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A Multistage Stochastic Program for the Design and Management of Flexible Infrastructure Networks

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Abstract

Throughout their lifetime, infrastructure network systems face unplanned events that impose pressures on their integrity, functionality, and ability to deliver value. Most of the existing infrastructure is designed to deal with the challenges imposed by uncertain external phenomena. Authors from different backgrounds have identified flexibility, changeability, and adaptability as key attributes that modern systems should have to face uncertain scenarios. Specifically, flexibility is an ability that allows a system to be *easily* adapted when necessary. The concept of flexibility is compelling, but it is not clear how to measure the value it may provide. Determining how much to pay to introduce flexibility is an essential aspect of designing flexible systems, but the dependence of this value on the future evolution of the system results in a complex decision process. The sequential nature of the process can be modeled using multistage stochastic programming. The model explicitly considers the flexibility built into the network components as a decision variable at the initial stage. The model is tested in a generic infrastructure network that must meet a stochastic demand. The results show the relationship between the value of flexibility and the life-cycle costs at the construction and operation stages.

Keywords: Flexibility, Multistage Stochastic Programming, Value of Flexibility, Infrastructure Networks

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1. Introduction

1.1. Background

Infrastructure networks, as any infrastructure system, are constantly exposed to ever-changing conditions. Changes in the demand, the supply sources (in the case of water and energy distribution systems), and regulations may impose large stresses on the system [1, 2, 3, 4]. This uncertainty can either challenge or boost the ability of the network to perform at required levels [5]. In the former case, networks usually rely on the robustness provided by over-dimensioned designs which can be largely inefficient, as in the case of sanitation systems [6]. In the latter case, even if the new conditions are favorable, the network may not have the tools to take advantage of the new opportunities [7]. To face these challenges, many authors have identified the need to develop flexible and adaptable systems, which have the potential to improve the sustainability and efficiency of the system under highly uncertain conditions [8, 9, 10, 11, 12].

Specifically, flexible systems are defined as systems with the ability to change as “easily” as possible [13, 14, 15, 4, 16]. This can be measured as the money or time investment required to modify the system. In the case of infrastructure networks, adaptations can be usually seen as the addition (or removal) of nodes and links or the expansion (or downgrade) of their capacity. Other changes, mostly managerial, may be possible (e.g. reversal of the flow direction). Flexibility is introduced by investing a number of resources during the design and construction phase to enable the option of deploying future adaptations at lower costs and shorter lead times. This investment is usually called the *value of flexibility* and is defined as the amount the system stakeholders’ are willing to pay to introduce flexibility into the system [7, 8]. It is assumed that the enabled adaptations are capped at a certain value that depends on the amount invested at the initial stages of the project [16]. This interpretation of flexibility is similar to the concept of shell capacity described by Angelus et al. [17].

1.2. Previous works and applications

While flexibility is not limited to capacity expansion, this type of problem is the most common and has been extensively studied in the manufacturing sector [18, 19, 20]. The first work comes from Manne [21], who modeled the optimal excess capacity of pipelines, highways, and steel plants subject to a random walk demand. Luss [18] further detailed the problem as determining the size, location, and timing of future adaptations while minimizing the discounted costs of the expansion

31 processes. This author also identified the modeling of the demand, the representation of the capacity
32 as a continuous or discrete variable, the selection of the discount rate, and the presence of economies
33 of scale as key elements of the problem formulation. Mathematical programming rapidly became
34 one of the preferred methods to solve this type of problem, as shown by the work of Rajagopalan
35 [22], who formulated a mathematical programming model to determine the optimal initial capacity
36 and the technology acquisition decisions, for increasing demands. Later came the work of Ahmed
37 and Sahinidis [23] who developed a multistage stochastic (MS) integer program to analyze a multi-
38 period investment model for capacity expansion under uncertain demand and costs, and including
39 the effect of economies of scale. Singh et al. [24] also developed an MS integer programming model
40 for planning discrete capacity expansion of production facilities. Similarly, Huang and Ahmed [25]
41 formulated a general MS capacity planning model with discrete capacity for the semiconductor
42 industry.

43 The prevalence of stochastic and dynamic programming models to solve capacity expansion
44 problems in the manufacturing sector led to the adoption of these techniques in other areas such
45 as planning and design of infrastructure networks. For instance, in the context of transportation
46 networks, Marín and Jaramillo [26] formulated a multi-period capacity expansion problem for rapid
47 transit network design. Karoonsoontawong and Waller [27] developed a robust optimization model
48 for the problem of continuous traffic network capacity expansion with dynamic traffic assignment
49 and traffic signal optimization. Gao et al. [28] considered the case where a road network is expanded
50 either by adding new links or by increasing the capacity of the existing links, integrated with the
51 road maintenance problem using a mixed-integer, non-linear, bi-level optimization program, using
52 multi-period decisions. In the area of water and sewage distribution systems, Mortazavi-Naeini
53 et al. [9] developed a multi-objective optimization approach for the planning of urban water system
54 expansions that considers the combined effect of operating rules and infrastructure conditions. Saif
55 and Almansoori [29] developed a model for the expansion of water desalination and power supply
56 infrastructure using a deterministic multi-period mixed-integer linear programming formulation.
57 Similarly, Fraga et al. [10] used dynamic programming to develop an integrated framework for
58 the optimization of water supply system expansions, considering short and long-term water supply
59 sources. In the case of energy production and transmission networks, Loureiro et al. [30] used the
60 concept of real options combined with a mixed-integer linear programming model for multistage
61 expansion planning. Cardin et al. [31] combined decision rules and stochastic programming to model

62 flexibility in a nuclear infrastructure system, considering a random demand and including the social
63 acceptance of nuclear power as a limitation to the system expansion capabilities. For networks in
64 general, Taghavi and Huang [32] applied the concepts of spot market and contract capacity combined
65 with an MS integer program to model a network with multiple sources of capacity.

66 In both manufacturing systems and infrastructure networks, the adaptation process requires
67 large capital investments (in proportion with the total system size), which are sometimes irre-
68 versible [33], and are susceptible to the negative consequences of large lead times. In both cases,
69 there is a contradictory effect from economies of scale, which may favor both large initial designs
70 and large future adaptations. However, infrastructure networks are subject to more strict gov-
71 ernmental regulations, where failure to comply with minimum levels of performance can result in
72 heavy societal costs. This poses an additional problem of strategically selecting the extent to which
73 flexibility should be incorporated. Besides, infrastructure networks are usually planned for longer
74 service horizons, which favors delayed deployments due to discounting [34]. Furthermore, the nature
75 of stakeholders affects the management and adaptation strategies. While manufacturing systems
76 are generally completely privately owned, infrastructure network ownership can range from com-
77 pletely public to completely private, which may create conflicting interests. For instance, in energy
78 infrastructure, maintaining a determined level of excess capacity may be valuable for society but
79 not for profit maximization [35]. Finally, the frequency of adaptation is considerably different: in
80 manufacturing systems is not uncommon to see changes every one or two years, while infrastructure
81 networks may see changes in periods of five to ten years.

82 *1.3. Objectives and scope*

83 The works discussed in the previous section show the potential of (and the preference for) MS
84 programming as a tool to model and solve complex sequential decision problems. Considering that
85 the problem of designing and managing flexible infrastructure networks is in itself a sequence of
86 decisions under uncertainty, this paper proposes a multistage stochastic program (MSP) to analyze
87 the problem of flexibility. Operational and maintenance costs are explicitly considered, to take
88 into account the relative importance of these costs in the systems modeled and the impact these
89 costs may have on the preference of the model for flexible solutions. The model uses scenario
90 trees generated by Monte-Carlo simulation and k-medoids clustering to simulate and discretize
91 the random process. The novelty of the model is that it explicitly considers the flexibility range

92 introduced by design as a decision variable, which will restrict all the future planned adaptations.
93 The proposed model results in an alternative methodology to real options as a tool to determine the
94 *value of flexibility*. By modeling flexibility as a decision variable, the program decides how much
95 should be paid at the initial stage to lower future expenses.

96 The article is organized as follows: 2 gives an overview of the concept of flexibility in infras-
97 tructure systems and its numerical representation. A review of the theory behind two-stage and
98 MSPs is presented in 3. 4 provides the formulation of the proposed MSP. 5 presents a methodology
99 to solve the MSP using an approximate approach. Finally, ?? provide a numerical example with a
100 generic network and a summary of the key results.

101 2. Designing for Flexibility

102 The concept of flexibility is considered to be not "academically mature" [14] due to the lack of a
103 precise and universally accepted definition. In this study, flexibility will be understood as defined
104 by [7, 15], and [16]. The authors define flexibility as the ability of a system to *easily* adapt any of
105 its components (or subsystems). The effort necessary to complete an adaptation is measured as the
106 number of required resources, which are usually represented as a monetary quantity. Furthermore,
107 any adaptation will be limited to a maximum value given by the system's context and the resources
108 invested to make the system flexible. These additional costs may be incurred at the initial stage
109 of the project to introduce the option to adapt some of the system components. These expenses
110 may come from research and development activities or by installing physical elements that will
111 facilitate future adaptations. For instance, the floating platforms used in offshore wind farms could
112 be designed to be expanded or be built larger-than-required to facilitate future adaptations to the
113 turbine sizes.

114 The problem of determining how much should be invested to have flexibility in the system is
115 usually known as measuring the *value of flexibility*. This is a complex problem because capturing the
116 value of having the option to modify the system under certain circumstances is not a straightforward
117 process [8]. Different approaches have been developed based on different assumptions and modeling
118 frameworks. For instance, the Real Options Analysis (ROA) method was developed by adapting the
119 concept of *options* in financial markets to physical assets [36, 37]. Under this approach, the value
120 of flexibility is measured as the pricing of an option that provides the right but not the obligation
121 to modify the system in some predefined way. In a different approach, Cardin et al. [7] defined the

122 value of flexibility as the difference between the expected net present value (ENPV) of a flexible
123 system and an inflexible system subjected to the same conditions. Similarly, Špačková and Straub
124 [15] defined it as the additional investment that should be made to have a flexible system compared
125 with the inflexible alternative, using Markov Decision Processes (MDP) to model the sequential
126 decision process. Following these works, the value of flexibility will be understood in this paper
127 as the maximum value that should be paid at the start of the system life-cycle to minimize the
128 discounted costs of future adaptations and maximize the received utility.

129 In the area of chemical processes, the concept of Flexibility Analysis (FA) has been proposed to
130 optimize the design and operation of chemical plants under uncertainty [38, 39, 23, 40]. The main
131 objective of this approach is to determine the optimal design and control variables that allow feasible
132 operation for the whole range of uncertain parameters. If feasible operation is achieved, then the
133 process is considered flexible enough. To guarantee feasibility, FA uses the worst-case approach [40]
134 as in the case of robust optimization. In fact, FA and robust optimization share many concepts
135 and methods, even if historically they evolved separately. One key difference, however, is the use of
136 recourse variables (the control variables) to modify the plant's response to particular realizations of
137 the uncertain parameters. While recourse variables are traditionally not used in robust optimization,
138 they are a key concept in stochastic optimization. By combining concepts from these optimization
139 approaches, FA can be used to determine the optimal design characteristics of flexible chemical
140 plants.

141 The formulation of the flexibility concept and the optimization models proposed in FA may
142 suggest that the approach can be extended to other engineering systems. Indeed, the design and
143 management of flexible infrastructure networks can be properly modeled using the representation
144 of design, state, and control variables from the FA approach. The recourse options provided via
145 control variables can represent adaptation decisions instead. In both cases, flexibility exists to
146 face the uncertainty in external phenomena. Despite these similarities, the concept of flexibility in
147 infrastructure systems (and networks) is more closely related to the concept of Real Options [36, 37],
148 while the definition of flexibility in FA is more similar to the concept of robustness in infrastructure
149 systems (as defined in [13]). The problem of how much should be paid today to have the option to
150 modify the system in the future is a very important notion in flexibility for infrastructure systems
151 that does not exist in FA. Furthermore, the changes implemented using control variables in chemical
152 processes are usually reversible, while modifying infrastructure systems involves a sequential decision

153 process that results in incremental modifications. The sequential nature of the decision process
 154 requires a modeling approach that considers the temporal interdependence of the decisions (such
 155 as dynamic or stochastic programming) for which the tools developed in FA are insufficient. For
 156 these reasons and despite the conceptual similarities, the FA approach is not used in this paper and
 157 rather an MS programming model is proposed.

158 2.1. Numerical representation of flexibility

159 To formulate and solve the problem of the value of flexibility, it is useful to have a numerical
 160 representation of flexibility. Unfortunately, the lack of consensus in the conceptual definition is
 161 also present in the formal description. Various authors have proposed an array of indices with
 162 different levels of complexity [38, 13, 41, 42] but none has been universally adopted. This study
 163 will measure flexibility using the *flexibility vector* defined by Torres-Rincón et al. [16], shown in
 164 Equation 1. This formulation explicitly represents two characteristics usually associated with the
 165 concept flexibility: the presence of the option to change, and a measurement of the effort required to
 166 complete the change. The flexibility vector for the design or operation variable i has two dimensions:
 167 the first dimension measures the effort necessary to complete an adaptation as the ratio between
 168 the unitary cost of modifying the component without flexibility (without being specifically designed
 169 to be adapted) and the unitary cost of performing an adaptation when flexibility was introduced
 170 $c_{nf,i}/c_{f,i}$; the second dimension measures the size of the available adaptation space as the ratio
 171 between the maximum value the design or operation property can take divided by its initial value
 172 $x_{max,i}/x_{0,i}$. Therefore, a system can increase its flexibility by increasing the number of states that
 173 can reach through low-cost adaptations, or by reducing the cost of such adaptations.

$$\mathbf{fv}_{i,t} = \left[\frac{c_{nf,i} - c_{f,i}}{c_{f,i}}, \frac{x_{max,t,i} - x_{0,i}}{x_{0,i}} \right] \quad (1)$$

174 For instance, Figure 1 shows two flexibility vectors of the same magnitude. Vector A, how-
 175 ever, has a larger cost component (horizontal axis), while vector B has a larger adaptation space
 176 component (vertical axis). Clearly, the expected behavior of both designs under the same external
 177 conditions will be different. The vectorial representation of flexibility allows capturing the complex
 178 nature of the property.

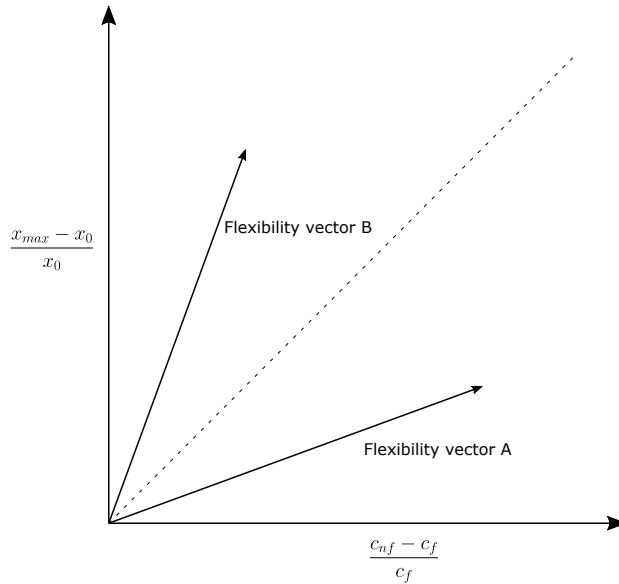


Fig. 1. Flexibility vector representation

179 *2.2. Managing flexibility through policies*

180 The elements of flexibility described previously are deeply interconnected with the physical
 181 characteristics of the system. A system is specifically designed and built to be flexible, which
 182 affects its morphology. Therefore, future management process to decide the optimal timing and
 183 magnitude of adaptations will be constrained by the limitations of the design [13, 41, 7, 31, 4, 16].
 184 The management process is also conditioned by external factors and the system’s current state,
 185 but also on particular preferences of the stakeholders, technical limitations, user requirements, and
 186 regulatory frameworks. For instance, risk-averse stakeholders may prefer to build large systems from
 187 the beginning with enough flexibility to perform small adaptations, while stakeholders with a higher
 188 risk tolerance may favor smaller initial systems with high flexibility to deploy large adaptations.
 189 These complex interactions between external elements and individual preferences can be modeled
 190 in the form of *policies*.

191 Policies are functions that map a set of states to decisions [31, 43]. Depending on the modeling
 192 framework, policies can be deterministic or stochastic with variable degrees of complexity. For
 193 many applications, however, a simplified framework can be formulated using “*if-then*” conditionals
 194 that instruct when to trigger an adaptation process [44]. Under this formulation, the conditional

195 threshold and the size of interventions become the main parameters to be defined. These elements
196 can be used to construct more complex policies. For instance, a more complex policy can be
197 constructed from the if-then conditional by defining the magnitude of the adaptation as the optimal
198 change that minimizes the expected present value of the associated costs and maximizes the expected
199 utility, while constraining the desired performance to remain at certain levels.

200 Generally, flexibility management requires not one but a complete sequence of decisions. The
201 sequence starts with an initial decision to define the design characteristics of the system, i.e., the
202 initial dimensions and the flexibility (maximum adaptation range and adaptation costs). Once
203 the system is fully commissioned and starts operation, a monitoring process is required to verify
204 the adaptation conditions defined by the policy. The successful implementation of any policy is
205 conditioned on an inspection and monitoring program. When the conditions are met, a decision is
206 made concerning the magnitude of the adaptation. The complexity of this problem lies in that every
207 decision made will affect the future state of the system and, in consequence, the input for the policy
208 in the future. Furthermore, the future state will also depend on uncertain external conditions that
209 may not be stationary (changes in traffic, demand, climate change, etc.) [12]. Thus, the problem of
210 managing flexibility is a classic example of sequential decision problems.

211 In summary, designing and managing flexible systems requires at least the following tasks (for
212 one flexible component):

- 213 1. determine the optimal initial design \mathbf{x}_0 ,
- 214 2. determine the optimal initial flexibility level \mathbf{f}_0 ,
- 215 3. define the policy π to represent the management decisions,
- 216 4. following π , determine the optimal timing τ_1, τ_2, \dots , and size $y_{\tau_1}, y_{\tau_2}, \dots$ of future adaptations.

217 The model presented in this paper addresses the first two items. Figure 2 presents a diagram that
218 synthesizes this design and management process.

219 *2.3. Sequential decision-making*

220 Typical decision-making models require an agent and an environment. At time t , the agent
221 observes the state of the environment s_t and selects an action a_t according to the policy π . Then,
222 the environment produces a reward r_t for the agent and evolves to a new state s_{t+1} following a
223 probabilist model that depends on the sequence of previous states $s_{1:t}$ and actions $a_{1:t}$ [45]. The

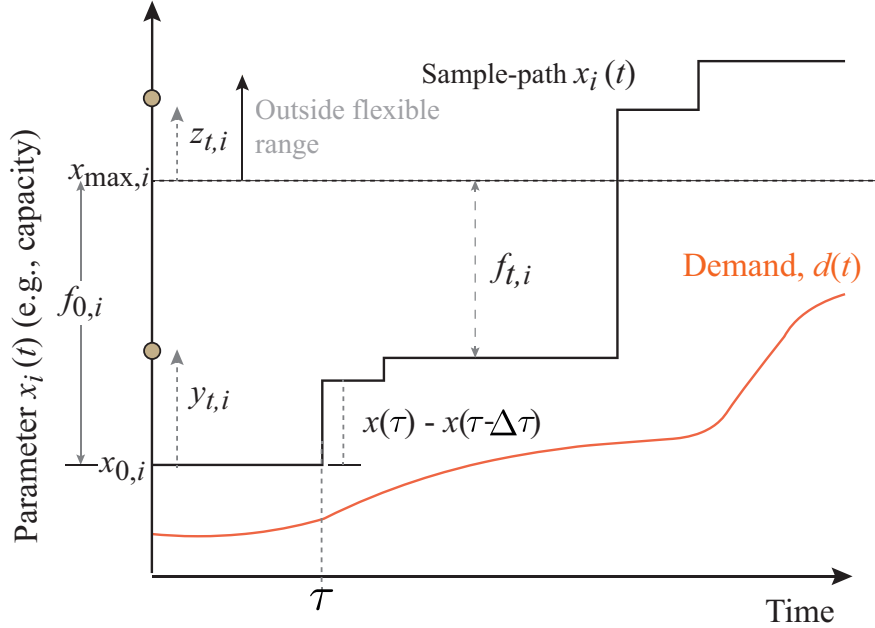


Fig. 2. Description of the main elements for flexible designs

224 agent observes this new state and selects another action a_{t+1} . If the new state only depends
 225 on the previous state and action, then the decision model is referred to as a Markov Decision
 226 Process (MDP). While the basic version assumes that the probabilistic model is known and the
 227 environment is fully observable, these conditions are not a requirement. For instance, Partially
 228 Observable Markov Decision Processes (POMDPs) are a generalized version of MDPs.

229 Solving a sequential decision problem is not a unique problem; each model may have many
 230 different elements that need to be determined, which requires different methods. For instance, a
 231 key problem is finding the optimal policy, i.e., the optimal function that selects an action a_t for
 232 each pair of action and state history $a_{1:t-1}, s_{1:t-1}$. Problems of optimal policy are usually solved
 233 using methods such as dynamic programming (DP), approximate dynamic programming (APD),
 234 reinforcement learning, genetic algorithms, and Monte Carlo tree searches. A different problem
 235 is finding the optimal initial decisions that optimize a certain performance measurement over the
 236 system lifetime, i.e., the optimal design that minimizes discounted adaptation costs. In this case,
 237 the optimal initial decisions are determined considering future sequences of decisions that may result
 238 from a particular decision rule and a probability model, even if the optimal sequence of decisions

239 is not determined. Decision problems focused on finding a robust, optimal initial decision can be
 240 solved using MS programming [46, 47, 48].

241 The problem of flexibility and the value of flexibility as presented before is better suited to be
 242 solved using MS programming. The next section presents an overview of the theory behind MSPs
 243 and 4 presents the proposed MS model for the problem of flexibility.

244 3. Stochastic Programming

245 3.1. Two-stage program with fixed recourse

246 The simplest stochastic programming model is the linear two-stages model with recourse shown
 247 in Equation 2 [48], where the objective is to minimize the costs at the initial stage $t = 0$ and at the
 248 recourse stage $t = 1$:

$$\begin{aligned}
 \min_{\mathbf{x}_0} f &= \underbrace{\mathbf{c}^T \mathbf{x}_0}_{\text{Initial stage costs}} + \underbrace{\mathbb{E}_{\boldsymbol{\xi}}[\min \mathbf{q}(\omega)^T \mathbf{x}_1(\omega)]}_{\text{Recourse costs}} \\
 \text{s.t. } \mathbf{A}\mathbf{x}_0 &= \mathbf{b} \\
 \mathbf{H}(\omega)\mathbf{x}_0 + \mathbf{J}\mathbf{x}_1(\omega) &= \mathbf{h}(\omega) \\
 \mathbf{x}_0 \geq 0, \mathbf{x}_1 &\geq 0
 \end{aligned} \tag{2}$$

249 At the initial stage, the decision $\mathbf{x}_0 \in \mathbb{R}^{n_1}$ is made considering that the first stage data is
 250 known: matrix $\mathbf{A} \in \mathbb{R}^{m_1 \times n_1}$ and vector $\mathbf{b} \in \mathbb{R}^{m_1}$. At the second stage, a random event $\omega \in \Omega$ is
 251 realized (Ω represents the set of all random outcomes) and the second stage information is revealed:
 252 technology matrix $\mathbf{H}(\omega) \in \mathbb{R}^{m_2 \times n_1}$, recourse cost vector $\mathbf{q}(\omega) \in \mathbb{R}^{n_2}$, and right-hand side vector
 253 $\mathbf{h}(\omega) \in \mathbb{R}^{m_2}$. Then, the second stage decision $\mathbf{x}_1(\omega) \in \mathbb{R}^{n_2}$, the *recourse decision*, is made. This
 254 model is classified as with fixed recourse because the recourse matrix $\mathbf{J} \in \mathbb{R}^{m_2 \times n_2}$ is assumed
 255 deterministic. This assumption implies that the effect of the recourse decision \mathbf{x}_1 in the constraints
 256 is known, which greatly simplifies the solution of the problem and still can be used to model a wide
 257 range of real life situations.

258 Each element in the random matrix and vectors $\mathbf{H}(\omega)$, $\mathbf{q}(\omega)$, and $\mathbf{h}(\omega)$ is a random variable.
 259 Therefore, the random vector $\boldsymbol{\xi} = (\mathbf{q}(\omega), \mathbf{h}(\omega), \text{vec}(\mathbf{H}(\omega)))$ with support $\Xi \subset \mathbb{R}^d$ can be constructed
 260 to represent all the data that depends on the random event ω . By taking the expectation with
 261 respect to $\boldsymbol{\xi}$, the estimation of the second stage costs is considering all possible realizations of ω (at
 262 least those with non-zero probability).

263 A key characteristic of stochastic programs (two-stages or multistage) is that the recourse de-
 264 cisions \mathbf{x}_1 will be different for each realization of the random event ω . Therefore, if the random
 265 parameter distribution is discretized into r intervals, there will be r different \mathbf{x}_1 solutions, which is
 266 not very useful from a planning perspective. The true value of stochastic programs lies in the ini-
 267 tial stage solutions \mathbf{x}_0 because they are unique for all trajectories of the random parameter. These
 268 decisions are being selected to guarantee that the expected value of future costs (which depend
 269 on $\boldsymbol{\xi}$ and \mathbf{x}_1) will be minimal. The next section presents how this model can be extended to the
 270 multistage case.

271 3.2. Multistage stochastic programs

272 Multistage stochastic programming is a modeling framework that generalizes the two-stage
 273 problem to a sequence of recourse decisions. In the two-stage problem, the realization of the
 274 uncertain parameter becomes known at the second stage and the decision-maker has the option to
 275 make a second decision \mathbf{x}_1 -the recourse- to adjust the initial decision \mathbf{x}_0 . When the approach is
 276 extended to the multistage case, every recourse decision \mathbf{x}_t will be selected based on the expected
 277 value of this decision for stage $t + 1$ assuming that every future decision will be optimal [48]. This
 278 results in a particular sequence of recourse decisions for every sequence of uncertain parameters.

279 The general MS programming model can be formulated as follows [47]:

$$\begin{aligned} \min_{\mathbf{x}_1 \in \mathcal{X}_1} \quad & f_1(\mathbf{x}_1) + \mathbb{E} \left[\inf_{\mathbf{x}_2 \in \mathcal{X}_2(\mathbf{x}_1, \boldsymbol{\xi}_2)} f_2(\mathbf{x}_2, \boldsymbol{\xi}_2) + \mathbb{E}[\dots \right. \\ & \left. + \mathbb{E} \left[\inf_{\mathbf{x}_T \in \mathcal{X}_T(\mathbf{x}_{T-1}, \boldsymbol{\xi}_T)} f_T(\mathbf{x}_T, \boldsymbol{\xi}_T) \right] \dots \right] \end{aligned} \quad (3)$$

280 where $\mathbf{x}_t \in \mathbb{R}^{n_t}$ is the vector of decisions variables at time t ; $\boldsymbol{\xi}_t \in \mathbb{R}^{M_t}$ is the vector of uncertain
 281 parameters at time t (with M_t as large as $n_t + m_t + n_t \times m_t$); functions $f_t(\mathbf{x}_t, \boldsymbol{\xi}_t)$ define the cost of
 282 making the decision \mathbf{x}_t ; and \mathcal{X}_t represent the set of constraints that define the feasibility regions.

283 By using an expectation functional in the recourse functions, it is being assumed that the
 284 decision-maker is risk-neutral. Risk-averse attitudes can be modeled by replacing the expectation
 285 functional with risk functionals that consider both the mean and the variance of the random out-
 286 come in the optimization process. Functionals such as mean-variance, semi-deviations, weighted
 287 mean deviations from quantiles, and average value at risk can be used to generate solutions with
 288 limited risk [47]. Nonetheless, for the case of infrastructure networks where the performance is con-
 289 sidered long-term and the structural integrity is not being subject to optimization, the expectation

290 functional can be used as long as the number of scenarios considered is large enough for the Law
291 of Large Numbers to apply. Other requirements defined by the decision-maker, e.g. performance
292 levels, can be specified either as hard constraints in the feasibility set \mathcal{X}_t or the deviations can be
293 penalized in the cost function.

294 In an MSP there may be four uncertain elements: future costs, right-hand side vector, technol-
295 ogy matrix, and recourse matrix. The right-hand side vector is usually associated with demand, the
296 technology matrix with the initial decision response, and the recourse matrix with the recourse de-
297 cisions response. For instance, the technology matrix could represent the productivity of the initial
298 conditions while the recourse matrix could represent the productivity of the recourse decisions, and
299 both affect the system's ability to serve the random demand given by the right-hand side vector.
300 Clearly, the decision of which elements are considered random in the model depends on the type of
301 system and the modeler's assumptions.

302 The decision on how to model the random parameters depends entirely on the type of problem.
303 Historical data, econometric projections, experts' opinions, all can be used to define the probability
304 distributions of the elements selected to be random. The scenario generation techniques discussed
305 in the following section are used to transform these probability distributions and the filtration
306 structure of the information into discrete approximations in the form of scenario trees that can be
307 used to solve numerically the MSP.

308 An important property of MSPs is that the information available is represented as σ -algebras
309 \mathcal{F}_t . As the stages move forward, the information available increases, which is modeled as a sequence
310 of increasing σ -algebras: $\mathcal{F}_t \subseteq \mathcal{F}_{t+1}$. This growing sequence of σ -algebras is known as a *filtration*.
311 This is an important property of the problem because it implies that decisions at each stage x_t can
312 only consider the information available up to that stage [49]. This requirement can be explicitly or
313 implicitly modeled in the constraint set and it is known as *non-anticipativity constraints*.

314 3.3. Challenges in stochastic programming

315 The MS programming approach, while useful to capture the presence of uncertainty in planning
316 problems, faces limitations due to its computational complexity. The first limitation is defined by
317 the representation of the uncertain parameters in the model. Regardless of whether the distribution
318 of the random processes is known or if only sample paths are available, this information has to be
319 transformed somehow to be entered as an input to the model. Scenario trees are the usual approach

320 to represent the uncertain elements in MSP and constructing them implies the finite discretization
321 of the original distributions (if they are continuous in the first place).

322 To properly represent the original distribution a large number of points are required. Further-
323 more, the nested structure of scenario trees results in exponential growth of the number of scenarios
324 with the number of stages. These two issues (many points and exponential growth) render the prob-
325 lem intractable if a detailed representation of the underlying distribution is desired in a model with
326 many stages. For this reason, methods for the generation and reduction of scenario trees constitute
327 a very active area of research. Høyland and Wallace [50] developed an approach based on mini-
328 mizing a distance measure (e.g. squared norm) between a predefined statistical property and the
329 statistical property from the approximation. This approach is known as moment matching and has
330 been further expanded in various works (e.g. [51]). However, as shown in [50], matching some mo-
331 ments of the distributions is not enough to guarantee that the solutions of the stochastic program
332 will be similar. For this reason, Heitsch and Römisch [52] developed scenario generation heuristics
333 based on the concept of stability. Stability is a property in stochastic programs that guarantees
334 that the optimal value does not change excessively by changing the scenario tree if both come from
335 the same distribution. The approaches developed by Heitsch and Römisch [52] recursively reduce
336 and bundle scenarios using the concept of filtration distance. The more recent approach developed
337 by Pflug and Pichler [53], based on the concept of nested distance, allows generating the structure
338 of the tree dynamically to meet a prescribed precision.

339 The second limitation is associated with the solution of the optimization problem. The nested
340 structure found in MSPs severely complicates the problem and special algorithms are sometimes
341 required to provide a solution. Two general approaches are usually recognized in the literature
342 [54, 55, 56]: Primal and Dual decomposition. In the primal decomposition approach, the problem
343 is divided into a master problem for the current stage and sub-problems for the previous stages.
344 During each iteration, cuts are generated to linearly approximate the recourse function and generate
345 a candidate solution \mathbf{x}^* [48]. Examples of primal decomposition algorithms can be found in the
346 works of Birge [57], Ruszczyński [58], and Ruszczyński [59]. In the dual decomposition approach,
347 non-anticipativity constraints are relaxed and moved to the Lagrangian. The division between a
348 master problem and many sub-problems also exists in this approach, but here the sub-problems
349 are connected with scenarios instead of stages. As in the case of primal decomposition, most of the
350 advances came during the late 80s and early 90s with the Progressive Hedging Algorithm [60] and

351 the Diagonal Quadratic Approximation [61]. Most recent efforts have focused on extending these
 352 approaches to mixed-integer problems, as in the Branch and Price approach from Lulli and Sen
 353 [62].

354 Nonetheless, not all the MSP required special algorithms to be solved. Alternative approaches
 355 exist to exploit particular structures of the problem. For instance, if Block-Separability Recourse
 356 (BSR) exists, the MSP can be analyzed as a two-stage problem with aggregate-level decisions being
 357 made at the first stage and planning-level decisions happening during the second stage. Commercial
 358 solvers are also capable of handling MSPs with linear constraints and non-linear objectives [63] while
 359 state-of-the-art solvers can handle problems in the deterministic equivalent formulation [56].

360 4. Multistage Stochastic Programming Model for Flexibility

361 4.1. Problem description

362 An important challenge faced by infrastructure network systems consists of managing changes
 363 in demand such as traffic, consumption, or population growth efficiently. This can be accomplished
 364 with sporadic adaptations in size, capacity, or any other performance parameter to maintain min-
 365 imum safety and operational standards. Decisions on the timing and sizes of these interventions
 366 should be carefully made to maintain an acceptable level of performance while keeping the costs
 367 minimal. In this section, an MSP has been adapted to model this problem.

368 Consider a network represented as a directed graph $G(E, V)$, where E is the set of edges and
 369 V a set of nodes, such that every edge $e_{uv} \in E$ connects nodes u and v with $u, v \in V$. Each edge
 370 is associated with a flow $w_t(u, v)$ occurring at time t . This flow can represent traffic, electricity,
 371 water, information, etc., moving through typical infrastructure network systems such as highways,
 372 power distribution networks, water supply and sewage distribution networks, and telecommunica-
 373 tion networks. Simplifying the notation, each edge e can be characterized by a vector of design and
 374 operational variables $\mathbf{x}_{t,e} \in \mathbb{R}^{n_e}$ that may change over time (e.g., number of lanes in a highway or
 375 capacity in a power transmission network), where n_e represents the number of variables considered
 376 for edge e . These potential changes may occur as a result of some external random phenomena
 377 (e.g., change in traffic demand or flow), described by event $\omega \in \Omega$, and are represented by the
 378 variables $\mathbf{y}_{t,e} \in \mathbb{R}^{n_e}$, in the case of flexible adaptations, and $\mathbf{z}_{t,e} \in \mathbb{R}^{n_e}$ for unplanned adaptations.

379 An additional element is the flexibility built into each design or operational variable, which
 380 is represented in the model as $\mathbf{f}_{0,e} \in \mathbb{R}^{n_e}$ (a decision variable), the maximum range of change

381 allowed by design for $\mathbf{x}_{t,e}$. This is not the flexibility vector \mathbf{fv} presented in Equation 1, but the
382 difference between x_{max} and x_0 . For simplicity, $\mathbf{f}_{0,e}$ will be regarded as the flexibility from now on.
383 The flexibility decisions are made at the initial stage, together with the decision concerning initial
384 design and operation properties $\mathbf{x}_{0,e}$, and their purpose is to restrict the flexible adaptations $\mathbf{y}_{t,e}$
385 that can occur in the future. These modeling decisions allow introducing the problem of balancing
386 three approaches: investing at the initial stage to reduce future adaptation costs, investing at
387 the initial stage to have robust/unchangeable systems, and not investing and risking larger future
388 adaptation costs. This is, deciding the optimal values of $\mathbf{x}_{0,e}$, $\mathbf{f}_{0,e}$, $\mathbf{y}_{t,e}$, and $\mathbf{z}_{t,e}$. It is expected that
389 most solutions will involve a combination of the three strategies.

390 4.2. Definition of costs

391 Each decision variable of the model has a cost function associated. These cost functions can
392 represent costs related with activities of design, construction, adaptation, operation, and mainte-
393 nance of the network edges. The model considers the initial stage costs (design and construction) as
394 known, while the costs associated with future activities (adaptation, operation, and maintenance)
395 can be affected by uncertainty. This distinction is represented in the following cost functions: for
396 the initial stage, a_e is the cost function of building edge e with the initial design parameters $\mathbf{x}_{0,e}$,
397 and b_e is the cost function of adding features to network edge e that facilitate future changes,
398 i.e., the cost of adding flexibility to the system. For example, b_e can represent the additional
399 construction costs for a larger foundation in a building to facilitate future expansions. For the
400 recourse stages, $c_e(\mathbf{y}_{t,e}, \omega)$ is the cost function of adapting the network edge parameters by $\mathbf{y}_{t,e}$
401 units within the flexible range $\mathbf{f}_{0,e}$ (i.e., the initial design value of variable i plus the sum of all the
402 flexible adaptations over the lifetime must be less or equal than the initial value plus the flexibil-
403 ity $x_{0,e}(i) + \sum_{t=1}^T y_{t,e}(i) \leq x_{0,e}(i) + f_{0,e}(i)$), and $d_e(\mathbf{z}_{t,e}, \omega)$ is the cost function of performing an
404 adaptation $\mathbf{z}_{t,e}$ outside the flexible range. This option is included in the model to represent the
405 fact that any infrastructure network can be modified if enough resources are invested; however, this
406 does not mean that the alternative will be efficient. Additional costs related with operation are the
407 flow cost $q(w_e, \omega)$, the operation and maintenance (O&M) cost $g_e(\mathbf{x}_{t,e}(i), \omega)$, and the function of
408 revenue received from exploiting the network $h_e(\omega)$.

409 The parameters of these cost functions depend on, first, the type of function used for the
410 representation and, second, the procedure performed to obtain the data. For many applications,

411 a linear or convex function is enough to represent the required behavior. However, to represent
412 specific conditions, e.g. economies of scale [7], concave functions may be needed. The values of
413 these parameters can be obtained from autoregressive models, reports in the literature, experts'
414 opinions, industry surveys, and sensitivity analyses. If the model considers random costs, then the
415 cost function parameters become random variables and additional parameters for their distributions
416 must be defined (using historical data, available literature, surveys, etc.).

417 In the model proposed, all the cost functions can be represented using convex functions. If a
418 typical linear cost representation is used, then the only cost parameter is the unitary cost coefficient.
419 In this case, the only requirement is that the unitary costs of functions d_e (unplanned adaptations)
420 must be larger than the unitary costs of functions c_e (planned adaptations); otherwise, the model
421 will never assign value to the flexible option. If a different convex function is used, then it must be
422 guaranteed that $d_e(x) > c_e(x) \forall x > 0$. This does not mean that flexibility will always be preferred
423 due to the additional cost be of introducing flexibility at the initial stage. However, if the condition
424 is not met, then flexibility will never be a viable option from a cost standpoint.

425 *4.3. MSP problem formulation*

426 The elements described in the previous sections are combined in the following objective function
427 for an MSP to analyze flexible infrastructure networks:

$$\begin{aligned}
& \min_{\mathbf{x}_0, \mathbf{f}_0} \underbrace{\sum_{e \in E} \sum_{i=1}^{n_e} a_e(\mathbf{x}_{0,e}(i))}_{\text{Initial design costs}} + \underbrace{\sum_{e \in E} \sum_{i=1}^{n_e} b_e(\mathbf{f}_{0,e}(i))}_{\text{Flexibility introd. cost}} \\
& + \mathbb{E}_{\xi_1} \left[\min \gamma_1 \left(\underbrace{\sum_{e \in E} \sum_{i=1}^{n_e} c_e(\mathbf{y}_{1,e}(i), \omega)}_{\text{Flexible adaptations cost}} + \underbrace{\sum_{e \in E} \sum_{i=1}^{n_e} d_e(\mathbf{z}_{1,e}(i), \omega)}_{\text{Unplanned adaptations cost}} + \underbrace{\sum_{e \in E} q(w_{1,e}, \omega)}_{\text{Flow cost}} \right. \right. \\
& \left. \left. + \underbrace{\sum_{e \in E} \sum_{i=1}^{n_e} g_e(\mathbf{x}_{1,e}(i), \omega)}_{\text{O\&M cost}} \pm \underbrace{h(\omega)}_{\text{Exploitation cost/revenue}} \right) + \dots \right. \\
& \left. + \mathbb{E}_{\xi_T} \left[\min \gamma_T \left(\sum_{e \in E} \sum_{i=1}^{n_e} c_e(\mathbf{y}_{T,e}(i), \omega) + \sum_{e \in E} \sum_{i=1}^{n_e} d_e(\mathbf{z}_{T,e}(i), \omega) + \sum_{e \in E} q(w_{T,e}, \omega) \right. \right. \right. \\
& \left. \left. \left. + \sum_{e \in E} \sum_{i=1}^{n_e} g_e(\mathbf{x}_{T,e}(i), \omega) \pm h(\omega) \right) \right] \dots \right] \tag{4}
\end{aligned}$$

428 where i indexes the design and operational parameters being considered for each edge e , n_e is
429 the number of design and operational parameters, and γ_t is the discount factor. To represent the
430 limitations and special conditions associated with the management of flexibility and the optimal
431 flow distribution inside the network, the following constraints are formulated:

432 First, the constraints for flow conservation between converging and diverging links at a node k
433 are:

$$\sum_v w_{(k,v)} - \sum_u w_{(u,k)} = \phi_k(\omega) \quad \forall u, v, k \in V, \tag{5}$$

434 where $\phi_k = 0$ if k is a transshipment node and $\phi_k \neq 0$ if k is either a supply or a demand node.

435 Second, the constraints for the minimum and maximum flow that can move through each link
436 e are:

$$\ell_{lower,t,e} \leq w_{t,e} \leq \ell_{upper,t,e} \quad t = 1 \dots T, \forall e \in E \tag{6}$$

437 where the lower and upper limits $\ell_{lower}, \ell_{upper}$ are part of the design and operational variables
438 vector $\mathbf{x}_{t,e}$ and may have a flexibility value associated (can be adapted).

439 Third, the constraints that control the trigger of an adaptation process:

$$V(\mathbf{x}_{t,e}, \xi_t) \geq \pi_{t,e} \quad t = 1 \dots T, \forall e \in E \tag{7}$$

440 where the performance function V provides a quality measure of the design and operation
 441 variables $\mathbf{x}_{t,e}$ in comparison with the pressure caused by the random parameters ξ_t . This set of
 442 constraints establishes that the performance measure must be larger than a certain value $\pi_{t,e}$ given
 443 by the management policy. In this way, the constraints defines the criteria to make a change. For
 444 example, it can define the maximum ratio flow/capacity in an edge before it has to be expanded.

445 Fourth, the constraints for the maximum planned adaptation in a design or operation variable
 446 in an edge are:

$$\sum_{j=1}^t \mathbf{y}_{j,e}(i) \leq \mathbf{f}_{0,e}(i) \quad t = 1, \dots, T, \quad i = 1, \dots, n_e, \quad \forall e \in E \quad (8)$$

447 These constraints establish that the sum of the total planned changes for a design/operation variable
 448 i in an edge e must be, at most, the flexibility built for that variable i and edge e .

449 Fifth, the constraints to update the state variables of an edge in accordance with the available
 450 actions are:

$$\mathbf{x}_{t,e}(i) = \mathbf{x}_{t-1,e}(i) + \mathbf{y}_{t-1,e}(i) + \mathbf{z}_{t-1,e}(i) \quad t = 1, \dots, T, \quad i = 1, \dots, n_e, \quad \forall e \in E \quad (9)$$

451 Other constraints can be added depending on the nature of the network, or to properly character-
 452 ize the adaptations (e.g. non-negativity constraints restrict adaptations to expansions; integrality
 453 changes the nature of the adaptations from modular to continuous). There are non-anticipativity
 454 constraints implicitly formulated in the other constraints to restrict the knowledge available to make
 455 a decision at each stage; this is, at each time t , the decisions can only depend on the information
 456 available up to t .

457 The decision stages t are selected considering the frequency of the decision process. This fre-
 458 quency may not coincide with the frequency of the random processes, in which case one decision
 459 stage may encompass many random process periods. For instance, it is not expected for an infras-
 460 tructure network to be expanded every year, even if the demand is reported every month. In such
 461 a case, the decision stage t can consider the average, the maximum, or a quantile of the demand
 462 periods clustered in t . It is also not required for the decision stages t_1, t_2, \dots to be equal in size.
 463 Considerations regarding the life-cycle of the network and the reliability of the uncertain data may
 464 result in a higher frequency of decisions during the first years of the network. It must also be con-
 465 sidered that increasing the frequency of the decision stages also increases the size of the problem.

466 In brief, it is a modeling decision that depends on the purpose of the model and the experience of
 467 the modeler.

468 5. Approximate Solution of MSPs

469 5.1. Criteria for approximating the solution

470 Solving a multistage stochastic program using the original distribution of the uncertain param-
 471 eters ξ_t may not be feasible in many real applications because the set of possible paths (realizations
 472 of the demand) is infinite. For this reason, several methods have been developed to approximate
 473 the distribution of the random phenomena [52, 49, 64]. Overall, the available methods focus on:
 474 minimizing some measure of the distance between the original and the approximated distribution
 475 (e.g. Wasserstein distance) [52, 65], matching moments between the distributions [50], and generat-
 476 ing samples (e.g. Monte-Carlo sampling [66]). As a result, the original distribution becomes a set of
 477 scenarios (each with an associated probability), usually organized in a tree structure. This scenario
 478 tree represents the increasing finite filtrations of available information, as shown in Figure 3.

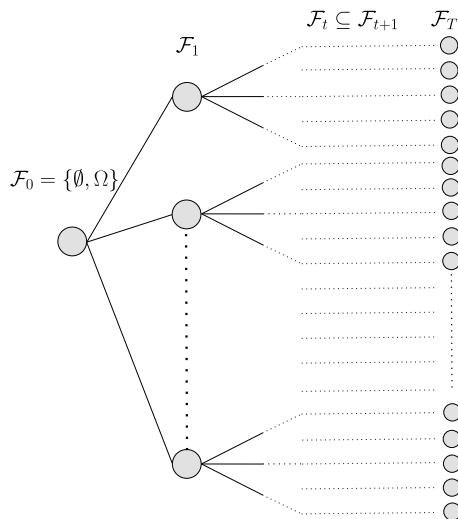


Fig. 3. Scenario tree representation of increasing finite filtrations

479 The accuracy of the stochastic program solution highly depends on the quality of the resulting
 480 discretization [67]. However, it is important to keep in mind that the main objective is to optimize
 481 the quality of the solution obtained by the stochastic program and not necessarily to produce an

482 optimal discretization of the original distribution. This can be achieved by having stability and
483 low bias in the solution [68]. Stability means that the variation in the optimal value is minimal
484 among different scenario trees (obtained from the same distribution). Low bias refers to a small
485 gap between the real (unobtainable) and the approximated solution. The most direct approach to
486 achieve both stability and low bias is to simulate a large number of scenarios [69, 68]. A trial and
487 error procedure is usually necessary to achieve these objectives with the smaller scenario tree as
488 possible.

489 5.2. Discretization method by clustering

490 As explained before, it is generally not possible to solve an MSP using the real distribution
491 of ξ_t ; instead, an approximation is required. Typical approximate distributions are represented
492 in the form of scenario trees. In this paper, the discretization process is performed by combining
493 Monte-Carlo sampling and a k-medoids clustering algorithm (see Figure 4).

494 The methodology requires listing the properties of the random process to be simulated, which
495 can include values such as the mean and the standard deviation, or additional shape parameters.
496 Then, the time mission is divided in a finite set of time steps (stages) $t = 1, \dots, T$, and a large
497 number N of random trajectories are generated (see Figure 4(a)). At the first time stage $t = 1$,
498 the coefficient k_1 is defined to represent the number of clusters in which the trajectories will be
499 aggregated; this coefficient is known as the *branching factor*. Then, the trajectories are combined
500 into k_1 clusters using a k-medoids clustering algorithm. At the next time stage, the trajectories
501 grouped inside each cluster are further divided into k_2 clusters using the same algorithm. This
502 process is repeated until the end of the time mission when a total of $K = \prod_{t=1}^T k_t$ clusters are
503 generated. This process results in a symmetric scenario tree.

504 This method requires a large number of trajectories per cluster to avoid clustering single tra-
505 jectories. Considering that the number of scenarios grows exponentially with the number of stages,
506 the number of required trajectories may be difficult to generate for large scale applications. While
507 it works well for problems with a number of scenario paths of order 10^5 to be clustered in trees
508 with a number of scenarios of order 10^3 , problems of larger scale may require more complex ap-
509 proaches. For instance, a larger number of clusters can be used at the initial stages to obtain a
510 finer discretization due to the importance of the prediction at these early stages (Figure 4(b)).
511 Fortunately, in most cases, flexibility problems in infrastructure networks usually involve a small

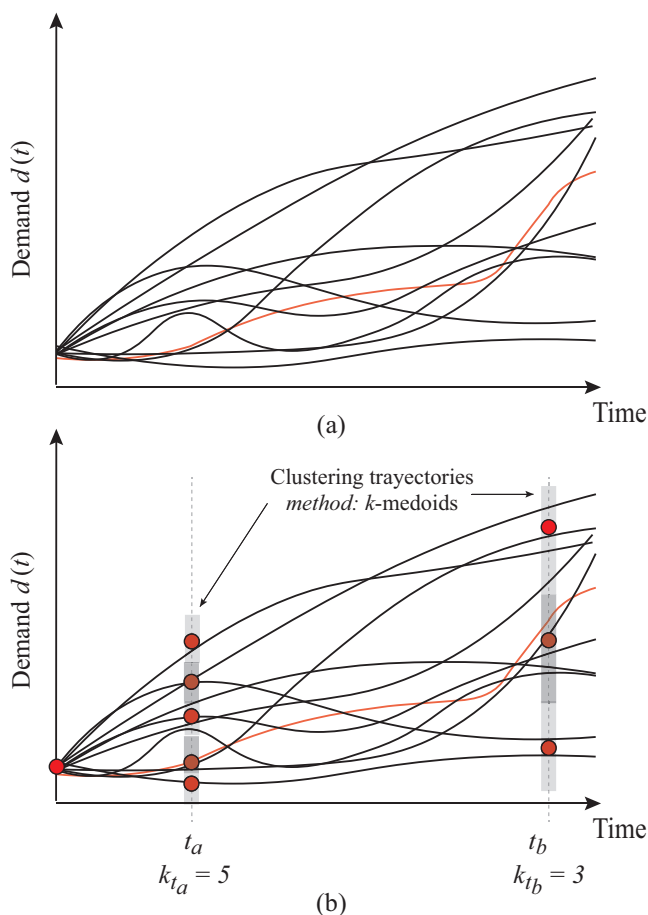


Fig. 4. (a) realization of system path trajectories; (b) clustering of sample paths at different stages.

512 number of variables and stages due to the limited number of characteristics that can be adapted
 513 and the low frequency of changes.

514 5.3. Approximated flexibility MSP model

515 The approximation of the distribution of ξ_t using a scenario tree can be combined with the
 516 original flexibility MS approach ((4)) to produce an approximated MSP that can be solved with
 517 numerical solvers. By replacing the probability model with an approximate, simpler, model with
 518 finite support [65], the original filtration becomes a finite filtration with a tree structure (scenario
 519 tree). This approximation leads to the formulation of an equivalent deterministic program, as shown
 520 in Equation 10:

$$\begin{aligned}
\min_{\mathbf{x}_0, \mathbf{f}_0} & \sum_{e \in E} \sum_{i=1}^{n_e} a_e(\mathbf{x}_{0,e}(i)) + \sum_{e \in E} \sum_{i=1}^{n_e} b_e(\mathbf{f}_{0,e}(i)) \\
& + \underbrace{\sum_{k=1}^s p_k}_{\text{Scenario probability}} \left[\gamma_1 \left(\sum_{e \in E} \sum_{i=1}^{n_e} c_{e,k}(\mathbf{y}_{1,e,k}(i)) + \sum_{e \in E} \sum_{i=1}^{n_e} d_{e,k}(\mathbf{z}_{1,e,k}(i)) + \sum_{e \in E} q_k(w_{1,k}) \right. \right. \\
& \left. \left. + \sum_{e \in E} \sum_{i=1}^{n_e} g_{e,k}(\mathbf{x}_{1,e,k}(i)) \pm h_{1,k} \right) + \dots \right. \\
& \left. + \gamma_T \left(\sum_{e \in E} \sum_{i=1}^{n_e} c_{e,k}(\mathbf{y}_{T,e,k}(i)) + \sum_{e \in E} \sum_{i=1}^{n_e} d_{e,k}(\mathbf{z}_{T,e,k}(i)) + \sum_{e \in E} q_k(w_{T,k}) \right. \right. \\
& \left. \left. + \sum_{e \in E} \sum_{i=1}^{n_e} g_{e,k}(\mathbf{x}_{T,e,k}(i)) \pm h_{T,k} \right) \right] \tag{10}
\end{aligned}$$

521 where s indicates the number of scenarios and p_k is the individual probability of each scenario.
522 Scenarios are understood as a complete path from the root node to a leaf node in the scenario tree.
523 This formulation of the objective function clearly shows that a particular solution is obtained for
524 each scenario k except for the initial stage variables (\mathbf{x}_0 and \mathbf{f}_0) which are the same for all scenarios.

525 Formulating the MSP in this form requires to explicitly develop the non-anticipativity con-
526 straints. The objective of these constraints is to ensure that scenarios with the same history up to
527 stage t are indistinguishable [47].

$$\begin{aligned}
\mathbf{x}_{t,e,j}(i) &= \mathbf{x}_{t,e,k}(i) \\
\mathbf{y}_{t,e,j}(i) &= \mathbf{y}_{t,e,k}(i) \\
\mathbf{z}_{t,e,j}(i) &= \mathbf{z}_{t,e,k}(i) \quad \forall j, k \text{ for which } \boldsymbol{\xi}_{t,j} = \boldsymbol{\xi}_{t,k}, \quad t = 1, \dots, T, \quad i = 1, \dots, n_e, \quad \forall e \in E
\end{aligned} \tag{11}$$

528 To illustrate the non-anticipativity conditions, Figure 5 shows a fragment of a scenario tree. The
529 figure shows that Scenario s_{1,t_r} and Scenario s_{2,t_r} are identical up to stage t_q , same as Scenario
530 s_{3,t_r} and Scenario s_{4,t_r} . Furthermore, the four scenarios are indistinguishable up to stage t_p . The
531 restrictions in Equation 11 ensure that the decision variables comply with the non-anticipativity
532 conditions given by the scenario tree of the random process.

533 The procedures described in Sections 5.1-5.3 to formulate and solve the proposed MSP are
534 summarized in the chart presented in Figure 6.

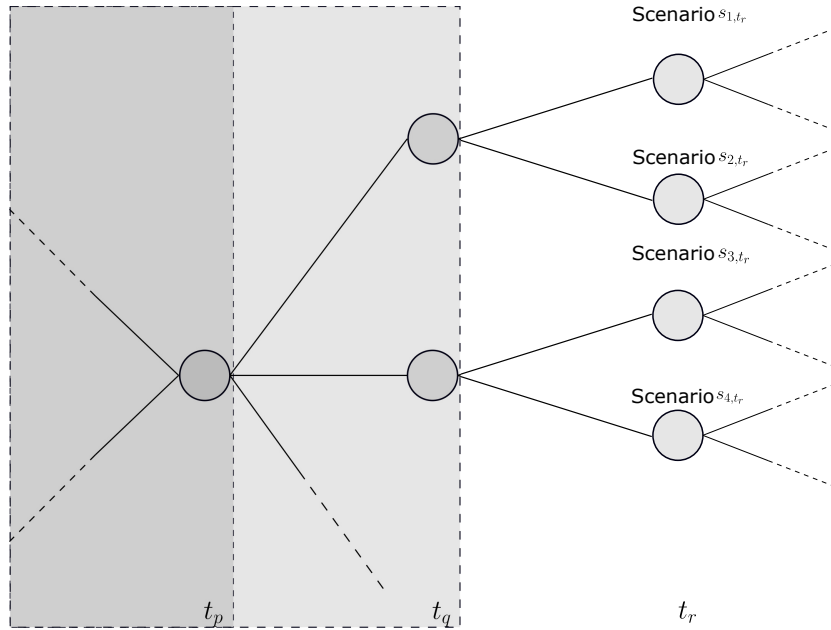


Fig. 5. Non-anticipativity conditions in scenario tree

535 *5.4. Model scalability*

536 The scalability of the proposed model can be analyzed for two elements: the scenario tree
 537 generation algorithm and the solution method. First, the scenario generation algorithm is based on
 538 the general approach described by Dupačová et al. [63] where a set of scenario paths are generated
 539 according to a probabilistic model (based on, for instance, historical data series), a scenario tree
 540 structure is predefined according to some heuristic (e.g. detailed branching for early stages and
 541 coarse branching for later stages), and a clustering algorithm is applied to the scenario paths based
 542 on some dissimilarity measure. The scenario path generation process, even for the multivariate
 543 case, scales linearly in the worst case, and it is never regarded in the literature as a worrisome
 544 source of computational complexity. The clustering algorithms, however, do not scale as well due
 545 to the required comparisons between all the data points (distance or dissimilarity measures).

546 The k-medoid algorithms used in this paper are the Partitioning Around Medoids (PAM) and
 547 the Clustering LARge Applications (CLARA) implementations in Matlab[®]. The CLARA algorithm
 548 is used to cluster the first 2-3 stages of the scenario tree where the number of data points is large
 549 (> 5000). This algorithm is of order $O(k^3)$ [70] and can be used for large data sets to be bundled in

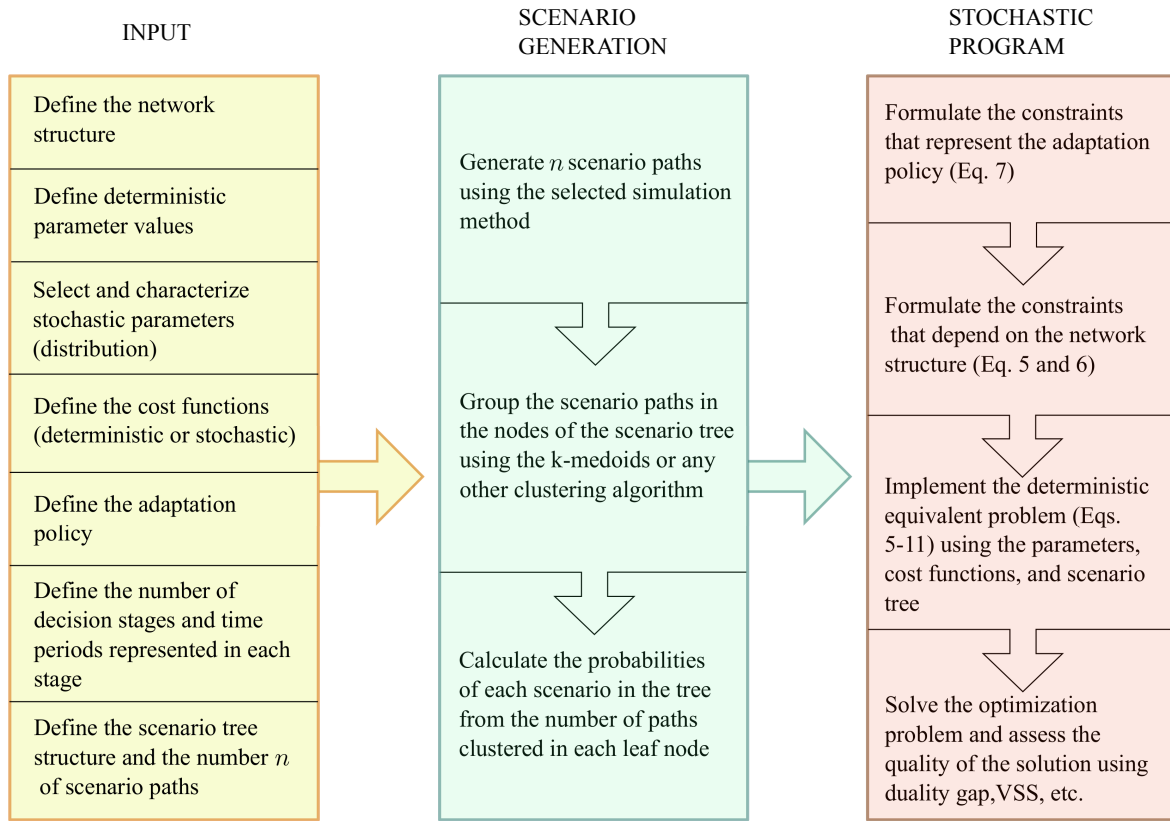


Fig. 6. General procedure to formulate and solve proposed MSP model

550 a small number of clusters. The subsequent stages require clustering progressively smaller number
 551 of points in an also small number of clusters, which can be done using the PAM algorithm of order
 552 $O(k(n - k)^2)$. While these clustering algorithms work well for a wide range of data set sizes and a
 553 limited number of branching factors, the structure of the scenario tree requires that the algorithms
 554 have to be executed an increasing number of times at each stage. This is, the number of calls for
 555 the clustering algorithms grows exponentially with the number of stages. Therefore, the scalability
 556 of the scenario generation procedure depends on both the scalability of the clustering algorithms
 557 and the size of the scenario tree.

558 The second element that affects the scalability of the model is the solution method. As explained
 559 in Section 3.3, solving an MSP is challenging due to the considerable number of variables, the
 560 interdependence between them, and possible non-linear constraints. While special methods have

561 been developed to solve MSPs, in some cases commercial solvers are enough to obtain a solution
 562 in reasonable times for problems with hundreds of thousands of variables. One of these cases is
 563 when the problem has a linear (or non-linear) objective and linear constraints [63]. By introducing
 564 the scenario tree representation to formulate the deterministic equivalent problem, the result is a
 565 large scale linear (or non-linear) optimization problem that can be solved efficiently by commercial
 566 solvers. The model presented in Section 5.3 has linear constraints and the cost functions can be
 567 properly represented as linear or quadratic functions for many applications. Therefore, commercial
 568 solvers can be used to solve the proposed model, as is shown in the example in Section 6.

569 Nonetheless, the exponential growth in the number of variables as the number of stages increases
 570 poses a significant limitation to the frequency of the decision process. This limitation can be
 571 circumvented as the nature of infrastructure systems makes monthly or even annual adaptation
 572 plans unrealistic due to the time and resources required to complete an adaptation. The decision
 573 stages for infrastructure adaptation problems can encompass years and, in consequence, it is possible
 574 to model the complete or a considerable portion of the system's lifetime with a limited number of
 575 stages.

576 In addition to the linearity of the constraints and the control to the scenario tree explosion by
 577 limiting the number of stages, the proposed model has two additional properties that reduce its
 578 computational complexity. The first property is the diagonal structure in the constraint matrices.
 579 Limiting the dependence of the decision variables to their immediate predecessor by adding state
 580 variables results in sparse matrices that are much faster to solve [46]. The second property is
 581 block-separable recourse. This property allows transforming an MSP into a two-stage problem by
 582 separating the aggregate level decision variables from the planning level variables. The aggregate
 583 level decisions can represent capacity expansion decisions while the planning level decisions can
 584 refer to the use of this capacity [48]. If the aggregate level decisions do not depend on the planning
 585 level decisions, then the former can be moved to the first stage while the latter are placed on the
 586 second stage. Specifically, block-separable recourse exists if i) at stage t the cost functions can be
 587 written as:

$$f_t(\mathbf{u}_t) + g_t(\mathbf{v}_t) \tag{12}$$

588 where \mathbf{u}_t represents the aggregated level decisions, \mathbf{v}_t the planning level decisions, and f_t and
 589 g_t are the respective cost functions. And ii) if the constraints can be written as:

$$\begin{pmatrix} T_t & 0 \\ S_t & 0 \end{pmatrix} \begin{bmatrix} \mathbf{u}_{t-1} \\ \mathbf{v}_{t-1} \end{bmatrix} + \begin{pmatrix} W_t & 0 \\ 0 & D_t \end{pmatrix} \begin{bmatrix} \mathbf{u}_t \\ \mathbf{v}_t \end{bmatrix} = \begin{bmatrix} \mathbf{a}_t \\ \mathbf{b}_t \end{bmatrix} \quad (13)$$

590 where T_t and S_t are submatrices of the technology matrix, W_t and D_t are submatrices of the
 591 recourse matrix, and \mathbf{a}_t and \mathbf{b}_t are subvectors of the right-hand side vector. This equation clearly
 592 shows that both current aggregate and current planning level decisions depend only on previous
 593 aggregate level decisions.

594 The model proposed in Equations 4-11 is block-separable for the following reasons: i) the cost
 595 functions are clearly segregated between the aggregate level variables (\mathbf{x}_t , \mathbf{y}_t , \mathbf{z}_t , and \mathbf{f}_0) and the
 596 planning variables \mathbf{w}_t ; ii) the constraints in Equations 5-9 and 11 are not enforcing any dependence
 597 between the set of aggregate decisions \mathbf{x}_t , \mathbf{y}_t , \mathbf{z}_t , and \mathbf{f}_0 and the decisions \mathbf{w}_{t-1} . Constraints 6 and
 598 7 do generate a dependence between \mathbf{w}_t and \mathbf{x}_{t-1} but this relationship is included in the definition
 599 of block-separability.

600 In summary, the proposed model has advantageous characteristics in its structure that can be
 601 exploited to vastly reduce its computational complexity and allow the use of commercial solvers
 602 to find a solution. The main bottleneck happens in the scenario tree generation procedure due to
 603 the combinatorial nature of the process. Despite these advantages, the number of variables in the
 604 problem still grow exponentially with the number of stages, and even if the MSP can be transformed
 605 into a large scale linear problem and a simple binary tree structure is used, having more than ~ 22
 606 stages implies hundreds of millions of variables.

607 **6. Numerical Example**

608 *6.1. Case description*

609 This section presents an example of modeling flexibility in network design and operation using
 610 the proposed MSP. The purpose of this example is threefold: i) to show how the model can be
 611 used to make decisions when managing flexible networks; ii) to study how the presence of flexibility
 612 affects the cost performance of the network; and iii) to identify the elements in the model that have
 613 the highest impact in the assessment of the value of flexibility.

614 The network used in the example is shown in Figure 7; it is a generic network that may describe
 615 telecommunication, energy, water distribution, or a transportation network. This example only
 616 considers one element in the random parameters vector $\boldsymbol{\xi}_t = \delta_t$ to represent the demand that enters

617 the network through node A and propagates throughout the network until it reaches node F , where
 618 it exits. This demand is modeled as a stochastic process defined by:

$$\delta(t) = \beta_1 t + e^{\beta_2 t} \beta_3 t \sin(\beta_4 t) + B(t) \quad (14)$$

619 where $\beta_1, \beta_2, \beta_3, \beta_4$ are normally distributed random variables and $B(t)$ is a Wiener process. Table
 620 1 shows the values of the parameters used in the example. Figure 8 shows five realizations of this
 621 process.

Table 1. Random demand process parameters

Parameter	Distribution(μ, σ^2)
β_1	$\mathcal{N}(450, 1.26 \times 10^4)$
β_2	$\mathcal{N}(-0.02, 2.5 \times 10^{-5})$
β_3	$\mathcal{N}(100, 625)$
β_4	$\mathcal{N}(0.9, 5.06 \times 10^{-2})$

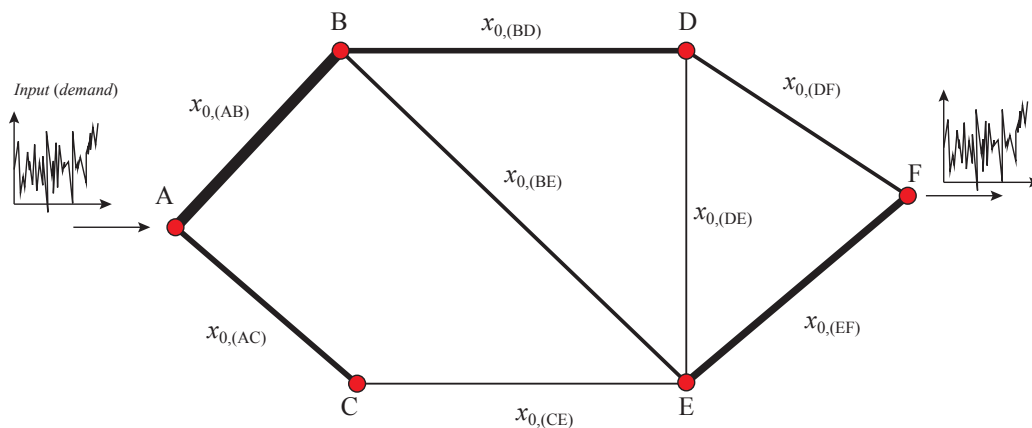


Fig. 7. Network used in the numerical example.

622 The design and operation variables vector $\mathbf{x}_{t,e}$ of each link in the network only contains the link
 623 capacity $x_{t,e}$. The initial capacity is notated as $x_{0,e}$, and together with the flexibility range $f_{0,e}$
 624 define the initial stage decisions. The maximum capacity that can be reached by flexibility for each
 625 link at any time is given by $x_{max,e} = x_{0,e} + f_{0,e}$. However, the system can be modified beyond

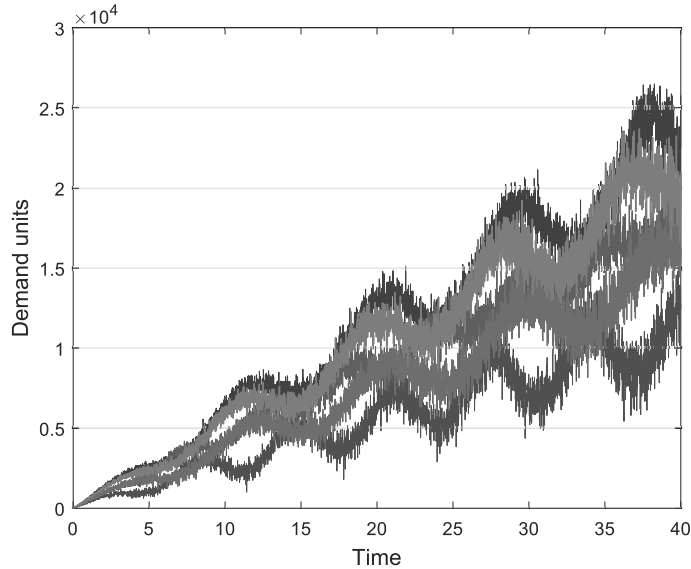


Fig. 8. Realizations of the stochastic process

626 that value but at a larger cost. This may happen, for instance, due to larger times required to
 627 modify the system, larger material and labor costs that result from last-minute contracts, and the
 628 related disturbances and damages caused by the unexpected partial or total cease of operation. In
 629 the example, only capacity expansions are considered.

630 The time mission of the network is $T = 40$ years starting from the date of commissioning. To
 631 reduce the size of the problem, it is assumed that adaptations can only occur at specific times:
 632 $[6, 12, 18, 24, 32, 40]$ (except for the sensitivity analysis in Section 7.1.2). This limitation allows
 633 downsizing the scenario tree, which highly impacts the computational performance. The reduction
 634 can be justified by the fact that infrastructure networks, and in general infrastructure systems,
 635 cannot be changed frequently in most of their dimensions without incurring high costs.

636 The model uses two families of cost functions. The initial design cost functions a_e , flexible
 637 adaptation cost functions c_e , and the unplanned adaptation cost functions d_e (see Equation 10) are
 638 assumed linear of the form $a_e = \alpha_0 \mathbf{x}_{0,e}$, $c_e = \alpha_{fa} \mathbf{y}_{t,e}$, and $d_e = \alpha_{ia} \mathbf{z}_{t,e}$, equal for all the edges.
 639 The linear cost assumption reduces the comparison to a straightforward relationship between the
 640 cost coefficients. Quadratic functions are used to represent the costs of introducing flexibility of the
 641 form $b_e = \alpha_f f_{0,e}^2$. This non-linear increasing function is used to consider the effect of dis-economies

642 of scale that may exist in systems that cannot be expanded indefinitely without restrictions, such
 643 as most of infrastructure networks.

644 Coefficient α_0 is equal to 5 cost units per capacity unit. Coefficients α_{fa} and α_{ia} vary for the
 645 analyses shown in Section 7.2. Operation and maintenance costs (O&M) g_e is assumed equal to
 646 2% of α_0 per unit of capacity installed, except for the last analysis. Flow costs q_e are function of
 647 O&M costs g_e as shown in Table 2.

Table 2. Flow costs summary

Link	Flow Cost
A-B	$0.05g_e$
A-C	$1.2q_{AB}$
B-D	$1.1q_{AB}$
B-E	q_{AB}
C-E	$1.2q_{AB}$
D-E	$1.3q_{AB}$
D-F	$1.1q_{AB}$
E-F	$1.2q_{AB}$

648 The flow costs vary between links to represent an existing minimal cost route and increase the
 649 stability of the solution by avoiding a random allocation of initial capacity and flexibility.

650 6.2. Discretization of the demand space

651 The first task is the discretization of the demand space using the sampling and clustering algo-
 652 rithm presented in Section 5.2. This requires defining first the scenario tree branching factors which
 653 vary between $[4\ 4\ 2\ 2\ 2\ 2]$ (256 scenarios) and $[4\ 4\ 4\ 4\ 2\ 2]$ (1024 scenarios). Then, the algorithm
 654 generates between 15000 and 25000 demand trajectories according to Equation 14 depending on the
 655 total number of scenarios. The clustering algorithm processes these inputs to generate a scenario
 656 tree whose nodes contain the demand values at the decision stages $[6, 12, 18, 24, 32, 40]$ years, with
 657 an associated probability that depends on the number of trajectories clustered in each node.

6.3. MSP solution

To obtain the optimal network initial conditions, $x_{0,e}^*$ and $f_{0,e}^*$ for every edge e in the network, it is necessary to solve the multistage stochastic problem described by Equation 10, subject to the restrictions presented in Equations 5 to 11. The model assumes that every time a flexible adaptation $y_{t,e}$ occurs, the flexibility built into the system at the initial stage $f_{0,e}$ is reduced until its depletion. Once this point is reached only unplanned adaptations $z_{t,(u,v)}$ can be deployed. Finally, adaptations (i.e., increments incapacity) can only occur at $t \in \{0, 6, 12, 18, 24, 32, 40\}$.

The evolution of the network capacity is controlled by a policy $\pi(\xi_t, x_{t,e})$ based on a safety criteria that establishes that the capacity of every edge at time t should be such that $x_{t,e} \geq SF w_{t,e}$, with $w_{t,e}$ the flow moving through link e , and SF some safety factor. This formulation keeps the constraints in Equation 7 linear. Additional flow conservation constraints are formulated for each network node according to the topology shown in Figure 7.

The numerical results were obtained using the CPLEX 12.8 API for Matlab® 2017b and the YALMIP toolbox as algebraic interpreter [71] with an Intel®Core™ i5-5200 2.20GHz processor. An average run for 1000 scenarios takes approximately 100 seconds of processing time, where 95% of the time is spent in the scenario tree generation algorithm and less than 5% in solving the optimization program.

The solver used (CPLEX) automatically selects the barrier optimizer due to the large size and sparsity of the problem. This algorithm stops iterating when the primal and dual solution are complementary, i.e., when a sum of products between the primal and dual solutions is smaller than a predefined tolerance. The tolerance of the complementary used in this numerical example is defined as 1^{-8} .

7. Network Design for Flexibility: Results

Solving the MSP defined by Equations 5 to 11 a results in an optimal (minimal) value of the total present costs C_{total}^* , an optimal value of the initial capacity $x_{0,e}^*$ for each link e , an optimal value of the flexibility $f_{0,e}^*$ for each link e , and optimal values each variable $y_{t,e,k}^*$ and $z_{t,e,k}^*$, that represent the evolution process of each network link, for every stage t and scenario k . Figure 9 shows three realizations of the demand process and the respective evolution of the whole network capacity.

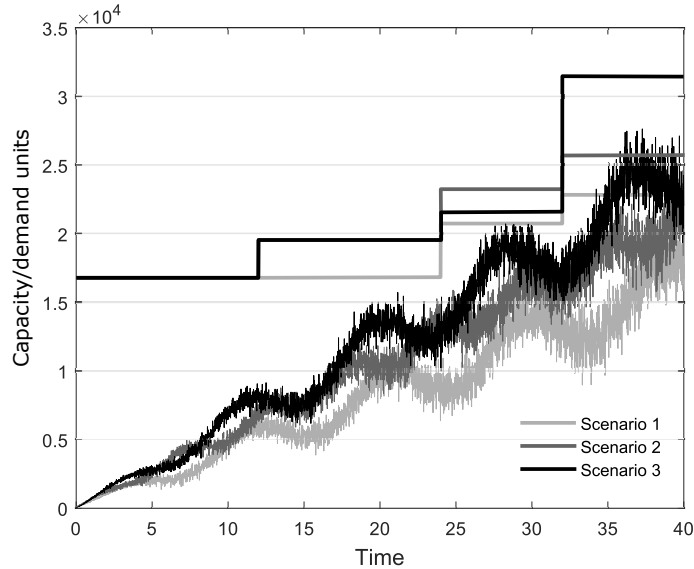


Fig. 9. Network capacity evolution for three demand scenarios

687 This figure illustrates one of the main properties of MSPs: The optimal solution for the initial
688 capacity, i.e. the initial stage decision, is the same for all scenarios while the recourse decisions
689 that expand the system are particular for each scenario. By averaging the optimal costs of all the
690 system's future responses, the model makes an initial decision that is optimal in average for all
691 scenarios.

692 Considering that the solutions of the recourse decisions $y_{t,e,k}^*$ and $z_{t,e,k}^*$ are generated for each
693 scenario, they are not suited for the comparisons intended in the forthcoming analyses. Only the
694 initial solutions $x_{0,e}^*$ and $f_{0,e}^*$ are the same for all scenarios. For this reason, only these solutions,
695 together with the optimal costs C_{total}^* , will be considered in the analysis of results.

696 7.1. Numerical stability of the model

697 7.1.1. Stability verification of the demand tree generation

698 Stability in the generation of the demand tree is essential for the accuracy of the model. If
699 the program is not stable, the solution can vary considerably between scenario trees. To verify
700 the stability of the solution, three scenario tree structures were generated using different branching
701 factors. These factors determine how many nodes grow from each node at each time stage and,

ultimately, the number of scenarios. Furthermore, the demand values in the nodes of the scenario tree vary due to the Monte-Carlo generation procedure. To address these sources of variability, the MSP was solved in batches of increasing size (from 2 solutions up to 40 solutions) and the resulting optimal costs were averaged for each batch. Figure 10 presents the coefficient of variation, CV , of the optimal costs for each batch size, and for three sets of branching factors (scenario tree structures).

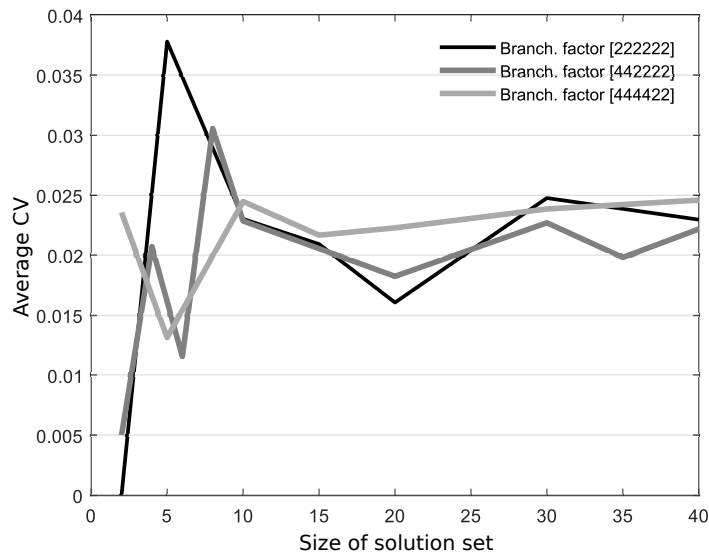


Fig. 10. Solution variability as function of number of scenarios and runs

The scenario tree structures compared in Figure 10 were selected starting by the smallest possible symmetrical tree [2 2 2 2 2]. Additional nodes were increasingly added to the initial stages to improve the quality of the short-term predictions. The largest scenario tree considered, [4 4 4 4 2 2], showed similar behavior with larger trees but with considerably lower computational times.

Figure 10 shows that there is variability due to the number of solutions being considered and due to the instability in the solution. As the size of the batch increases, the first source decreases and only instability remains. It is observed that the optimal costs are relatively stable with a $CV = 2.5\%$. It is also observed that a larger number of scenarios reduces the number of solutions required to reach a stable CV (branching factor [4 4 4 4 2 2]). This result is expected as a larger number of scenarios represent better the underlying distribution, achieving higher stability as a

Table 3. Sensitivity analysis results

Decision stages	Optimal value	$f_{0,AB}^*$	$x_{0,AB}^*$	Processing time(s)
4	361619	2546	14787	45
5	340954	2657	13341	62
6	334664	2614	12798	85
7	339480	2661	12894	95
8	329768	2793	11175	92
9	341655	2664	12618	90
10	341035	2693	12961	110

718 result. Generally, only the optimal costs are compared in stability analyses and not the optimal
719 solutions due to the typical flat objective functions found in MSPs [72].

720 7.1.2. Planning horizon discretization into decision stages

721 The results from the previous section assumed that the planning horizon is divided into 6 decision
722 stages. This section presents a sensitivity analysis to determine how this modeling decision affects
723 the MSP results.

724 To this end, the planning horizon is divided into 4, 5, . . . , 10 decision stages, and the resulting
725 MSP is solved. In the cases when the planning horizon is not a multiple of the number of stages, the
726 initial stages encompass fewer years than later stages, but the purpose is to cover the entirety of the
727 planning horizon as uniformly as possible. The scenario trees are designed to have the same number
728 of scenarios (2^{10}) which requires using variable branching factors. Therefore, the discretization of
729 the probability space for the intermediate stages will not be equal for all cases, with a difference of
730 a factor of 2 at most in the number of scenario tree nodes for similar time instants. For instance,
731 for 4 decision stages, there are 64 nodes representing year 20, while for 5 stages there are 32 nodes
732 at year 16. Table 3 shows the optimal value, the optimal flexibility solution $f_{0,AB}^*$ for link AB ,
733 the optimal initial capacity solution $x_{0,AB}^*$, and the processing time obtained for each number of
734 decision stages.

735 The results in Table 3 show that when the number of stages that divide the planning horizon is
736 5 or larger, only the processing times are affected. For 4 decision stages, the model solutions seem

737 to be significantly different, beyond what could be explained by the random variation present in the
738 solutions for the remaining number of stages. This tendency was found in multiple executions. This
739 difference could be explained by the relationship between the frequency of the required adaptations
740 and the frequency of the decision stages. As shown in Figure 9, the time between adaptations can
741 vary between 8 and 12 years on average, with longer times during the first half of the planning hori-
742 zon. This means that during the system’s lifetime only 3 or 4 adaptations are expected. Therefore,
743 when the planning horizon has 4 decision stages or fewer, it can be expected that an adaptation
744 will take place at every stage and that the magnitude of these adaptations (and the initial capacity)
745 will be larger to accommodate the changes in the demand during such long time periods.

746 In summary, the discretization of the planning horizon into decision stages can affect the solution
747 when there is insufficient opportunity to use the recourse to respond to the changes in the external
748 conditions.

749 7.1.3. *Out-of-sample behavior of the model*

750 The verification of the out-of-sample behavior consists in solving the optimization program with
751 a particular tree and using the optimal solution, i.e., $x_{0,e}^*$ and $f_{0,e}^*$, as the initial solution for MSPs
752 with different scenario trees of the same size and generated from the same stochastic process. Figure
753 11 compares the optimal costs obtained in the baseline program (horizontal line), with the optimal
754 costs found in 10 out-of-sample programs. This figure shows that 70% of the out-of-sample costs
755 have a difference of 1.2% or less with the base case, while the remaining 30% has, at most, a
756 difference of 3.3%. The results from the stability analysis and the out-of-sample analysis suggest
757 that even if there is some instability because the scenario tree is not infinite, it is under control
758 and the program exhibits both *in* and *out* sample stability, which is a requirement for low bias and
759 general stability.

760 7.1.4. *Value of the stochastic solution*

761 A second key element in the analysis of MSPs is the value of the stochastic solution (VSS).
762 This metric compares how much is gained by solving the stochastic program instead of solving a
763 deterministic program where the scenario tree is replaced with the expected value of the random
764 process. To calculate the VSS, first, the expected value problem is solved and the optimal solution
765 is used as the initial solution for an MSP. The optimal costs obtained for this new program are

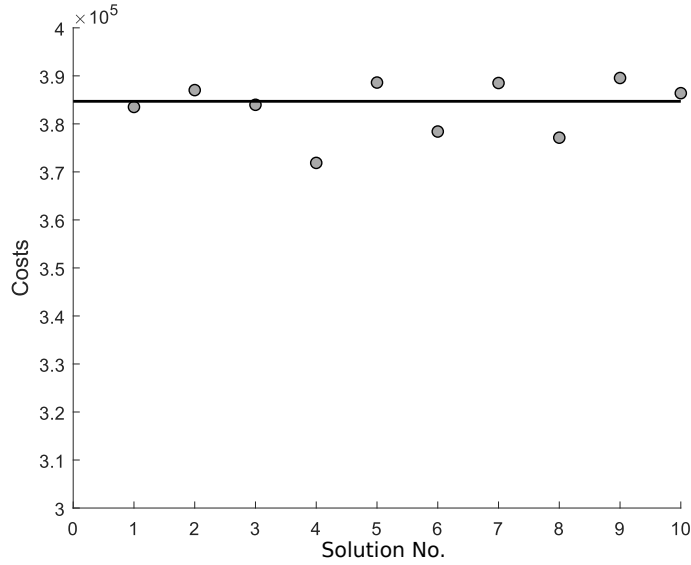


Fig. 11. Out-of-sample solution comparison

766 known as the *expected value* solution (EV). The VSS is obtained as the difference between the
 767 original MSP optimal costs and the EV.

768 Using the scenario tree structure regarded as the most stable from the analysis in Section 7.1.1,
 769 [4 4 4 4 2 2], generating 50 scenario trees and averaging the VSS, it was obtained a value that is
 770 16% of the average optimal costs. This result is not negligible and suggests that the VSS is large
 771 enough to justify solving a stochastic program instead of the much simpler expected value problem.

772 *7.2. The value of flexibility*

773 The third element of the analysis focuses on the effects of the cost functions on the value of
 774 flexibility. Different relationships between α_0 , α_{fa} , and α_{ia} are considered to determine the effect
 775 on: i) the optimal initial capacity $x_{0,e}^*$, ii) the optimal flexibility $f_{0,e}^*$, and iii) the optimal costs.

776 Figures 12 and 13 show, respectively, the evolution of the optimal solution for the capacity $\mathbf{x}_{0,e}^*$
 777 and the flexibility $\mathbf{f}_{0,e}^*$ in each link for different ratios α_{fa}/α_0 . The critical route is highlighted
 778 with the dotted lines and light shadow. As expected, the larger the cost of future adaptations α_{fa}
 779 compared with the cost of building the required capacity from the beginning α_0 , the less flexibility
 780 is introduced, and the system relies more on the initial capacity to serve the demand. This means

781 that reducing α_0 or increasing α_{fa} decreases the value associated with flexibility, and the system
 782 manager is willing to pay less to have it in the network.

783 It is interesting to observe in Figures 12 and 13 that the optimal solution for the initial capacity
 784 $\mathbf{x}_{0,e}^*$ results in larger values for the links in the minimum cost route, while the optimal solution for
 785 flexibility $\mathbf{f}_{0,e}^*$ is similar for all the links in the network, excluding DE. This suggests that the short
 786 and medium-term demand is mostly met by the minimum cost route until a point where it is less
 787 expensive to use the flexibility built in other links to expand them and reroute part of the demand.
 788 Besides, the quadratic cost function used for the introduction of flexibility discourages the approach
 789 of adding too much flexibility into one link.

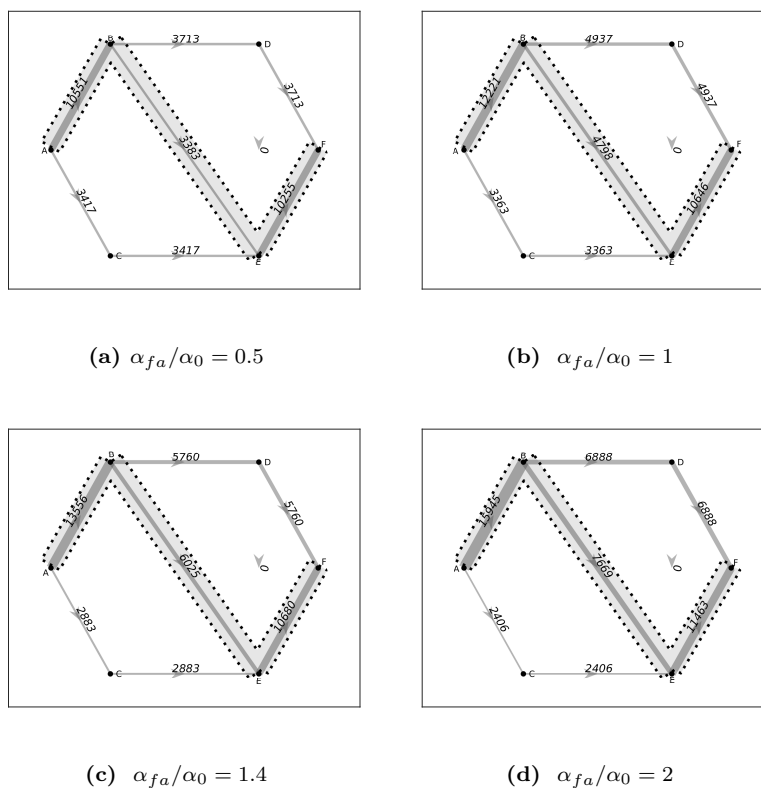


Fig. 12. Initial capacity evolution for different ratios α_{fa}/α_0

790 Figure 14 compares the optimal costs obtained for different ratios α_{fa}/α_0 . The figure includes
 791 in the comparison the flexibility vectors \mathbf{fv} (see Equation 1) for each link of the network. The
 792 horizontal component of the vector \mathbf{fv} depends on the ratio between planned and unplanned adap-

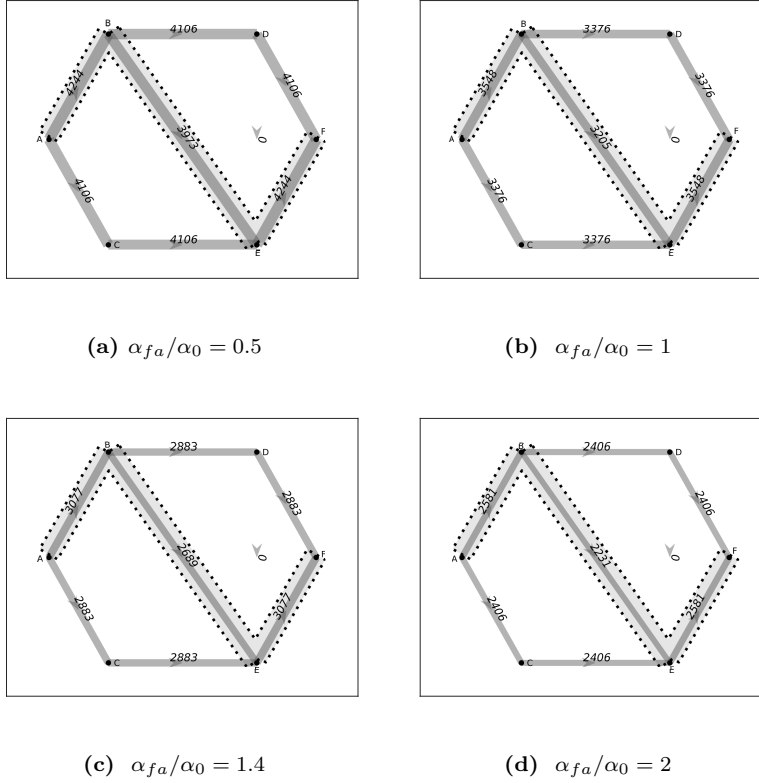


Fig. 13. Initial flexibility for different ratios α_{fa}/α_0

793 tations, while the vertical component depends on the ratio between the flexibility $\mathbf{f}_{0,e}$ and the initial
 794 design/operation conditions $\mathbf{x}_{0,e}$. It is first observed the increment in optimal costs by increasing
 795 the ratio α_{fa}/α_0 ; expectedly, increasing the flexible adaptation costs increases the overall costs. As
 796 shown in Figures 12 and 13, the model reacts by the decreasing the flexibility $\mathbf{f}_{0,e}$ and increasing
 797 the initial capacity $\mathbf{x}_{0,e}$; nonetheless, the model cannot completely offset the increments in costs
 798 because flexibility is still a cost-efficient strategy. This behavior is replicated in the flexibility vec-
 799 tors, whose vertical component decreases as α_{fa}/α_0 increases (the horizontal component decreases
 800 by definition).

801 It is interesting to observe that the decrement is not uniform for all links. For $\alpha_{fa}/\alpha_0 = 0.5$,
 802 the vectors are divided into two groups (vector for link DE has a zero vertical component): i)
 803 links AB, EF , which are part of the critical route, and ii) the remaining. Links AB, EF are key
 804 for the entrance and exit of the demand to the network, and the model privileges initial capacity,

805 which results in \mathbf{fv} with a lower vertical component. Because the demand is free to flow through all
 806 the remaining links, the program can assign more flexibility. As α_{fa}/α_0 increase to 1 and 1.4, the
 807 vertical component of the \mathbf{fv} keeps decreasing non-uniformly, resulting now in three distinct groups:
 808 i) key input and output links of the critical route AB, EF , with the lowest vertical component; ii)
 809 internal transit links BD, BE, EF whose vertical component keeps decreasing as flexibility becomes
 810 less attractive and it is replaced by adding more initial capacity; and iii) the less used links AC, AE ,
 811 which start transporting flow later on the network life-cycle and can keep relying on flexibility.
 812 This tendency is clearly observed when $\alpha_{fa}/\alpha_0 = 2$ and the vertical component of \mathbf{fv} decreases
 813 considerably for all links except AC, AE .

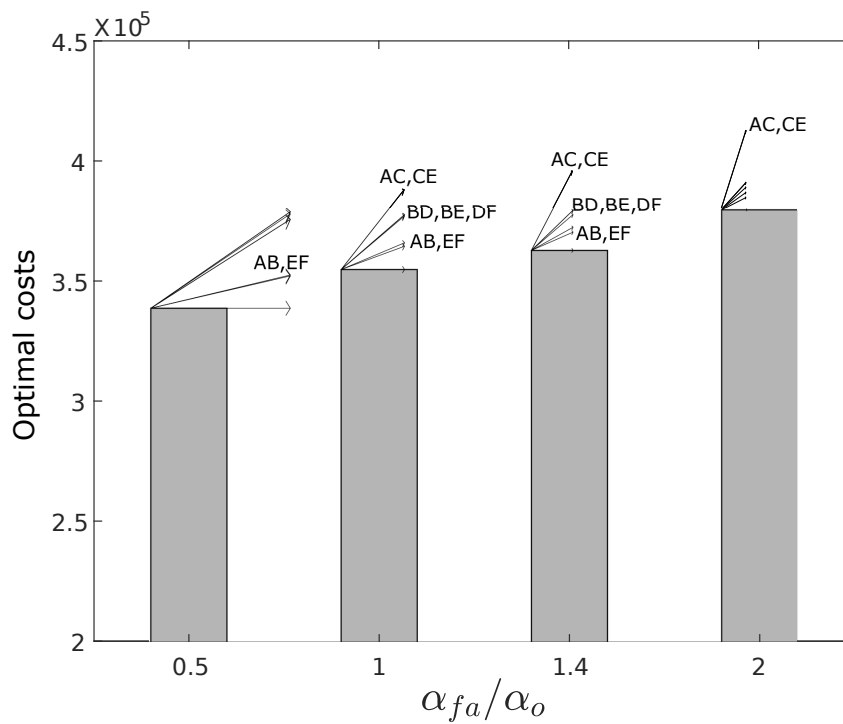


Fig. 14. Optimal costs and flexibility vectors \mathbf{fv} for different ratios α_{fa}/α_0

814 *7.3. Effect of the discount rate*

815 A similar analysis was conducted to examine the effect of the discount rate on the optimal
816 values $x_{0,e}^*$ and $f_{0,e}^*$. Figure 15 compares the capacity and flexibility of link AB (other links follow
817 a similar tendency) for three discount rates: $dr = \{3\%, 5\%, 8\%\}$ as a function of α_{fa}/α_0 . While the
818 values of flexibility are mostly unaffected by the change in the discount rate dr , the initial capacity
819 presents a clear decreasing tendency with an increment in the rate. In this case, larger discount
820 rates make future adaptations (inside or beyond the flexible range) more attractive in the medium
821 and long-term.

822 The decision of adding flexibility depends both on the cost of adding flexibility, which is incurred
823 at the initial stage and it is not affected by the discount rate, and the costs of performing adap-
824 tations, which are discounted. Increasing the discount rate reduces the present value of all future
825 cost making both planned and unplanned adaptations more attractive, or at least moves forward
826 the time stage where they are the optimal option. As a result, the short and medium-term demand
827 is served using planned adaptations (flexibility) and long-term demand is served with unplanned
828 adaptations, in detriment of the initial capacity. Therefore, it is expected that for typical ranges of
829 the discount rate ($\sim 3 - 10\%$) flexibility will be considered valuable. However, it could be argued
830 that for unrealistically large discount rates the model would rely entirely on unplanned adaptations,
831 except for the initial demand, abandoning completely the option of flexibility.

832 *7.4. Costs of adaptation beyond the flexible range*

833 In this analysis, the effect of the cost of unplanned adaptations α_{ia} is determined by using
834 different ratios α_{fa}/α_{ia} and α_0/α_{ia} to solve the MSP. Figure 16 shows a similar tendency as Figure
835 15, with the optimal initial capacity and flexibility decreasing as the cost of unplanned adaptations
836 α_{ia} decreases, and the initial capacity being affected by the increasing discount rate, which further
837 bolsters the cost-effectiveness of unplanned adaptations. However, in this case, a reversing tendency
838 is detected for the 8% discount rate. Upon closer inspection, it is revealed that there is a point
839 where the short-term demand must be met before unplanned adaptation can take place. At this
840 point it does not matter how much further α_{ia} can be reduced; there is a minimum demand that
841 must be met.

842 For lower discount rates it was cost-effective to use more flexibility to meet this short-term
843 demand; however, for large discount rates, the initial cost of flexibility plus the adaptation cost

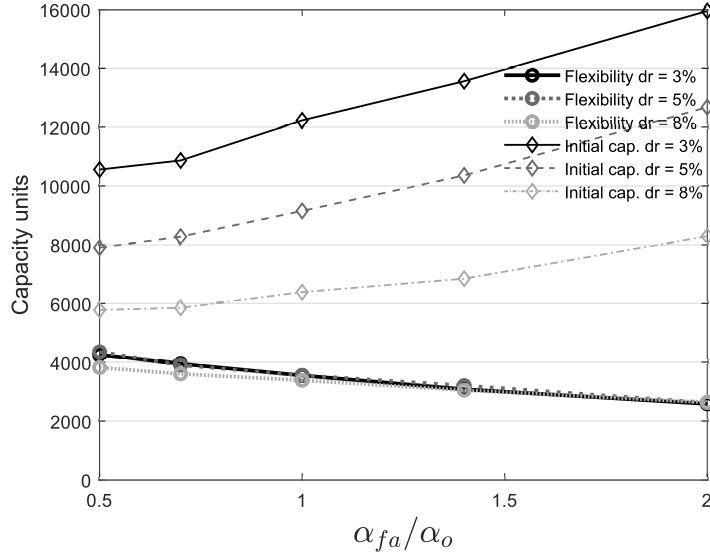


Fig. 15. Installed capacity and flexibility in link AB as function of ratio α_{fa}/α_o and discount rate

844 is not as cost-effective as just relying on the initial capacity for the short-term and unplanned
 845 adaptations for the medium and long-term. This could explain the monotonic decreasing tendency
 846 for flexibility while the initial capacity behaves asymptotically. This means that reducing the cost
 847 of unplanned adaptations can only reduce the dependence on initial capacity up to the point where
 848 there is enough capacity to attend the short-term demand that could not be served in any other
 849 way. This point is dictated by the initial demand and its growth during the first decision period.

850 A final analysis was conducted to examine the effect of the unitary O&M costs on the optimal
 851 initial capacity and flexibility. By increasing the unitary O&M costs from 2% of α_o to 10% of α_o ,
 852 the optimal costs increase approximately 20%. The optimal initial capacity in all links is reduced by
 853 approximately 12% which is compensated by a similar increment in the optimal installed flexibility.
 854 If the O&M costs are further increased to 16% of α_o , the optimal initial capacity decreases another
 855 9% with a similar increment in the optimal flexibility. Increasing the O&M costs discourage large
 856 system configurations in the short and medium-term, which makes flexibility more valuable.

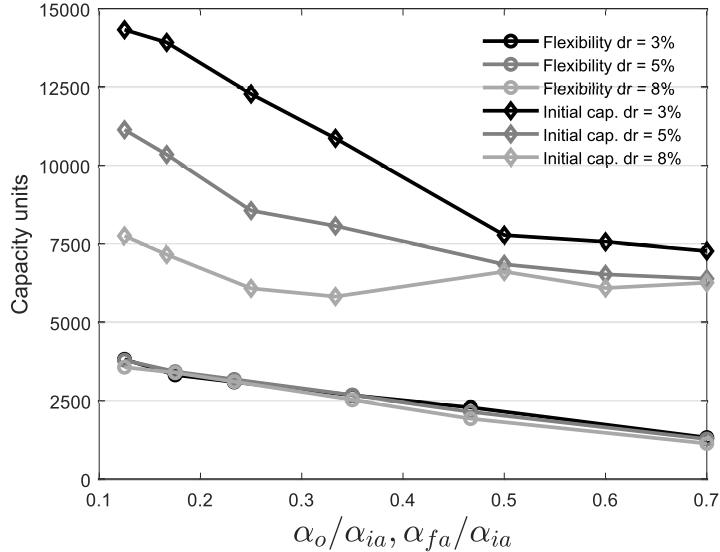


Fig. 16. Installed capacity and flexibility in link AB as function of ratios α_{fa}/α_{ia} , α_o/α_{ia} and discount rate

857 **8. Summary and Conclusions**

858 This paper presented a multistage stochastic programming model for the design and management
 859 of flexible infrastructure networks. The model considers the initial design characteristics of each
 860 network link for different design elements as an initial decision that can be modified in future
 861 stages by implementing planned and unplanned adaptations to face random external conditions.
 862 By modeling the flexibility built at the initial stage as a decision variable that has an associate cost
 863 and that limits future planned adaptations, the program can determine the optimal configuration
 864 that minimizes costs. This formulation also provides an indirect measure of the value of flexibility
 865 by finding the optimal *amount* of flexibility that is cost-effective to introduce.

866 An approximate solution method was implemented using Monte-Carlo simulation and k-medoids
 867 clustering to generate the discrete representation of the stochastic process (scenario tree). The de-
 868 terministic equivalent program was formulated and tested for a generic network with one input and
 869 one output node for a stochastic demand. The numerical solution showed stability for reasonably
 870 sized scenario trees and a not negligible VSS of around 16%. Furthermore, the results showed how
 871 the value of flexibility is affected by the discount rate and the ratios between all the costs involved,
 872 not just the cost of deploying planned adaptations. If the ratios α_{fa}/α_0 , and α_{ia}/α_0 are too large,

873 the system will prefer to meet the short and medium-term demand with the initial capacity, lowering
874 the value of flexibility. As these ratios increase, the adaptations are further pushed into the future,
875 especially for low discount rates. In the case where the ratio α_{fa}/α_{ia} is large, unplanned adapta-
876 tions become cost-effective, also decreasing the value of flexibility. In contrast, for a given α_{fa} , as
877 the ratio α_{ia}/α_0 grows, flexibility becomes more desirable. These examples show how the value of
878 flexibility constantly evolves due to complex decisions that happen at different time instants.

879 The proposed model and the procedure implemented in the numerical example assumed that
880 all decision and state variables are continuous. A mixed-integer formulation may require specific
881 algorithms (e.g. primal decomposition) to find a solution efficiently. Furthermore, all cost functions
882 were assumed convex, which is a reasonable assumption for most applications, but if concave func-
883 tions are required, the solution procedure may not efficient nor may guarantee to find a solution.
884 The scenario generation procedure can be extended to the multivariate case (for independent vari-
885 ables) but the combinatorial nature of the problem restricts the number of decision stages than can
886 be modeled. The formulation and solution of the MSP is also restricted by the exponential scala-
887 bility. The expectation functional used in the model (Equation 4) enforces a risk-neutral approach
888 that may not be appropriate for some applications. Further works should consider a formulation of
889 the model with risk-averse functionals.

890 9. Acknowledgements

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892 the Regional Council of ‘Pays de la Loire’ within the framework of the BUENO 2018-2021 research
893 program (Durable Concrete for Offshore Wind Turbines) is gratefully acknowledged.

894 List of symbols and abbreviations

895 a_e Cost function of building edge e with intial parameters $\mathbf{x}_{0,e}$

896 α_0 Unitary cost of building initial design $\mathbf{x}_{0,e}$

897 α_{fa} Unitary cost of performing flexible adaptation $\mathbf{y}_{t,e}$

898 α_{ia} Unitary cost of performing unplanned adaptation $\mathbf{z}_{t,e}$

899 b_e Cost function of adding $\mathbf{f}_{0,e}$ flexibility to edge e

900 c_e Cost function of changing edge e by $\mathbf{y}_{t,e}$

901 **CV** Coefficient of variation
902 d_e Cost function of changing edge e by $\mathbf{z}_{t,e}$
903 γ_t Discount factor at time t
904 G Directed graph
905 dr Discount rate
906 e Edge/link in directed graph G
907 $\mathbf{f}_{0,e}$ Flexibility range of design/operation parameters for edge e
908 $w_{t,e}$ Flow through edge e at time t
909 \mathbf{fv} Flexibility vector
910 g_e Operation and maintenance cost function for edge e
911 $h(\boldsymbol{\xi}_t)$ Revenue function
912 **MDP** Markov decision process
913 **MS** multistage stochastic
914 **MSP** multistage stochastic program
915 **O&M** Operation and Maintenance
916 p_k Probability of scenario k
917 π Policy function
918 q_e Flow cost function for edge e
919 $\boldsymbol{\xi}_t$ Vector of random, external parameters at time t
920 **ROA** real options analysis
921 **VSS** Value of stochastic solution
922 $\mathbf{x}_{t,e}$ Vector of design/operation parameters at time t for edge e
923 $\mathbf{x}_{0,e}$ Vector of initial design/operation parameters for edge e
924 $\mathbf{y}_{t,e}$ Vector of flexible adaptations at time t for edge e
925 $\mathbf{z}_{t,e}$ Vector of unplanned adaptations at time t for edge e

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