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"Mean fractal dimension" of blood ultrasonic Doppler signals

Jean-Marc Girault, *Member, IEEE*, Denis Kouamé, *Member* and Frédéric Patat

Abstract— This paper deals with the characterization of blood ultrasonic Doppler signals via the computation of the "mean fractal dimension". We show and we discuss the existing links between the "mean fractal dimension" and the well-known mean frequency and the bandwidth usually used to characterize the blood Doppler signals. To finish, we calculate and we discuss all these parameters in the case of *in vivo* blood Doppler signals.

Keywords— Fractal dimension, Length, Ultrasound Doppler signal.

I. INTRODUCTION

THE main purpose that we wish to give to this work is to study the contribution of the fractal dimension estimation to characterize narrow band stochastic signals like the blood ultrasonic Doppler signals.

The idea which underlies this work is based on the fact that the $1/f^\alpha$ noises, characterized by fractal dimension, are obtained for example by low-pass filtering of a white noise. We think that the blood ultrasonic Doppler signal which can be obtained by filtering band pass of a white noise, can also be characterized by fractal dimension.

This work, which continues the idea of Wang [1] on the characterization of blood ultrasonic Doppler signals, by the computation of fractal dimension, will try to answer the questions left outstanding by the author.

More precisely, our study will consist in on the one hand justifying the computation of fractal dimension of *a priori* non fractal signals and on the other hand to clarify in details the similarity between the temporal evolution in the short time of fractal dimension and the mean frequency (or maximum) of the Doppler spectrum.

To achieve our goal we must answer the following question: can the fractal theory be applied within the framework of our study?

To answer it, it is initially necessary to point out what are the blood ultrasonic Doppler signals and fractal signals.

Another question is to know which are the parameters which make it possible indeed to characterize them.

We will initially give some brief replies with regard to the Doppler signals then in a second time the signals of fractal character.

II. THE BLOOD DOPPLER SIGNAL

Because of the random spatial distribution of red blood cells and the dynamics of the cardiovascular system, the Doppler signal is considered as a time-varying random signal. This signal which has a non-stationary character in

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frequency can be considered stationary on a time interval of duration lower or equal to 10 ms.

This signal can be modeled coarsely by a narrow band random signal modulated at the same time in amplitude and frequency:

$$z(t) = \rho(t)e^{j\phi(t)}, \quad (1)$$

where $\rho(t)$ is the instantaneous magnitude of the analytic signal, indicating the amplitude modulation and $\phi(t)$ is the instantaneous phase specifying the Doppler frequency modulation. Indeed, both instantaneous energy and frequency of the Doppler signal are very dependent on the various phases on the cardiac cycle.

In the literature, it was shown that Doppler signals could be modeled by parametric models as for example AR (autoregressive) model. The modeled signals are nothing other than colored noises obtained by narrow band AR filters [2] or by other types of filtering[3].

Let us recall that this aspect of filtering, is a key point since, as we will see it thereafter, the fractal signals can also be obtained by filtering.

III. FRACTAL SIGNALS

A signal can be described as fractal if it is continuous and non-derivable and if it presents:

- deterministic or statistical self-similarities,
- deterministic or statistical self-affinities,

and this on all the scales and thus at all the frequencies.

Let us specify that there exists in the nature of the signals which present a fractal character only in one restricted range of frequency. Moreover, let us add that it is also the case in practice, a numerical signal will be fractal only on one range of limited scale: the smallest scale will be defined by the sampling period and the largest by the size of the analyzed signal.

Furthermore, let us specify that if we do not have *a priori* knowledge on the fractal character of a random signal, nothing enables us to affirm that it has or not fractal properties.

To characterize such signals, there is a certain number of parameters. The simplest and the most commonly used is the fractal dimension D . The latter makes it possible to evaluate the degree of regularity or self-affinity present in the signal. Let us note that a very irregular signal, will fill all space and fractal dimension is worth $D \approx 2$ whereas a regular signal rather has a value of fractal dimension near to the unity: $D \approx 1$.

Another aspect developed for better understanding the fractal signals is the implementation of mathematical mod-

els, as for examples the fractional brownian motion [4], known as $1/f^\alpha$ noises, where $\alpha = (5 - D)/2$ and where D is fractal dimension. Let us note that a white noise is a $1/f^0$ noise.

Such noises can be obtained for example starting from the fARIMA model [5] or by summation of several signals resulting by the filtering of a white noise through low-pass filters of different cut-off frequencies [6].

Here, as we will see it thereafter, it is rather the opposite operation which will interest us, in the sense where it is rather the filter or the system characteristics which we will try to evaluate starting from the filtered stochastic signal.

IV. MEASURE OF THE FRACTAL DIMENSION

After having introduced the various models of fractal signals, let us clarify the technique chosen to evaluate fractal dimension such signals.

In the case of signals analysis (in opposite to the study of graphs or images), the technique which seems the best adapted is that of the length measure. It is Mandelbrot [7] with the help of the Richardson study [8] that showed the length could be expressed according to fractal dimension D by:

$$L(k) = L_0 k^{1-D}, \quad (2)$$

where k is a gauge which plays the role of a zoom and where L_0 is a constant.

In the case of the study of numerical signals, Eq.(2) becomes:

$$L(K) = T^D K^{1-D}, \quad (3)$$

where $K = kT_s$ is the experimental gauge, T_s is the sampling period and T is the duration of the window of analysis, duration which will be selected to ensure the stationarity of the studied signal. In our case this value will be normalized and will be worth the unity. On the formula above, one sees that fractal dimension can be simply obtained while calculating, by linear regression, the slope of the curve log-log of according to K .

After having introduced the link between the length and the fractal dimension, focus to a length estimator. Let us note that there exist several length estimators as for example the one proposed by [9]. Here we only present the simplest.

The presented estimator was introduced by Burlaga and Klein [10] to measure the length of signals resulting from the interplanetary magnetic fields. This estimator based on the "L1 norm" is expressed by:

$$L_{BK}(n, k) = \frac{1}{k} \sum_{n=1}^{N-k} |z(nT_s) - z((n+k)T_s)|, \quad (4)$$

where n is the current time, k is called gauge, N is the number of points of the studied signal.

This length estimator is a generalized length estimator, in the sense that instead of having one sampling period T_s between two consecutive points (as it is the case for the Euclidean length), it is rather a multiple of the sampling period kT_s with $k = 1, 2, \dots, k_{max}$. This later point indicates

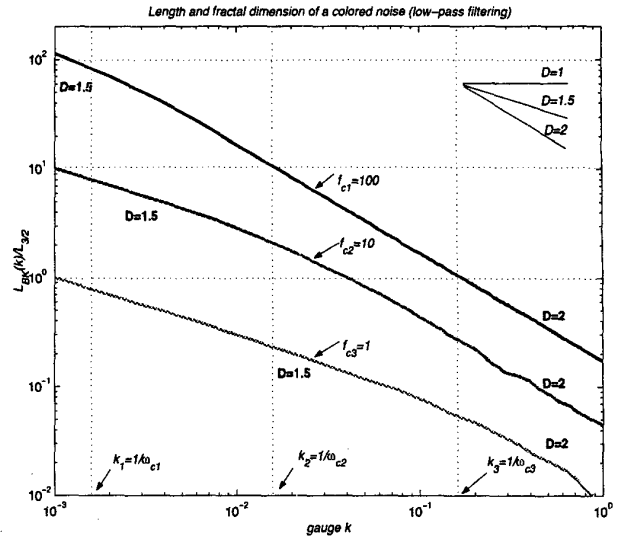


Fig. 1.

Length of colored noises obtained by a first order low-pass filter of the Lorentz type for various values of the cut-off frequency ($f_{c1} = 100$, $f_{c2} = 10$ and $f_{c3} = 1$), the sampling frequency is $f_s = 1000$.

that the estimated length is not the length of the studied signal but rather the length of a filtered signal since this one results from a decimation and an interpolation of first-order.

V. MEASURE OF THE LENGTH AND FRACTAL DIMENSION OF FILTERED RANDOM SIGNALS

Having all tools necessary for our study, let us focus to the length, then to the fractal dimension in the case of colored signals by low-pass filtering, and in the second time in the case of colored signals by filtering band pass.

A. Low-pass Filter

Let us consider a simple first order low-pass filter (Lorentz type) defined by the square modulus of its transfer function:

$$|H_0(\omega)|^2 = \frac{1}{1 + (\omega/\omega_c)^2} \quad (5)$$

where ω_c is the cut-off frequency of the filter.

It was shown by [6] that the length of the noise thus filtered, which is named "a random walk" (or first-order Markov process), is proportional to the cut-off frequency in a restricted range of frequency.

As it is indicated in table (I), there are three jointed areas where fractal dimension (slope of the curve) take different values. Remark, as it is illustrated on the Fig. (1), that there are only two observed areas, the first one corresponds to $D = 1.5$ and the second to $D = 2$, the third could not be reach. It is clear in this case that the gauge values at

TABLE I

gauge	$k \gg \sqrt{\frac{\omega_c}{2}}$	$\sqrt{\frac{\omega_c}{2}} > k > \frac{1}{\omega_c}$	$k < \frac{1}{\omega_c} < \sqrt{\frac{\omega_c}{2}}$
$L_{BK}(k)$	$L_1 k^{1-D}$	$L_2 \sqrt{f_c} k^{1-D}$	$L_{3/2} f_c k^{1-D}$
D	1	2	1.5
King of signal	continuous signal	white noise	random walk

Length and fractal dimension according to the various values of gauge k ; L_1 , L_2 and $L_{3/2}$ are tree constants.

the jointed areas points ($k_1 = 1/\omega_1$, $k_2 = 1/\omega_2$, $k_3 = 1/\omega_3$) are also functions of the cut-off frequency.

If one calculates the average slope of the logarithm length according to the logarithm of the gauge, one sees that because of the various slopes, the associated fractal dimension corresponds to an average value of the fractal dimension.

B. Band pass filter

With an aim of simplicity, suppose that the narrow band is sufficiently weak to assimilate the filtered noise as a sinusoidal signal. The modulus of the transfer function of the band pass filter is written in this case, i.e. when $\omega_0/\omega_c \ll 1$:

$$|H(\omega)| \approx |H_0(\omega - \omega_0)| + |H_0(\omega + \omega_0)|, \quad (6)$$

where ω_0 is the central frequency and $2\omega_c$ is the bandwidth.

The length measure highlights several significant points illustrated on the Fig.(2):

- firstly when the gauge is a multiple of the period of the sinusoidal signal, the evaluated length presents local minima. Indeed let us suppose that the sinusoidal signal period, $T_0 = 1/f_0$, that is observed over one duration equals to $10T_0$, then if the gauge is worth $k = T_0$, the length is worth $10T_0$, whereas if the gauge takes its maximum value $k = 10T_0$ the length is worth $10T_0$ too ;
- secondly when the gauge takes its minimum value, $k = T_s$, the length takes its maximum value;
- thirdly, when the gauge takes a multiple value of the half-period $k = T_0/2$ then the length presents local maxima which follow the same slope as that of a white noise. Be careful, starting from the graphs of the Fig.(2), one would not have to conclude too hastily only the Doppler signal has a fractal character. Indeed, although these successive points have a slope indicating a fractal dimension $D = 2$, it seems that this behavior results simply from the decimation and the periodicity of the Doppler signal.

If one measures signal fractal dimension on all the scale range, as it is the case in work of Wang [1], this one seems being dependent on the central frequency of the filter and its band-width. Here, in opposite to the Wang study, we show (see Figs.(2) and (3)) and demonstrate its dependence (see below). This fractal dimension does not correspond any more to the usual sense of the term but rather to an average fractal dimension.

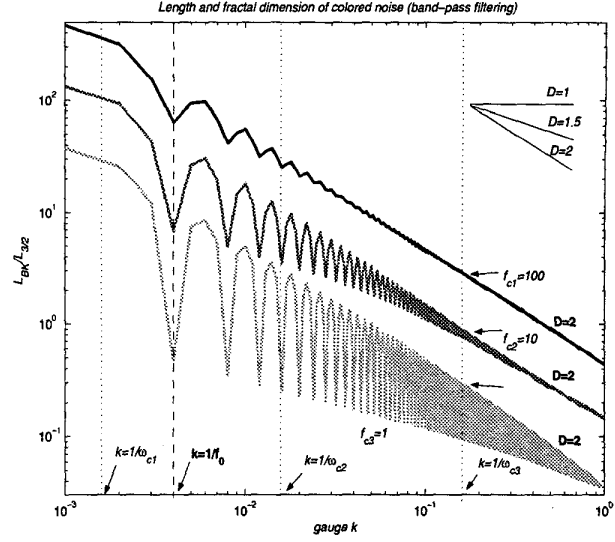


Fig. 2.

Length of filtered noises obtained by a band pass filter for a constant value of the central frequency $f_0 = 250$ and for different values of the cut-off frequency $f_{c1} = 100$, $f_{c2} = 10$ and $f_{c3} = 1$, the sampling frequency is $f_s = 1000$.

In fact, one can show that the length of a narrow band signal is connected to the central frequency [11] and its bandwidth by the following relation:

$$L_{BK} = \beta \sqrt{\kappa \omega_c^2 + \omega_0^2} \quad (7)$$

where κ and β are constants. Here the magnitude of the analyzed signal will be set to one. Note that in [11], it is proposed a new method to evaluate the mean frequency. Under these conditions the mean fractal dimension can be expressed as a function :

$$D = 1 - g \left(\log(\omega_c^2 + \omega_0^2) \right), \quad (8)$$

where g is a function related to the derivative of the logarithm of the gauge according to the logarithm of the gauge.

In short, one highlighted that the measured length is dependent on filtering used. In the case of low-pass filters, the dependence appears with the cut-off frequency whereas in the case of band pass filters the dependence appears at the same time with the cut-off frequencies but also with the central frequency.

VI. FRACTAL DIMENSION OF BLOOD ULTRASOUND DOPPLER SIGNALS

Focus now to *in vivo* signals and let us evaluate the mean fractal dimension of such signals. We have illustrated on the Fig.(4) a typical encountered Doppler signal (carotid artery Doppler signal). As it has been already introduced, it can be observed, from the Fig.(4b), the time-varying behavior (frequency non-stationarities) of the Doppler signal

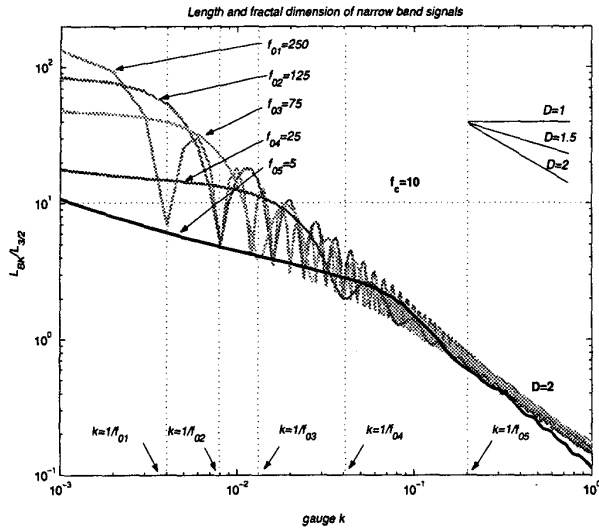


Fig. 3. Length of filtered noises obtained by a band-pass filter for different values of the central frequency $f_{01} = 250$, $f_{02} = 125$, $f_{03} = 75$, $f_{04} = 25$, $f_{05} = 5$ and for a constant cut-off frequency $f_c = 10$, the sampling frequency is $f_s = 1000$.

at the different cardiovascular phases (the cardiac cycle is about 1 beat per second). From the time-frequency representation, we can extract frequency parameters to characterize for example of arterial stenosis; the most commonly used are the mean frequency and the bandwidth (see Fig.(4c)). On Fig.(4d) we show that the mean fractal dimension follows the cardiac cycle as expected. The latter fluctuates around values ranging between 1 and 2 what corresponds well to our waitings.

We think that the mean fractal dimension is an interesting parameter in the sense where it provides a synthetic parameter which describes at the same time its dependence according to the central frequency and of the band-width. However this advantage can be regarded as a disadvantage especially if we want to follow the evolution of only one of the two parameters.

VII. CONCLUSION

In this study, after having recalled the theoretical aspects, we have in a first time demonstrated the link between the mean fractal dimension and the characteristic parameters of the filter and in a second step we have discussed and demonstrated why it is significant.

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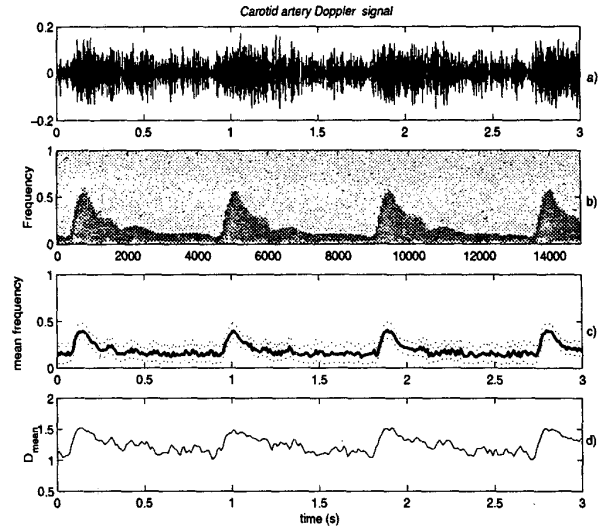


Fig. 4. Fractal dimension of blood ultrasound Doppler signals from a carotid artery. a) Doppler signal, b) spectrogram of the Doppler signal, c) mean frequency ($\omega_0 + | - \omega_c / 2$ and d) mean fractal dimension.)

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