Sizing of a heterogeneous fleet of robots in a logistics warehouse

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***Abstract*— The purpose is to determine the size of the robot fleet, that is, to determine the optimal types and number of robots for each type. We first consider the case where loads are all of the same type (homogeneous) before generalizing to several types of loads (heterogeneous). In both cases, we show that the problem can be formulated by an integer linear program. In the case of homogeneous loads, we consider a relaxation of the problem where the number of robots can be a real number. For this approximation, we show that it is optimal to use a single type of robot. It is also near-optimal when the number of robots must be an integer number.**

I. INTRODUCTION

Robotics has become an essential concept in industry today and a major concern in research [1]. Indeed, it allows to increase robustness, flexibility and efficiency of manufacturing systems. In addition, the existence of robotic industrial platforms in an industrial unit reflects the level of its performance and reliability. After a remarkable evolution of industrial robotics, the concept of Multi-Robot System (MRS) appeared in 1989 in order to replace a large robot by a set of small robots [2]. Logistics warehouses are essential in any supply chain and are a factor of success or failure of any business [3]. Finally, MRS are considered an essential tool for any logistics warehouse to have a high performance for its industrial future [4].

Today, through MRS, we are talking about the automation of logistics systems. Indeed, this concept has today become an essential concern for any warehouse wishing to have high performance in its industrial future. In addition, the automation of a logistics warehouse makes it possible to optimize the overall operations (speed, precision, organization, etc.) and to guarantee very high industrial efficiency through rapid processing of orders, high precision in the stock management and increased storage capacity and space.

The literature on MRS in warehouses has since been enriched by addressing several problems such as: guide-path design, sizing of vehicle fleets, planning of transport operations, positioning, battery management, conflict management (see e.g. [5]). The number of vehicles plays a very important role in the performance of MRS. We focus on the fleet sizing problem, which consists in determining the optimal number of vehicles capable of carrying out requested transport operations in a given time limit at minimum cost [6].

In the literature, several works deal with the fleet sizing problem with different methods. There are several classifications of these works as [7] and [8]. According to [7], the design models treated in the literature can be classified into four categories:

1) Discrete and continuous event simulation models [9]–[13];
2) Calculus approaches including a set of simple computations [4], [6], [14]–[18];
3) Deterministic operations research models including linear and integer programming [19]–[21];
4) Stochastic models including queuing theory [22]–[25].

The seminal paper of Egbelu [14] calculates the number of robots in the fleet by dividing the total operating time by the time limit. Although this model takes into account several constraints (empty circulation of vehicles, number of loading / unloading stations per zone, etc), it does not take the integer values into consideration which strongly impacts the optimality of the solution.

[7] presents an analytical model to estimate the size of an AGV fleet in a Closed Queuing Network (CQN) system. This model is based on linear programming in order to minimize the number of robots required. A comparison part was presented in order to compare the analytical results with those found by simulation. In addition, [4] treats the problem of sizing an AGV fleet in a warehouse. The goal is to solve this problem while ensuring efficient assignment of tasks and minimum operational cost. The study is carried out on two warehouse configurations. It consists in initially estimating the number of robots to calculate the total service time, the distance traveled by each robot and the number of tasks for each robot. Then a meta-heuristic on the initial solution is implemented in order to optimize the size of the fleet.

More recently, [17] deals with the problem of sizing a robot system in a FMS (Flexible Manufacturing System) with the aim of optimizing the fleet size of robots for FMS with different dimensions. The authors compare the results obtained by an analytical model and by a Grey wolf optimization algorithm for three different sizes of FMS. [18] tackle the same problem with a particle swarm optimization algorithm.

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Contributions: All papers deal with the problem of sizing a homogeneous fleet of robots, that is, a fleet of identical robots. The notion of heterogeneity has not been addressed in any of these works. Although the notion of heterogeneity requires a greater effort in terms of fleet control, it remains very interesting and important compared to a homogeneous system. Indeed, a heterogeneous fleet allows the system to easily adapt to any situation of such a dynamic system : new and unpredicted needs [26]. Through this paper, we deal with the problem of sizing a fleet of heterogeneous robots in a logistics warehouse.

In section II, we treat the problem with a set of homogeneous loads to be transported and show that it is near-optimal to use a single type of robots. In section III, we assume that the loads are heterogeneous. In both models, we formulate our problem as an Integer linear program (ILP) to optimize the fleet size based on the cost expressions of each configuration.

II. TRANSPORT OF HOMOGENEOUS LOADS

In this part, we want to determine the size of a heterogeneous fleet of robots allowing the transport of homogeneous loads from a zone A to a zone B. Figure 1 shows an example of studied model with 3 types of robots.

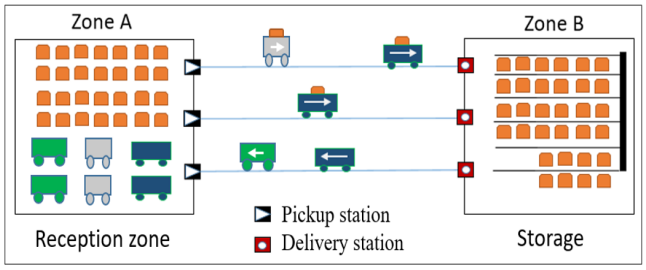

Fig. 1. Example of studied model with 3 types of robots

Assumptions

We will make the following assumptions:

- The robots are stored at point A;
- The transport is unitary: each robot carries at most a single load at a given time;
- The loads to be transported are always available and there is no waiting time before loading and unloading;
- Deadlock during travel and idleness of robots are not taken into account;
- The battery charging problem is not taken into account.

Notations

We will use the following notations :

- n: number of homogeneous loads to transport
- D: round trip distance between A and B
- T: time limit
- K: number of robots types
- For a k type of robot:
  - m_k: number of robots of type k
  - n_k: number of loads carried by robots of type k
  - v_l^k: travel speed of a robot of type k carrying a load
  - v_e^k: travel speed of an empty robot of type k
  - t_l^k: loading time of a load carried by a robot of type k
  - t_u^k: unloading time of a load carried by a robot of type k
  - p_k : transport cycle time of a load from point A to point B by a robot of type k (loading, loaded travel, unloading and empty travel)

$$p_k = \frac{D}{2} \cdot \left(\frac{1}{v_l^k} + \frac{1}{v_e^k} \right) + t_l^k + t_u^k$$

- Cost structure :
  - α_k : fixed cost per robot linked to the hardware part of the system (connectors, chargers, batteries, etc).
  - β_k : fixed cost linked to the type k of robots used, independent of the number of robots. This is the software part of the system that manages the robots operations (planning, routing, etc).
  - γ_k : cost per distance traveled by a robot of type k, including all variable costs (maintenance, energy consumption, depreciation, etc).

The cost of a fleet of m_k robots of type k to carry n_k loads is

$$C_k(m_k, n_k) = \alpha_k m_k + \beta_k \mathbb{1}\{m_k > 0\} + \gamma_k n_k D$$

The cost of a fleet of K types of robots is then

$$\begin{aligned} C(\overrightarrow{m}, \overrightarrow{n}) &= \sum_{k=1}^{K} C_k(m_k, n_k) \\ &= \sum_{k=1}^{K} (\alpha_k m_k + \beta_k \mathbb{1}\{m_k > 0\} + \gamma_k n_k D) \end{aligned}$$

where $\overrightarrow{m} = (m_1, \cdots, m_K)$ and $\overrightarrow{n} = (n_1, \cdots, n_K)$

A. ILP formulation

We use the following indexes :

- $i = 1, \cdots, n$: loads
- $j = 1, \cdots, n$: robots (there are at most n robots)
- $k = 1, \cdots, K$: robots types

We can formulate our problem by an ILP. Using the following binary variables:

$$x_{ijk} = \begin{cases} 1 & \text{if the load } i \text{ is carried by the robot } j \\ & \text{of type } k \\ 0 & \text{otherwise} \end{cases}$$

$$y_{jk} = \begin{cases} 1 & \text{if the robot } j \text{ is of type } k \text{ and is used} \\ 0 & \text{otherwise} \end{cases}$$

$$z_k = \begin{cases} 1 & \text{if a robot of type } k \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

Therefore, we have :

$$m_k = \sum_{j=1}^{n} y_{jk}$$
$$n_k = \sum_{i=1}^{n}\sum_{j=1}^{n} x_{ijk}$$

The optimization problem can then be written as an ILP :

$$C^* = \min \left[\sum_{k=1}^{K} \left(\alpha_k \sum_{j=1}^{n} y_{jk} + \beta_k z_k + \gamma_k D \sum_{i=1}^{n}\sum_{j=1}^{n} x_{ijk} \right) \right]$$

s.t.

$$\sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} = 1 \qquad \forall i \qquad (1)$$

$$\sum_{k=1}^{K}\sum_{i=1}^{n} x_{ijk} \cdot p_k \leq T \qquad \forall j \qquad (2)$$

$$x_{ijk} + x_{i'jk'} \leq 1 \qquad \forall i\ ,\ \forall i'\ ,\ \forall j\ ,\ \forall k \neq k' \qquad (3)$$

$$y_{jk} \geq x_{ijk} \qquad \forall i\ ,\ \forall j\ ,\ \forall k \qquad (4)$$

$$z_k \geq x_{ijk} \qquad \forall i\ ,\ \forall j\ ,\ \forall k \qquad (5)$$

$$x_{ijk} \in \{0,1\}\ ,\ y_{jk} \in \{0,1\}\ ,\ z_k \in \{0,1\} \qquad (6)$$

Constraints meaning :

- Constraint (1) : each load must be carried by only one robot
- Constraint (2) : each robot must be able to transport all the loads assigned to it within the time limit T
- Constraint (3) : each robot j is of a single type k
- Constraint (4) : if a load i is carried by a robot j of type k, this robot is used
- Constraint (5) : if a load i is carried by a robot of type k, robots of type k are used

B. Heuristic : Using a single type of robot

As a simple heuristic, we propose- to use a single type of robot, the one that minimizes the cost. If we only use robots of type k, the optimal cost is, according to [26], $C_k^* = C_k(m_k^*, n)$ with

$$m_k^* = \left\lceil \frac{n}{\left\lfloor \frac{T}{p_k} \right\rfloor} \right\rceil .$$

It follows that

$$C_k^* = \alpha_k \left\lceil \frac{n}{\left\lfloor \frac{T}{p_k} \right\rfloor} \right\rceil + \beta_k + \gamma_k n D$$

The heuristic consists in choosing a type of robot k' minimizing C_k^*, i.e. $k' \in \arg\min_{k\in\{1,\cdots,K\}} C_k^*$. The heuristic cost is therefore :

$$C^+ = \min_{k\in\{1,\cdots,K\}} C_k^*.$$

We can, through a simple example, show that this heuristic is not always optimal. Table I presents a counterexample with 2 possible types of robots and 3 loads.

TABLE I

EXAMPLE WHERE THE HEURISTIC IS NOT OPTIMAL

$(n = 3, T = 10, K = 2)$

k	p_k	α_k	β_k	γ_k
1	6	3	0	0
2	4	5	0	0

Table II presents the optimal solutions according to authorized types of robots.

TABLE II

OPTIMAL SOLUTIONS ACCORDING TO AUTHORIZED TYPES OF ROBOTS

	m_1	m_2	n_1	n_2	total cost
Type 1 only	3	0	3	0	9
Type 2 only	0	2	0	3	10
Types 1 and 2 authorized	1	1	1	2	8

In the above example, the optimal solution is to use both types of robots.

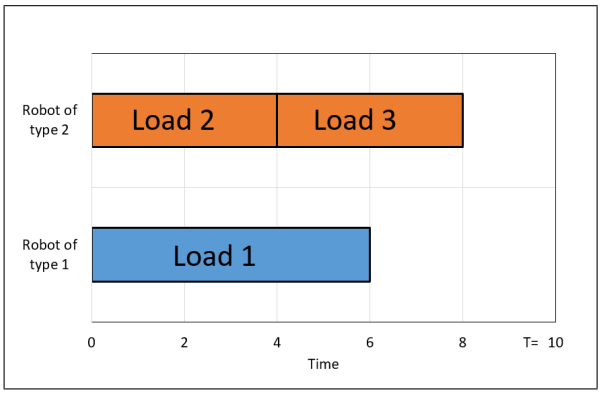

Fig. 2. Optimal solution of the instance with two types of robots

C. Relaxation

If we relax the integrity constraint on the number of robots and loads carried by a robot, we show that it is then optimal to use only a single type of robot. If the number of loads and robots can be real numbers, the number of robots m_k can be expressed as a function of the number of loads carried by robots of type k:

$$m_k = \frac{n_k \cdot p_k}{T} \qquad (7)$$

The cost of a fleet with m_k robots of type k can then be expressed as

$$C_k(m_k) = \left(\alpha_k + \gamma_k D \frac{T}{p_k}\right) m_k + \beta_k z_k$$
$$= \delta_k m_k + \beta_k z_k$$

where $\delta_k = \alpha_k + \gamma_k D \frac{T}{p_k}$.

The optimization problem can then be formulated as a MIP (Mixed Integer Programming) :

$$C^- = \min \quad \sum_{k=1}^{K} \delta_k m_k + \beta_k z_k \tag{8}$$

$$s.t. \quad \sum_{k=1}^{K} \frac{m_k}{p_k} \geq \frac{n}{T} \tag{9}$$

$$n z_k \geq m_k \qquad \forall k = 1, \cdots, K \tag{10}$$

$$m_k \geq 0, z_k \in \{0,1\} \qquad \forall k = 1, \cdots, K \tag{11}$$

Constraints meaning :

- Constraint (9) : the number of robots must carry all the loads in the time limit T
- Constraint (10) : if a load is carried by a robot of type k, robots of type k are used

If we use robots of type k only, the optimal number of robots is

$$m_k^* = \frac{n p_k}{T}$$

and the minimum cost is then

$$\begin{aligned} C_k^* &= C_k\left(m_k^*\right) \\ &= \frac{n}{T}\delta_k p_k + \beta_k \end{aligned}$$

The following theorem establishes that it is optimal to have a homogeneous fleet of the type minimizing C_k^*.

Theorem 1: In the relaxed problem, it is optimal to use a single type of robots and the optimal cost is

$$C^- = \min_{k \in \{1,\cdots,K\}} \frac{n}{T}\delta_k p_k + \beta_k.$$

Proof : *:* We first prove the result in the case where $\beta_k = 0$ before relaxing this assumption.

$\underline{\beta_k = 0}$: Suppose that $\beta_k = 0$ for all k. Let $u \in \arg\min_k C_k^*$ and let $\overrightarrow{m} = (m_1, \cdots, m_K)$ be a feasible solution such that $m_u > 0$ and $m_v > 0$ with $v \neq u$.

By definition we have $C_u^* \leq C_v^*$. This implies that $\delta_u p_u \leq \delta_v p_v$. Note also that robots of type v carry $n_v = \frac{m_v T}{p_v}$ loads.

Let $\overrightarrow{m}' = (m_1', \cdots, m_K')$ be another feasible solution, identical to $\overrightarrow{m}$ except that the n_v loads carried by robot of types v are now carried by robots of type u. More precisely :

$$\begin{aligned} m_v' &= 0 \\ m_u' &= m_u + n_v \frac{p_u}{T} = m_u + \frac{p_u}{p_v} m_v \\ m_k' &= m_k, \quad \forall k \neq u, v \end{aligned}$$

The cost of solution $\overrightarrow{m'}$ is then

$$C(\overrightarrow{m}') = C(\overrightarrow{m}) + \delta_u \frac{p_u}{p_v} m_v - \delta_v m_v \tag{12}$$

$$= C(\overrightarrow{m}) + (\delta_u p_u - \delta_v p_v) \frac{m_v}{p_v} \tag{13}$$

$$\leq C(\overrightarrow{m}) \tag{14}$$

since $\delta_u p_u \leq \delta_v p_v$.

We have therefore built a solution $\overrightarrow{m}'$ of lower cost than $\overrightarrow{m}$, not using v robots. We can repeat the above process for all types of robots. We deduce that it is optimal to use only robots of type u.

$\underline{\beta_k \geq 0}$: Consider the more general case. Let a feasible solution $\overrightarrow{m} = (m_1, \cdots, m_K)$ with $m_u > 0$ and $m_v > 0$, $u \neq v$, and as $\delta_u p_u \leq \delta_v p_v$. The cost associated with robots of types u and v is then

$$C_u(m_u) + C_v(m_v) = \delta_u m_u + \delta_v m_v + \beta_u + \beta_v$$

From what has been shown previously ($\beta_k = 0$), there is a solution using m_u' robots of type u, and no robots of type v, of lower variable cost ($\delta_u m_u' \leq \delta_u m_u + \delta_v m_v$). We have then

$$\begin{aligned} C_u(m_u) + C_v(m_v) &\geq \delta_u m_u' + \beta_u + \beta_v \\ &\geq \delta_u m_u' + \beta_u \\ &= C_u(m_u') \end{aligned}$$

So we built a solution using only one type of robot and lower cost. We conclude that there is an optimal solution using a single type of robot.

□

D. Heuristic asymptotic optimality

We show that the heuristic introduced in Section II-B works very well when it is necessary to use a large number of robots and each robot is carrying a lot of loads. We first remind some notations :

- C^* : optimal cost
- C_k^* : optimal cost if we restrict ourselves to robots of type k
- $C^+ = \min_k C_k^*$: heuristic cost
- C^- : optimal cost in continuous approximation

The heuristic solution being a feasible solution of the ILP, the cost of the heuristic is an upper bound of the optimal cost. The continuous approximation also provides a lower bound of the optimal cost. So we have $C^- \leq C^* \leq C^+$ which implies that

$$1 \leq \frac{C^+}{C^*} \leq \frac{C^+}{C^-}. \tag{15}$$

Remind that

$$C^- = \min_k \alpha_k \frac{n p_k}{T} + \beta_k + \gamma_k n D \tag{16}$$

$$C^+ = \min_k \alpha_k \left\lceil \frac{n}{\left\lfloor \frac{T}{p_k} \right\rfloor} \right\rceil + \beta_k + \gamma_k n D \tag{17}$$

If T is a multiple of p_k and n is a multiple of $\frac{T}{p_k}$, then $C^- = C^+ = C^*$ and the heuristic is optimal.

Otherwise, we can upper-bound the error by using the fact that $x - 1 < \lfloor x \rfloor \leq x$ and that $x \leq \lceil x \rceil < x + 1$. We have then

$$\frac{np_k}{T} \leq \left\lceil \frac{n}{\left\lfloor \frac{T}{p_k} \right\rfloor} \right\rceil \leq \frac{n}{\frac{T}{p_k} - 1} + 1$$

which can be rewritten as

$$\frac{np_k}{T} \leq \left\lceil \frac{n}{\left\lfloor \frac{T}{p_k} \right\rfloor} \right\rceil \leq \frac{np_k}{T}\left(\frac{1}{1 - \frac{p_k}{T}} + \frac{T}{np_k}\right) \tag{18}$$

Note that the term $\left(\frac{1}{1-\frac{p_k}{T}} + \frac{T}{np_k}\right)$ tends to 1 when simultaneously $\frac{p_k}{T}$ tends to 0 and $\frac{T}{np_k}$ tends to 0.

Noting $p_{min} = \min_k p_k$ and $p_{max} = \max_k p_k$ and using the equations (15)-(18), we have

$$\frac{C^+}{C^*} \xrightarrow[\frac{np_{min}}{T} \to \infty]{\frac{T}{p_{max}} \to \infty} 1$$

Consequently, the heuristic cost will be close (in relative value) to the optimal cost if the following conditions are met:

- the number of robots required is large : $\frac{np_{min}}{T} >> 1$;
- Each robot can carry many loads within the time horizon: $T >> p_{max}$.

III. TRANSPORT OF HETEROGENEOUS LOADS

In this part, we extend the approach to the case of heterogeneous loads. Figure 3 shows an example of studied model with 3 types of robots and 2 types of loads.

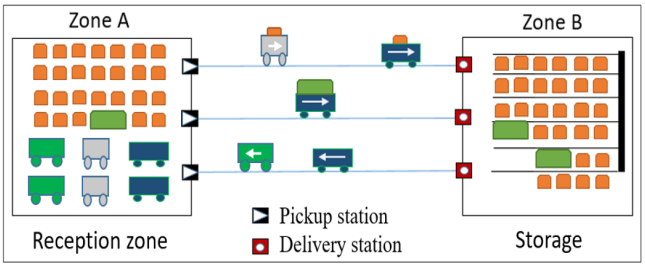

Fig. 3. Example of studied model with 3 types of robots and 2 types of loads

A. Assumptions and notations

We keep the same assumptions as in the previous section, but in this part, we suppose that :

- All types of robots can carry any type of load.
- The speed of a loaded robot depends on the type of the carried load.

Using the second added assumption, we define v_l^{gk} as the travel speed of a robot of type k carrying a load of type g.

Then, we define the transport cycle time of a load of type g by a robot of type k (loading, loaded travel, unloading and empty travel) :

$$p_{gk} = \frac{D}{2} \cdot \left(\frac{1}{v_l^{gk}} + \frac{1}{v_e^k}\right) + t_l^k + t_u^k$$

We have the following relations for the numbers of loads:

$$n = \sum_{g=1}^{G} n_g = \sum_{k=1}^{K}\sum_{g=1}^{G} n_{gk}$$

and

$$n_g = \sum_{i=1}^{n} w_{ig}$$

where w_{ig} is a parameter which is equal to 1 if the load is of type g.

For this transport model, we keep the same cost modeling as in the previous section. We therefore have the following expression describing the total cost :

$$C(\vec{m}, \vec{n}) = \sum_{k=1}^{K}\left(\alpha_k m_k + \beta_k \mathbb{1}\{m_k > 0\} + \gamma_k \sum_{g=1}^{G} n_{gk} D\right)$$

We use the following indexes :

- $i = 1, \cdots, n$: for loads
- $j = 1, \cdots, n$: for robots
- $k = 1, \cdots, K$: for robots types
- $g = 1, \cdots, G$: for loads types

B. ILP formulation

We can formulate our ILP using the following binary variables :

$$x_{ijk} = \begin{cases} 1 & \text{if the load } i \text{ is carried by the robot } j \\ & \text{of type } k \\ 0 & \text{otherwise} \end{cases}$$

$$y_{jk} = \begin{cases} 1 & \text{if the robot } j \text{ is of type } k \text{ and is used} \\ 0 & \text{otherwise} \end{cases}$$

$$z_k = \begin{cases} 1 & \text{if a robot of type } k \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

Our optimization problem is then written :

$$\min \sum_{k=1}^{K}\left(\alpha_k \sum_{j=1}^{n} y_{jk} + \beta_k z_k + \gamma_k D \sum_{i=1}^{n}\sum_{j=1}^{n} x_{ijk}\right) \tag{19}$$

$$s.t. \sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} = 1 \qquad \forall i \tag{20}$$

$$\sum_{k=1}^{K}\sum_{g=1}^{G}\sum_{i=1}^{n} x_{ijk} \cdot p_{gk} \cdot w_{ig} \leq T \qquad \forall j \tag{21}$$

$$x_{ijk} + x_{i'jk'} \leq 1 \qquad \forall i, i', \forall j, \forall k' \neq k \tag{22}$$

$$y_{jk} \geq x_{ijk} \qquad \forall i, \forall j, \forall k \tag{23}$$

$$z_k \geq x_{ijk} \qquad \forall i, \forall j, \forall k \tag{24}$$

$$x_{ijk} \in \{0,1\}, y_{jk} \in \{0,1\}, z_k \in \{0,1\} \tag{25}$$

Constraints meaning :

- Constraint (20) : each load must be carried by only one robot
- Constraint (21) : each robot must be able to transport all the loads assigned to it in the time limit T
- Constraint (22) : each robot j is of a single type k
- Constraint (23) : if a load i is carried by a robot j of type k, this robot is used
- Constraint (24) : if a load i is carried by a robot of type k, robots of type k are used

IV. CONCLUSIONS

This paper presents a fleet-sizing problem with several types of robots in a logistics warehouse. We show that the problem can be formulated by an integer linear program. In the case of homogeneous loads, we consider a relaxation of the problem under which it is optimal to use a single type of robot. We also show that it is near-optimal to use a single type of robots when the number of required robots is large and each robot can carry many loads within the time horizon. After that, we have implemented and tested our mathematical model with Cplex Optimizer.

An avenue of research would be to consider several storage zones and stochastic cycle times.

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