem becomes well-posed, however strong oscillations at the corners remain. Dissipation al-
 539 lows to *attenuate the black-hole waves*, $s \notin H_{\text{loc}}^1(\mathbb{R}^2)$ being replaced by $s^\gamma \in H_{\text{loc}}^1(\mathbb{R}^2)$, and selects
 540 the *outgoing* ones (limiting absorption principle), where the outgoing wave is the one traveling
 541 towards the corners (as reference to their names). Observed peaks then correspond to attenu-
 ated black-hole waves going towards the corners. Similarly, Poynting fluxes get *smoothed out*

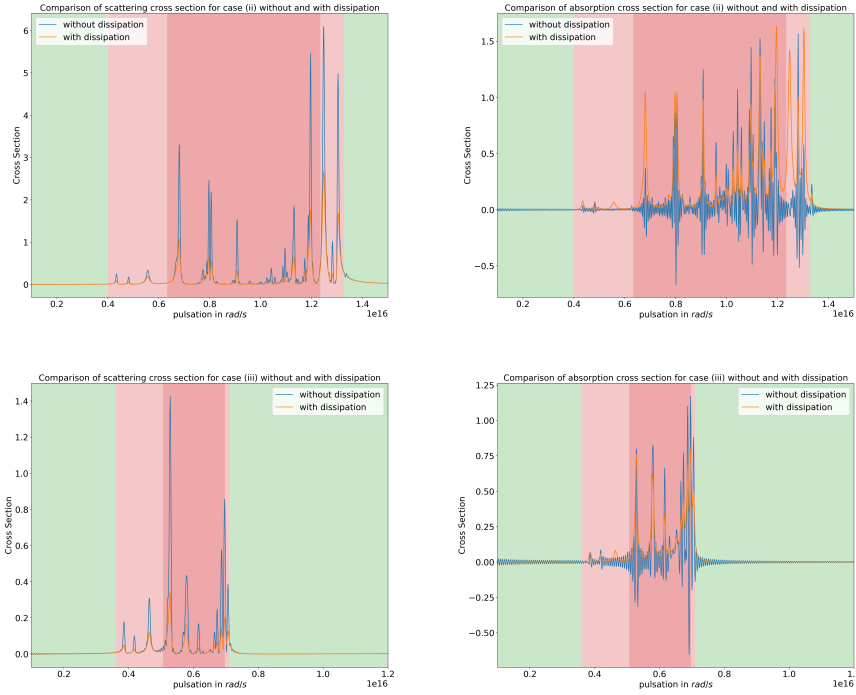


Figure 16: Comparison of cross sections for cases (ii) (top row) - (iii) (bottom row) with and without dissipation: scattering C_{sca} (left), absorption C_{abs} (right).

542
 543 by dissipation, and most of the energy fluxes come from the corners at critical pulsations: this
 544 corresponds to *attenuated* black-hole resonances contributions.

545 **Remark 12.** *As explained in Section 3.1, the frequency theory also allows to characterize the*
 546 *singularities as odd or even coupled plasmons depending on the surface plasmon frequency. Due*
 547 *to the chosen non symmetric configuration, we expect that the excitation of odd plasmons will be*
 548 *favoured under the surface plasmon frequency, whereas the excitation of even plasmons will be*
 549 *favoured above the surface plasmon frequency. One can identify a change of behavior in C_{sca} ,*
 550 *where the scattering cross section vanishes for $\omega = \omega_{sp}$.*

551 To sum up, studying the limit non lossy models allows to explain underlying resonances from
 552 physical lossy configurations.

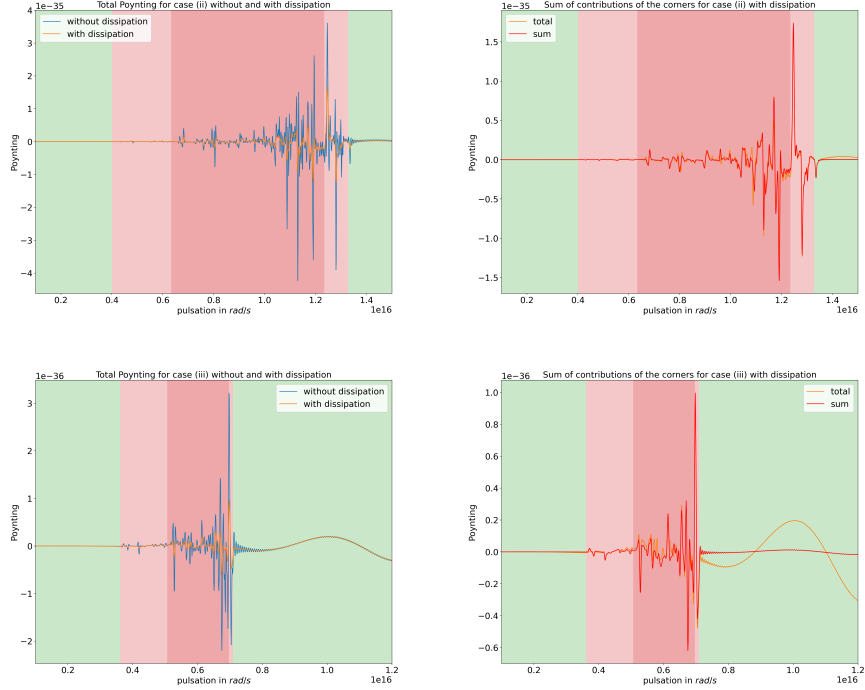


Figure 17: Left: comparison of Poynting fluxes with and without dissipation: case (ii) (top row), case (iii) (bottom row). Right: comparison of total Poynting fluxes and the sum of the Poynting fluxes at the corners: case (ii) with dissipation (top row), case (iii) with dissipation (bottom row).

553 5.3. Corner effects

554 It is well known via Mie theory that dissipative subwavelength cylindrical scatterers exhibit
 555 one resonance located at the surface plasmons frequency ω_{sp} . This resonance is called a dipole
 556 resonance. This result is in accordance with the fact that the critical interval reduces to exactly
 557 $\{\omega_{sp}\}$ for smooth interfaces. We simply provide below illustrations of the above statement, using
 558 the same material properties and for \mathbb{D} a disk with same perimeter as the considered triangle.
 559 Figure 18 shows that only one resonance at ω_{sp} is observed. This also allows to additionally
 560 validate our approach by recovering a known result.

561 On the other hand, from Section 5.2 we identify multiple resonances at critical pulsations,
 562 and those resonances are related to specific surface plasmons (called in the limit case black-hole
 563 waves). In other words, this single subwavelength structure with corners allows to produce mul-
 564 tipolar resonances (quadripolar, octopolar, etc...). Furthermore, the level of intensity of these
 565 multiple resonances is equivalent to the level of the dipolar resonance that could be obtained
 566 with a cylinder with equivalent section perimeter (see Figure 19). The resonance obtained with
 567 a cylinder is however broader. Thus, it is possible to use triangular scatterers rather than circular
 568 ones to obtain: (i) multiple resonances with one single structure, (ii) sharper resonances of equiv-
 569 alent intensity than the single dipolar resonance of a cylindrical structure of equivalent perimeter.
 570 Polygonal interfaces then offer a larger range of possible light enhancements.

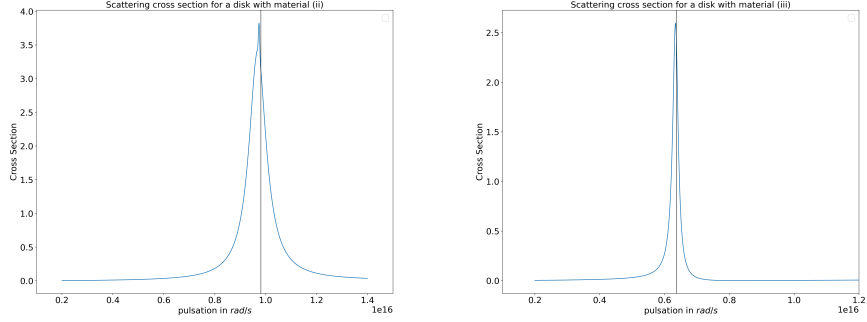


Figure 18: Scattering cross sections for a disk made of a Drude material (ii) and (iii) (no dissipation). The 2D section of the cylinder (a disk) has the same perimeter as the triangle section used in this work. The maximum is achieved at $\omega = 9.74 \times 10^{15} \text{rad.s}^{-1}$ for case (ii) (0.6% relative error to ω_{sp}) and $\omega = 6.34 \times 10^{15} \text{rad.s}^{-1}$ for case (iii) (0.5% relative error to ω_{sp}).

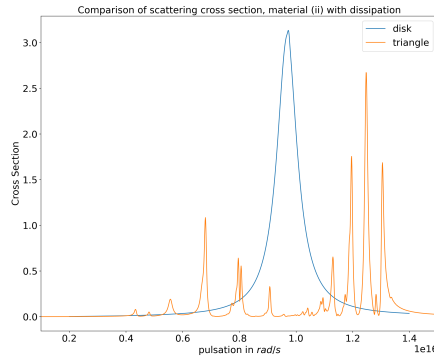


Figure 19: Comparison of scattering cross sections for a disk made of a Drude material (ii) with dissipation using the triangular section and a disk section with same perimeter as the triangle section.

571 6. Conclusion

572 In this paper we provided a systematic numerical approach to identify if the limiting ampli-
 573 tude principle holds in ideal plasmonic structures that is, non lossy plasmonic structures with
 574 corners, and identified the underlying causes when it does not. Moreover, a study of cross sec-
 575 tions and Poynting fluxes revealed that the underlying resonances appearing at critical pulsations
 576 are related to localized surface plasmons at the corners called black-hole waves. We found that
 577 those characterized behaviors are intrinsic to the problem, as being captured with or without
 578 dissipation. Overall, this first work provides an interesting framework to investigate unexplored
 579 models and configurations, where no theory is available. One can for example now investigate
 580 the fully three-dimensional case, where the associated critical interval is not explicitly known
 581 in general, and test other plasmonic models such as Drude-Lorentz or more generalized models
 582 (such as those in [31]). In particular, future work will include the study of non-local effects.

583 **Acknowledgment**

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585

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