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# The effect of greenness- and price-based competition on a product's environmental 

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#### Abstract

This study considers two retailers that offer substitutable products and compete in a greennessand price-sensitive market. The retailers rely on supply chains that have different costs, replenishment times and carbon intensities. The demand is stochastic, and the inventory replenishment time is also stochastic. Each retailer decides the greenness of its product (carbon intensity), the price and the order size to maximize its expected profit under a service level constraint. The mean demand for each retailer decreases in its price and carbon intensity and increases in other retailer's price and carbon intensity. The customers' switchovers are, thus, governed by disparities in carbon intensity and price. The retailers may have different market powers since the total demand is not necessarily equally shared between them when they offer the same price and greenness level. We derive analytically the best response of each retailer to other retailer's decisions at the Nash equilibrium. We determine the market conditions that lead the competing retailer to offering a greener or a dirtier product than the product it would have offered in a monopolistic situation. We explain how the retailer's market power impacts on its greenness strategy under different market conditions. We also investigate insights into the impact of disparities in transportation carbon emissions, the order size decisions, and the retailers' optimal profit under the different competition scenarios.


Keywords: Supply chain management; Greenness- and price-based competition; Environmental performance; Nash equilibrium; Retail management.

## The effect of greenness- and price-based competition on a product's environmental performance

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## 1. Introduction

The environmental performance of a product, often assessed by the amount of carbon emissions released during the production and transportation phases, is becoming an important purchase criterion for many customers (Palacios-Argüello et al. 2020, Hammami et al. 2018, Borin et al. 2013). According to a survey of 6,000 consumers in 11 countries across North America, Europe and Asia, conducted by

Accenture in April 2019, 72\% of consumers said they are currently buying more environmentally-friendly products than they were five years ago, and $81 \%$ said they expect to buy more over the next five years. Moreover, more than half of consumers would pay more for environmentally-friendly products (Accenture, 2019). Many firms have developed a thorough understanding of customers' preferences to environmentally-friendly (green) products and their willingness to pay and, consequently, have adapted their strategies to offer greener alternatives to their customers. There are different business modes of green product development in practical situations. According to many authors (Hong et al. 2019, Agi and Yan, 2020) product grenning may be led by the manufacturer or the retailer. Examples of manufacturers that initiate green product development include PepsiCo and Coca-Cola (Hong et al. 2019). It is also a common practice among retailers to pressure suppliers and push them to greening their products (Alibeiki et al. 2020, Hong et al. 2019, Agi and Yan, 2020). This is particularly relevant for supply chains involving powerful retailers and relatively small suppliers. Examples of retailer-led green product development involve some retail giants, like Carrefour and Walmart (Hong et al. 2019). For instance, in the case of Walmart and for many product categories, it tells the manufacturers which products to develop (Alibeiki et al. 2020).

This study considers the case of retailers that lead the products' greening and pricing decisions and compete in a greenness- and price-sensitive market. There are different retailer-led greening approaches in practice. Some retailers asked their suppliers to reduce carbon emissions in production and transportation phases to offer greener products (Ramanathan et al. 2012). For instance, Walmart has set in October 2016 an emissions-reduction plan and invited the suppliers to reduce greenhouse gas emissions resulting from their operations and supply chains (SCs) (Walmart, 2017). Bestseller, one of the leading fashion retailers in Denmark, has committed to continuously improve the environmental footprint of its products and gives its customers an opportunity to buy a wide range of environmentally-friendly products (Bestseller, 2018). To our knowledge, the above retailers, however, are not engaged in a green labeling effort. The use of green labels is not only a regulation implemented by the government but is also a
chance for retailers to attract consumers and be successful in green-market competitions (Hong et al. 2019). Green labeling allows cunstomers to compare products with respect to their environmental performances and, thus, to transform their environmental awareness into purchase decisions. Benjaafar et al. (2013) explained that various retailers are starting to attach carbon footprint labels to their products and position these products as greener alternatives. Two leading retailers, Tesco in the UK and Casino in France, have already embarked on aggressive labeling efforts (Benjaafar et al. 2013, Hammami et al. 2018). Different products were the object of carbon footprinting such as fruit smoothies, shoes, and beer (Hammami et al. 2018, Benjaafar et al. 2013).

The main objective of this study is to investigate how the greenness- and price-based competition between retailers impact a product's environmental performance. We consider two retailers that offer two substitutable products and compete in a greenness- and price-sensitive market. Greenness is measured by the carbon intensity which refers here to the amount of carbon emissions during production and transportation activities per unit of product. Each retailer has its own supplier and, thus, relies on a different SC that has specific costs, replenishment time and transportation carbon emissions. Demand is random and the replenishment time of each retailer's stock is also random. The mean demand of each retailer's product decreases in its carbon intensity and price, and increases in other product's carbon intensity and price. The customers' switchovers (from a given retailer's product to the other retailer's product) are, thus, governed by disparities in carbon intensity and price. Moreover, the retailers may have different market powers since we model the demand function in a way that the total demand is not necessarily equally shared between the two retailers when they offer the same price and carbon intensity. The carbon emissions in transportation depend mainly on the SC structure (locations of suppliers and retailers), so are considered as given input parameters. The carbon emissions in production can be reduced, but this implies a higher purchasing cost for the retailer as it leads to an increase in the supplier's cost (Conrad, 2005).

In the general case (full reaction), each retailer decides the price of its product, the carbon intensity, and the order size (stock level) to maximize its expected profit under a service level constraint while considering other retailer's decisions. We derive analytically the best greenness, pricing and inventory decisions of each retailer to other retailer's decisions at the Nash equilibrium. We also consider other competition scenarios that fit with practical situations in which there are an established retailer in the market and a new retailer entering the market. We study the cases where the established retailer does not react to new retailer's decisions or react partially by adjusting its price and inventory policy, but without updating the carbon intensity. We use our analytical results to derive insights into the effect of green- and price-based competition on products' greenness decisions, pricing and retailers' profits. Our study makes the following main contributions:

- This is the first study to investigate the effect of greenness- and price-based retail competition on the environmental performance of products while comparing different competition scenarios to the monopolistic case. Another new feature of this work is the study of the impact of retailer's market power on the greenness decisions. Our analysis is based on stochastic models that capture real-world complexities faced by retailers as we consider random demand and random inventory replenishment time.
- We provide the closed-form expressions of optimal solutions and derive new managerial insights that have not been reported in the related literature. Some of our findings are not intuitive. For instance, we show under some market conditions that greenness- and price-based competition leads the retailer to offering a dirtier product than the product it would have offered in a monopolistic situation. We also explain how the market power of the retailer impacts on its greenness strategy under different market conditions.

Section 2 gives an overview of the related literature. In Section 3, we develop the modeling framework and discuss the main assumptions. In Section 4, we determine analytically the optimal strategy of each retailer under different competition scenarios. In Section 5, we analyze the optimal strategies and
derive managerial insights into the effect of greenness- and price-based competition on products' environmental performance, pricing decisions and retailers' profits. The practical implications of our findings are discussed in Section 6. Finally, we conclude the paper and provide future work directions.

## 2. Literature review

There are valuable studies that focused on environmental considerations in operations management and SC literature (see Chelly et al. 2019, Nouira et al. 2016). Our study relates to the analytical research in this field and, particularly, those studies that focus on greenness-based competition in SCs. In this section, we review the extant research in these areas and highlight the contributions of our work.

In the context of a single product offered to a price- and greenness-sensitive market, Hammami et al. (2018) studied the joint and alternative effect of the customers' environmental awareness (CEA) and environmental regulations on the production policy, price, and greenness level of a product offered by a manufacturer. The authors considered linear and exponential demand functions that decrease in price and carbon intensity. Reducing the carbon intensity leads to increasing the production cost. The main results indicated that CEA is an efficient driver for better environmental performance, acting as a substitute for a carbon tax but, unlike a carbon tax, leading to a lower price for the customers. In the same research stream, many studies consider a SC that consists of one manufacturer and one retailer. These studies typically consider a linear demand function. Ghosh and Shah (2012) investigated the impact of greening decisions when the manufacturer and the retailer cooperate or act individually. Under the decentralized policy, the manufacturer moves first and decides the greenness level and the wholesale price. Improving the greenness requires an investment that is modeled as a quadratic cost in greenness improvement. The retailer reacts by deciding the final price. Under the cooperative policy, the retailer and manufacturer first bargain on the greenness level; the manufacturer then decides the wholesale price and the retailer reacts by deciding the final price. The study showed how greenness levels, prices and profits are influenced by channel structures. Ghosh and Shah (2015) studied a quite similar problem while focusing on a cost
sharing contract setting where the retailer shares a proportion of the greening investment cost with the manufacturer or both players bargain on the cost sharing. Compared to the decentralized setting of Ghosh and Shah (2012), cost sharing contracts result in higher greening levels, higher profits for individual firms and an increase in SC profits. Heydari et al. (2019) considered a three-tier dual-channel SC where a distributor buys a product from the manufacturer and sells it to customers via two available channels: the direct (e-retailing) channel (controlled by the distributor) and the classical retail channel (controlled by the retailer). The demand of each channel is sensitive to price and greenness level. The greenness level is decided by the manufacturer. The distributor and the retailer respectively decide the e-channel price and the classical channel price. The authors investigated three decision-making structures including the decentralized setting, the centralized setting, and a coordination setting that leads to increasing all individual profits with comparison to the decentralized setting.

Still in the context of single product-related decisions, some papers investigated the product positioning and pricing problem when a green version of an already existing product is to be launched. For instance, Hong et al. (2019) studied a manufacturer-retailer SC that offers a green product to compete with an exiting traditional product when taking customers' reference behavior and government regulation into consideration. Customers' reference behavior reflects how customers base their purchasing decision, between the green product and the traditional one, on comparing these two products in price, functional quality, and environmental concerns. The manufacturer determines the product's greenness degree and the retailer determines the retail price under three decision-making structures: manufacturer-led SC, retailerled SC, and coordinated SC with revenue-sharing contract. The main result showed that consumers' reference behaviors significantly influence the product's greenness and pricing decisions. Notice that the characteristics of the traditional product are exogenous input parameters in this study. Agi and Yan (2020) studied the pricing strategies in a manufacturer-retailer SC when a green version of an existing conventional product is launched. The model assumes a market segmented based on the consumers' willingness to pay for the green feature of the product and incorporates a fixed cost related to launching
the green product. The authors showed that a manufacturer-led SC is better prepared than a retailer-led SC to overcome the fixed cost and launch the green product. Notice that this study focuses only on pricing decisions while considering the greenness level as an exogenous input parameter.

Some studies include the effect of the green marketing effort on sales. For instance, Ma et al. (2013) investigated the coordination of the SC when the demand linearly decreases in the price and increases in the greening effort made by the manufacturer and the marketing effort made by the retailer. A higher marketing effort means a higher investment which is modeled as a quadratic cost function. The manufacturer acts as the Stackelberg leader. The main finding indicated that the best coordination of the SC is obtained when the greenness improvement cost and marketing effort cost are shared between the retailer and the manufacturer. Within the same context, Hong and Guo (2019) studied cooperation contracts and investigated their environmental effect. In the first case, the manufacturer decides the greenness level and the wholesale price, and the retailer decides the green marketing effort and the final price. The authors also investigated the cases where the green marketing effort (cost) is shared between the retailer and the manufacturer, and where the SC is fully coordinated (centralized setting). This study showed that cooperation is necessary to improve the environmental performance of a SC.

The research works presented above consider a single product offered to the customers. Another stream of research considered a SC of one manufacturer and one retailer that offers two substitutable products differentiated in terms of greenness level and price, but without competition between retailers. In these works, the demand is typically linear with substitution. For instance, Zhang et al. (2015) analyzed the impact of CEA on order quantities and channel coordination under a newsvendor scheme in centralized and decentralized settings. Although there are two products differentiated in terms of price and greenness, the greenness level is not a decision variable in this study. The retailer determines the order quantity, and the manufacturer decides the wholesale price for each product. Under the decentralized setting, the authors showed that the retailer is the puller of the SC in developing the market for eco-friendly products. Basiri and Heydari (2017) added the green marketing effort as a third
differentiation criteria, besides price and greenness. The characteristics of the traditional product are fixed. For the green product, the manufacturer decides the greenness level, while the retailer decides the price and marketing efforts. Numerical illustrations showed that the collaboration is capable of enhancing the SC profit fairly close to the centralized model and also ensures higher profits for both channel members.

Unlike the works cited above, we study the case where products are offered by different competing retailers. There are valuable works on retailers' competition. Most of these studies, however, do not consider environmental aspects. For instance, Cachon (2001) studied competitive and cooperative situations in a SC of one supplier and N retailers. Demand is stochastic but exogenous. Inventory is hold by the different players. The objective is to find the optimal reorder points to minimize the cost. Wu et al. (2012) studied a SC that consists of 2 competing retailers and one common supplier under a deterministic setting. The retailers offer substitutable products and compete on prices in a price-sensitive market. Chakraborty et al. (2019) considered a SC that consists of two manufacturers who sell the products through a common retailer and compete on prices and quality of the products. The authors studied different pricing and quality improvement strategies of the channel members under both non-collaborative and collaborative quality improvement scenarios. Recently, Alibeiki et al. (2020) developed competitive models between two independent, asymmetric retailers in making variety and quantity/price decisions either simultaneously or sequentially. The authors assessed the effect of retail powers advantages on the product assortment choice while consideing two sorts of retail powers: price leadership and cost advantage.

More related to our work, a few studies dealt with the competition between retailers under environmental considerations. Qi et al. (2017) investigated the effects of a carbon cap regulation on the pricing decisions in a two-stage make-to-order SC that is composed of one supplier and two retailers. The retailers sell substitutable products and compete on price. Under the decentralized setting, the supplier, as a leader, decides the wholesale prices. The retailers react by setting their own prices. The demand of each
retailer linearly decreases in its price and increases in other retailer's price. In the centralized system, the supplier and two retailers determine the retail prices together. The authors analyzed the performance of different scenarios for the SC members and provided the appropriate range of a carbon cap for the policy maker to effectively reduce carbon emissions. Zhou et al. (2018) investigated the effect of a carbon tax policy on the pricing decisions and social welfare in a two-stage SC composed of one common supplier and N retailers that sell substitutable products. The system consists of three players, the government, which determines the carbon tax rate to maximize the social welfare, the manufacturer, which moves second and determines the wholesale prices, and the retailers which finally react and determine the retail prices. The retailers compete only on prices. The authors found that the retail price and wholesale price increase with the carbon tax rate and the optimal carbon tax rate is increasing in retail competition intensity. These studies by Qi et al. (2017) and Zhou et al. (2018), however, do not consider a greennesssensitive demand and do not investigate the greenness-based competition (in both studies, the greenness level is not a decision variable). Moreover, they assume a deterministic setting which does not fit with most retail operations. Recently, Xu et al. (2020) studied the price-based competition between a high carbon SC and a low carbon SC selling substitutable products. Each SC consists of one manufacturer and one retailer. The government subsidizes consumers who buy low carbon products but imposes a carbon tax on the manufacturer producing high carbon products. The main result indicates that under government intervention, the integration of both manufacturers or both retailers has no effect on social welfare. Notice that this study assumes a deterministic setting and does not consider greenness decisions.

Finally, the study that comes closest to our work is Liu et al. (2012). This study investigated the impact of CEA and competition intensity on the profits of the decision-makers in different SC structures (one manufacturer and one retailer, two manufacturers and one retailer, and two manufacturers and two retailers). The manufacturer decides the greenness level and the wholesale price. The retailer decides its selling price. A Stackelberg game is used to model the problems with the manufacturer as a first-mover and the retailer as a follower. The demand of each product linearly decreases in its price and increases in
its greenness level; it also increases in the price and decreases in the greenness level of the other product. The main findings suggested that, as CEA increases, retailers and manufacturers with superior ecofriendly operations will benefit, while the profitability of the inferior eco-friendly firm will tend to increase if the production competition level is low and decrease if the production competition level is high. Our study differs from that of Liu et al. (2012) in many aspects. First, we focus on the retailer's problem and consider that the retailer undertakes all decisions (greenness level, price, and inventory decisions) while Liu et al. (2012) focused on the manufacturer's problem and assumed that the retailer's role is just to price the product. Second, we consider inventory decisions for the retailers in a context of stochastic demand and stochastic inventory replenishment time. Liu et al. (2012), however, did not consider inventories and, thus, ignored replenishment aspects and service level constraints, which are crucial factors in the retail context. Third, our main objective is to investigate the impact of the greennessand price-based competition on a product's environmental performance, while Liu et al. (2012) focused on the profits of the different SC actors.

## 3. Modeling framework

We consider two retailers that sell substitutable products, differentiated in terms of greenness level (i.e., carbon intensity) and price, in a greenness- and price-sensitive market (see Figure 1 below). Customers arrive according to a Poisson process with mean arrival rate $\lambda_{\mathrm{i}}$ for retailer $i$. The demand function will be presented later in this section. Each retailer has its own supplier. The product sold by retailer $i$ is denoted by $\mathcal{P}_{i}$. The main sources of carbon emissions are the transportation and production activities. The transportation emissions per unit of product, denoted by $e_{i}$ for retailer $i$, depend on the distance between the supplier's site and the retailer's warehouse. As we do not deal with supplier selection, $e_{i}$ is not a decision variable. Without loss of generality, we ignore the transportation emissions from the retailers' warehouses to end customers, which implicitly assumes that the retailers are located close to the demand zone. As for production emissions, we consider a context where the retailer can choose the production emission level and, thus, ask the supplier to manufacture the product accordingly.

This context fits with the practical examples provided in Section 1 to illustrate how leading retailers, such as Walmart and Bestseller, are asking their suppliers to produce greener products. We let $x_{0}$ denote the amount of production emissions per unit of the standard variety of the product. A greener variety (with lower production emissions) implies a higher purchasing cost for the retailer as this incurs a higher manufacturing cost. We let $x_{i}$ denote the amount of production emissions per unit of product $\mathcal{P}_{i}$. The carbon intensity of $\mathcal{P}_{i}$ is thus given by $e_{i}+x_{i}$. The unitary purchasing cost of $\mathcal{P}_{i}$ is given by $c_{i}+$ $b\left(x_{0}-x_{i}\right)^{2}$, where $c_{i}$ is the unitary cost of the standard product and $b$ is the cost factor for production emissions reduction. We consider a quadratic cost function as usual in the related literature (e.g., Liu et al. 2012, Ghosh and Shah 2012, 2015).

The replenishment order size of retailer $i$ is denoted by $S_{i}$. The service time to refill the stock is exponentially distributed with mean rate $\mu_{i}$ for retailer $i$. The replenishment time does not depend on the order size since the products are assumed to be always available at the supplier's site, which is a common assumption (Zhu, 2015, Hammami and Frein, 2014a). Thus, the replenishment time corresponds basically to preparation and transportation activities. To simplify the analysis, we assume that the retailer places a replenishment order when the standard stock is empty, and demand is satisfied from a safety stock during the replenishment time. We assume that the safety stock is large enough to satisfy all demand in the vast majority of cases. The management of the safety stock is out of the scope of this study. The average inventory level for retailer $i$ can thus be approximated by $\frac{s_{i}}{2}$. We let $2 h$ denote the unit inventory cost. Thus, the inventory cost for retailer $i$ is $h S_{i}$. To avoid unrealistic inventory levels, the probabitity of satisfying demand from the standard stock, denoted by $1-\psi_{i}$ for retailer $i$, must be greater than a predetermined minimum service level $1-r$. Notice that $r$ represents the maximum allowed level of stockout probability. Hence, the service level constraint for retailer $i$ is given by $1-\psi_{i} \geq 1-r$, which is equivalent to $\psi_{i} \leq r$. Notice that $\psi_{i}$ represents the probability of having an empty standard stock when a customer arrives (i.e., probability of serving this customer from the safety stock). Since customers' arrival and inventory refilling service time follow the exponential distribution with mean rates $\lambda_{i}$ and $\mu_{i}$,
respectively, our stochastic system is a Markovian process. We can, thus, use continuous Markov chain process properties and deduce that $\psi_{i}=\left(1+\frac{\mu_{i} S_{i}}{\lambda_{i}}\right)^{-1}$ (see Appendix A). Hence, the service level constraint for retailer $i$ is given by $\left(1+\frac{\mu_{i} S_{i}}{\lambda_{i}}\right)^{-1} \leq r$.


Figure 1. General framework

Without loss of generality, we let retailer 1 represents the retailer that has the farthest supplier. Therefore, we have $e_{1} \geq e_{2}$ and $\mu_{1} \leq \mu_{2}$. We now present the demand function.

We recall that customers arrive according to a Poisson process with mean arrival rate $\lambda_{\mathrm{i}}$ for retailer $i$. Our demand is linear with substitution. The mean demand of each product is decreasing in its carbon intensity and price and increasing in other product's carbon intensity and price. In practice, the retailers may have different market powers and, thus, do not attract the same number of customers even when the same price and greenness are offered by both retailers. This is a typical situation when there is an established leading retailer and a new retailer entering the market since the new retailer generally has a smaller market power. We let $\omega \in[0,1]$ denote the market share of retailer 1 and, thus, $(1-\omega)$ is the market share of retailer 2 , when both retailers offer the product with the same price $p$ and carbon intensity $x+e$. The market potential is denoted by $A$. We let $\alpha_{p}$ and $\beta_{e}$ respectively denote the market sensitivity
to price and carbon intensity. As for switchovers, we respectively denote by $\alpha_{s}$ and $\beta_{s}$ the sensitivity of switchover toward price difference and carbon intensity difference. The mean demand rates are given as follows.
$\lambda_{1}=\omega\left[A-\alpha_{p} p_{1}-\beta_{e}\left(x_{1}+e_{1}\right)\right]+\alpha_{s}\left(p_{2}-p_{1}\right)+\beta_{s}\left(\left(x_{2}+e_{2}\right)-\left(x_{1}+e_{1}\right)\right)$
$\lambda_{2}=(1-\omega)\left[A-\alpha_{p} p_{2}-\beta_{e}\left(x_{2}+e_{2}\right)\right]+\alpha_{s}\left(p_{1}-p_{2}\right)+\beta_{s}\left(\left(x_{1}+e_{1}\right)-\left(x_{2}+e_{2}\right)\right)$

Notice that $\lambda_{1}+\lambda_{2}=A-\omega \alpha_{p} p_{1}-(1-\omega) \alpha_{p} p_{2}-\omega \beta_{e}\left(x_{1}+e_{1}\right)-(1-\omega) \beta_{e}\left(x_{2}+e_{2}\right)$. This means that the total mean demand is sensitive to prices and greenness. If the same price $p$ and same carbon intensity $x+e$ are offered by both retailers, then the mean demand is $\omega\left(A-\alpha_{p} p-\beta_{e}(x+e)\right)$ for retailer 1 and $(1-\omega)\left(A-\alpha_{p} p-\beta_{e}(x+e)\right)$ for retailer 2 . The case where both retailers have the same market power, which is the typical case studied in the literature, corresponds to $\omega=0.5$. In our model, $\omega$ represents the market power of retailer 1 . If $\omega>0.5$, then retailer 1 has more market power than retailer 2 , and vice versa. With the consideration of $\omega \in[0,1]$, we generalize the linear demand with substitution that is usually adopted in the related literature. For further clarity, notice that retailer 1 can attract $\omega \alpha_{p}+\alpha_{s}$ more customers with one unit decrease in price and $\omega \beta_{e}+\beta_{s}$ more customers with one unit decrease in carbon intensity. Only a part of these customers represent a new created demand in the market $\left(\omega \alpha_{p}\right.$ new customers for one unit decrease in price and $\omega \beta_{e}$ new customers for one unit decrease in carbon intensity), and the other attracted customers are just switching from the other retailer ( $\alpha_{s}$ switching customers for a unit decrease in price and $\beta_{s}$ switching customers for a unit decrease in carbon intensity). We can do a similar analysis for the demand of retailer 2 while replacing $\omega$ with $1-\omega$.

Since $e_{1} \geq e_{2}$ and both of them are fixed parameters, we can normalize $e_{2}$ to 0 without loss of generality and, thus, $e_{1}$ can be interpreted as the difference in transportation emissions. To simplify notation, we let $A_{1}=\omega A-\theta_{1} e_{1}, A_{2}=(1-\omega) A+\beta_{s} e_{1}, \delta_{1}=\omega \alpha_{p}+\alpha_{s}, \delta_{2}=(1-\omega) \alpha_{p}+\alpha_{s}, \theta_{1}=$ $\omega \beta_{e}+\beta_{s}$ and $\theta_{2}=(1-\omega) \beta_{e}+\beta_{s}$. The mean demand rates are finally given as follows.

$$
\begin{align*}
& \lambda_{1}=A_{1}-\delta_{1} p_{1}+\alpha_{s} p_{2}-\theta_{1} x_{1}+\beta_{s} x_{2}  \tag{3}\\
& \lambda_{2}=A_{2}-\delta_{2} p_{2}+\alpha_{s} p_{1}-\theta_{2} x_{2}+\beta_{s} x_{1} \tag{4}
\end{align*}
$$

The notation used in this paper is recapitulated in Table 1.
Table 1. Notation

| Parameters |  |
| :--- | :--- |
| $A$ | Market potential. |
| $\omega$ | Market share of retailer 1 and, thus, $(1-\omega)$ is the market share of retailer 2. |
| $\beta_{e}$ | Market sensitivity to carbon intensity. |
| $\beta_{s}$ | Sensitivity of switchover toward carbon intensity difference. |
| $\alpha_{p}$ | Market sensitivity to price. |
| $\alpha_{s}$ | Sensitivity of switchover toward price difference. |
| $\delta_{1}$ | $=\omega \alpha_{p}+\alpha_{s}$. |
| $\delta_{2}$ | $=(1-\omega) \alpha_{p}+\alpha_{s}$. |
| $\theta_{1}$ | $=\omega \beta_{e}+\beta_{s}$. |
| $\theta_{2}$ | $=(1-\omega) \beta_{e}+\beta_{s}$. |
| $\mu_{i}$ | Mean service time rate to refill the stock of retailer $i$. |
| $c_{i}$ | Unit cost of standard product $\mathcal{P} i$. |
| $2 h$ | Unit inventory cost. |
| $b$ | Cost factor for production emissions reduction. |
| $r$ | Maximum allowed level of stockout probability. |
| $e_{i}$ | Amount of transportation emissions per unit of product $\mathcal{P}_{i .}$. |
| $x_{0}$ | Amount of production emissions per unit of the standard variety of the product. |
| $A_{1}$ | $=\omega A-\theta_{1} e_{1}$ |
| $A_{2}$ | $=(1-\omega) A+\beta_{s} e_{1}$ |
| $A_{1}^{\prime \prime}$ | $=\omega A-\theta_{1}\left(e_{1}+x_{1}\right)$ |
| $A_{2}^{\prime \prime}$ | $=(1-\omega) A+\beta_{s}\left(e_{1}+x_{1}\right)$ |
| Variables |  |
| $x_{i}$ | Amount of production emissions per unit of product $\mathcal{P}_{i}$. |


| $p_{i}$ | Price of product $\mathcal{P}_{i .}$ |
| :--- | :--- |
| $S_{i}$ | Replenishment order size for retailer $i$. |
| $\lambda_{i}$ | Mean demand rate for retailer $i$. |
| $\psi_{i}$ | Stockout probability for retailer $i$. |
| $\pi_{i}$ | Profit for retailer $i$. |

In the next section, we first solve the case of a single monopolistic retailer. This model will serve as a benchmark model to understand the effect of competition. We then study different competition scenarios. In the general competition model, each retailer decides the greenness level of its product, the price and the order size to maximize its expected profit under the service level constraint while considering other retailer's decisions. We consider a non-cooperative game and determine the optimal strategy of each retailer at the Nash equilibrium. We also study two other particular settings that are particularly relevant when there is an established retailer (let us say, retailer 1) and a new retailer that enters the market (let us say retailer 2). In the first particular setting, retailer 2 enters the market and decides $x_{2}, p_{2}$ and $S_{2}$ whereas retailer 1 does not react. In the second setting, retailer 1 reacts by adjusting price $p_{1}$ and order size $S_{1}$, but without changing the emission level $x_{1}$.

## 4. Models and solutions

We first solve the benchmark model (monopoly situation) and then turn to the competition models.

### 4.1. Benchmark model ( $M_{0}$ )

In this model, retailer $i$ is alone in the market. The problem consists in deciding price $p_{i}$, carbon emission $x_{i}$, and order size $S_{i}$ to maximize the expected profit. As a single product is offered to customers in this case, the mean demand rate is given by $\lambda_{i}=A-\alpha_{p} p_{i}-\beta_{e}\left(x_{i}+e_{i}\right)$. Since there is only one retailer and $e_{i}$ is fixed, we normalize $e_{i}$ to 0 without loss of generality. The model is given below.
$\left(\boldsymbol{M}_{\mathbf{0}}\right) \max _{x_{i} p_{i}, S_{i}} \pi=\left(p_{\mathrm{i}}-\left(c_{i}+b\left(x_{0}-x_{i}\right)^{2}\right)\right) \lambda_{i}-h S_{i}$
(5)

Subject to
$\frac{\lambda_{i}}{\lambda_{i}+\mu_{i} s_{\mathrm{i}}} \leq r$
$\lambda_{i}=A-\alpha_{p} p_{i}-\beta_{e} x_{i}$
(7)
$x_{\mathrm{i}}, p_{\mathrm{i}}, S_{i} \geq 0, \quad x_{i} \leq x_{0}, \quad \lambda_{i} \geq 0$.

The objective is to maximize the total expected profit given in equation (5). This profit is equal to the revenue (i.e., $\left.p_{\mathrm{i}} \lambda_{i}\right)$ - the procurement cost (i.e., $\left.\left(c_{i}+b\left(x_{0}-x_{i}\right)^{2}\right) \lambda_{i}\right)$ - the inventory cost (i.e., $\left.h S_{i}\right)$. Constraint (6) is the service level constraint. The mean demand rate is given in equation (7). To solve this model, we first transform it into a single-variable model thanks to Lemma 1 and 2 given below. All proofs are given in the appendix.

Lemma 1. For given values of $p_{i}$ and $x_{i}$, service level constraint (6) is binding and the optimal order size is $S_{i}^{*}\left(p_{i}, x_{i}\right)=\frac{(1-r)\left(A-\alpha_{p} p_{i}-\beta_{e} x_{i}\right)}{r \mu_{i}}$.

We, thus, replace $S_{i}^{*}$ with its expression given in Lemma 1 and obtain the following equivalent formulation of model $\left(M_{0}\right)$ with only two variables $p_{i}$ and $x_{i}$.
$\max _{0 \leq x_{i} \leq x_{0}, p_{i} \geq 0} \pi=\left(p_{\mathrm{i}}-\left(c_{i}+b\left(x_{0}-x_{i}\right)^{2}+\frac{h(1-r)}{r \mu_{i}}\right)\right)\left(A-\alpha_{p} p_{i}-\beta_{e} x_{i}\right)$
Subject to $A-\alpha_{p} p_{i}-\beta_{e} x_{i} \geq 0$

In Lemma 2, we determine the optimal price for a given carbon emission $x_{i}$.

Lemma 2. For a given $x_{i}$, the optimal price $p_{i}^{*}\left(x_{i}\right)=\frac{\alpha_{p} b x_{i}^{2}-\left(2 \alpha_{p} b x_{0}+\beta_{e}\right) x_{i}+A+\alpha_{p}\left(c_{i}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{i}}\right)}{2 \alpha_{p}}$.

We can now replace the price with its expression given in Lemma 2 and obtain the following equivalent formulation of model $\left(M_{0}\right)$ with only one variable $x_{i}$.
$\max _{0 \leq x_{i} \leq x_{0}} \pi=\frac{\left(-\alpha_{p} b x_{i}^{2}+\left(2 \alpha_{p} b x_{0}-\beta_{e}\right) x_{i}+A-\alpha_{p}\left(c_{i}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{i}}\right)\right)^{2}}{4 \alpha_{p}}$

We finally solve this single-variable model and derive the optimal solution in Proposition 1.

## Proposition 1. The optimal solution of benchmark model $\left(M_{0}\right)$ is the following.

$x_{i}^{*}=x_{0}-\frac{\beta_{e}}{2 \alpha_{p} b}, p_{i}^{*}=\frac{A-\beta_{e} x_{0}+\frac{3 \beta_{e}^{2}}{4 \alpha_{p} b}+\alpha_{p}\left(c_{i}+\frac{h(1-r)}{r \mu_{i}}\right)}{2 \alpha_{p}}$, and $S_{i}^{*}=\frac{(1-r)\left(A-\beta_{e} x_{0}+\frac{\beta_{e}^{2}}{4 \alpha_{p} b}-\alpha_{p}\left(c_{i}+\frac{h(1-r)}{r \mu_{i}}\right)\right)}{r \mu_{i}}$.

### 4.2. General competition model with full reaction ( $M_{1}$ )

We now consider the problem of two competing retailers that offer two substitutable products. Each retailer decides its price $p_{i}$, carbon emission $x_{i}$ and order size $S_{i}$ to maximize its expected profit, while taking into account other retailer's decisions. We consider a non-cooperative game in which there is no dominant retailer (player, from a game theory perspective). Our purpose is to determine the optimal strategy of each retailer at the Nash equilibrium, i.e., the set of optimal decisions such as no player can benefit by changing its own decisions while the other player keeps its decisions unchanged. We recall that the mean demand rates are explained in Section 3. The optimization model for each retailer is given in the following.

## Retailer 1's model ( $\mathbf{M}_{1.1}$ )

$\max _{x_{1}, S_{1}, p_{1}} \pi_{1}=\left(p_{1}-\left(c_{1}+b\left(x_{0}-x_{1}\right)^{2}\right)\right) \lambda_{1}-h S_{1}$

Subject to
$\frac{\lambda_{1}}{\lambda_{1}+\mu_{1} S_{1}} \leq r$
$\frac{\lambda_{2}}{\lambda_{2}+\mu_{2} S_{2}} \leq r$
$\lambda_{1}=A_{1}-\delta_{1} p_{1}+\alpha_{s} p_{2}-\theta_{1} x_{1}+\beta_{s} x_{2}$
$\lambda_{2}=A_{2}-\delta_{2} p_{2}+\alpha_{s} p_{1}-\theta_{2} x_{2}+\beta_{s} x_{1}$
$x_{1}, S_{1}, p_{1} \geq 0, \quad x_{1} \leq x_{0}, \quad \lambda_{1} \geq 0$.
$x_{2}, S_{2}, p_{2} \geq 0, \quad x_{2} \leq x_{0}, \quad \lambda_{2} \geq 0$.

To find the Nash equilibrium strategies, we first suppose that retailer 2's decisions are known and determine retailer 1's best response, which means that we solve model ( $M_{1.1}$ ). Then, we consider that retailer 1's decisions are known and solve model $\left(M_{1.2}\right)$ to obtain retailer 2's best response. We finally determine the Nash equilibrium of the game.

We now determine the best strategy of retailer 1 given retailer 2's decisions. Thus, we need to solve model $\left(M_{1.1}\right)$ for given values of $p_{2}, x_{2}$, and $S_{2}$. In this case, we use the same methodology of Lemma 1 to prove that the service constraint is tight at optimality and, thus, the optimal order size is $S_{1}^{*}\left(p_{1}, x_{1}\right)=$ $\frac{(1-r)\left(A_{1}-\delta_{1} p_{1}+\alpha_{s} p_{2}-\theta_{1} x_{1}+\beta_{s} x_{2}\right)}{r \mu_{1}}$. We replace $S_{1}^{*}$ with its expression and obtain the following equivalent formulation of model $\left(M_{1.1}\right)$.
$\max _{0 \leq x_{1} \leq x_{0}, p_{1} \geq 0} \pi_{1}=\left(p_{1}-\left(c_{1}+b\left(x_{0}-x_{1}\right)^{2}+\frac{h(1-r)}{r \mu_{1}}\right)\right)\left(A_{1}-\delta_{1} p_{1}+\alpha_{s} p_{2}-\theta_{1} x_{1}+\beta_{s} x_{2}\right)$

Subject to $A_{1}-\delta_{1} p_{1}+\alpha_{s} p_{2}-\theta_{1} x_{1}+\beta_{s} x_{2} \geq 0$

The above formulation of model $\left(M_{1.1}\right)$ comprises two variables $x_{1}$ and $p_{1}$. In the following lemma, we determine the optimal value of $p_{1}$ for a given $x_{1}$. This will enable to simplifying the model.

Lemma 3. For a given strategy of retailer 2 (i.e., given $p_{2}, x_{2}$, and $S_{2}$ ), the optimal retailer 1 's price in function of $x_{1}$ is $p_{1}^{*}\left(x_{1}\right)=\frac{\delta_{1} b x_{1}{ }^{2}-\left(\theta_{1}+2 \delta_{1} b x_{0}\right) x_{1}+A_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}+\delta_{1}\left(c_{1}+b x_{0}{ }^{2}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \delta_{1}}$.

Thanks to Lemma 3, we obtain the following equivalent formulation of model ( $M_{1.1}$ ) with only one variable $x_{1}$.
$\max _{0 \leq x_{1} \leq x_{0}} \pi_{1}=\frac{\left(-\delta_{1} b x_{1}^{2}-\left(\theta_{1}-2 \delta_{1} b x_{0}\right) x_{1}+A_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}-\delta_{1}\left(c_{1}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{1}}\right)\right)^{2}}{4 \delta_{1}}$

We solve this single-variable model and obtain the optimal value of $x_{1}$ in function of retailer 2 's decisions. The result is given in the following Lemma.

Lemma 4. For a given strategy of retailer 2 (i.e., given $p_{2}, x_{2}$, and $S_{2}$ ), the optimal carbon emission of retailer 1's product $\left(\mathcal{P}_{1}\right)$ is $x_{1}^{*}=x_{0}-\frac{\theta_{1}}{2 \delta_{1} b}$.

Consequently, the optimal price of $\mathcal{P}_{1}$ is $p_{1}^{*}\left(p_{2}, x_{2}\right)=\frac{A_{1}+\frac{3 \theta_{1}^{2}}{4 \delta_{1} b}-\theta_{1} x_{0}+\alpha_{s} p_{2}+\beta_{s} x_{2}+\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \delta_{1}}$.

We now assume that retailer 1's decisions (i.e., $x_{1}, p_{1}$ and $S_{1}$ ) are known and solve model ( $M_{1.2}$ ) for retailer 2. It is noted that models $\left(M_{1.1}\right)$ and $\left(M_{1.2}\right)$ are symmetric. Therefore, we use the same approach to solve model $\left(M_{1.2}\right)$. For given $x_{1}, p_{1}$ and $S_{1}$, we deduce that $S_{2}^{*}\left(p_{2}, x_{2}\right)=$ $\frac{(1-r)\left(A_{2}-\delta_{2} p_{2}+\alpha_{s} p_{1}-\theta_{2} x_{2}+\beta_{s} x_{1}\right)}{r \mu_{2}}$, and the optimal values of $x_{2}$ and $p_{2}$ are given in Lemma 5.

Lemma 5. For a given strategy of retailer 1 (i.e., given $x_{1}, p_{1}$ and $S_{1}$ ), the optimal carbon emission of retailer 2's product $\left(\mathcal{P}_{2}\right)$ is $x_{2}{ }^{*}=x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}$.

Consequently, the optimal price of $\mathcal{P}_{2}$ is $p_{2}^{*}\left(x_{1}, p_{1}\right)=\frac{A_{2}+\frac{3 \theta_{2}^{2}}{4 \delta_{2} b}-\theta_{2} x_{0}+\alpha_{s} p_{1}+\beta_{s} x_{1}+\delta_{2}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \delta_{2}}$.

Based on the analysis presented above, we can now derive the best response of each retailer to other retailer's decisions. We substitute $x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}$ for $x_{2}$ into the optimal expression of $p_{1}$ given in Lemma 4, and deduce that retailer 1's best pricing response is the following.

$$
\begin{equation*}
p_{1}^{*}\left(p_{2}\right)=\frac{\alpha_{s}}{2 \delta_{1}} p_{2}+\frac{A_{1}+\left(\beta_{s}-\theta_{1}\right) x_{0}+\frac{3 \theta_{1}{ }^{2}}{4 \delta_{1} b}-\frac{\beta_{s} \theta_{2}}{2 \delta_{2} b}+\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \delta_{1}} \tag{12}
\end{equation*}
$$

Similarly, we substitute $x_{0}-\frac{\theta_{1}}{2 \delta_{1} b}$ for $x_{1}$ into the optimal expression of $p_{2}$ given in Lemma 5, and deduce that retailer 2's best pricing response is the following.

$$
\begin{equation*}
p_{2}^{*}\left(p_{1}\right)=\frac{\alpha_{s}}{2 \delta_{2}} p_{1}+\frac{A_{2}+\left(\beta_{s}-\theta_{2}\right) x_{0}+\frac{3 \theta_{2}{ }^{2}}{4 \delta_{2} b}-\frac{\beta_{s} \theta_{1}}{2 \delta_{1} b}+\delta_{2}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \delta_{2}} \tag{13}
\end{equation*}
$$

The response of each player is, thus, linear in other player's decision. Consequently, the intersection of the two best response curves is the Nash equilibrium, as illustrated in Figure 2. We see that when retailer 1 chooses a given price different from the intersection point, retailer 2 reacts according to its response curve and chooses a new best price. Since the price of retailer 2 changed, retailer 1 reacts again and chooses a new price according to its response curve, and so on until both retailers converge to the intersection point. At the intersection point, each retailer does not gain anything from deviating from this position, assuming the other retailer also keeps its position unchanged, which corresponds to the Nash equilibrium.


Figure 2. Illustration of the Nash equilibrium

Based on the above analysis, we finally derive in the following Proposition the optimal strategy of each retailer at the Nash equilibrium.

Proposition 2. The optimal strategy of each retailer at the Nash equilibrium is the following.

For retailer $1, p_{1}^{*}=\frac{2 \delta_{2}\left(2 \delta_{1} z_{1}+\alpha_{s} z_{2}\right)}{4 \delta_{1} \delta_{2}-\alpha_{s}^{2}}, x_{1}^{*}=x_{0}-\frac{\theta_{1}}{2 \delta_{1} b}$, and
$S_{1}^{*}=\frac{(1-r)}{r \mu_{1}}\left(A_{1}+\frac{2 \delta_{1}\left(-2 \delta_{1} \delta_{2}+\alpha_{s}^{2}\right) z_{1}+2 \alpha_{s} \delta_{1} \delta_{2} z_{2}}{4 \delta_{1} \delta_{2}-\alpha_{s}^{2}}-\omega \beta_{e}\left(x_{0}-\frac{\theta_{1}}{2 \delta_{1} b}\right)+\frac{\beta_{s}}{2 b}\left(\frac{\theta_{1}}{\delta_{1}}-\frac{\theta_{2}}{\delta_{2}}\right)\right)$.

For retailer $2, p_{2}^{*}=\frac{2 \delta_{1}\left(2 \delta_{2} z_{2}+\alpha_{s} z_{1}\right)}{4 \delta_{1} \delta_{2}-\alpha_{s}{ }^{2}}, x_{2}{ }^{*}=x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}$, and

$$
S_{2}^{*}=\frac{(1-r)}{r \mu_{2}}\left(A_{2}+\frac{2 \delta_{2}\left(-2 \delta_{1} \delta_{2}+\alpha_{s}^{2}\right) z_{2}+2 \alpha_{s} \delta_{1} \delta_{2} z_{1}}{4 \delta_{1} \delta_{2}-\alpha_{s}^{2}}-(1-\omega) \beta_{e}\left(x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}\right)+\frac{\beta_{s}}{2 b}\left(\frac{\theta_{2}}{\delta_{2}}-\frac{\theta_{1}}{\delta_{1}}\right)\right) .
$$

$$
\text { where } z_{1}=\frac{A_{1}+\left(\beta_{s}-\theta_{1}\right) x_{0}+\frac{3 \theta_{1}{ }^{2}}{4 \delta_{1} b} \frac{\beta_{s} \theta_{2}}{2 \delta_{2} b}+\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \delta_{1}} \text { and } z_{2}=\frac{A_{2}+\left(\beta_{s}-\theta_{2}\right) x_{0}+\frac{3 \theta_{2}{ }^{2}}{4 \delta_{2} b}-\frac{\beta_{s} \theta_{1}}{2 \delta_{1} b}+\delta_{2}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \delta_{2}}
$$

### 4.3. Particular competition scenarios

In the previous section, we have solved the general competition problem where both retailers undertake price, carbon emission, and inventory decisions. In practice, there are many situations in which an existing retailer (let us say retailer 1) is already operating in the market, and a new retailer (let us say retailer 2) enters the market and offers a substitutable product. In this case, two situations are relevant to study:

- Competition without reaction. The existing retailer does not react to the new retailer's decisions because, for instance, it has a much higher market power.
- Competition with partial reaction. The existing retailer just updates its price and order size but does not change the carbon intensity as this requires new deals with the supplier.

With respect to solving approaches, the case of competition without reaction is similar to the benchmark case, and the case of competition with partial reaction is a particular case of our general competition model. We present in what follows the optimal solutions in these two particular cases, but do not provide the details of modeling and solving approaches to avoid redundancy.

### 4.3.1. Competition without reaction

In the case of competition without reaction, retailer 1 's strategy is fixed (i.e., $x_{1}, p_{1}$ and $S_{1}$ are known) and retailer 2, as a new player in the market, chooses its best strategy (i.e., determines the optimal values of $x_{2}, p_{2}$ and $S_{2}$ ) to maximize its expected profit. Since $x_{1}$ and $p_{1}$ are known, we let $A_{2}^{\prime}=$ $(1-\omega) A+\alpha_{s} p_{1}+\beta_{s}\left(x_{1}+e_{1}\right)$ to simplify notation. Thus, we can write $\lambda_{2}=A_{2}^{\prime}-\delta_{2} p_{2}-\theta_{2} x_{2}$. The formulation of retailer 2's model, denoted here by $\left(M_{2}\right)$, is thus similar to that of the benchmark model ( $M_{0}$ ) while replacing $\lambda_{2}$ with $A_{2}^{\prime}-\delta_{2} p_{2}-\theta_{2} x_{2}$. The optimal solution is provided in Proposition 3.

Proposition 3. In competition without reaction, the optimal decisions for retailer 2 are the following.

$$
x_{2}^{*}=x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}, p_{2}^{*}=\frac{A_{2}^{\prime}-\theta_{2} x_{0}+\frac{3 \theta_{2}^{2}}{4 \delta_{2} b}+\delta_{2}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \delta_{2}}, \text { and } S_{2}^{*}=\frac{(1-r)\left(A_{2}^{\prime}-\theta_{2} x_{0}+\frac{\theta_{2}^{2}}{4 \delta_{2} b}-\delta_{2}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)\right)}{2 r \mu_{2}} .
$$

### 4.3.2. Competition with partial reaction

In this case, retailer 2's decisions are price $p_{2}$, carbon emission $x_{2}$ and order size $S_{2}$. Under the competition of retailer 2 , retailer 1 's decisions are price $p_{1}$ and order size $S_{1}$, but carbon emission $x_{1}$ remains unchanged. Since $x_{1}$ is fixed, we let $A_{1}^{\prime \prime}=\omega A-\theta_{1}\left(e_{1}+x_{1}\right)$ and $A_{2}^{\prime \prime}=(1-\omega) A+\beta_{s}\left(e_{1}+x_{1}\right)$ to simplify notation. Thus, the optimization model for each retailer is given as follows.
$\left(M_{3.1}\right) \max _{S_{1}, p_{1}} \pi_{1}=\left(p_{1}-c_{1}\right) \lambda_{1}-h S_{1}$

Subject to
$\frac{\lambda_{1}}{\lambda_{1}+\mu_{1} S_{1}} \leq r$
$\lambda_{1}=A_{1}^{\prime \prime}-\delta_{1} p_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}$
$S_{1}, p_{1} \geq 0, \quad \lambda_{1} \geq 0$,
$\left(M_{3.2}\right) \max _{x_{2}, S_{2}, p_{2}} \pi_{2}=\left(p_{2}-\left(c_{2}+b\left(x_{0}-x_{2}\right)^{2}\right)\right) \lambda_{2}-h S_{2}$

Subject to

$$
\frac{\lambda_{2}}{\lambda_{2}+\mu_{2} S_{2}} \leq r
$$

$\lambda_{2}=A_{2}^{\prime \prime}-\delta_{2} p_{2}+\alpha_{s} p_{1}-\theta_{2} x_{2}$
$x_{2}, S_{2}, p_{2} \geq 0, \quad x_{2} \leq x_{0}, \quad \lambda_{2} \geq 0$.

We provide in Proposition 4 the optimal strategy of each retailer under the Nash equilibrium.

Proposition 4. The optimum strategy of each retailer in the partial competition case is the following.

- For retailer 1, $p_{1}^{*}=\frac{2 \delta_{2}\left(2 \delta_{1} z_{1}^{\prime}+\alpha_{s} z_{2}^{\prime}\right)}{4 \delta_{1} \delta_{2}-\alpha_{s}^{2}}$, and
$S_{1}^{*}=\frac{(1-r)}{r \mu_{1}}\left(A_{1}^{\prime \prime}+\frac{2 \delta_{2}\left(-2 \delta_{1}^{2}+\alpha_{S}^{2}\right) z_{1}^{\prime}+2 \alpha_{S} \delta_{1}\left(-\delta_{1}+2 \delta_{2}\right) z_{2}^{\prime}}{4 \delta_{1} \delta_{2}-\alpha_{s}^{2}}+\beta_{s}\left(x_{0}-\frac{\theta_{1}}{2 \delta_{1} b}\right)\right)$.
- For retailer 2, $p_{2}^{*}=\frac{2 \delta_{1}\left(2 \delta_{2} z_{2}^{\prime}+\alpha_{s} z_{1}^{\prime}\right)}{4 \delta_{1} \delta_{2}-\alpha_{s}^{2}}, x_{2}{ }^{*}=x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}$, and
$S_{2}^{*}=\frac{(1-r)}{r \mu_{2}}\left(A_{2}^{\prime \prime}+\frac{2 \alpha_{s} \delta_{2}\left(-\delta_{2}+2 \delta_{1}\right) z_{1}^{\prime}+2 \delta_{1}\left(-2 \delta_{2}^{2}+\alpha_{S}^{2}\right) z_{2}^{\prime}}{4 \delta_{1} \delta_{2}-\alpha_{s}^{2}}-\left((1-\omega) \beta_{e}+\beta_{s}\right)\left(x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}\right)\right)$,
where $z_{1}^{\prime}=\frac{A_{1}^{\prime \prime}+\beta_{s}\left(x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}\right)+\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \delta_{1}}$ and $z_{2}^{\prime}=\frac{A_{2}^{\prime \prime}-\theta_{2} x_{0}+\frac{3 \theta_{2}^{2}}{4 \delta_{2} b}+\delta_{2}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \delta_{2}}$.


## 5. Analysis and managerial insights

In this section, we investigate the impact of competition on products' environmental performance as well as the effect of transportation carbon emissions on retailers' strategies. We also discuss the optimal order size decisions. Most of our results are analytically-driven. For a smoother reading of this section, all
proofs are given in the appendix. The set of data used in the different illustrations is the following: $A=$ $1200, c_{1}=10, c_{2}=12, x_{0}=100, e_{1}=20, b=0.01, h=2, \mu_{1}=30, \mu_{2}=45$, and $r=0.05$. For the neutral market, $\alpha_{p}=6, \alpha_{s}=2, \beta_{e}=3$, and $\beta_{s}=1$. For the GDS market, $\alpha_{p}=6, \alpha_{s}=2, \beta_{e}=3$, and $\beta_{s}=2$. For the PDS market, $\alpha_{p}=3, \alpha_{s}=2, \beta_{e}=6$, and $\beta_{s}=2$.

### 5.1. Insights into the impact of competition on products' environmental performance

To understand the impact of competition on greenness, we compare the greenness level of product $\mathcal{P}_{2}$ in competition scenarios to that obtained in the monopoly situation. We recall that the greenness level is measured in terms of carbon intensity and that the carbon intensity of $\mathcal{P}_{2}$ is given by $x_{2}$ (since $e_{2}$ has been normalized to 0 , without loss of generality). We focus on retailer 2's strategy because it is the retailer that undertakes all decisions (i.e., greenness, price and inventory) in all studied situations.

It is first noted that in all competition scenarios (regardless of whether and how retailer 1 reacts), retailer 2 offers $\mathcal{P}_{2}$ with $x_{2}^{*}=x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}=x_{0}-\frac{(1-\omega) \beta_{e}+\beta_{s}}{2 b\left((1-\omega) \alpha_{p}+\alpha_{s}\right)}$. It is interesting to figure out that the carbon emission reduction (i.e., $x_{0}-x_{2}^{*}$ ) depends only on market characteristics and emission reduction cost, but does not depend on other retailer's strategy. The price and inventory decisions, however, depend on whether the other retailer reacts or not and the nature of decisions it makes. Hence, the retailer chooses its greenness strategy based on market characteristics and reacts to other retailer's decisions only by adjusting pricing and inventory policies.

We now compare the carbon intensity obtained in the monopoly situation for retailer 2 (i.e., $x_{2}^{*}=$ $\left.x_{0}-\frac{\beta_{e}}{2 \alpha_{p} b}\right)$ to that resulting from the different competition scenarios (i.e., $\left.x_{2}^{*}=x_{0}-\frac{(1-\omega) \beta_{e}+\beta_{s}}{2 b\left((1-\omega) \alpha_{p}+\alpha_{s}\right)}\right)$. We obtain the following main result presented in Proposition 5.

## Proposition 5.

When the switchovers are more governed by greenness differentiation (i.e., $\frac{\alpha_{s}}{\alpha_{p}}<\frac{\beta_{s}}{\beta_{e}}$ ), the competition leads to offering a greener product.

- When the switchovers are more governed by price differentiation (i.e., $\frac{\alpha_{s}}{\alpha_{p}}>\frac{\beta_{s}}{\beta_{e}}$ ), the competition leads to offering a dirtier product.

In a neutral market (i.e., $\frac{\alpha_{s}}{\alpha_{p}}=\frac{\beta_{s}}{\beta_{e}}$, the competition does not affect the greenness of the product. The retailer offers the product with the same greenness level in both monopoly and competition situations.

Compared to the monopoly situation, one may expect that greenness- and price-based competition will lead to enhance the product's greenness. However, our results show that this holds only for some specific market conditions while, for other types of markets, competition either has no effect or even deteriorates the product's greenness. As we will see afterwards, the market characteristics influence many of our results, so they deserve deeper investigation. We first recall that a unit decrease in carbon intensity of $\mathcal{P}_{2}$ generates customers at a rate of $\beta_{e}+\beta_{s}$ for $\mathcal{P}_{2}$, out of which $\beta_{e}$ are "new" attracted customers and $\beta_{s}$ are "cannibalized" customers, substituting $\mathcal{P}_{2}$ for $\mathcal{P}_{1}$. Similarly, a unit price decrease generates customers at a rate of $\alpha_{p}+\alpha_{s}$ for $\mathcal{P}_{2}$, out of which $\alpha_{p}$ are "new" attracted customers and $\alpha_{s}$ are cannibalized from $\mathcal{P}_{1}$.

In the case of $\frac{\alpha_{s}}{\alpha_{p}}<\frac{\beta_{s}}{\beta_{e}}$, a unit decrease in carbon intensity leads to a higher proportion of cannibalized customers (with respect to "new" customers) than the proportion resulting of a unit decrease in price. Thus, the switchovers are more governed by greenness differentiation rather than price differentiation, which will henceforth be referred to as a greenness-driven switchovers (GDS) market. Notice that GDS condition (i.e., $\frac{\alpha_{s}}{\alpha_{p}}<\frac{\beta_{s}}{\beta_{e}}$ ) is also equivalent to $\frac{\alpha_{s}}{\alpha_{p}+\alpha_{s}}<\frac{\beta_{s}}{\beta_{e}+\beta_{s}}$, which means that the fraction of "cannibalized" demand (with respect to the total demand generated) is higher for a carbon emission reduction rather than a price reduction. Therefore, in GDS market, the switchovers effect is stronger for carbon emission. With the same analysis, we deduce in the case of $\frac{\alpha_{s}}{\alpha_{p}}>\frac{\beta_{s}}{\beta_{e}}$ that the switchovers are more governed by price differentiation, which will henceforth be referred to as a price-driven switchovers
(PDS) market. Finally, when $\frac{\alpha_{s}}{\alpha_{p}}=\frac{\beta_{s}}{\beta_{e}}$, price and greenness disparities have the same importance with respect to switchovers, which is therefore referred to as a neutral market. The above market segregation builds an analysis framework that helps to understand the results of Proposition 5. It will also be used later to explain other results.

Back to the findings of Proposition 5. The results indicate that whether switching of customers is influenced more by the disparity in prices or carbon emissions, governs how the competition influences the optimal greenness strategy. The main difference between the competition case and the monopoly case is the structure of demand. In competition, the retailer's demand is sensitive not only to its own price and carbon intensity level, but also to the price and carbon intensity differentiations with the other retailer as this determines the number of customers switching from one retailer to another. When the switchovers are more governed by greenness differentiation (i.e., in GDS market), there is a fierce competition on greenness and the retailer should therefore capitalize more on greenness performance, which leads to offering a greener product with comparison to the product offered in the monopoly case. In PDS market, in which the switchovers are more governed by price differentiation, the retailer should offer a more competitive price which requires to reducing the cost and, consequently, reducing the greenness performance. This leads to offering a dirtier product (i.e., greater carbon intensity). Finally, in the neutral market, the switchovers are equally governed by price and carbon emission disparities. The impact of competition on greenness is, thus, neutralized, and the retailer offers the product with the same greenness level of the monopoly case.

The above discussion explains how competition impacts the greenness level with comparison to the monopoly case. In what follows, we focus on the impact of the market power on the greenness strategy of the retailer.

[^0]We recall that $\omega$ represents the market power of retailer 1 and, thus, $(1-\omega)$ is the market power of retailer 2 . One may expect that a retailer that sees a decrease in its market power (i.e., for retailer $1, \omega$ decreases and, for retailer $2, \omega$ increases) will decrease its price and, thus, will offer a dirtier product. However, our results indicate that the retailer can have two different reactions according to market characteristics. These results are illustrated in Figure 3. We see that a decrease in the market power of retailer 1 (i.e., decrease in $\omega$ ) leads to a decrease in $x_{1}^{*}$ (i.e., more greenness) in GDS market and an increase in $x_{1}^{*}$ (i.e., less greenness) in PDS market. For retailer 2, a decrease in the market power (i.e., increase in $\omega$ ) leads a decrease in $x_{2}^{*}$ (i.e., more greenness) in GDS market and an increase in $x_{2}^{*}$ (i.e., less greenness) in PDS market.


Figure 3. Optimal carbon emissions in function of market power

Proposition 6 provided insights into the greenness strategy of a retailer that has less market power than what it had before (this does not necessarily mean that this retailer will have less market power than the other retailer). In what follows, we explain the retailer's strategy under each market structure to understand the result of Proposition 6. In GDS market, a retailer with decreasing market power needs to offer a greener product in order to attract more switching customers since switchovers are here governed by greenness. Although offering a greener product increases the unit cost, it is the best strategy here
because the priority is given to maintaining a profitable amount of demand. In PDS market, a retailer with decreasing market power needs to offer a cheaper product in order to attract more switching customers since switchovers are here governed by price. Offering a cheaper product is obtained by decreasing the unit cost which also implies reducing the greenness level.

We now compare the optimal carbon emissions $x_{1}^{*}$ and $x_{2}^{*}$ to determine which retailer buys the greener product (i.e., asks its supplier to provide the greener product). Notice that $x_{2}^{*}>x_{1}^{*}$ means that retailer 1 buys a greener product than the product bought by retailer 2. However, this does not necessarily mean that retailer 1 offers a greener product to its customers because we must add the transportation carbon emissions $e_{1}$ for $\mathcal{P}_{1}$ while we have $e_{2}=0$ for $\mathcal{P}_{2}$. We consider here the general competition model, in which both retailers undertake greenness decisions. The result is given in Proposition 7.

Proposition 7. The retailer that has a smaller market power buys a greener product (than the product bought by the other retailer) under GDS market, and a dirtier product (than the product bought by the other retailer) under PDS market.

This result can be observed in Figure 3 above. For instance, when retailer 1 has less market power (i.e., $\omega<0.5$ ), we see in Figure 3 that $x_{1}^{*}<x_{2}^{*}$ in GDS market and $x_{1}^{*}>x_{2}^{*}$ in PDS market. We now provide a qualitative explanation of Proposition 7. If a given retailer (let us say retailer 2) has a smaller market power, then it needs to increase its market share. Thus, in GDS market, retailer 2 buys a greener product than the product bought by retailer 1 in order to convince more customers to substitute $\mathcal{P}_{2}$ for $\mathcal{P}_{1}$ since switchovers are, in this case, more governed by greenness. In PDS market, retailer 2 adopts an aggressive pricing strategy to attract more switching customers. Consequently, retailer 2 needs to decrease the cost and, thus, reduce its greenness level, which results in buying a dirtier product than the product bought by retailer 1. The result of Proposition 7 also means that the retailer that has a larger market power (let us say retailer 1) buys a dirtier product in GDS market and a greener product in PDS market.

In the case where retailer 2 enters a market in which retailer 1 is already established, we can imagine that retailer 2 will start with a smaller market power. In this case, retailer 2 will buy a greener product than retailer 1's product under GDS market and a dirtier product under PDS market. Thus, a new retailer entering a greenness- and price-sensitive market will not always buy a greener product than the product bought by the existing retailer, but this depends on its market power and the market type (GDS market or PDS market). This is an interesting result because one may expect that a new retailer will always have a more aggressive greenness strategy.

### 5.2. Insights into the effect of disparity in transportation carbon emissions on retailers' strategies

We now investigate the impact of disparity in transportation carbon emissions. We recall that retailer 1 has the farthest supplier and, thus, generates more carbon emissions in transportation than retailer 2. Hence, without loss of generality, we normalized $e_{2}$ to 0 and, thus, the transporation emissions per unit of $\mathcal{P}_{1}$, namely $e_{1}$, refers to the disparity in transportation carbon emissions. Moreover, given that retailer 1 relies on a longer SC (more distant supplier), we have $\mu_{1} \leq \mu_{2}$. In the previous analysis, we did not impose any condition on base costs $c_{1}$ and $c_{2}$. Here, we make the realistic assumption that the retailer with a longer SC has a smaller base cost and, thus, a smaller purchasing cost if both products have the same emissions intensity (we recall that the purchasing costs are $c_{1}+b\left(x_{0}-x_{1}\right)^{2}$ and $c_{2}+b\left(x_{0}-\right.$ $\left.x_{2}\right)^{2}$ ). Therefore, we have here $c_{1} \leq c_{2}$. This is the typical situation when one retailer relies on a low-cost abroad supplier whereas the other retailer relies on a local but more expensive supplier, which is a common practical situation (Kandil et al. 2020, Aissaoui et al. 2007, Hammami and Frein 2014b). We consider here the general competition model. We have seen that the transportation emissions do not affect $x_{1}^{*}$ and $x_{2}^{*}$. However, they impact the pricing decisions. Hence, we study the impact of disparity in transportation carbon emissions on pricing. The main result is presented in Proposition 8.

Proposition 8. When both retailers have the same market power, the retailer that relies on a shorter SC offers a higher price if and only if the difference in transportation carbon emissions (i.e., $e_{1}$ ) is higher than a threshold value $e_{0}=\frac{\left(\frac{\alpha_{p}}{2}+\alpha_{s}\right)\left(\left(\begin{array}{ll}\frac{1}{\mu_{1}} & \left.\frac{1}{\mu_{2}}\right)\end{array}\right) \frac{(1-r) h}{r}-\left(c_{2}-c_{1}\right)\right)}{\left(\frac{\beta_{e}}{2}+2 \beta_{s}\right)}$.

As long as the disparity in transportation emissions (i.e., $e_{1}$ ) is smaller than $e_{0}$, the higher the disparity, the smaller the price differentiation between products. When the disparity in transportation emissions is greater than $e_{0}$, the higher the disparity, the higher the price differentiation between products.

One may expect that the retailer that has a shorter SC and bears a higher base cost (here, retailer 2) will always offer a higher price than the other retailer. Our findings indicate that retailer 2 offers a higher price than retailer 1 when $e_{0} \leq 0$, which is equivalent to $\left(\frac{1}{\mu_{1}}-\frac{1}{\mu_{2}}\right) \frac{(1-r) h}{r} \leq\left(c_{2}-c_{1}\right)$. In other words,
the retailer that has a shorter SC will always offer a higher price when the difference in cost is too high (i.e., greater than $\left(\frac{1}{\mu_{1}}-\frac{1}{\mu_{2}}\right) \frac{(1-r) h}{r}$ ). However, if the difference in cost is not that big (i.e., $c_{2}-c_{1} \leq$ $\left.\left(\frac{1}{\mu_{1}}-\frac{1}{\mu_{2}}\right) \frac{(1-r) h}{r}\right)$, then $e_{0}$ is positive. In this case, we may have two scenarios, $e_{1} \leq e_{0}$ or $e_{1}>e_{0}$. In the case of $e_{1}>e_{0}$, retailer 2 still offers a higher price than retailer 1 . However, in the case of $e_{1} \leq e_{0}$, retailer 2 offers a smaller price despite its higher base cost. The qualitative explanation is the following. A small disparity in transportation carbon emissions implies that retailer 2 should do more effort to offer a product with an attractive carbon intensity level that enables to attract a profitable amount of greennesssensitive customers. However, this increases the cost and reduces the net margin. Hence, it becomes more profitable for retailer 2 to capitalize on price-sensitive customers and, thus, offer a lower price than retailer 1.

As for the impact of the disparity in transportation carbon emissions on the price differentiation between products, it can be verified that $p_{1}^{*}$ is decreasing in $e_{1}$, whereas $p_{2}^{*}$ is increasing in $e_{1}$. When $e_{1}<$ $e_{0}, \mathcal{P}_{2}$ is offered at a lower price than $\mathcal{P}_{1}$. A higher disparity in transportation emissions (i.e., an increase in $e_{1}$ ) will increase $p_{2}^{*}$ and decrease $p_{1}^{*}$, which closes the price gap. However, when $e_{1}>e_{0}, \mathcal{P}_{2}$ is offered at a higher price than $\mathcal{P}_{1}$, which implies that a higher disparity in transportation emissions will increase the price gap. This is illustrated in Figure 4.


### 5.3. Insights into order size decisions

We have obtained complex expressions of the optimal order size for each retailer. Theses expressions show that the inventory policy is impacted by all the input parameters of the problem (market characteristics, cost factors, supply characteristics, etc.). It is known in the inventory literature that the order size of a given retailer highly depends on its inventory refilling rate. In our context, we observe that the order size also depends on the refilling rate of the other retailer, as highlighted in the following proposition.

Proposition 9. An increase in the inventory refilling rate of one retailer not only leads to reducing the optimal order size of this retailer but also to reducing the optimal order size of the other retailer.

In Figure 5, we increase retailer 1's refilling rate $\left(\mu_{1}\right)$ and represent the optimal order size of each retailer. This provides an illustration of Proposition 9. Note that we obtain the same behavior when we increase $\mu_{2}$.


Figure 5. Effect of $\mu_{1}$ on retailers' optimal order sizes

An increase in the inventory refilling rate of a given retailer implies a shorter expected lead time between the moment when the retailer places the order and when it receives the products from the supplier. This means that the retailer can still satisfy the service level constraint (expressed in terms of maximum stock out probability level) while holding a lower stock. It is therefore expected that this retailer will reduce its order size as this allows to reducing the inventory cost. However, the inventory refilling rate of the other retailer does not change and, thus, it is not intuitive that this second retailer will also reduce its order size (even though this order size reduction is relatively small). This reaction is due to the price competition. Indeed, with an increase in its inventory refilling rate, the first retailer will decrease its price as its inventory cost goes down. This leads the other retailer to react and decrease its price (to limit the switchovers) but this price reduction is smaller than that of the first retailer. Hence, the second retailer will necessarily lose demand. Thus, it will need less inventory level which implies a smaller order size.

### 5.4. Insights into the impact of competition on retailers' optimal profits

In this section, we aim to compare the optimal retailers' profits under the different competition scenarios. Given the complexity of the expressions of optimal profits, we base our analysis on numerical experiments. We conducted experiments to assess how the optimal profit of each retailer varies in function of the market power under the different competition scenarios. To obtain robust results, we considered different values of models parameters under the different market types (PDS, GDS, and neutral market). We tested 10 instances for each market type. In all cases, we obtained the same behavior for each retailer. This behavior is illustrated in Figures 6 and 7 for the existing retailer and the new retailer, respectively.


Figure 6. Existing retailer's profit under the different competition scenarios


Figure 7. New retailer's profit under the different competition scenarios

When a new retailer enters the market, the existing retailer obtains the highest profit if it adopts a full reaction by adjusting its pricing, greenness and inventory policies (see Figure 6). We also observe that the partial reaction scenario is preferred to the scenario without reaction. All these observations are expected. As for the new retailer entering the market, we see in Figure 7 that this retailer makes the highest profit when the existing retailer does not react, which is also expected.

Less intuitive is that a full reaction of the existing retailer can be more beneficial for the new retailer than a partial reaction (see Figure 7). This result deserves a deeper investigation. Under partial reaction, the existing retailer reacts by adjusting its price and order size. Thus, as the existing retailer does not change its product's greenness level, this retailer is obliged to adopt an aggressive pricing strategy to avoid a big loss of demand. This leads the new retailer to react by reducing its price, and finally leads to a significant price decrease for the new retailer at the equilibrium. This has a significant impact on new retailer's profit since its market potential is relatively low. However, under a full reaction scenario, the price reduction performed by the existing retailer is not that high, which incurs a smaller price reduction for the new retailer and leads to a relatively higher profit.

## 6. Practical implications and examples

This section discusses the practical implications of our main findings to the different stakeholders. Most of our results depend on the market type (PDS or GDS). We recall that a PDS market characterizes the case where switchovers are more governed by price differentiation than greenness differentiation, which does not necessarily mean that customers are less sensitive to greenness but may refer to situations where it is not possible for the customers to compare the greenness levels of products. For instance, for many products, there is no green labeling or the labeling does not allow customers to make reliable comparisons, such as when firms just highlight that the product is made from sustainable materials (e.g., Matt \& Nat handbags and wallets, some products offered by H\&M and Zara) or with an environmentallyfriendly process (e.g., Lobodis coffee). In these cases, the customers can compare the products only in terms of price, and switchovers will then be more governed by price differentiation (PDS market) even in the presence of environmentally-conscious customers. However, in a market characterized by a high customers' environmental awareness (e.g., agribusiness) and the possibility for customers to compare the greenness levels of products, switchovers can be more governed by greenness differentiation (GDS market). This motivated many agribusiness companies to focus on offering greener products while simplifying the greenness comparison for consumers. For instance, the British firm Innocent indicates the
carbon footprint of some fruit smoothies on the packaging (Hammami et al. 2018). A GDS market may also correspond to products for which the greenness level implies a lower use cost (e.g., energy saving). The example of the environmentally-friendly bulb of the Philips-Carrefour SC in Europe, provided by Hong et al. (2019), is a good one. For this product, greenness is associated with energy labeling, which is a color-coded performance scale from G to A+++. The customers use this scale to compare green lamps with the conventional ones, and this comparison significantly influences their purchasing decisions (Hong et al. 2019).

This study showed that greenness- and price-based competition leads the retailer to offering a greener product in GDS market than the product it would have offered in a monopolistic situation, and to offering a dirtier product in PDS market. This leads to the following practical implication. A retailer that faces a greenness- and price-based competition will not offer a greener product than a monopolistic retailer if customers are not able to compare objectively the environmental performances of competing products. This raises the role of policy-makers. In fact, if the policy-maker imposes green labeling that can allow customers to compare products, then this can transform the customers' environmental awareness into a real purchase decision and, thus, transform the market into a GDS market. Thus, even in the presence of environmentally-conscious customers, government intervention is required to make the competition a driver for a better environmental performance.

We also found that the size of the retailer in the market plays a key role and determines its greenness strategy. In PDS markets, the power of retailers is a driver for a better greenness performance. Hence, the powerful retailers will be the ones that lead the the market transformation towards greener products. This corresponds to many practical situations as described earlier (Walmart, Bestbuy, Carrefour, etc.). However, if the customers have the possibility to compare the products' greenness, then we can move from PDS to GDS markets. In this case, small entrant retailers will have more interest in offering greener products.

Finally, we showed that existing retailers should fully react to new retailers by adjusting price, greenness, and inventory strategies. In addition, this full reaction of the existing retailer leads to a better profit for the new retailer than a price- and inventory-based reaction that keeps the products' greenness unchanged. Hence, it is in the interest of new retailers that existing retailers are able to improve the greenness of their products.

## 7. Conclusion

While considering two competing retailers that offer substitutable products, this study focused on the effect of greenness- and price-based competition on the environmental performance of products. We found that when switchovers are more governed by greenness differentiation (GDS market), the competition leads to offering a greener product (compared to the monopoly case). However, when switchovers are more governed by price differentiation (PDS market), the competition leads to offering a dirtier product. Moreover, a retailer that gains more market power will decrease its product's greenness in GDS market and increase it in PDS market. Our results also indicated that the retailer that has a smaller market power will buy a greener product (than the product bought by the other retailer) when switchovers are more governed by greenness differentiation, and will buy a dirtier product when switchovers are more governed by price differentiation. Hence, a new retailer entering a greenness- and price-sensitive market will not always buy a greener product than the product bought by the existing retailer, but this depends on its market power. We investigated the impact of disparity in transportation carbon emissions on the optimal strategy of retailers. We found that when both retailers have the same market power, the retailer that relies on a shorter supply chain offers a higher price if and only if the difference in transportation carbon emissions is higher than a given threshold value.

Our modeling effort and analysis come with limitations that can provide directions for further research in the area. We assumed that the demand occurring during the replenishment time is delivered from a safety stock but did not manage this safety stock. Future research can improve our inventory policy by accounting for the safety stock and its replenishment. We also assumed that the replenishment
time does not depend on the order size. Another direction for future research is to consider that a higher order size leads to a longer replenishment time. Furthermore, we considered random demand and random replenishment time with specific distribution functions. It may be interesting to test other distribution functions. Our results are also based on the assumption of linear demand with substitution, which is a common assumption in the related literature. Other demand functions may change some of our results. Finally, future research can investigate the case of a single retailer that offers two substitutable products and compare the results to those obtained in this study.

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## Appendices

Appendix A. We consider the inventory system of a given retailer $i$, but remove subscript $i$ to simplify notation. The different inventory states are illustrated in the following figure.


Let's define $\psi_{j}$ which stands for the probability of system in state $j$ (i.e., probability of having $j$ items in the stock). Since customers' arrival and inventory refilling service time follow the exponential distribution with mean rates $\lambda$ and $\mu$, respectively, our stochastic system is a Markovian process. Thus, for each state $j$, the rate out of state $j$ is equal to rate into state $j$. Therefore we have for state $0, \mu \psi_{0}=$ $\lambda \psi_{1} \leftrightarrow \psi_{1}=\frac{\mu}{\lambda} \psi_{0}$. For state $j \in[1, S-1], \lambda \psi_{j}=\lambda \psi_{j+1} \leftrightarrow \psi_{i}=\psi_{j+1}$. For state $S, \mu \psi_{0}=\lambda \psi_{S} \leftrightarrow \psi_{S}=$ $\frac{\mu}{\lambda} \psi_{0}$. The summation of all states probabilities must be equal to 1 . Therefore, $\sum_{j=0}^{S} \psi_{j}=1 \leftrightarrow \psi_{0}+$ $\frac{\mu}{\lambda} \psi_{0}+\frac{\mu}{\lambda} \psi_{0}+\cdots+\frac{\mu}{\lambda} \psi_{0}=1 \leftrightarrow \psi_{0}\left(1+S \frac{\mu}{\lambda}\right)=1 \leftrightarrow \psi_{0}=\left(1+S \frac{\mu}{\lambda}\right)^{-1}$.

Appendix
B. $\quad R_{1} \geq x_{0} \leftrightarrow x_{0}+\frac{-\beta_{e}+\sqrt{\Delta_{2}}}{2 \alpha_{p} b} \geq x_{0} \leftrightarrow-\beta_{e}+\sqrt{\Delta_{2}} \geq 0 \leftrightarrow 4 \alpha_{p} b\left(A-\beta_{e} x_{0}-\alpha_{p}\left(c_{i}+\right.\right.$ $\left.\left.\frac{h(1-r)}{r \mu_{i}}\right)\right) \geq 0$. The latter condition is verified since $A$ is sufficiently large.
$R_{2} \leq 0 \leftrightarrow x_{0}-\frac{\beta_{e}+\sqrt{\Delta_{2}}}{2 \alpha_{p} b} \leq 0 \leftrightarrow \sqrt{\Delta_{2}} \geq 2 \alpha_{p} b x_{0}-\beta_{e} \leftrightarrow 4 \alpha_{p} b\left(A-\alpha_{p} b x_{0}{ }^{2}-\alpha_{p}\left(c_{i}+\frac{h(1-r)}{r \mu_{i}}\right)\right) \geq 0$. The latter condition is verified since $A$ is sufficiently large.

Proof of Lemma 1. For given $p_{i}$ and $x_{i}$, the objective function is linearly decreasing in $S_{i}\left(\frac{\partial \pi}{\partial S_{i}}=-h<\right.$ 0). Hence, the optimal order size is the smallest feasible one. Constraint (6) imposes that $\frac{\lambda_{i}}{\lambda_{i}+\mu_{i} S_{\mathrm{i}}} \leq r$, which is equivalent to $S_{i} \geq \frac{1-r}{r \mu_{i}}\left(A-\alpha_{p} p_{i}-\beta_{e} x_{i}\right)$. Therefore, the optimal order size $S_{i}^{*}\left(p_{i}, x_{i}\right)=$ $\frac{(1-r)\left(A-\alpha_{p} p_{i}-\beta_{e} x_{i}\right)}{r \mu_{i}}$

Proof of lemma 2. For a given $x_{i}, \frac{\partial^{2} \pi}{\partial p_{i}^{2}}=-2 \alpha_{p}<0$. The objective function is therefore concave in $p_{i}$. According to the first derivative condition, $\pi\left(p_{i}\right)$ reaches its maximum in $p_{i}^{\max }=$ $\frac{\alpha_{p} b x_{i}^{2}-\left(2 \alpha_{p} b x_{0}+\beta_{e}\right) x_{i}+A+\alpha_{p}\left(c_{i}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{i}}\right)}{2 \alpha_{p}}$. Thus, if the following conditions $(a)$ and $(b)$ hold, then $p_{i}^{\text {max }}$ is feasible, which implies that $p_{i}^{\text {max }}$ is the optimal price. Condition $(a) \leftrightarrow p_{i}^{\max } \geq 0$, and condition $(b) \leftrightarrow$ $\lambda_{i}\left(p_{i}^{\max }\right)=A-\alpha_{p} p_{i}^{\max }-\beta_{e} x_{i} \geq 0$.
(a) $p_{i}^{\text {max }} \geq 0 \leftrightarrow \alpha_{p} b x_{i}^{2}-\left(2 \alpha_{p} b x_{0}+\beta_{e}\right) x_{i}+A+\alpha_{p}\left(c_{i}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{i}}\right) \geq 0$. We obtain by standard calculus the discriminant of this quadratic equation, $\Delta_{1}=-4 \alpha_{p} b\left(A-\beta_{e} x_{0}-\frac{\beta_{e}{ }^{2}}{4 \alpha_{p} b}+\alpha_{p}\left(c_{i}+\frac{h(1-r)}{r \mu_{i}}\right)\right)$. The market potential $A$ is very large in practice and typically assumed to be sufficiently large in the related literature. We therfore have $A>\beta_{e} x_{0}+\frac{\beta_{e}{ }^{2}}{4 \alpha_{p} b}-\alpha_{p}\left(c_{i}+\frac{h(1-r)}{r \mu_{i}}\right)$ in the relevant case. Thus, $\Delta_{1}<$ 0 and, consequently, $\alpha_{p} b x_{i}^{2}-\left(2 \alpha_{p} b x_{0}+\beta_{e}\right) x_{i}+A+\alpha_{p}\left(c_{i}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{i}}\right) \geq 0$. Hence, condition (a) holds.
(b) $\lambda_{i}\left(p_{i}^{\max }\right) \geq 0 \leftrightarrow-\alpha_{p} b x_{i}^{2}+\left(2 \alpha_{p} b x_{0}-\beta_{e}\right) x_{i}+A-\alpha_{p}\left(c_{i}+b x_{0}^{2}+\frac{h(1-r)}{r \mu_{i}}\right) \geq 0$. The discriminant of this quadratic equation is $\Delta_{2}=4 \alpha_{p} b\left(A-\beta_{e} x_{0}+\frac{\beta_{e}{ }^{2}}{4 \alpha_{p} b}-\alpha_{p}\left(c_{i}+\frac{h(1-r)}{r \mu_{i}}\right)\right)$. Given that $A$ is sufficiently large (as explained earlier), we obtain $\Delta_{2} \geq 0$ and, thus, the above equation has two roots,
$R_{1}=x_{0}+\frac{-\beta_{e}+\sqrt{\Delta_{2}}}{2 \alpha_{p} b}$ and $R_{2}=x_{0}-\frac{\beta_{e}+\sqrt{\Delta_{2}}}{2 \alpha_{p} b}$. Condition (b) holds if and only if $R_{2} \leq x_{i} \leq R_{1}$. We prove in Appendix $B$ that $R_{1} \geq x_{0}$ and $R_{2} \leq 0$. Therefore, condition (b) holds for any $x_{i}$ in $\left[0, x_{0}\right]$.

Thus, we verified that conditions (a) and (b) hold, which demonstrates that $p_{i}^{\max }$ is feasible and, consequently, optimal

Proof of proposition 1. It can be verified by standard calculus that the objective function (equation 9) is equal to $\frac{\lambda_{i}{ }^{2}}{\alpha_{p}}$. Since $\lambda_{i} \geq 0$ (as guaranteed by the optimal price identified in Lemma 2), maximizing $\frac{\lambda_{i}{ }^{2}}{\alpha_{p}}$ is equivalent to maximizing $\lambda_{i}$. We have $\frac{\partial^{2} \lambda_{i}}{\partial x_{i}{ }^{2}}=-\alpha_{p} b<0$, which means that $\lambda_{i}$ is concave in $x_{i}$. Hence, we use the first derivative condition and deduce that $\lambda_{i}$ reaches its maximum in $x_{i}^{m a x}=x_{0}-\frac{\beta_{e}}{2 \alpha_{p} b}$. Obviously, we have $x_{i}^{\max } \leq x_{0}$. Moreover, the case of $x_{i}^{\max }<0$ (i.e., $x_{0}<\frac{\beta_{e}}{2 \alpha_{p} b}$ ) does not worth study since this implies that $x_{i}^{*}=0$. Hence, we have $0 \leq x_{i}^{\max } \leq x_{0}$ in the relevant case and, thus, $x_{i}^{*}=x_{0}-$ $\frac{\beta_{e}}{2 \alpha_{p} b}$.

By substituting $x_{i}^{*}$ into the formula of price and stock (see Lemma 2 and 1, respectively), we obtain by standard calculus the final expressions of $p_{i}^{*}$ and $S_{i}^{*}$ as given in Proposition 1 -

Proof of Lemma 3. For given $p_{2}, x_{2}, S_{2}$, and $x_{1}$ we have $\frac{\partial^{2} \pi_{1}}{\partial\left(p_{1}\right)^{2}}=-2 \delta_{1}<0$. Thus, $\pi_{1}$ is concace in $p_{1}$. According to the first derivative condition, we deduce that $\pi_{1}$ is maximized in $p_{1}^{\max }=$ $\frac{\delta_{1} b x_{1}{ }^{2}-\left(\theta_{1}+2 \delta_{1} b x_{0}\right) x_{1}+A_{1}+\alpha_{s} p_{2}+\beta_{s} x_{2}+\delta_{1}\left(c_{1}+b x_{0}{ }^{2}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \delta_{1}}$. Therefore, if $p_{1}^{\text {max }}$ verifies $p_{1}^{\text {max }} \geq 0$ and $\lambda_{1}\left(p_{1}^{\text {max }}\right) \geq 0$, then it is the optimal price. With the same approach used in the proof of Lemma 2 , we demonstrate that these two conditions hold for any $x_{1}$ in $\left[0, x_{0}\right]$ (for a concise presentation, we do not give the detailed proof). Hence, $p_{1}^{\max }$ is the optimal price

Proof of Lemma 4. It can be verified by standard calculus that objective function (11) is equal to $\frac{\lambda_{1}^{2}}{\delta_{1}}$. Since $\lambda_{1} \geq 0$ (as guaranteed by the optimal price identified in Lemma 3), maximizing $\frac{\lambda_{1}^{2}}{\delta_{1}}$ is equivalent to maximizing $\lambda_{1}$. We have $\frac{\partial^{2} \lambda_{1}}{\partial x_{1}{ }^{2}}=-\delta_{1} b<0$, which means that $\lambda_{1}$ is concave in $x_{1}$. Hence, we use the first derivative condition and deduce that $\lambda_{1}$ reaches its maximum in $x_{1}^{\max }=x_{0}-\frac{\theta_{1}}{2 \delta_{1} b}$. Obviously, we have $x_{1}^{\max } \leq x_{0}$. Moreover, the case of $x_{1}^{\max }<0$ (i.e., $x_{0}<\frac{\theta_{1}}{2 \delta_{1} b}$ ) does not worth study since this implies that $x_{1}^{*}=0$. Hence, we have $0 \leq x_{1}^{\max } \leq x_{0}$ in the relevant case and, thus, $x_{1}^{*}=x_{0}-\frac{\theta_{1}}{2 \delta_{1} b}$. By substituting $x_{1}^{*}$ into the formula of price (see Lemma 3), we obtain the expression of $p_{1}^{*}$ in function of retailer 2's decisions

Proof of Lemma 5. Same approach as Lemma 3 and 4

Proof of Proposition 2. We know that the optimal prices $p_{1}^{*}$ and $p_{2}^{*}$ are obtained at the intersection of the two best response curves. If we take the optimal price $p_{1}^{*}$, we can determine its associated $p_{2}^{*}\left(p_{1}^{*}\right)$ by using equation (13). The optimal price of $\mathcal{P}_{1}$ that is associated with this $p_{2}^{*}\left(p_{1}^{*}\right)$ is, on the one hand, obtained by equation (12) and, on the other hand, equal to $p_{1}^{*}$ since we are at the intersection point. Consequently, it comes that $\frac{\alpha_{S}}{2 \delta_{1}}\left[\frac{\alpha_{S}}{2 \delta_{2}} p_{1}^{*}+\frac{A_{2}+\left(\beta_{s}-\theta_{2}\right) x_{0}+\frac{3 \theta_{2}{ }^{2}}{4 \delta_{2} b}-\frac{\beta_{s} \theta_{1}}{2 \delta_{1} b}+\delta_{2}\left(c_{2}+\frac{h(1-r)}{r \mu_{2}}\right)}{2 \delta_{2}}\right]+$ $\frac{A_{1}+\left(\beta_{s}-\theta_{1}\right) x_{0}+\frac{3 \theta_{1}{ }^{2}}{4 \delta_{1} b}-\frac{\beta_{s} \theta_{2}}{2 \delta_{2} b}+\delta_{1}\left(c_{1}+\frac{h(1-r)}{r \mu_{1}}\right)}{2 \delta_{1}}=p_{1}^{*}$. We then deduce by standard calculus that $p_{1}^{*}=\frac{2 \delta_{2}\left(2 \delta_{1} z_{1}+\alpha_{s} z_{2}\right)}{4 \delta_{1} \delta_{2}-\alpha_{s}{ }^{2}}$ and, consequently, $p_{2}^{*}=\frac{2 \delta_{1}\left(2 \delta_{2} z_{2}+\alpha_{s} z_{1}\right)}{4 \delta_{1} \delta_{2}-\alpha_{s}{ }^{2}}$.

The optimal carbon emissions are given in Lemma 4 and 5. The optimal order sizes are deduced from the formula $S_{1}^{*}\left(p_{1}, x_{1}\right)=\frac{(1-r)\left(A_{1}-\delta_{1} p_{1}+\alpha_{s} p_{2}-\theta_{1} x_{1}+\beta_{s} x_{2}\right)}{r \mu_{1}}$ and $S_{2}^{*}\left(p_{2}, x_{2}\right)=\frac{(1-r)\left(A_{2}-\delta_{2} p_{2}+\alpha_{s} p_{1}-\theta_{2} x_{2}+\beta_{s} x_{1}\right)}{r \mu_{2}}$ by replacing $p_{1}, x_{1}, p_{2}$ and $x_{2}$ with their optimal expressions

Proof of Proposition 3. Deduced from the general model.

Proof of Proposition 4. Deduced from the general model.

Proof of Proposition 5. The difference between the optimal carbon emissions of retailer 2 in competition and monopoly is given by $\left(x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}\right)-\left(x_{0}-\frac{\beta_{e}}{2 \alpha_{p} b}\right)=\frac{1}{2 b}\left(\frac{\beta_{e}}{\alpha_{p}}-\frac{\theta_{2}}{\delta_{2}}\right)=\frac{1}{2 b}\left(\frac{\beta_{e}}{\alpha_{p}}-\frac{(1-\omega) \beta_{e}+\beta_{s}}{(1-\omega) \alpha_{p}+\alpha_{s}}\right)=$ $\frac{1}{2 b}\left(\frac{\beta_{e} \alpha_{S}-\alpha_{p} \beta_{s}}{\alpha_{p}\left((1-\omega) \alpha_{p}+\alpha_{s}\right)}\right)$. Since $\alpha_{p}\left((1-\omega) \alpha_{p}+\alpha_{s}\right)>0$, the sign of $\frac{1}{2 b}\left(\frac{\beta_{e} \alpha_{s}-\alpha_{p} \beta_{s}}{\alpha_{p}\left((1-\omega) \alpha_{p}+\alpha_{s}\right)}\right)$ is given by the sign of $\beta_{e} \alpha_{s}-\alpha_{p} \beta_{s}$. The results of Proposition 5 follow immediately.

Proof of Proposition 6. For retailer 2, we have $x_{2}^{*}=x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}=x_{0}-\frac{(1-\omega) \beta_{e}+\beta_{s}}{2 b\left((1-\omega) \alpha_{p}+\alpha_{s}\right)}$. Retailer 2 has more market power when $\omega$ decreases. We have $\frac{\partial x_{2}^{*}}{\partial \omega}=\frac{\left(\alpha_{s} \beta_{e}-\alpha_{p} \beta_{s}\right)}{2 b\left((1-\omega) \alpha_{p}+\alpha_{s}\right)^{2}}$. Therefore, $\frac{\partial x_{2}^{*}}{\partial \omega}<0$ in GDS market (i.e., when $\frac{\alpha_{s}}{\alpha_{p}}<\frac{\beta_{s}}{\beta_{e}}$ ) and $\frac{\partial x_{2}^{*}}{\partial \omega}>0$ in PDS market. Hence, an increase in market power for retailer 2 leads to an increase in $x_{2}^{*}$ in GDS market and a decrease in $x_{2}^{*}$ in PDS market, as stated in Proposition 6. For retailer $1, x_{1}^{*}=x_{0}-\frac{\theta_{1}}{2 \delta_{1} b}=x_{0}-\frac{\omega \beta_{e}+\beta_{s}}{2 b\left(\omega \alpha_{p}+\alpha_{s}\right)}$. Retailer 1 has more market power when $\omega$ increases. We have $\frac{\partial x_{1}^{*}}{\partial \omega}>0$ in GDS market and $\frac{\partial x_{1}^{*}}{\partial \omega}<0$ in PDS market. The results for retailer 1 follow immediately.

Proof of Proposition 7. We have $x_{2}^{*}-x_{1}^{*}=\left(x_{0}-\frac{\theta_{2}}{2 \delta_{2} b}\right)-\left(x_{0}-\frac{\theta_{1}}{2 \delta_{1} b}\right)=\frac{1}{2 b}\left(\frac{(1-2 \omega)\left(\alpha_{p} \beta_{s}-\beta_{e} \alpha_{s}\right)}{\left(\omega \alpha_{p}+\alpha_{s}\right)\left((1-\omega) \alpha_{p}+\alpha_{s}\right)}\right)$. We know that $\left(\omega \alpha_{p}+\alpha_{s}\right)\left((1-\omega) \alpha_{p}+\alpha_{s}\right)>0$. Hence, the sign of $x_{2}^{*}-x_{1}^{*}$ is given by the sign of $(1-2 \omega)\left(\alpha_{p} \beta_{s}-\beta_{e} \alpha_{s}\right)$. Notice that $\omega>\frac{1}{2}$ (respectively, $\omega<\frac{1}{2}$ ) means that retailer 1 has more power (respectively, less power). Thus, for $\omega>\frac{1}{2}$, we have $x_{2}^{*}>x_{1}^{*}$ if $\frac{\alpha_{s}}{\alpha_{p}}>\frac{\beta_{s}}{\beta_{e}}$ (PDS market) and $x_{2}^{*}<$ $x_{1}^{*}$ if $\frac{\alpha_{s}}{\alpha_{p}}<\frac{\beta_{s}}{\beta_{e}}\left(\right.$ GDS market). for $\omega<\frac{1}{2}$, we have $x_{2}^{*}>x_{1}^{*}$ if $\frac{\alpha_{s}}{\alpha_{p}}<\frac{\beta_{s}}{\beta_{e}}$ (GDS market) and $x_{2}^{*}<$ $x_{1}^{*}$ if $\frac{\alpha_{s}}{\alpha_{p}}>\frac{\beta_{s}}{\beta_{e}}$ (PDS market). This demonstrates the results of Prorposition 7.

Proof of Proposition 8. $p_{2}^{*}>p_{1}^{*} \leftrightarrow \frac{2 \delta_{1}\left(\alpha_{s} z_{1}+2 \delta_{2} z_{2}\right)}{4 \delta_{1} \delta_{2}-\alpha_{S}{ }^{2}}>\frac{2 \delta_{2}\left(2 \delta_{1} z_{1}+\alpha_{s} z_{2}\right)}{4 \delta_{1} \delta_{2}-\alpha_{S}{ }^{2}} \leftrightarrow \frac{z_{2}}{z_{1}}>\frac{2 \delta_{1} \delta_{2}-\delta_{1} \alpha_{s}}{2 \delta_{1} \delta_{2}-\delta_{2} \alpha_{s}}$. When both retailers have the same market power, we have $\omega=0.5$ and $\delta_{1}=\delta_{2}$. Therefore, the above condition can be simplified as $\frac{z_{2}}{z_{1}}>1$, which is equivalent to $A_{2}-A_{1}+\delta_{2}\left(c_{2}-c_{1}+\frac{(1-r) h}{r}\left(\frac{1}{\mu_{2}}-\frac{1}{\mu_{1}}\right)\right)>0$. We know that $A_{2}-A_{1}=\left(\omega \beta+2 \beta_{s}\right) e_{1}$. Hence, the above inequality is equivalent to $\left(0.5 \beta_{e}+2 \beta_{s}\right) e_{1}+\delta_{2}\left(c_{2}-c_{1}+\frac{(1-r) h}{r}\left(\frac{1}{\mu_{2}}-\frac{1}{\mu_{1}}\right)\right)>0, \quad$ which $\quad$ is equivalent to $e_{1}>e_{0}=\frac{\left(0.5 \alpha_{p}+\alpha_{s}\right)\left(\frac{(1-r) h}{r}\left(\frac{1}{\mu_{1}} \frac{1}{\mu_{2}}\right)-\left(c_{2}-c_{1}\right)\right)}{\left(0.5 \beta_{e}+2 \beta_{s}\right)}$.

## Proof of Proposition 9.

The first derivatives of $S_{1}^{*}$ and $S_{2}^{*}$ with respect to $\mu_{1}$ are presented in the following.

We have $\frac{\partial S_{1}^{*}}{\partial \mu_{1}}==-\frac{(1-r)}{r \mu_{1}{ }^{2}}\left(\lambda_{1}+\frac{h \delta_{1}(1-r)\left(-2 \delta_{1} \delta_{2}+\alpha_{s}{ }^{2}\right)}{r \mu_{1}\left(4 \delta_{1} \delta_{2}-\alpha_{S}{ }^{2}\right)}\right)<0$ and $\frac{\partial S_{2}^{*}}{\partial \mu_{1}}=-\frac{h(1-r)^{2}}{r^{2} \mu_{1}{ }^{2} \mu_{2}}\left(\frac{2 \alpha_{S} \delta_{1} \delta_{2}}{4 \delta_{1} \delta_{2}-\alpha_{s}{ }^{2}}\right)<0$.

In addition, $\frac{\partial S_{1}^{*}}{\partial \mu_{2}}=-\frac{h(1-r)^{2}}{r^{2} \mu_{2}^{2} \mu_{1}}\left(\frac{2 \alpha_{s} \delta_{1} \delta_{2}}{4 \delta_{1} \delta_{2}-\alpha_{s}^{2}}\right)<0$ and $\frac{\partial S_{2}^{*}}{\partial \mu_{2}}=-\frac{(1-r)}{r \mu_{2}{ }^{2}}\left(\lambda_{2}+\frac{h \delta_{1}(1-r)\left(-2 \delta_{1} \delta_{2}+\alpha_{s}^{2}\right)}{r \mu_{2}\left(4 \delta_{1} \delta_{2}-\alpha_{s}{ }^{2}\right)}\right)<0$.


[^0]:    Proposition 6. A retailer that sees a decrease in its market power will improve its product's greenness in GDS market and reduce it in PDS market.

