



SPIE Photonics Europe 2020 - Digital Forum

Paper 11351-32

Building an inverse approach for the reconstruction of in-line holograms: a parallel with Fienup's phase retrieval technique

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Thursday 9 April 2020

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2.2 - The phase retrieval problem

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5 - Let's cross the 'IP' line

6 - Discussion & conclusion

Preamble

This presentation highlights a tutorial recently published in JOSA A :

F. Momey, L. Denis, T. Olivier, and C. Fournier,
"From Fienup's phase retrieval techniques to regularized inversion for in-line holography : tutorial,"
J. Opt. Soc. Am. A 36,
D62-D80 (2019).

[Link OSA](#)

[Link HAL](#)

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2.1 - Physical principle

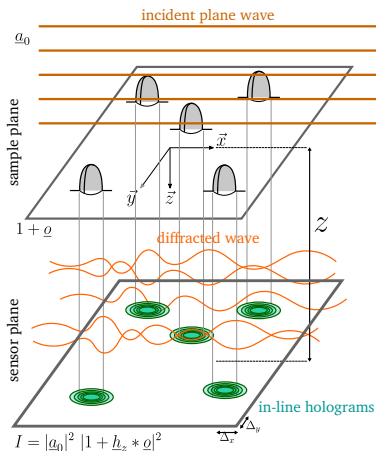
- ▶ The "object" :

$$t(\mathbf{r}) = 1 + \underline{o}(\mathbf{r}) = \rho(\mathbf{r})e^{i\varphi(\mathbf{r})}$$

- ▶ Hologram formation :

$$I(\mathbf{r}) = |a_0|^2 |1 + \underline{h}_z * \underline{o}|^2$$

- ▶ \underline{h}_z is a propagation kernel (convolutive) :
Rayleigh-Sommerfeld, Fresnel, angular spectrum.



In-line DH principle.

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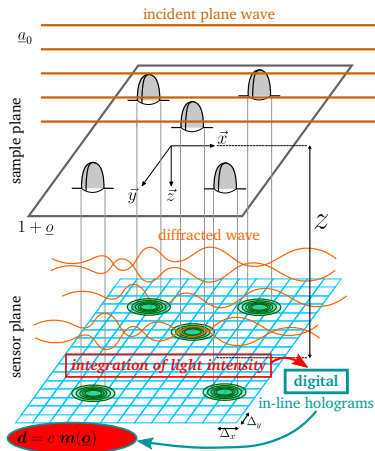
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- "Algebraic" in-line hologram (**data**) formation model :

$$d = c m(\underline{o}) = c |\mathbf{1} + \underline{\mathbf{H}}_z \underline{o}|^2$$

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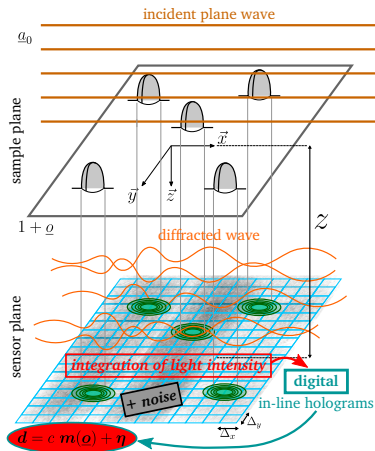
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- ▶ "Algebraic" in-line hologram (**data**) formation model :

$$d = c m(\mathbf{o}) + \eta = c |\mathbf{1} + \underline{\mathbf{H}}_z \mathbf{o}|^2 + \eta$$

- ▶ η = electronic noise (hardware : sensor), modeling approximations (physical & mathematical : $m(\mathbf{o})$).

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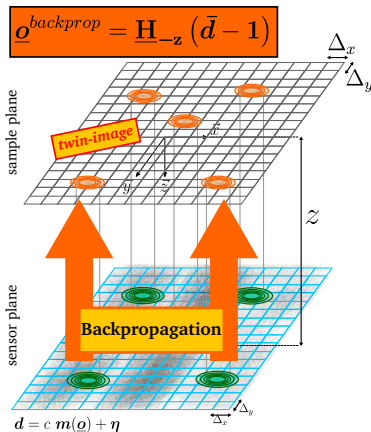
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- ▶ Simple backpropagation of the normalized data image \bar{d} :

$$\underline{o}^{backprop} = \underline{H}_{-z}(\bar{d} - 1)$$

- ▶ under hypothesis : $\underline{H}_z \underline{H}_{-z} \approx \mathbf{I}$.

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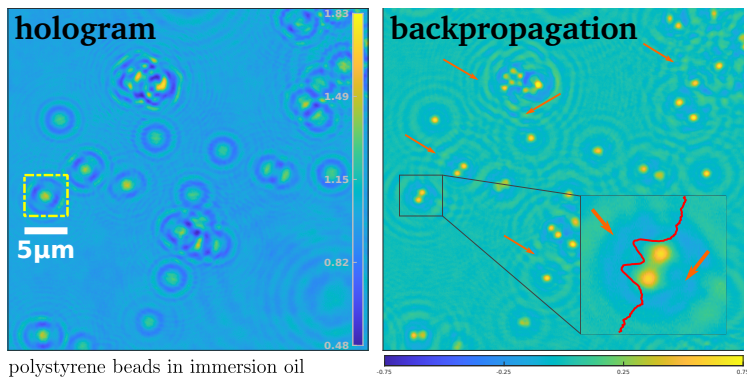
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polystyrene beads in immersion oil

- illumination wavelength: $\lambda = 532\text{nm}$
- refractive index of the medium: $n_0 = 1.52$
- pixel size: $\Delta_x = \Delta_y = 2.2\mu\text{m}$

- propagation distance: $z = 12.5\mu\text{m}$
- refractive index of the beads: $n_0 = 1.59$
- magnification factor: $\text{mag} = 56.7$

- ▶ The "sadly famous" *twin-image* problem induced by the loss of phase information on the sensor.
- ▶ "Rise" of advanced *phase retrieval* reconstruction techniques.

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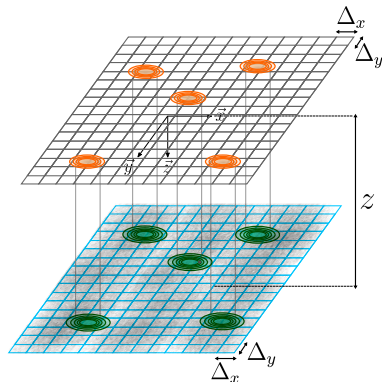
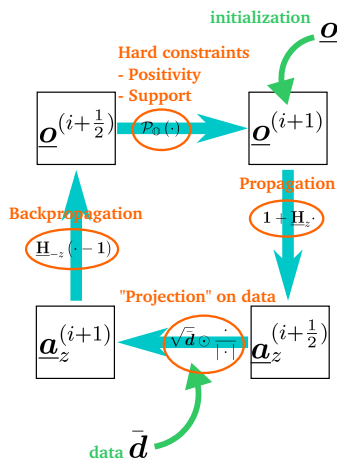
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- ▶ Alternated projections approaches :
 - initiated by **Gerchberg-Saxton (1972)**, **Fienup (1978, 1982)**.
 - popularized in the 2000s by **Latychevskaïa (2007)**.



Fienup's Error Reduction algorithm.

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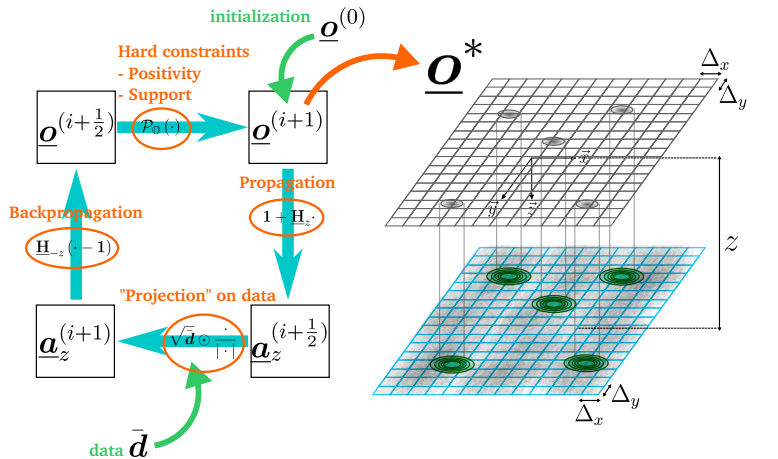
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- ▶ Alternated projections approaches :
 - iterative procedure involving a projection step on a feasible domain $\underline{o} \in \mathcal{O}$ (e.g. positive values of the phase-shift / support constraint) to reduce/suppress twin-image artifacts.



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▶ Advantages :

- ▶ **Intuitive** : propagations/backpropagations ; constraints enforcement.
- ▶ **Ease of implementation.**
- ▶ **Convergence** ensured to a local minimum.

▶ Drawbacks :

- ▶ Mandatory data **preprocessing** (normalization, background subtraction).
- ▶ **Full confidence to the data** : uncertainties due to noise are not taken into account.
- ▶ **Not flexible** in terms of :
 - ▶ modeling :
 - strong hypothesis $\underline{\mathbf{H}}_z \underline{\mathbf{H}}_{-z} \approx \mathbf{I}$ (wrong because of field truncation).
 - difficult to take other **instrumental effects** into account (calibration, coherence, aberrations, etc.).
 - ▶ prior knowledge : hard constraints only (not necessarily applicable, e.g. support constraints).

▶ Proposition :

- ▶ switch to the **Inverse Problems (IP)** framework.
- ▶ *Alternating projections methods are in fact part of it ...*

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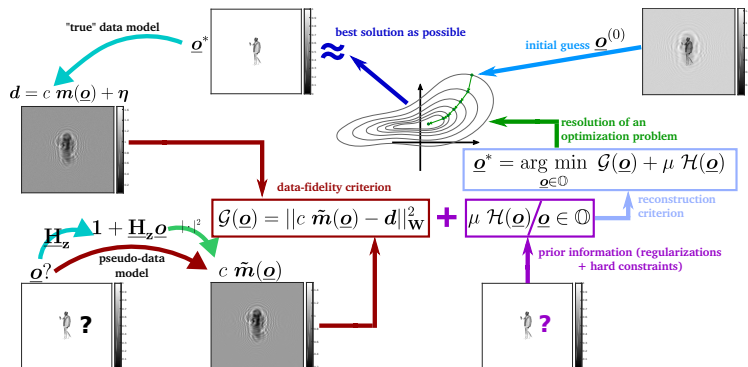
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- Let's have an overview of the "concept".

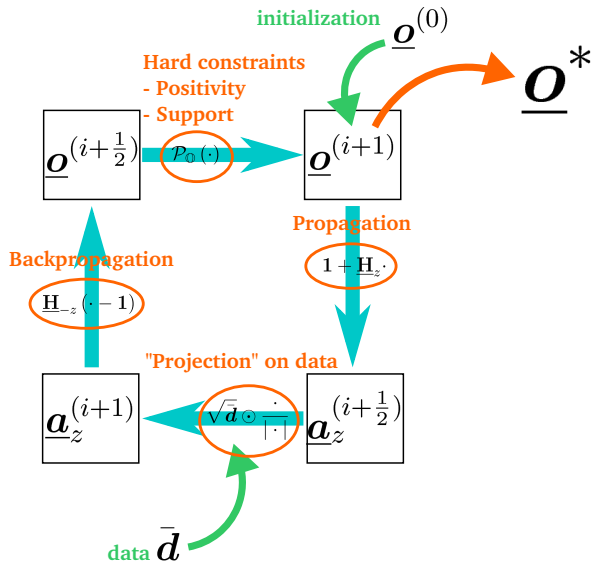


This strategy departs from the classical phase retrieval principle, as it aims to reconstruct the targeted information \underline{o} from the intensity measurements, instead of trying to retrieve the missing phase information on the sensor plane.

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- ▶ Let's "equationize" Fienup's ER algorithm :



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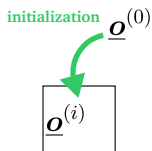
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► Let's "equationize" Fienup's ER algorithm :

→ initialize with first guess $\underline{\mathbf{o}}^{(i=0)}$.



$\underline{\mathbf{o}}^{(i)}$

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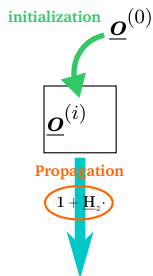
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- ▶ Let's "equationize" Fienup's ER algorithm :
- forward propagation of $\underline{o}^{(i)}$...



$$1 + \underline{H}_z \underline{o}^{(i)}$$

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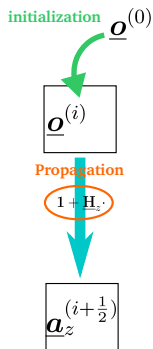
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► Let's "equationize" Fienup's ER algorithm :

→ ... to get a "pseudo-hologram" $\underline{a}_z^{(i+1/2)}$.



$$\underline{a}_z^{(i+1/2)}$$

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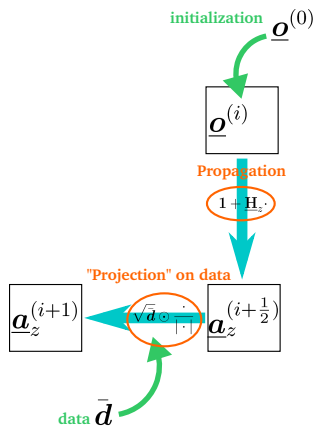
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► Let's "equationize" Fienup's ER algorithm :

→ replace amplitude of $\underline{a}_z^{(i+1/2)}$ with the recorded one, to get $\underline{a}_z^{(i+1)}$.



$$\left(\sqrt{\bar{d}} \odot \frac{\underline{a}_z^{(i+1/2)}}{|\underline{a}_z^{(i+1/2)}|} \right)$$

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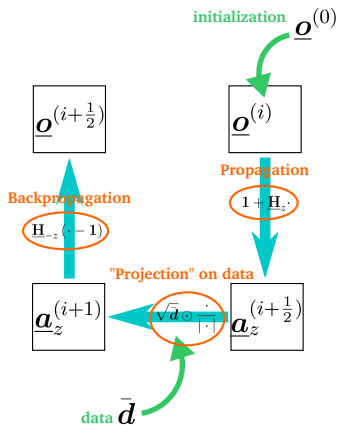
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► Let's "equationize" Fienup's ER algorithm :

→ backpropagation of $\underline{a}_z^{(i+1)}$ to get a new estimate $\underline{o}^{(i+1/2)}$.



$$\underline{H}_{-z} \left(\sqrt{\bar{d}} \odot \frac{\underline{a}_z^{(i+1/2)}}{|\underline{a}_z^{(i+1/2)}|} - \mathbf{1} \right)$$

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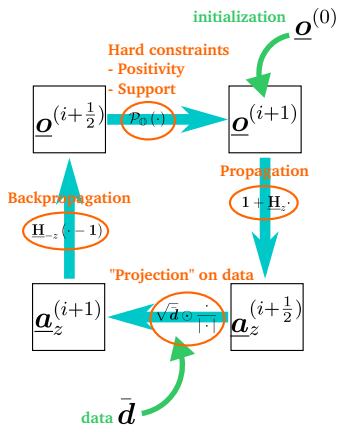
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► Let's "equationize" Fienup's ER algorithm :

→ project $\underline{o}^{(i+1/2)}$ on constraints to get the new estimate $\underline{o}^{(i+1)}$.



$$\underline{o}^{(i+1)} \leftarrow \mathcal{P}_{\mathcal{O}} \left(\underline{H}_{-z} \left(\sqrt{\bar{d}} \odot \frac{\underline{a}_z^{(i+1/2)}}{|\underline{a}_z^{(i+1/2)}|} - \mathbf{1} \right) \right)$$

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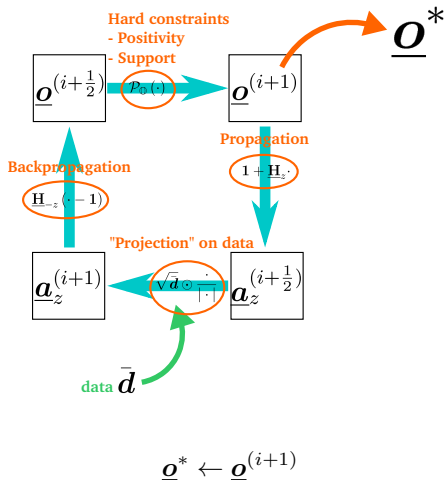
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► Let's "equationize" Fienup's ER algorithm :

→ iterate until reaching a fixed maximum of iterations or a convergence criterion, to get the solution \underline{o}^* .



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- ▶ Let's "equationize" Fienup's ER algorithm :

$$\underline{\mathbf{o}}^{(i+1)} \leftarrow \mathcal{P}_0 \left(\underline{\mathbf{H}}_{-z} \left(\sqrt{\underline{\mathbf{d}}} \odot \frac{\underline{\mathbf{a}}_z^{(i+1/2)}}{|\underline{\mathbf{a}}_z^{(i+1/2)}|} - \mathbf{1} \right) \right)$$

- ▶ Some mathematical rearrangements + 1 hypothesis ($\underline{\mathbf{H}}_{-z} \underline{\mathbf{H}}_z = \underline{\mathbf{H}}_z^\dagger \underline{\mathbf{H}}_z \approx \mathbf{I}$) :

$$\underline{\mathbf{o}}^{(i+1)} \approx \mathcal{P}_0 \left[\underline{\mathbf{o}}^{(i)} - \underline{\mathbf{H}}_z^\dagger \underbrace{\left(\frac{\underline{\mathbf{a}}_z^{(i+1/2)}}{|\underline{\mathbf{a}}_z^{(i+1/2)}|} \odot \left(|\underline{\mathbf{a}}_z^{(i+1/2)}| - \sqrt{\underline{\mathbf{d}}} \right) \right)}_{\nabla \mathcal{G}(\underline{\mathbf{o}}^{(i)})} \right]$$

where $\nabla \mathcal{G}(\underline{\mathbf{o}}^{(i)})$ corresponds to the gradient (derivative) ∇ of a data-fidelity criterion \mathcal{G} calculated for $\underline{\mathbf{o}}^{(i)}$.

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- ▶ One iteration gives :

$$\underline{\boldsymbol{o}}^{(i+1)} \approx \mathcal{P}_{\mathbb{O}} \underbrace{\left[\underline{\boldsymbol{o}}^{(i)} - \nabla \mathcal{G}(\underline{\boldsymbol{o}}^{(i)}) \right]}_{\underline{\boldsymbol{o}}^{(i+1/2)}}$$

which alternates between

- ▶ a gradient descent step of the criterion \mathcal{G} at current point $\underline{\boldsymbol{o}}^{(i)}$,
- ▶ a projection step $\mathcal{P}_{\mathbb{O}}$ on the domain \mathbb{O} , called proximal mapping of the indicator function $\iota_{\mathbb{O}}$ at point $\underline{\boldsymbol{o}}^{(i+1/2)}$:

$$\iota_{\mathbb{O}}(\underline{\boldsymbol{o}}) = \begin{cases} 0 & \text{if } \underline{\boldsymbol{o}} \in \mathbb{O} \\ +\infty & \text{otherwise} \end{cases}$$

This resolution method is an instance of the ISTA algorithm (Iterative Shrinkage Thresholding Algorithm).

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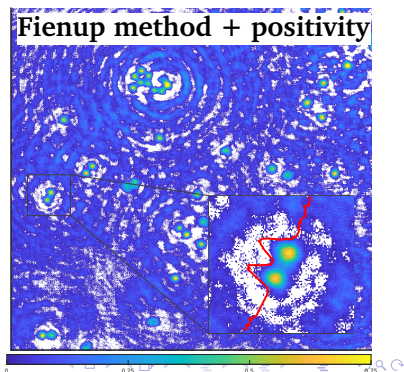
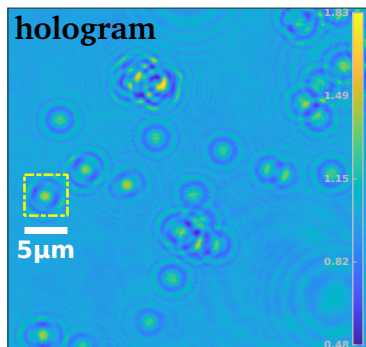
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- Fienup's ER algorithm is an **Inverse Problems** approach that uses a particular optimization strategy for minimizing the following criterion :

$$\underline{o}^* = \arg \min_{\underline{o}} \underbrace{\left\| |\mathbf{1} + \mathbf{H}_z \underline{o}| - \sqrt{d} \right\|_2^2}_{\text{data-fidelity } \mathcal{G}(\underline{o}) \text{ (smooth)}} + \underbrace{\iota_0(\underline{o})}_{\mathcal{H}(\underline{o}) \text{ (non smooth)}}$$



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We do not pretend to have discovering it ! This has already been noted in previous works \Rightarrow see our JOSA A tutorial for more details.

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Alternating projections methods such Fienup ER can then benefit from this rigorous framework to gain flexibility and reconstruction quality.

A more appropriate direct image formation model $\tilde{m}(\underline{o})$

- ▶ modeling the "full" process of image formation (not necessarily invertible) :
 - ▶ inclusion of more physical and/or instrumental effects.

$$\tilde{m}(\underline{o}) = \mathcal{T} \left(\mathcal{C} \left(|\mathbf{1} + \underline{\mathbf{H}}_z \underline{o}|^2 \right) \right)$$

→ $\mathcal{C}(\cdot)$ could be (for example) an operator that take into account the effects of the **coherence** of the source in the model.

→ $\mathcal{T}(\cdot)$ can take into account field truncation.

- ▶ The unique requirement is that this model must be derivable (smooth) and that the numerical calculation of its derivative (its gradient) is possible.

⇒ **auto-calibration** of model parameters made possible.

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Alternating projections methods such Fienup ER can then benefit from this rigorous framework to gain flexibility and reconstruction quality.

Improve the data-fidelity term

- ▶ taking into account data uncertainties (statistical considerations about the noise in the criterion) :

$$\mathcal{G}(\underline{\mathbf{o}}) = \|c^*(\underline{\mathbf{o}}) \tilde{\mathbf{m}}(\underline{\mathbf{o}}) - \mathbf{d}\|_{\mathbf{W}}^2 = \sum_q w_q (\tilde{m}_q(\underline{\mathbf{o}}) - d_q)^2$$

→ e.g. **weighted (W) least-squares criterion** under assumption of Gaussian errors.

- ▶ no preprocessing of the data : optimal scaling $c^*(\underline{\mathbf{o}})$ (noted c^*) ⇒ see our JOSA A tutorial for more details.

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ISTA strategy : the "alternating projections" point of view

- ▶ The "new" ISTA iteration using the proposed data-fidelity :

$$\underline{\mathbf{o}}^{(i+1)} = \mathcal{P} \left(\underline{\mathbf{o}}^{(i)} - 2 t \underbrace{c^* \underline{\mathbf{H}}_z^\dagger \mathbf{W} (c^* \tilde{\mathbf{m}}(\underline{\mathbf{o}}^{(i)}) - \mathbf{d})}_{\nabla \mathcal{G}(\underline{\mathbf{o}}^{(i)})} \right)$$

→ with $\tilde{\mathbf{m}}(\underline{\mathbf{o}}) = |\mathbf{1} + \underline{\mathbf{H}}_z \underline{\mathbf{o}}|^2$.

→ $\underline{\mathbf{H}}_z^\dagger = \underline{\mathbf{H}}_{-z}$, the hermitian transpose of $\underline{\mathbf{H}}_z$ (backpropagator), which appears to be the "Jacobian operator" of $\underline{\mathbf{H}}_z$ involved in the gradient $\nabla \mathcal{G}$.

→ the **backpropagation step** constitutes the **gradient descent step**.

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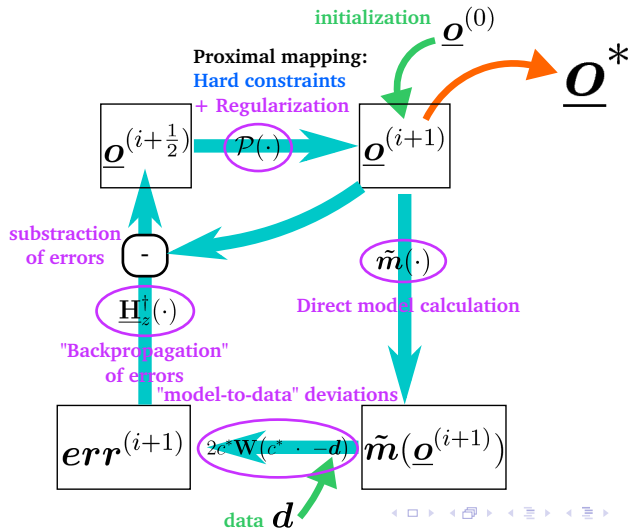
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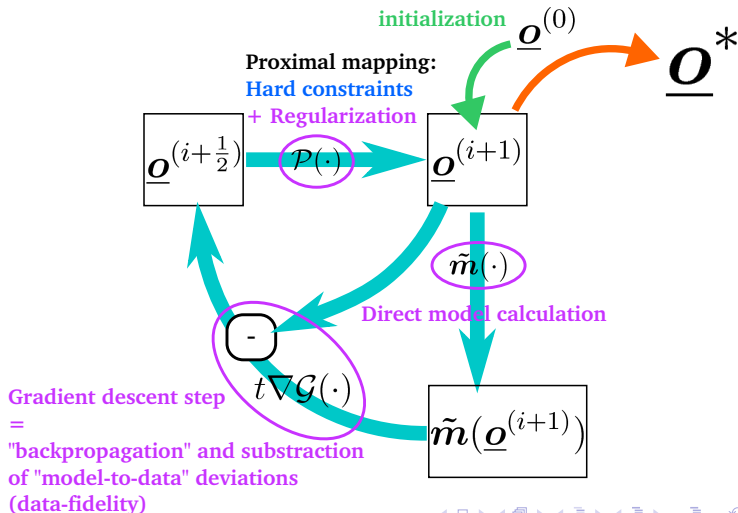
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Prior knowledge enforcement strategies
(regularization + constraints)

- ▶ Another very popular instance of **ISTA** [Beck & Teboulle, 2009] :

$$\underline{\mathbf{o}}^{(i+1)} = \mathcal{T}_{\mu t} \left(\underline{\mathbf{o}}^{(i)} - t \nabla \mathcal{G}(\underline{\mathbf{o}}^{(i)}) \right)$$

$\mathcal{T}_{\alpha}(\mathbf{o})_q = \max(0, o_q - \alpha)$ is the soft-thresholding operator = proximal mapping of the l_1 regularization (sparsity constraint).

- ▶ Equivalent criterion to minimize :

$$\underline{\mathbf{o}}^* = \arg \min_{\underline{\mathbf{o}} \geq \mathbf{0}} \underbrace{\mathcal{G}(\underline{\mathbf{o}})}_{\text{(smooth)}} + \underbrace{\mu \|\underline{\mathbf{o}}\|_1}_{\mathcal{H}(\underline{\mathbf{o}}) \text{ (non smooth)}}$$

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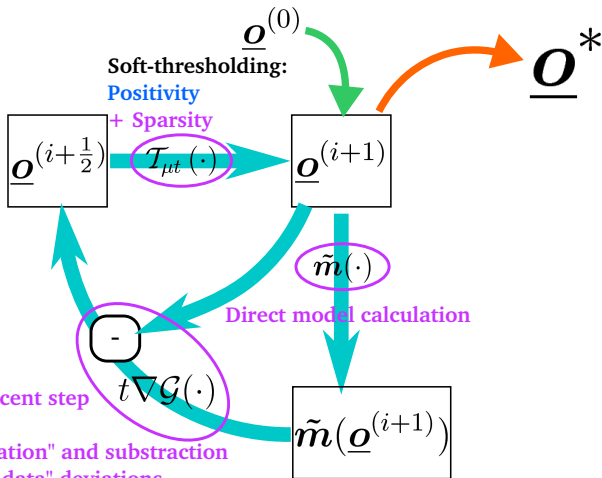
4 - Towards inverse approaches

5 - Let's cross the "IP" line

6 - Discussion & conclusion

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Prior knowledge enforcement strategies
(regularization + constraints)



Gradient descent step
=
"backpropagation" and subtraction
of "model-to-data" deviations
(data-fidelity)

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Prior knowledge enforcement strategies
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- ▶ Inclusion of a new **regularizer**, e.g. **edge-preserving** :

$$\underline{\mathbf{o}}^* = \arg \min_{\underline{\mathbf{o}} \geq \mathbf{0}} \underbrace{\|c^* \tilde{\mathbf{m}}(\underline{\mathbf{o}}) - \mathbf{d}\|_{\mathbf{W}}^2 + \mu_{\text{TV}} \sum_q \sqrt{\|\nabla_q \underline{\mathbf{o}}\|_2^2 + \epsilon^2}}_{\mathcal{G}(\underline{\mathbf{o}}) \text{ (smooth)}} + \underbrace{\mu_{l_1} \|\underline{\mathbf{o}}\|_1}_{\mathcal{H}(\underline{\mathbf{o}}) \text{ (non smooth)}} .$$

→ $\nabla_q \underline{\mathbf{o}}$ corresponds to the finite difference operator of $\underline{\mathbf{o}}$ at pixel q .

- ▶ Resolution with ISTA :

$$\underline{\mathbf{o}}^{(i+1)} = \mathcal{T}_{\mu_{l_1}} t \left(\underline{\mathbf{o}}^{(i)} - 2 t c^* \underline{\mathbf{H}}_z^\dagger \mathbf{W} (c^* \tilde{\mathbf{m}}(\underline{\mathbf{o}}^{(i)}) - \mathbf{d}) - t \mu_{\text{TV}} \sum_q \nabla_q^T \left(\frac{\nabla_q \underline{\mathbf{o}}^{(i)}}{\sqrt{\|\nabla_q \underline{\mathbf{o}}^{(i)}\|_2^2 + \epsilon^2}} \right) \right) .$$

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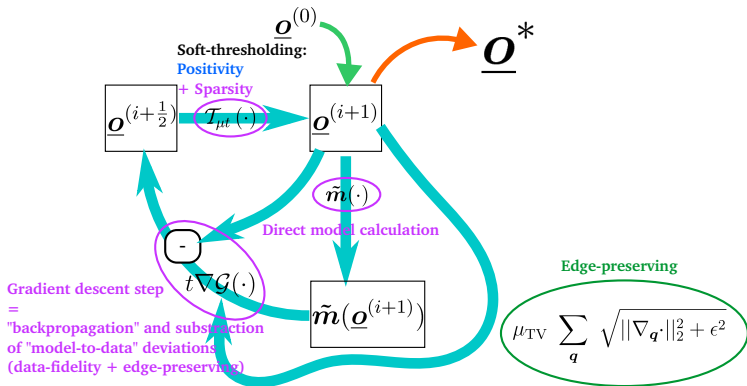
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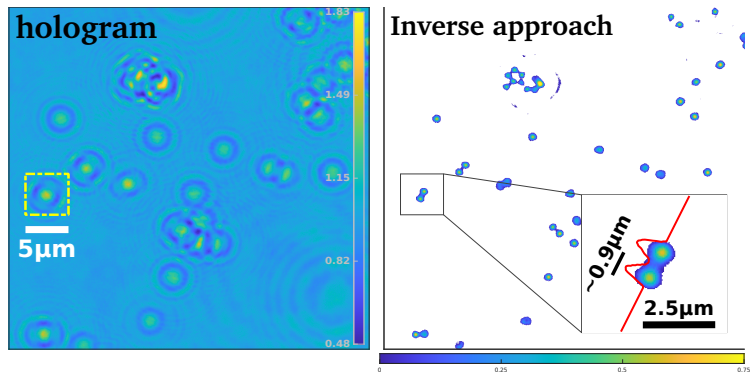


. A **Matlab™** code implementing this algorithm will be available soon for download at this link : https://github.com/fabienmomey/Inline_Hologram_Reconstruction_by_Regularized_Inversion.git.

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Prior knowledge enforcement strategies
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- ▶ The **sparsity constraint** limits the size of the objects support by favoring reconstructions with many pixels at zero.
- ▶ The **edge-preserving regularizer** enforces some kind of "piecewise smoothness" of the image.

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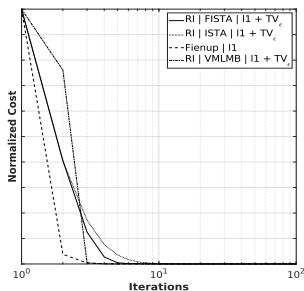
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Optimization method

- ▶ **ISTA** constitutes a particular **optimization method**, with equivalent computational burden than Fienup ER.
- ▶ Others can be used :
 - ▶ "Accelerated" ISTA : **FISTA** [Beck & Teboulle, 2009].
 - ▶ For smooth criterions : **VMLMB** (Variable Metric Limited Memory with Bound constraints).
 - ▶ For more complex criterions : **ADMM** [Jolivet *et al.*, 2018].



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For details, let's have a look at our JOSA A tutorial!

- ▶ **Inverse problems** constitutes a rigorous framework for solving image reconstruction and/or parameter estimation tasks :
 - ▶ Modeling flexibility : only requires a direct model and the calculation of its gradient.
 - ▶ Using more relevant data-fidelity terms taking into account statistical aspects on data and model uncertainties.
 - ▶ More choice in prior knowledge enforcement : hard constraints, regularizations. *At the cost of more tuning parameters ...*
 - ▶ More appropriate optimization methods.
- ▶ **Alternating projections strategies such the Fienup method falls into this framework!**

A **MatlabTM** code implementing the last example - **FISTA + weighted least-squares + sparsity + edge-preserving** - will be available soon for download at this link :

https://github.com/fabienmomey/Inline_Hologram_Reconstruction_by_Regularized_Inversion.git

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