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## GENERAL BLOCH MODELING OF PIECEWISE PERIODIC STRUCTURES

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### ABSTRACT

*Bloch theory is broadly used to determine local wave dispersion characteristics of periodic media, which are key vibroacoustic indicators in a number of research fields such as metamaterials. Nonetheless, numerical frameworks based on Bloch theory, such as the Wave Finite Element method, can also be used to compute the dynamic response of large-scaled waveguides' assemblies. This formalism was used to tackle a variety of problems such as damage localization, material identification or vibroacoustic optimization of periodic media. The focus of this work is therefore to review and generalize the wave-finite element framework, as a tool for exploring wave dispersion, scattering or dynamic response in piecewise periodic structures.*

## 1 INTRODUCTION

The need to de-carbonize our economy has led to an urgent strive for lightweight alternatives to various structural components used in the transportation industry. Combined with increasingly challenging acoustic comfort and safety EU regulations, the compromise between enhanced vibroacoustic performances and lightweight design is often achieved at the cost of an increased topological and material complexity. In the mid-frequency range (generally the low-medium audible bandwidth), this complexity involves a variety dynamic behaviors which may co-exist at different scales (e.g. see. [1]) In terms of modeling, this can be tackled by using refined finite element descriptions of these structures. Therefore, tremendous progresses had to be made on the acoustic modeling and optimization of complex metamaterials and meta-structures, which are generally relying on model order reduction schemes or the introduction of relevant yet simple vibroacoustic indicators. Noteworthy, most of these indicators (e.g. transmission loss, dispersion curves, scattering coefficients) have the remarkable advantage of being retrieved from a single unit-cell model by exploiting multi-scale or periodic structure theories (see [2, 3]). For acoustic applications, one can mention the succession of works [4–8] only to cite a few dedicated to retrieve the sound transmission loss of periodic waveguides. Although it is now possible to design and produce meta-structures with desirable vibroacoustic performances within targeted bandwidths, these components still rarely appear among the most critical parts of a structure. Most metamaterials are therefore considered as "add-ons" to an existing structure whose integrity is guaranteed or monitored using state-of-the-art inspection techniques. A further embedding of lightweight meta-structures in the design process of critical transportation components will undeniably raise the following question: can we achieve the structural health monitoring of such complex structures?

In the presence of variable or uncertain boundary conditions, wave-based inspection techniques can prove efficient to perform damage inspection on specific regions of a waveguide. Periodic structure theory can also help to compute the dynamic response, hence provides an efficient numerical framework for modeling large-scaled slender structures such as pipes, ducts, blades, etc. The so-called "Wave Finite Element" method have been used in a variety of contexts relevant to SHM/NDE applications. Kessentini et al. [9] used the diffusion matrix method along with the WFE framework to study the transmission and reflection of acoustic waves in ducts. Mitrou et al. [10] investigated the scattering produced by infinite plates' junctions using the same method. Gras et al. [11] extended a formulation for railways with periodic supports and several applications were proposed to compute the forced response and scattering matrices of two waveguides connected by a junction, an external load or a damage/delamination [12]. An overview of the Bloch modeling framework is therefore proposed in this work.

## 2 METHOD AND RESULTS

The above formalism was implemented in many ways, including various edges or boundary conditions, the coupling of two waveguides with a middle junction or external loads. First, we propose an overview of Bloch modeling framework for 1D structures with recent developments on resolution techniques. Our aim is then to present a simple formalism addressing a wide range of possible waveguide configurations, where Bloch theory can be used to reduce considerably the modeling and computational efforts. Note that in many cases, a structure can be decomposed into a combination of periodic parts, separated by joints, coupling and loading regions, so that it can eventually be described as a piecewise periodic structure. The displacement  $\mathbf{u}$  at any unit-cell position  $p$  of a  $N$ -cell waveguide section can be expressed using the following Bloch

expansion:

$$\mathbf{u}_p = \sum_{i=1}^n \lambda_i^p \Psi_i^+ \mathbf{Q}_i^+ + \lambda_i^{N-p} \Psi_i^- \mathbf{Q}_i^-, \quad (1)$$

where  $\mathbf{Q}_i^+$  and  $\mathbf{Q}_i^-$  are the respective wave amplitudes while the  $n$  positive  $\Psi_i^+$  and negative  $\Psi_i^-$  eigenvectors and the propagation constants  $\lambda_i$  are solution of the quadratic eigenvalue problem Eq.(2), which is function of the unit-cell's dynamic stiffness sub-matrices  $\mathbb{D}_{ab}$ :

$$\left( \frac{1}{\lambda_i} \mathbb{D}_{RL} + (\mathbb{D}_{LL} + \mathbb{D}_{RR}) + \lambda_i \mathbb{D}_{RL} \right) \Psi_i = 0. \quad (2)$$

A simple validation is shown in Figure 1. The structure is made of four different regions name 'A', 'B', 'C', and 'D', while four loads are applied, including a load splitting the region 'B' in two. The framework provides good results although some numerical issues have to be discussed.

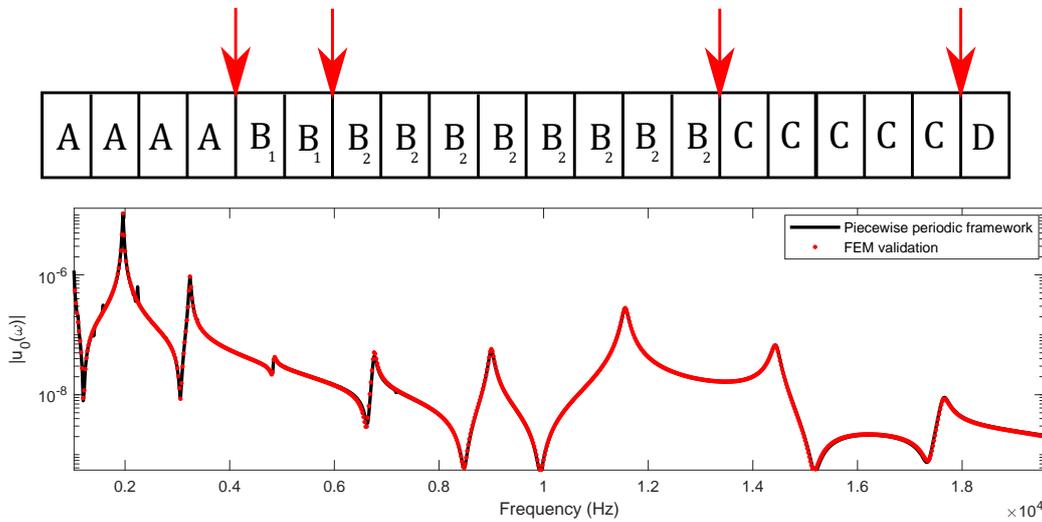


Figure 1. Forced response of a piece-wise periodic structure with irregular punctual loads

### 3 PERSPECTIVES

The framework is generalized and applied to various examples relevant to SHM and in a context damage scattering analysis.

A typical case scenario is shown in Figure 2, where different FE models of structural damages can be considered in a large-scaled duct whose periodicity is of equivalent or higher dimension that those considered damages or defects. Note that this reduced models can be efficiently used for sub-structuring purposes, while different 'damages' can be analyzed with remarkable computational efficiency (i.e. 2-4 orders of magnitude compared with FEM).

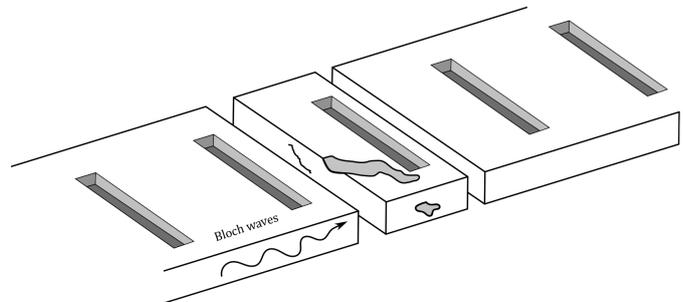


Figure 2. Scattering of Bloch waves in a periodic waveguide.

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## REFERENCES

- [1] Pascal Fossat, Claude Boutin, and Mohamed Ichchou. Dynamics of periodic ribbed plates with inner resonance: Analytical homogenized model and dispersion features. *Int. J. Solids Struct.*, 152:85–103, 2018.
- [2] D.J. Mead. A general theory of harmonic wave propagation in linear periodic systems with multiple coupling. *J. Sound Vib.*, 27(2):235–260, 1973.
- [3] Y.L.Y.K. Yong and Y.K. Lin. Propagation of decaying waves in periodic and piecewise periodic structures of finite length. *J. Sound Vib.*, 129(1):99–118, 1989.
- [4] A. Parrinello and G.L. Ghiringhelli. Transfer matrix representation for periodic planar media. *J. Sound Vib.*, 371:196–209, 2016.
- [5] J.-L. Christen, M. Ichchou, A. Zine, and B. Troclet. Wave finite element formulation of the acoustic transmission through complex infinite plates. *Act. Acust Ac*, 102(6):984–991, 2016.
- [6] E. Deckers, S. Jonckheere, L. Van Belle, C. Claeys, and W. Desmet. Prediction of transmission, reflection and absorption coefficients of periodic structures using a hybrid wave based–finite element unit cell method. *J. Comp. Phys.*, 356:282–302, 2018.
- [7] Y. Yang, B.R. Mace, and M.J. Kingan. Vibroacoustic analysis of periodic structures using a wave and finite element method. *J. Sound Vib.*, 457:333–353, 2019.
- [8] F. Errico, M. Ichchou, F. Franco, S. De Rosa, O. Bareille, and C. Droz. Schemes for the sound transmission of flat, curved and axisymmetric structures excited by aerodynamic and acoustic sources. *J. Sound Vib.*, 456:221 – 238, 2019.
- [9] A. Kessentini, M. Taktak, M.A.B. Souf, O. Bareille, M.N. Ichchou, and M. Haddar. Computation of the scattering matrix of guided acoustical propagation by the wave finite element approach. *App. Ac.*, 108:92–100, 2016.
- [10] G. Mitrou, N. Ferguson, and J. Renno. Wave transmission through two-dimensional structures by the hybrid fe/wfe approach. *J. Sound Vib.*, 389:484–501, 2017.
- [11] T. Gras, M.-A. Hamdi, M.B. Tahar, O. Tanneau, and L. Beaubatie. On a coupling between the finite element (fe) and the wave finite element (wfe) method to study the effect of a local heterogeneity within a railway track. *J. Sound Vib.*, 429:45–62, 2018.
- [12] C. Droz, O. Bareille, J.-P. Lainé, and M.N. Ichchou. Wave-based shm of sandwich structures using cross-sectional waves. *Struct. Contr. Health Monit.*, 25(2):e2085, 2018.