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SUB-WAVELENGTH DAMAGE DETECTABILITY ASSESSMENT IN PERIODIC ASSEMBLIES USING A BLOCH MODELLING FRAMEWORK

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Abstract. *In this work, we develop a numerical framework for analyzing Bloch waves and their interactions with localized damages. A reduced Bloch expansion technique is first used to sub-structure the waveguide's healthy part, while a detailed finite element description of the damaged regions can be applied. A remarkable strength of this modelling technique lies in the possibility to compute the dynamic response of the overall structure (finite or infinite) subjected to harmonic or transient loads. The damage model and its location on the waveguide can both be updated with almost negligible additional computational effort. The performances compared with standard finite element analysis of a similar problem are outstanding (i.e. faster by at least 3 orders of magnitude). Two types of indicators can therefore be computed over a large number of possible waveguide-damage configurations: (i) the local transmission, reflection and conversions of Bloch waves at the interfaces of the damaged region, and (ii) the global harmonic or transient response of a loaded waveguide. This approach is used to extend the detectability of small-scaled damages in large-scaled periodic waveguides by exploiting the frequency-conversions of the Bloch scattering coefficients. These so-called 'diffusion features' are eventually used to improve sub-wavelength damage quantification and localization.*

1 INTRODUCTION

Guided wave testing (GWT) is a reliable, long-range and highly sensitive damage inspection technique. It is already well documented and extensively used for structural health assessment of various transportation, aerospace and offshore components. Although embedded GWT solutions are still seldom in the transportation industry, the upcoming IoT revolution will undoubtedly stimulate this field and replace many periodic maintenance inspections by embedded monitoring devices. On the other hand, many structural parts are becoming increasingly complex in this industry, to tackle combined needs for lightweight structures and enhanced vibroacoustic performances. Despite their many advantages, these lightweight structures exhibit various scattering behaviors (e.g. local resonances, Bragg effects, frequency-conversion) which considerably alter the broadband dispersion features and overall applicability of GWT strategies. The question addressed in this work states as follows: how can advanced waveguide modelling techniques be used to detect small damages in large-scaled periodic assemblies?

If the structure has a canonical form, explicit homogenized models can be used to describe the local dynamics (for stiffened plates, see Fossat et al. [1]). In more complex assemblies such as typical lightweight structures, numerical methods (or semi-analytical [2]) are needed, generally involving a refined finite element description of the structure and associated with model order reduction schemes [3–5] to reduce the computational cost involved. Since it was shown that that high-order guided resonances tend to have a superior sensitivity to small scaled damages [6], the selective generation of guided resonances (or any form of Floquet solutions [7, 8]) can be seen as a possible way to perform multi-modal pre-assessment of a damage’s scattering properties. These indicators have already been used in axisymmetric structures (see for ex. [9–11]), however such analyses require extensive computation efforts and wave-based methodologies. This paper presents a computationally efficient modeling scheme able to predict both the wave dispersion characteristics, the finite or semi-infinite dynamic response and local damage scattering properties.

2 METHOD

2.1 Wave Finite Element Method

Consider a waveguide made of identical consecutive sub-structures of length d . Denoting \mathbb{M} , \mathbb{C} , \mathbb{K} the generalized finite element mass, damping and stiffness matrices of the structure and \mathbf{u} the displacements, the governing equation of the free waveguide writes:

$$(\mathbb{K} + j\omega\mathbb{C} - \omega^2\mathbb{M})\mathbf{u} = \mathbf{0}. \quad (1)$$

The Floquet theory gives the form of the solutions $U(x)$ of a linear differential equation $H(x)U(x) = 0$ where H is a periodic operator such as $H(x + d) = H(x)$. The solution can therefore be written as a superposition of Floquet vectors using a d -periodic invertible matrix \mathbb{V} and a constant matrix κ as:

$$U(x) = \mathbb{V}(x)e^{\kappa x} \quad (2)$$

Defining the complex propagation constants $\lambda = e^{\kappa d}$, the Floquet theory gives the relation between the displacements of two consecutive unit-cells as $u_{i+1} = \lambda u_i$. In the direct WFEM form, the generalized dynamic stiffness matrix is decomposed into its left and right-sided DOFs located at the junction between the unit-cells, while the inner DOFs can be condensed. Introducing the Floquet theory in the force equilibrium equation yields the direct form of the dispersion

relation as the following quadratic eigenvalue problem:

$$\left(\lambda \mathbb{D}_{LR} + (\mathbb{D}_{LL} + \mathbb{D}_{RR}) + \frac{1}{\lambda} \mathbb{D}_{RL} \right) \phi = \mathbf{0}.$$

Alternatively, the dispersion relation can be written without dynamic condensation with the form:

$$\mathbf{D}(\lambda) \begin{Bmatrix} \mathbf{u}_L \\ \mathbf{u}_I \end{Bmatrix} = \begin{Bmatrix} \mathbf{0} \\ \mathbf{0} \end{Bmatrix},$$

where:

$$\mathbf{D}(\lambda) : \mathbf{D} \longrightarrow \begin{bmatrix} \mathbf{D}_{RL} & \mathbf{D}_{RI} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \frac{1}{\lambda} + \begin{bmatrix} \mathbf{D}_{RR} + \mathbf{D}_{LL} & \mathbf{D}_{LI} \\ \mathbf{D}_{IL} & \mathbf{D}_{II} \end{bmatrix} + \begin{bmatrix} \mathbf{D}_{LR} & \mathbf{0} \\ \mathbf{D}_{IR} & \mathbf{0} \end{bmatrix} \lambda.$$

The wave-mode WFEM reduction was used to reduce the computational effort (e.g. [4, 5] for details). The propagating solutions are selected among the complex solutions $\lambda = e^{-jkd}$. Solutions are discriminated into positive Ψ^+ and negative Ψ^- wave amplitudes. A wavematching procedure is used to track frequency-dependent waves and plot the dispersion curves.

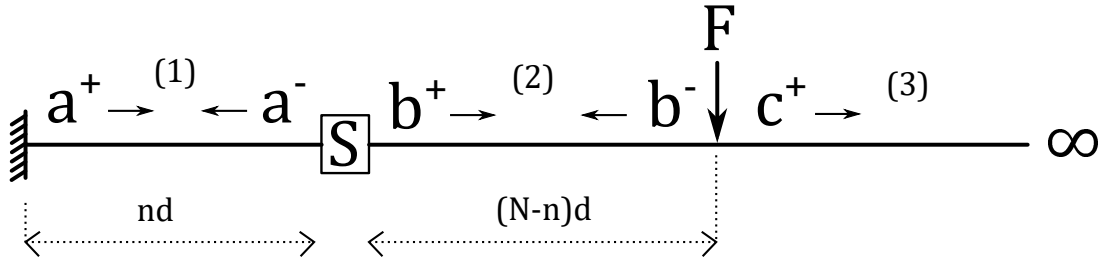


Figure 1: Semi-infinite piece-wise periodic waveguide configuration.

2.2 Diffusion properties

The presence of a coupling region, defined by the scattering matrix \mathbb{S} in the waveguide results in the introduction of two coupling conditions at both sides of the coupling element. Using the above mentioned Floquet expansion gives, for any incident wave source terms (α^+, β^+) [12], the reflection and transmission coefficients as:

$$\begin{Bmatrix} R \\ T \end{Bmatrix} = - \begin{bmatrix} \mathbb{D}_{RL}\Psi^- \Lambda + (\mathbb{D}_{RR} + \mathbb{S}_{LL})\Psi^- & \mathbb{S}_{LR}\Psi^+ \\ \mathbb{S}_{RL}\Psi^- & (\mathbb{S}_{RR} + \mathbb{D}_{LL})\Psi^+ + \mathbb{D}_{LR}\Psi^+ \Lambda \end{bmatrix}^{-1} \begin{Bmatrix} \alpha^+ \\ \beta^+ \end{Bmatrix}$$

The above relation allows the computation of diffusion coefficients from a single incident wavetype for any given scattering section \mathbb{S} .

2.3 Forced response of the piecewise periodic structure

The following configuration is considered: the piecewise waveguide is composed of three sections as shown in Figure 1. Section 1 contains N_1 cells. It is clamped on the left side and connected to section 2 by a scattering region described by matrix \mathbb{S} . An external load is applied at a distance N_2 from the coupling region and defines the semi-infinite section 3. Using

the Floquet expansion, the generalized governing equation can be rewritten under the reduced form:

$$\mathcal{H} \begin{Bmatrix} \mathbf{a}^+ \\ \mathbf{a}^- \\ \mathbf{b}^+ \\ \mathbf{b}^- \\ \mathbf{c}^+ \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ F \\ 0 \end{Bmatrix},$$

where $\mathcal{H} = \mathcal{H}(\mathbb{D}, \mathbb{S}, \Psi^+, \Psi^-, \{\Lambda^k\}_{k=[0..N]})$ is a Floquet decomposition of the dynamic stiffness matrix projected on the extremities of each waveguide section (interested readers are referred to [13] for detailed methodology). Note that the maximal dimension of this sparse linear problem is equal to the number of DOFs in a single unit-cell's edge. In general, a reduction to a few propagating and evanescent waves can provide an additional reduction of the dimension by several orders of magnitude. In the example below, the total number of DOFs in sections (1) and (2) is 100k DOFs including the scattering element. The Floquet expansion yields a reduced system of size 250.

3 APPLICATION

The methodology is then used to compute the diffusion indicators, displayed as a set of three frequency-dependent coefficients: the Transmission (T) and Reflection (R), as well as a Diffusion (D) term defined as in [12].

3.1 Description of the model

The considered waveguide is an aluminium framed structure of unit-cell's length $l_x=5$ cm meshed using 3D block elements and involving three different internal damages as described in fig. 2. The loading and boundary conditions are the ones of an attempted inspection of the region between the actuated region and the clamped end, considering a semi-infinite part on the other side (no reflected waves). The configuration therefore corresponds the one shown in fig. 1. The damage model is parametrized and consists in a local reduction of the stiffness on the walls of the damaged unit-cell along three possible direction (see fig. 2).

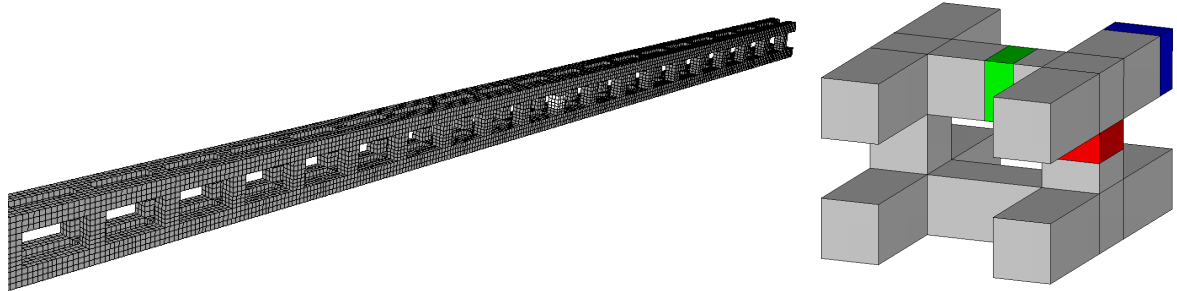


Figure 2: (a) FEM of a finite section of the framed structure. (b) Example of damage scenario along x (blue), y (green) and z (red).

3.2 Diffusion features

The diffusion of all propagating waves produced by a x-oriented damage are computed using Eq. 2.2. The dispersion curves of the first 9 propagating waves is shown in fig. 4. One

can identify the Bragg bandgap affecting most transverse waves between 8-13 kHz. The three coefficients displayed in fig. 3 over frequency for each wave indicate the expected sensitivity of the wave to a given damage. It can be assumed that the more sensitive the wave to the damage is, the more altered the overall dynamic response will be once subjected to this wavetype's loading. For a sake of clarity, the non-propagative regions (e.g. bandgaps) are hidden, based on the wave's spatial attenuation criterion (i.e. above 5% amplitude decay per unit-cell).

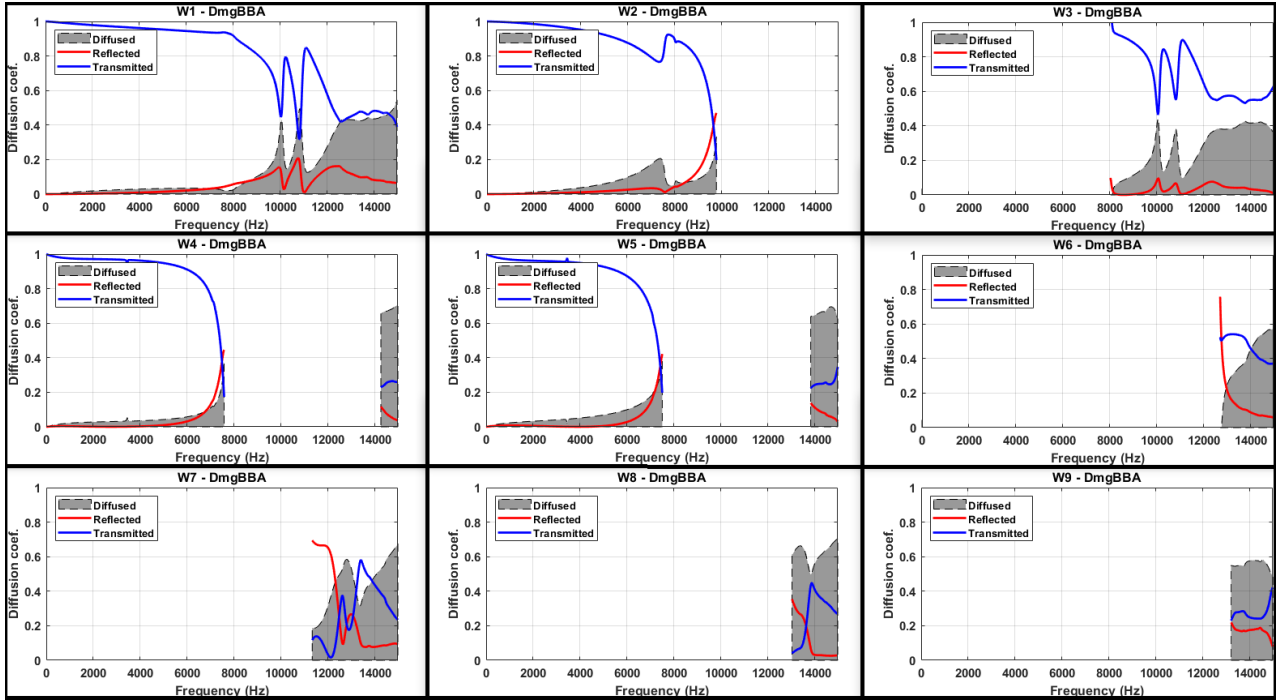


Figure 3: Transmission, Reflection, and Diffusion coefficients of all propagating waves in the considered bandwidth, for a damage of type 'longitudinal'.

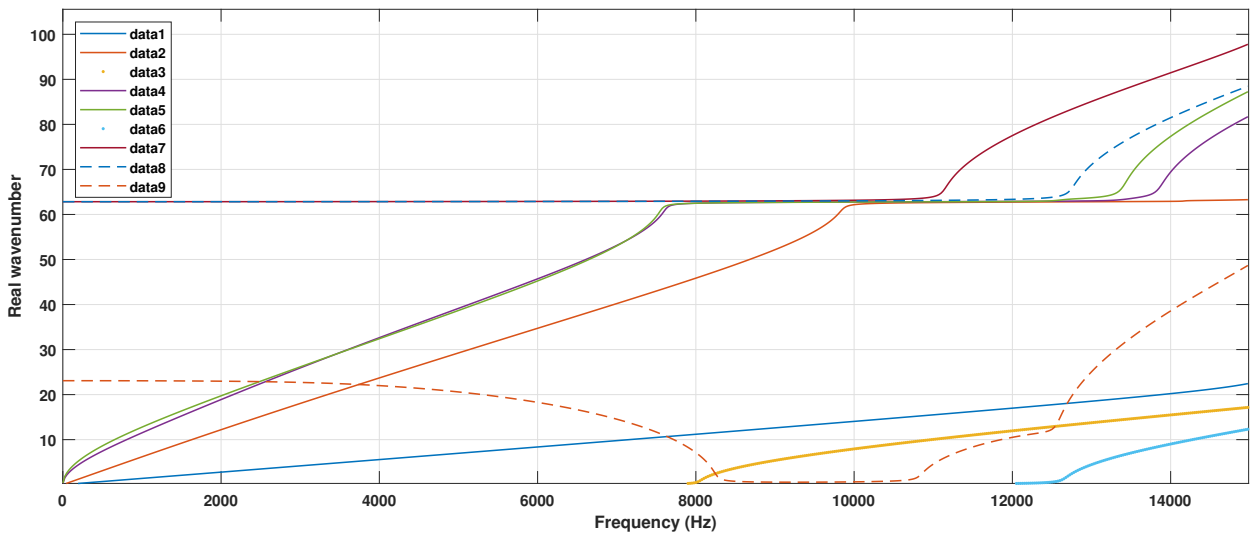


Figure 4: Dispersion curves in the framed structure.

Note that the sensitivities tend to increase close to the non-propagating regions (cut-on, Bragg, local resonances etc.). For illustration purpose, one can compare the diffusion plots of waves 5 (middle) and 7 (bottom left). It is clearly visible that wave 5 is almost insensitive to the damage, except near the bandgap, and is mainly diffused above 14 kHz. On the other hand, wave 7 cuts-on at 12 kHz but exhibits a higher reflection (above 60%) between 12 and 13 kHz.

3.3 Forced response

The hypothesis that the overall dynamic response of the semi-infinite structure will be more sensitive by applying wave 7 than wave 5 is tested in the below. In the loading region, a guided mode actuation is assumed to produce a frequency-dependent edge unit-cell forced appropriation of a selected wave. The co-localized forced response is calculated in fig. 5 for both wavetype actuations, each comparing the healthy and damaged cases. It shows that the diffusion analysis gave a fair prediction, as the dynamic response is more sensitive under normalized load to wave 7 than wave 5, despite its reduced propagative bandwidth. In addition, the reduced wave model described above gives the full dynamic response solution to a given loading case in less than a second.

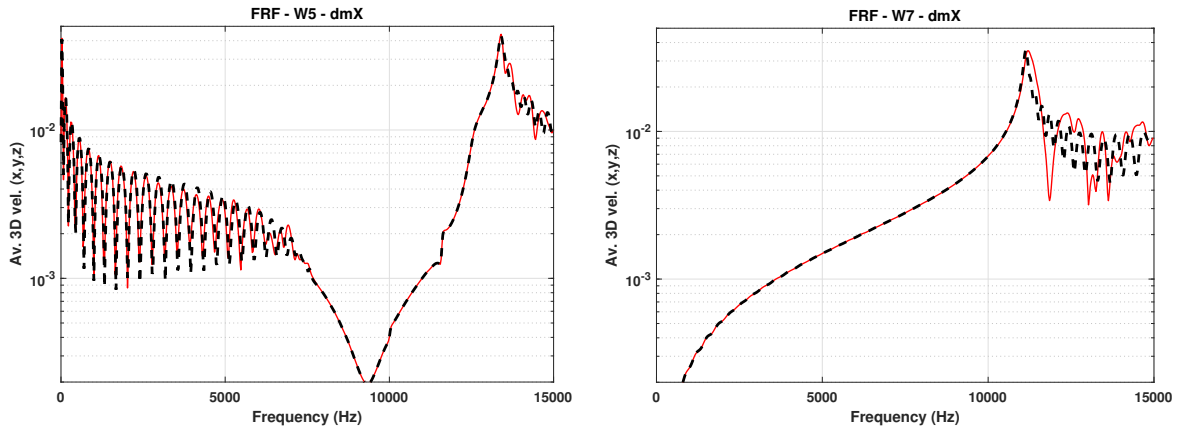


Figure 5: Forced response of the semi-infinite structure under wave-mode appropriation index 5 (a) and (7). The co-localized response is computed considering an healthy (dashed) and damaged (continuous) region with a longitudinal damage type.

4 CONCLUSIONS

The conclusions of this work can be stated as follows:

- A generalized Floquet decomposition was used to derive the dispersion characteristics, the damage scattering information and the forced response from a single unified numerical framework.
- The method was coupled with a model order reduction scheme, providing outstanding computation performances.
- These methodologies were proved of remarkable efficiency to model and predict the detectability of small-scaled damages in a framed structure using both dynamic response and diffusion analysis.

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