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► **To cite this version:**

Saïd Souam, Yacine Belarbi, Faycal Hamdi, Abderaouf Khalfi. Growth, institutions and oil dependence: a buffered threshold panel approach. *Economic Modelling*, 2021, 99. hal-03148732

HAL Id: hal-03148732

<https://hal.science/hal-03148732v1>

Submitted on 24 Apr 2023

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Growth, institutions and oil dependence: a buffered threshold panel approach

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Abstract

We examine the combined effects of oil dependence and the quality of institutions on economic growth. To do so, we introduce a new buffered threshold panel data model and apply it to 19 oil rent-dependent countries over the period 1996-2017. We show that the relationship between growth and oil-dependence is not linear. More precisely, three categories of oil-dependent countries are identified. Only countries with high-quality institutions are very stable. All the other countries have experienced a transition into a buffer zone and are potentially in a transition between two different regimes. When considering oil dependence as a threshold variable, it appears that the quality of institutions has a positive and significant effect on growth when dependence is either low or high. More interestingly, for countries with intermediate levels of oil-dependence, the quality of the institutions negatively impacts growth. Some of these countries have experienced something of an oil-dependence trap.

Keywords: Economic growth, oil dependence, quality of institutions, buffered threshold panel data model, nonlinear modelling of panel data.

1. Introduction

Dependence on natural resources is the subject of a wide debate in the analysis of economic growth in rentier states. However, in the empirical studies, there is no clear consensus on the negative impact of resource rents on long-term growth (Havranek et al., 2016). In practice, rentier states are characterized by important heterogeneity in their economic performance. The quality of institutions is one major explanation that has been advanced in the literature to explain these disparities. As is now well documented in the literature, natural resource dependence has given rise to some negative phenomena that could hinder growth (rent-seeking behaviours, the contraction of non-resource production activities, corruption, the voracity effect, civil conflicts, social pressure for additional redistribution, increases in public spending in less productive sectors, etc.). In fact, a diversification of the economy and an improvement in the quality of the institutions in natural resource exporting countries seem to be efficient tools for enhancing their growth performance. Indeed, these countries could reach such high levels of dependency that it would become very difficult to sustain good economic or institutional reforms. From an economic policy point of view, it is thus important to understand how institutional reforms could impact economic growth while interacting with natural resource dependence. Indeed, the economic cost of ameliorating the quality of the institutions could be very high before having a positive effect on economic growth.

The empirical literature studying the relationships among natural resources, the quality of institutions and economic growth has not brought about a consensus. Such studies can be roughly classified into three categories. In the first category, natural resources are found to have a negative effect on growth when they are associated with weak institutions (see for example, Leite and Weidman, 1999, Acemoglu et al., 2001, 2002, and Sala-i-Martin and Subramanian, 2013). The second category shows that natural resources interact with the quality of institutions, and their combined effects on growth depend on the nature of their combination (Mehlum et al. 2006*a, b*, Boschini et al., 2007, Arezki and Van der Ploeg, 2011). The third and last category shows that the observed heterogeneity in economic growth between rentier states is not explained by institutions (Sachs and Warner, 1999, Brunnschweiler, 2008 and Alexeev and Conrad, 2009). It is worth noting that this literature generally assumes linearity in the dynamics to address these rather complex relationships. Only a few contributions have

insisted on their nonlinearity (Leite and Weidmann, 1999 and Sala-i-Martin and Subramanian, 2013).

Since their introduction by Tong (1978), threshold models have been considered to be a very useful and sophisticated way to take into account the nonlinearity exhibited in several financial and macroeconomic phenomena. Indeed, they provide a simplified formulation to mimic nonlinear stylized facts and, more precisely, the dynamics of regime changes. Their structure has been widely used by econometricians in time series analysis. However, many extensions and mathematical developments of threshold models, in particular the panel data treatment framework, have been adopted for the analysis of other data structures. Hansen (1999) proposed a panel threshold regression (*PTR*) model for the nondynamic panel case. His main contribution lies in the possibility of allowing the individuals constituting the panel to be in different regimes during a given period. This enables the heterogeneity in the panel to be better captured and allows for a visualization of the nonlinearity in the interaction between the dependent variable and the explanatory variables for each panel's component. However, the sudden change in regime that characterizes Hansen's formulation may be problematic in some situations in which the transition is smooth. To capture the absence of sudden jumps, Gonzalez et al. (2017) develop a non-dynamic panel smooth transition regression (*PSTR*) model with individual fixed effects. The parameters are allowed to change smoothly as a function of the threshold variable. The performance of this model may depend on the choice of the transition function for the studied phenomenon. Overall, this form of modelling turns out to be useful when the number of regimes is sufficiently high.

In some circumstances, an interesting phenomenon happens when a past temporary change in a relevant forcing variable leads to a change in the economic behaviour of the analysed variable but a return to the initial value of the forcing variable does not induce a return to the initial behaviour (i.e., the state of a system is dependent on its history). This so-called hysteresis phenomenon, originally stemming from physics, has been widely used in labour theory and foreign trade to explain the persistent effects of temporary stimuli (see, e.g., Göcke, 2002). For example, in foreign trade, temporary exchange rate shocks could induce persistent consequences for quantities and prices due to sunk market-entry costs. Indeed, to sell in a foreign market, a firm incurs some entry costs that cannot be recovered after exit (e.g., distribution and service networks). If the domestic currency temporarily depreciates, entering this foreign market becomes profitable for some domestic firms. However,

even if the exchange rate regains its initial level, it is still profitable to sell in the foreign market if the variable costs are recovered. This simple microeconomic hysteresis can thus be aggregated and gives rise to a continuous macro-level loop in overall exports (Borgersen and Göcke, 2007). This effect has been widely documented in the empirical literature (see Belke and Kronen, 2019, for a recent study).

This smooth switching between different equilibria (finite configurations or finite states) of the studied variable (system) could thus be usefully mobilized to analyse the dynamics of its evolution. This is why in this article, we propose an alternative model based on this idea of hysteresis by defining a new smooth and flexible regime switching mechanism. To illustrate and highlight this point, let us limit ourselves, without loss of generality, to the case of a two-regime model. Instead of assuming a single threshold parameter, we consider an interval consisting of a lower and an upper threshold that acts as a buffer zone. If the threshold variable is below the lower boundary of the buffer zone, then the observation is from the first regime. Conversely, the observation comes from the second regime when the threshold variable is above the upper boundary. When the threshold variable falls within the buffer zone, the regime indicator keeps the value of its most recent past. This makes the transition dynamics smoother and more flexible than those of the classical *PTR* model. Even though this idea is still in its infancy, it provides a new way to understand and explain the nonlinearity observed in the data. In addition to this new modelling approach, our paper provides several interesting empirical results. First, we clearly show that the relationships between growth and dependence on oil rents are not linear: there is a gradual positive impact as the quality of institutions is enhanced. More precisely, our analysis identifies three categories of oil-dependent countries with respect to the quality of institutions. It is worth noting that except for three countries in the sample with high-quality institutions that are very stable, all the other countries have experienced a transition into a buffer zone. They are thus potentially in a transition between two different regimes, and the impact of oil resource dependence on their growth has not yet stabilized. Moreover, while considering dependence on oil rents as a threshold variable, it appears that the quality of institutions has a positive and significant effect on growth when dependence is low or high. More interestingly, it turns out that for intermediate oil-dependent countries, the quality of their institutions negatively impacts growth. Some of these countries have experienced something of an oil-dependence trap.

The remainder of the paper is organized as follows. In Section 2, we provide the analytical framework for our model: the definition of our buffered threshold panel data (*BTPD*) model, our estimation methodology, different test procedures and a simulation study. In Section 3, we first provide and discuss the results of our empirical study, which is devoted to the analysis of the combined interaction effects of natural resource dependence and the quality of institutions on economic growth in rentier countries. We thus compare our results to those provided by some alternative models and show how our model gives better results. Section 4 concludes.

2. Analytical framework

In this section, we first provide a precise definition of our *BTPD* model. We thus describe the general outlines of the proposed least squares estimation of the model. We afterwards lay down the general principles of our procedures for testing the number of regimes. We finally discuss the main results of our simulation study of the finite sample properties of these procedures.

Our model is inspired by the buffered process developed by Li et al. (2015) for time series analysis, the “hysteresis autoregressive time series model”. It is worth noting that this approach has been used in the recent literature exclusively to analyse time series.¹ It seems quite natural to extend this approach to phenomena characterized by a smooth transition in their dynamics in addition to having an individual dimension. This is the purpose of our model.

2.1. The buffered threshold panel data model

We consider the following balanced panel $\{Y_{i,t}, q_{i,t}, X_{i,t} : 1 \leq i \leq N, 1 \leq t \leq T\}$, where i and t denote the individual and temporal indices, respectively. The dependent variable $Y_{i,t}$ is scalar. The threshold variable $q_{i,t}$ is scalar. $X_{i,t}$ is an m -vector of control variables. We first present the case of two regimes. The observed data are generated from a nondynamic two-regime *BTPD* model with fixed effects if they satisfy the following regression

¹Lo et al. (2016) used buffered threshold GARCH models to study closing prices, adjusted for dividends and splits. Chen and Truong (2016) developed a double hysteretic heteroscedastic model to analyse the relationship between the underlying stock markets and index futures markets. Zhu et al. (2017) used a buffered autoregressive model with conditional heteroscedasticity to study exchange rates.

model:

$$Y_{i,t} = \mu_i + X_{i,t}\beta_1\mathbf{1}_{(R_{i,t}=1)} + X_{i,t}\beta_2\mathbf{1}_{(R_{i,t}=2)} + \varepsilon_{i,t} \quad (1)$$

where μ_i is the fixed effect for individual i and $\varepsilon_{i,t}$ is the error term, which is assumed to be independent and identically distributed with mean zero and positive finite variance σ^2 . β_k , $k = 1, 2$, represents the slope coefficients, and $\mathbf{1}_A$ is the indicator function for the set A . $R_{i,t}$ is the regime indicator defined as follows:

$$R_{i,t} = \begin{cases} 1 & \text{if } q_{i,t} \leq r_{L,1} \\ R_{i,t-1} & \text{if } r_{L,1} < q_{i,t} \leq r_{U,1} \\ 2 & \text{if } r_{U,1} < q_{i,t} \end{cases}$$

where $r_{L,1}$ and $r_{U,1}$ ($r_{L,1} \leq r_{U,1}$) are the boundary parameters that constitute the buffer or hysteresis zone. In model (1), the transition mechanism is modelled in the same way as for the buffered threshold in a time series (see Li et al., 2015). The originality lies in the representation of a panel as several distinct regimes. Each regime is characterized by a different regression slope (β_1 or β_2), and the transition between the two regimes is gradual rather than abrupt. This proposal is justified by the fact that in practice, regime $R_{i,t}$ may not shift immediately, and there could be a buffer region in which the regime of $Y_{i,t}$ depends on the regime of $Y_{i,t-1}$. In addition, the contribution of the individual dimension to the smooth transition mechanism is to group the individuals following the same linear regression model on a given date into the same regime. However, the individuals composing this regime are likely to evolve smoothly over time, since the transition variable also depends on the temporal dimension. In our empirical analysis, $Y_{i,t}$ is the real *GDP* growth rate (*GDPG*) of country i at time t . $X_{i,t}$ is a vector of explanatory variables. The threshold variable $q_{i,t}$ is oil rents or the quality of the institutions. μ_i is the fixed effect of country i .

Note that from formulation (1), we can obtain the classical *PTR* model of Hansen (1999) as a special case by setting $r_{L,1} = r_{U,1} = r$. However, it is worth mentioning that the transition is abrupt in Hansen (1999). Indeed, a country can switch from one regime to another if the status of the threshold variable oil rents (or quality of the institutions) $q_{i,t}$ crosses up or down the threshold r . More precisely, if the variable oil rents drops below the threshold ($q_{i,t} \leq r$), even slightly, *GDPG* is described by the first regime with slope coefficients β_1 . Conversely, when this variable exceeds r , *GDPG* is described by the second regime with slope coefficients β_2 . In other words, the regime indicator $R_{i,t}$ depends only on whether the oil rent value is smaller or larger

than r . In our proposed model, the regime indicator may depend on the infinitely far away past of the regime indicators (Li et al., 2015).

This two-regime *BTPD* model (1) can be easily extended to a three-regime model. Indeed, suppose there are two buffer zones $(r_{L,1}, r_{U,1}]$ and $(r_{L,2}, r_{U,2}]$ with

$$-\infty = r_{U,0} < r_{L,1} \leq r_{U,1} < r_{L,2} \leq r_{U,2} < r_{L,3} = +\infty.$$

Then, the three-regime *BTPD* model can be defined as follows:

$$Y_{i,t} = \mu_i + \sum_{k=1}^3 X_{i,t} \beta_k \mathbf{1}_{(R_{i,t}=k)} + \varepsilon_{i,t}, \quad (2)$$

where the regime indicator $R_{i,t}$ divides the observations into three regimes characterized by three different regression slopes β_k , $k = 1, 2, 3$. Note that the regime indicator $R_{i,t-1}$ is equal to k if the variable oil rents falls into $(r_{U,k-1}, r_{L,k}]$, and *GDPG* (more generally, the data-generating process, or *DGP*) will stay in the same regime at time t if the variable oil rents increases towards the buffer zone $(r_{L,k}, r_{U,k}]$ or decreases towards $(r_{L,k-1}, r_{U,k-1}]$. In general, when the variable oil rents falls into the buffer zone $(r_{L,l}, r_{U,l}]$ with $l > k$ or $l < k - 1$, the regime indicator $R_{i,t}$ is set to l or $l + 1$, respectively (see Figure 1).

2.2. Least squares estimation

The first step of the estimation process consists of eliminating the permanent differences that exist between individuals over the period, which could skew the estimates. Eliminating individual fixed effects involves removing specific individual means. This step is standard in linear models (within transformation). However, it requires more careful processing in the context of threshold models and particularly in the *BTPD* model. Indeed, the individual effects depend on our knowledge about the buffer zone and must therefore be recalculated for each candidate zone. Another problem arises when the values of the threshold variables for the first observations may fall within one of the buffer zones. This can make the identification of the regime indicators difficult. To address this problem, we propose setting the regime indicator for each individual component i of the first observation as a realization of a uniform random variable on $\{k, k + 1\}$ when its threshold variable value falls within the buffer zone $(r_{L,k}, r_{U,k}]$, i.e., when $r_{L,k} < q_{i,1} \leq r_{U,k}$.

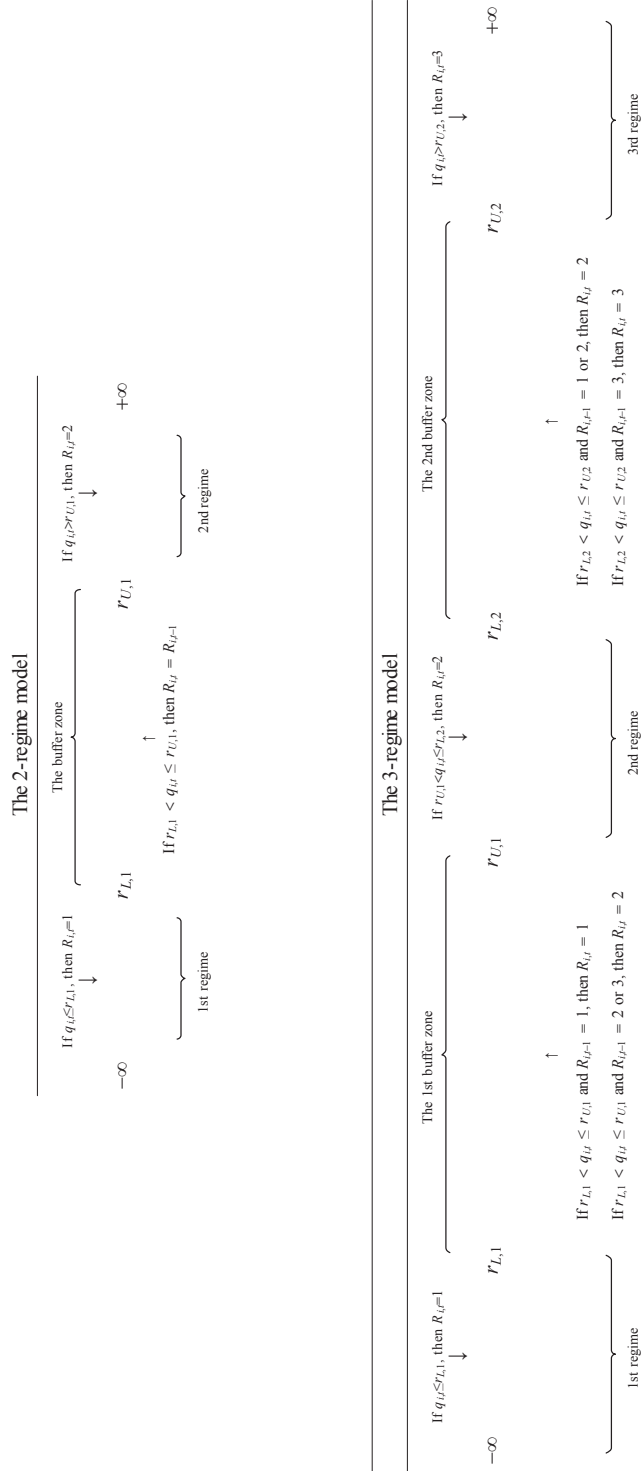


Figure 1: Diagram illustrating the buffered transition mechanism.

The regime indicator sequences $\{R_{i,t}, i = 1, \dots, N \text{ and } t = 2, \dots, T\}$ are subsequently generated.

To eliminate the individual effect, we need the following compact representation of the *BTPD* model:

$$Y_{i,t} = \mu_i + \mathbf{X}_{i,t}\beta + \varepsilon_{i,t}, \quad (3)$$

where $\mathbf{X}_{i,t} = (X_{i,t}\mathbf{1}_{(R_{i,t}=1)}, \dots, X_{i,t}\mathbf{1}_{(R_{i,t}=K)})$ and $\beta = (\beta'_1, \dots, \beta'_K)'$, for $K \in \{2, 3\}$. Let us define the centred individual variables $\mathbf{Y}_i^* = (Y_{i,1}^*, \dots, Y_{i,T}^*)'$, $\mathbf{X}_i^* = (\mathbf{X}_{i,1}^{*'}, \dots, \mathbf{X}_{i,T}^{*'})'$ and $\varepsilon_i^* = (\varepsilon_{i,1}^*, \dots, \varepsilon_{i,T}^*)$ constructed from $\mathbf{Y}_i = (Y_{i,1}, \dots, Y_{i,T})'$, $\mathbf{X}_i = (\mathbf{X}_{i,1}', \dots, \mathbf{X}_{i,T}')'$ and $\varepsilon_i = (\varepsilon_{i,1}, \dots, \varepsilon_{i,T})$, respectively, by removing the corresponding individual means. We are thus able to rewrite model (3) in the following compact form

$$\mathbf{Y}^* = \mathbf{X}^*\beta + \varepsilon^*,$$

where $\mathbf{Y}^* = (\mathbf{Y}_1^{*'}, \dots, \mathbf{Y}_N^{*'})'$, $\mathbf{X}^* = (\mathbf{X}_1^{*'}, \dots, \mathbf{X}_N^{*'})'$ and $\varepsilon^* = (\varepsilon_1^{*'}, \dots, \varepsilon_N^{*'})'$. Hence, assuming that the number of regimes is known and, for a given configuration of the buffer zones $\gamma = (r_{L,1}, r_{U,1}, \dots, r_{L,K-1}, r_{U,K-1})$, that the $NT \times Km$ matrix $\mathbf{X}^*(\gamma)$ has full column rank, the ordinary least squares (*OLS*) estimator of the slope coefficients is given by $\hat{\beta}(\gamma) = ((\mathbf{X}^*(\gamma))' \mathbf{X}^*(\gamma))^{-1} (\mathbf{X}^*(\gamma))' \mathbf{Y}^*$. Once $\hat{\beta}(\gamma)$ is obtained, the individual fixed effects are then estimated by $\hat{\mu}_i = \bar{Y}_i - \bar{\mathbf{X}}_i(\gamma) \hat{\beta}(\gamma)$, for $i = 1, \dots, N$. We can also obtain the residual vector as follows $\hat{\varepsilon}(\gamma) = \mathbf{Y}^* - \mathbf{X}^*(\gamma) \hat{\beta}(\gamma)$. This enables the calculation of the sum of the squared errors $S(\gamma) = \hat{\varepsilon}(\gamma)' \hat{\varepsilon}(\gamma)$. We can thus estimate the residual variance as follows $\hat{\sigma}^2(\gamma) = \frac{1}{NT-N} S(\gamma)$.

As previously discussed, the estimation of the slope coefficients requires knowledge of the buffer zones γ that we should estimate. To do this, we explore a set of buffer zone candidates constructed from quantiles of the observed values of the threshold variable. To ensure a minimum number of observations in each regime, we can - for example, when $K = 2$ - take quantiles from $a\%$ to $b\%$ of each threshold value and sample within a carefully chosen interval $[a, b]$, and we generate all ordered 2-vectors of γ such that $\hat{r}_{L,1} < \hat{r}_{U,1}$. However, when $K = 3$, we require all buffer zone parameters to lie within the bounded subset $[r_0, r_1]$ of the threshold variable sample space, and we choose the 4-vector $\hat{\gamma} = (\hat{r}_{L,1}, \hat{r}_{U,1}, \hat{r}_{L,2}, \hat{r}_{U,2})$ that minimizes the sum of the squared errors, i.e.,

$$\hat{\gamma} = \arg \min_{\gamma \in \Gamma} (S(\gamma)),$$

where $\Gamma = \{(\hat{r}_{L,1}, \hat{r}_{U,1}, \hat{r}_{L,2}, \hat{r}_{U,2}) \in [r_0, r_1]^4 \mid r_{L,1} \leq r_{U,1} < r_{L,2} \leq r_{U,2}\}$. Note that our Γ is larger than the one used in Hansen (1999). Therefore, our exploration of buffer zone candidates requires more computational effort.

2.3. Linearity test and determination of the number of regimes

We now deal with the issue of inference in *BTPD* models. As a priority, we present the tests of linearity and the determination of the number of regimes. These tests occupy a prominent place and guide us in the choice of specification to take into account the nonlinearities.

Consider again a *BTPD* model with two regimes ($K = 2$) as given by (2). As in Hansen (1999), one can use a likelihood ratio (*LR*) test to detect the effect of this nonlinearity. The null hypothesis (no buffered threshold effects) and the alternative hypothesis (the existence of buffered threshold effects) are given by

$$H_0 : \beta_1 = \beta_2 \text{ and } H_1 : \beta_1 \neq \beta_2.$$

This testing problem corresponds to the famous Davies problem (1977, 1987) and has been investigated by Andrews and Ploberger (1994) and Hansen (1996). The bootstrap procedure suggested by Hansen (1996) and used in Hansen (1999) to simulate the corresponding distribution of the *LR* test can be adapted for our framework.

Under H_0 , the *OLS* method provides an estimation $\hat{\beta}_1$ for β_1 and the residuals $\hat{\varepsilon}_{i,t}^*$. The *LR* test statistic is then defined by

$$F_{1,2} = \frac{S_1 - S_2(\hat{\gamma}_2)}{\hat{\sigma}_2^2(\hat{\gamma}_2)},$$

where S_1 and $S_2(\hat{\gamma}_2)$ are the residual sums of squared errors obtained from (1) without and with buffered threshold effects, respectively; $\hat{\sigma}_2^2(\hat{\gamma}_2)$ is the residual variance of the two-regime *BTPD* estimation.

For the special case in which $r_{L,1} = r_{U,1}$, the asymptotic distribution of $F_{1,2}$ is nonstandard (Hansen, 1999). Hence, the critical values and p -values cannot be tabulated. However, this problem can be solved by referring to Hansen's (1996, 1999) bootstrap methodology. The same logic can be applied to our buffered threshold model. The asymptotic p -value can be approximated with the following bootstrap procedure.

Algorithm 1.

1. Treat the regressors $X_{i,t}$ and the threshold jump variable $q_{i,t}$ as given, and their values remain fixed during repeated bootstrap simulations.
2. Recover the regression residuals obtained under the null hypothesis $\widehat{\varepsilon}_{i,t}^*$ and group them by individual $\widehat{\varepsilon}_i^* = (\widehat{\varepsilon}_{i,1}^*, \widehat{\varepsilon}_{i,2}^*, \dots, \widehat{\varepsilon}_{i,T}^*)$. Treat the sample $\{\widehat{\varepsilon}_1^*, \widehat{\varepsilon}_2^*, \dots, \widehat{\varepsilon}_N^*\}$ as the empirical distribution to be used for bootstrapping.
3. Draw (with replacement) a sample of size N from the empirical distribution and use these errors to create a bootstrap sample under H_0 .
4. Using the bootstrap sample, estimate the model under the null hypothesis and the alternative hypothesis and thereby calculate the bootstrap value of $F_{1,2}$.
5. Repeat this procedure a large number of times and calculate the percentage of draws for which the simulated statistic exceeds the observed statistic $F_{1,2}$. This percentage represents the p -value of $F_{1,2}$ under H_0 .

Once the linearity hypothesis is rejected, the question is whether all the nonlinearities in the observations have been taken into account or, in other words, whether we should use a buffered threshold model containing three regimes. The tests to determine the optimal number of regimes are an extension of the linearity tests and allow us to answer this question. Indeed, to test whether the model has two regimes, i.e., $H_0 : \beta_3 = 0$, or at least three regimes, i.e., $H_1 : \beta_3 \neq 0$, the following LR test statistic must be applied:

$$F_{2,3} = \frac{S_2(\widehat{\gamma}_2) - S_3(\widehat{\gamma}_3)}{\widehat{\sigma}_3^2(\widehat{\gamma}_3)}.$$

The null hypothesis of a single buffer zone is rejected in favour of at least two buffer zones if the value of $F_{2,3}$ is greater than the critical value simulated by bootstrapping. It is worth noting that the bootstrap approach to approximating the asymptotic critical values is very close to that presented in Algorithm 1. The only significant change is in Step 2, in which the bootstrap errors used are no longer those obtained under the null hypothesis but under the alternative hypothesis.

2.4. Simulation study

We conducted a Monte Carlo experiment to evaluate the performance of the OLS estimation under different settings. We generated samples from (2) using different values of γ and $\beta = (\beta'_1, \beta'_2, \dots, \beta'_K)'$, where $\beta_k = (\beta_{k,1}, \beta_{k,2}, \dots, \beta_{k,m})'$.

$X_{i,t}$, $q_{i,t}$ and $\varepsilon_{i,t}$ are generated from different independent distributions (normal distribution, log-normal distribution, uniform distribution). For the number of individuals N and the time period T , we use combinations of $T = 10, 20, 50$, and 100 and $N = 10, 20, 50$, and 100 . For each set of generated sample observations, we calculate the *OLS* estimator. We do this 1000 times with different values of the model parameters and for different choices of N and T . The finite sample properties of the estimators are summarized in Tables 10-13 provided in the appendix. For each case, we report the true values (True) of the parameters for each of the considered *BTPD* models, the empirical mean (Mean) and the empirical standard deviation (Std).

The estimates of the *BTPD* structural coefficients based on the *OLS* method display reasonable biases, which decrease as either N or T becomes large. Moreover, the Stds of some parameters are relatively large, but they rapidly decrease with either the number of individuals or the time period. The desirable consistency property of the *OLS* estimators is thus empirically satisfied, and the proposed estimation procedure provides good results.

We now examine the empirical distribution of the estimator for the slope parameters. Figures 6-8 in the appendix depict the sample histograms for the estimated first parameters ($\beta_{1,1}$ or $\beta_{2,1}$) for three configurations of the *BTPD* model that differ by the distribution of the error $\varepsilon_{i,t}$. In the first case, the error term has a uniform distribution, while in the second and third cases, the distribution is normal. It is important to note that the *DGPs* are the same and are given in Tables 11-13. The Jarque-Bera test (*JB* in Figures 6-8) shows that when the cross-sectional and temporal dimensions of the panel are small, the empirical distribution is far from normal. Indeed, the corresponding p -values are lower than 0.1. We thus reject the normality hypothesis at the 1% significance level. However, when the number of individuals and/or the time period is at least equal to 50, we approach asymptotic normality. Indeed, according to the Jarque-Bera test, we do not reject the normality of the empirical distribution at any reasonable level. It is worth noting that the same kind of results hold for the other slope parameters.

We finally investigate the finite sample performance of our proposed procedure for testing linearity in the *BTPD* framework. For that purpose, we consider two *DGPs*. The first is used to assess the size of the test, while the second is used to study its power. The number of replications is fixed at 1000. In each replication, the model is estimated and analysed using the proposed bootstrap-based test of linearity.

The rejection frequencies at the nominal levels of 1, 5 and 10 percent

Algeria	Cameroon	Ecuador	Kuwait	Qatar
Australia	Canada	Egypt, Arab Rep	Nigeria	Saudi Arabia
Bolivia	Colombia	Gabon	Norway	Vietnam
Brunei Darussalam	Côte d’Ivoire	Indonesia	Oman	

Table 1: Sample countries.

are presented in Table 14. It is clear that the rejection frequencies are quite close to the nominal sizes. Furthermore, the power of the test increases with the cross-sectional and temporal dimensions of the panel. In summary, our extension of the bootstrap-based test proposed by Hansen (1999) to investigate linearity seems to work very well in our framework.

3. Empirical results

3.1. The BTPD model

In this section, we study the combined effects of the interaction between natural resource dependence and the quality of institutions on economic growth for a panel of 19 countries for the period 1996-2017. Through the buffered regime switching mechanism, we analyse the heterogeneity in the studied panel and how the interaction between natural resources and the quality of institutions impacts the economic growth of rentier states. The countries in our sample are given in Table 1.

To control for dependence on natural resources and the effects of the quality of institutions, we introduce the variables ‘oil rents’ and ‘rule of law’, respectively. The interaction effect can be analysed by using these variables as explanatory and transition variables at the same time. We add the most used variables in the traditional empirical literature on the macroeconomic determinants of growth to our econometric specification. According to several studies (e.g., Barro, 1991, Barro and Sala-i-Martin, 2003 and Jones, 2001), trade openness, fixed investment, moderate inflation and output volatility, and a better educated workforce have helped countries achieve a higher rate of growth. All the variables used in our study are taken from the World Development Indicators, the World Government-Based Indicators and the International Financial Statistics databases. A short description of these variables is provided in Table 2.

Table 3 provides some descriptive statistics of the data used. It appears that the mean growth rate of GDP in our sample decreased between 1996 and 2017. The indicator of the quality of institutions (rule of law) increased

Variable	Description
GDPG	The growth rate of GDP.
QINST	Rule of law is a governance indicator developed by the World Bank. It includes several indicators that measure the confidence in and respect for the laws and rules of society. Its value varies between -2.5 and 2.5 . A high value indicates a favourable institutional environment and a low value indicates the opposite.
DEP	Dependence on natural resources is represented by oil rents as a percentage of GDP. Oil rents are the difference between the value of crude oil production at world prices and the total costs of production.
INFL	Macroeconomic stability as measured by the inflation rate.
INVEST	Gross fixed capital formation (GFCF) is measured as a percentage of GDP.
OPEN	Trade openness is the sum of exports and imports of goods and services relative to GDP.
POPG	Population growth is measured as the annual rate of population growth.

Table 2: Description of the variables used.

	Mean		Max		Min		Std	
	1996	2017	1996	2017	1996	2017	1996	2017
GDPG	4.13	2.337	9.34	7.70	0.61	-2.87	2.23	2.55
QINST	-0.026	0.019	1.92	2.02	-1.44	-1.20	1.00	0.97
DEP	14.37	8.52	40.96	36.61	0.60	0.21	14.25	9.86
INFL	7.95	4.16	29.27	29.50	0.496	-0.83	8.84	7.17
INVEST	23.45	27.26	41.31	48.40	12.11	15.27	8.14	8.77
OPEN	0.67	0.66	1.206	2.00	0.36	0.23	0.231	0.39
POPG	1.84	1.89	3.08	4.67	0.51	0.84	0.628	0.90

Source: Constructed using World Bank and IMF datasets

Table 3: Descriptive statistics.

very slightly. Moreover, we observe a significant decrease in the mean level of oil rents as a percentage of *GDP*, from 14.37% to 8.52%. Inflation also decreased significantly, from approximately 7.95% to 4.16%. However, the means of investment and population growth increased during this period, while openness decreased very slightly. These tendencies are globally and clearly observed in comparing the maximum and minimum values between these two years.

We investigate two models to explain the interaction between natural resources and the quality of institutions and their impact on *GDP* growth in our sample. In the first model, we consider the quality of institutions to be a threshold variable and dependence on natural resources to be an explanatory variable. In the second model, dependence on natural resources is considered the threshold variable, and the quality of institutions is considered an explanatory variable.

We first test for linearity against a two-regime *BTPD* model and then a two-regime model against a three-regime model.² The results of these tests are reported in Table 4. In both models, we reject linearity at the 1% level. We also reject the two-regime model in favour of the three-regime model

²It is important to note that in practice it is sufficient to consider only the cases $K = 2$ and $K = 3$ to capture the nonlinearities due to regime switching. This is why our analysis is limited to a model with at most three regimes. In our case, this is mainly due to the computational costs of the estimation. Indeed, for our case we have $N \times T = 19 \times 22 = 418$ observations and if the values of a and b are the 10th and 90th percentiles of the data respectively, we would need to estimate 324540216 candidate models when $K = 4$. Note that the the creation of arrays with this dimension (324540216×4) causes MATLAB to become unresponsive.

Linearity against a 2-regime <i>BTPD</i> model.		
	Model (<i>I</i>)	Model (<i>II</i>)
$\hat{\gamma}_2$	[0.1490, 0.9584]	[0.9941, 6.3392]
$S(\hat{\gamma}_2)$	3503.1316	3760.4068
$F_{1,2}$	55.5199	52.6517
p -value	0.0020	0.0000
2-regime <i>BTPD</i> model against a 3-regime <i>BTPD</i> model.		
	Model (<i>I</i>)	Model (<i>II</i>)
$\hat{\gamma}_3$	[-1.0506, 0.0720, 0.1490, 0.9584]	[0.9941, 6.3392, 35.9635, 36.6427]
$S(\hat{\gamma}_3)$	3263.9008	3324.4110
$F_{2,3}$	29.2451	52.3288
p -value	0.0020	0.0000

Table 4: Results of the tests of the *BTPD* models.

at the 1% level. We hereafter discuss the implications of this model for explaining the evolution of the growth of these oil rent-dependent countries. The results of our regressions are given in Table 5.

We first discuss Model (*I*), in which the threshold variable is the quality of institutions. Three regimes are clearly identified.³ In the lower one (regime 1 with low values for the rule of law), it appears that oil rents and openness have no significant effect on the growth of the economy, while inflation has a negative impact and investment and population growth have a positive impact. Using a *VAR* model, Antonakakis et al. (2016) showed that in developing and medium-high income countries with weak political institutions, oil dependence is not growth-enhancing. In the middle regime (regime 2 with intermediate values for the rule of law), oil rents have a positive and significant impact on growth as well as investment and population growth. Openness still has no significant effect. In the upper regime (regime 3 with high values for the rule of law), oil rents have an even greater positive and significant effect on growth. Moreover, openness has a strong and significant positive effect, while population growth and inflation have a negative impact. It thus seems that the stronger the rule of law is, the higher the positive impact of oil rents on economic growth. This gradual positive impact of the

³Figures 2 and 3 give the different regimes of the countries according to both models. A number in black means that the country is clearly in a given regime. When the colour is different, the country is in a buffer zone (i.e., in a possible transition to another regime).

Model Endogenous variable Threshold variable	Model (I)			Model (II)		
	GDPG					
	QINST			DEP		
	Lower	Middle	Upper	Lower	Middle	Upper
QINST	—	—	—	3.0868** (2.0027)	−3.3466*** (3.7693)	4.3239*** (2.5206)
DEP	0.0550 (1.0948)	0.1389*** (3.5044)	0.3144*** (5.4706)	—	—	—
OPEN	−1.5412 (0.7802)	−0.5650 (0.3474)	5.6116** (2.0016)	1.7676 (0.5819)	2.5070* (1.7458)	5.2574* (1.7982)
INF	−0.2442*** (5.3949)	−0.0093 (0.4126)	−0.5179** (2.1196)	−0.3091*** (5.6253)	−0.0142 (0.6850)	−0.0321 (0.1759)
INVEST	0.0798* (1.9024)	0.1457*** (3.1595)	−0.0069 (0.1090)	0.2213*** (3.5098)	−0.0025 (0.0847)	0.1247 (1.2769)
POPG	3.0376*** (3.7167)	0.4452*** (3.4131)	−1.2796*** (3.9515)	1.1948 (1.3368)	0.9298*** (7.4040)	−0.6637*** (2.7045)
$\hat{\gamma}_3$	[−1.0506, 0.0720, 0.1490, 0.9584]			[0.9941, 6.3392, 35.9635, 36.6427]		
AIC	2.1783			2.1966		
BIC	2.3327			2.3511		

The t -statistics are given in parentheses.

***Significant at the 0.01 level, **at the 0.05 level, and *at the 0.10 level.

Table 5: GDP growth, the quality of institutions and oil dependence: estimated three-regime *BTPD* model.

	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Algeria	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Australia	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
Bolivia	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1
Brunei Darussalam	3	3	3	3	3	3	3	3	3	3	2	2	2	2	2	2	2	2	2	2	2	2
Cameroon	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Canada	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
Colombia	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Cote d'Ivoire	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Ecuador	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1
Egypt	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Gabon	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Indonesia	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Kuwait	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2	2	2
Nigeria	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Qatar	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	3	3	3	3	3
Norway	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
Oman	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Saudi Arabia	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Vietnam	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

Figure 2: GDP growth and quality of institutions (as the threshold variable): regime indicator ($R_{i,t}$) values obtained from the estimated three-regime *BTPD* model.

	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Algeria	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Australia	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Bolivia	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	1	1
Brunei Darussalam	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Cameroon	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Canada	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Colombia	1	1	1	1	1	1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2
Cote d'Ivoire	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Ecuador	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Egypt	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Gabon	2	2	2	2	3	2	2	2	2	3	3	2	3	2	2	2	2	2	2	2	2	2
Indonesia	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	1	1
Kuwait	3	3	2	2	3	3	2	3	3	3	3	3	3	3	3	3	3	3	3	3	2	2
Nigeria	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Qatar	2	2	2	2	3	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Norway	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
Oman	3	2	2	2	3	3	2	2	3	3	3	3	3	2	2	3	3	3	3	2	2	2
Saudi Arabia	2	2	2	2	3	2	2	3	3	3	3	3	3	2	3	3	3	3	3	2	2	2
Vietnam	1	1	1	1	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

Figure 3: GDP growth and oil dependence (as the threshold variable): regime indicator ($R_{i,t}$) values obtained from the estimated three-regime *BTPD* model.

quality of the institutions has also been shown in a *PTR* model by Abdulahi et al. (2019).

In practice, our model shows that most of the countries (14 of 19) did not experience a switch in their growth regimes during the period of study. We can classify the countries in our sample into the three regimes as follows. Regime 1: Algeria, Cameroon, Colombia, Côte d'Ivoire, Gabon, Indonesia and Nigeria. Regime 2: Egypt, Oman, Saudi Arabia and Vietnam. Regime 3: Australia, Canada and Norway. However, five countries in our sample did experience a change in their growth regime. The situation for four of them worsened: Bolivia and Ecuador (from regime 2 to regime 1) and Brunei Darussalam and Kuwait (from regime 3 to regime 2). Only Qatar improved its situation (from regime 2 to regime 3). It is worth noting that for most of the period, these countries were in buffer zones (see Figure 2). They can be considered to still be in transition, and their regime shift is not definitive. More generally, it is important to mention that except for the countries in Regime 3, which were very stable during the whole period of study, all the other countries experienced, to different degrees, a transition into the buffer zones.

For Model (II), the three regimes show very different impacts of the quality of institutions on economic growth. In the lower and upper regimes (regimes 1 and 3), the quality of institutions has a positive and significant effect on growth (even though the impact is greater in the upper regime, in which the oil rents are very high). In the middle regime (regime 2, in

which the oil rents have intermediate values), the effect of the quality of institutions is negative and significant. This means that enhancing the rule of law damages the economic growth of the countries in this regime. One can assume that some of these countries have somehow experienced an oil-dependence trap. To the best of our knowledge, only Belarbi et al. (2016) mentioned such a result when considering the quality of institutions and growth in rentier states. In regime 1, we have Australia, Canada and Côte d'Ivoire (see Figure 3). The countries belonging to regime 2 are Algeria, Brunei Darussalam, Cameroon, Ecuador, Egypt, Nigeria and Norway. All the other countries experienced a shift from one regime to another during the period of study. For example, Colombia (in 2011) and Vietnam (in 2000) moved from regime 1 to regime 2. In these countries, the quality of institutions exerts a negative impact on economic growth (beginning in the transition year), while their oil rents significantly increased. Other countries experienced back-and-forth changes: between regimes 1 and 2 (Bolivia and Indonesia) and between regimes 2 and 3 (Gabon, Kuwait, Oman, Qatar and Saudi Arabia). It is worth noting that no switch between the extreme regimes 1 and 3 took place during the whole period of study.

3.2. Comparison with alternative models

For the sake of comparison, we first provide the same kind of analysis with a *PTR* model à la Hansen (1999). Table 6 provides the results of the tests of linearity vs. a one-threshold model and a one-threshold vs. two-threshold model. Our results clearly reject linearity in favour of the existence of a threshold at the 5% level. Moreover, the one-threshold model is rejected in favour of the two-threshold model at the 1% level. Table 7 provides the results of the estimation of a two-threshold Hansen (1999) model with a 0.1% step.

Concerning Model (*I*), in which the transition variable is the quality of institutions, the three regimes provide results that differ substantially from ours, particularly in terms of the impact of natural resource dependence on economic growth. Indeed, in regimes 1 and 3, the impact is positive and significant, while in regime 2, the impact is negative and significant. However, it is worth noting that countries are very rarely in regime 2 in our sample (only five times during the whole period of study; see Figure 4). Moreover, it appears that there was some switching between the extreme regimes 1 and 3, in contrast to the results of the *BTPD* model. This is explained by the presence of the buffer zone, which makes the transitions

Linearity against a one-threshold model.		
	Model (I)	Model (II)
$\hat{\gamma}_2 = \hat{r}_1$	0.4079	36.7892
$S(\hat{\gamma}_2)$	3798.6519	3855.9799
$F_{1,2}$	20.1600	41.4572
p -value	0.0360	0.0000
One-threshold model against a 2-threshold model.		
	Model (I)	Model (II)
$\hat{\gamma}_3 = (\hat{r}_1, \hat{r}_2)$	[0.0757, 0.1037]	[27.3327, 28.6872]
$S(\hat{\gamma}_3)$	3376.3984	3441.7750
$F_{2,3}$	52.4932	48.0182
p -value	0.0000	0.0000

Table 6: Results of the tests of the *PTR* model.

Model	Model (I)			Model (II)		
	GDPG					
Endogenous variable	QINST			DEP		
Threshold variable	Lower	Middle	Upper	Lower	Middle	Upper
QINST	—	—	—	0.4499 (0.5717)	−99.9189*** (2.6600)	2.2351 (1.5584)
DEP	0.1035*** (2.4978)	−4.3509*** (3.4777)	0.1884*** (4.0568)	—	—	—
OPEN	−1.7152 (1.1812)	152.2385*** (2.5700)	−3.0108 (0.9635)	−0.3238 (0.2492)	781.1823*** (3.0363)	−0.1336 (0.0579)
INF	−0.0576*** (2.9157)	4.3268*** (5.6810)	−0.0037 (0.0319)	−0.0636*** (3.2244)	21.0582*** (4.0837)	0.2342 (1.6050)
INVEST	0.1016*** (2.7473)	−3.2426*** (2.5130)	0.0731 (1.6057)	0.0347 (1.1076)	−21.3537*** (3.0790)	0.0647 (0.9093)
POPG	0.6112 (0.9315)	50.9949*** (3.4523)	0.6168*** (4.4795)	0.4377* (1.7740)	13.2968*** (3.6394)	0.4620*** (2.7784)
$\hat{\gamma}_3 = (\hat{r}_1, \hat{r}_2)$	[0.0757, 0.1037]			[27.3327, 28.6872]		
AIC	2.2122			2.2313		
BIC	2.3666			2.3858		

Table 7: GDP growth, quality of institutions and oil dependence (step = 0.1%): estimated three-regime *PTR* model.

between the regimes smoother. More generally, it appears that our model allows us to clearly classify countries into three categories, which is not the case with simple threshold regression models. The same kind of results are found for Model (II). Regime 2 is very different from the others but is very rarely occupied (only five times, see Figure 5). Even in threshold regressions with a 1% step, regime 2 is not significant. Our model thus better captures the evolution of the dynamics in the studied sample. This PTR approach has recently been used by Abdulahi et al. (2019) for 14 resource-rich sub-Saharan countries. They have results similar to ours: a three-regime model with a central regime that is also very rarely occupied, and their estimated thresholds are very close. The use of a buffer zone is thus more strongly indicated even for their data.

Fundamentally, it is worth noting that the classical *PTR* model is a special case of our *BTPD* model that is generated by setting $r_{L,k} = r_{U,k} = r_k$ for all $k = 1, \dots, K - 1$ (i.e., the buffering regions are absent). Therefore, when we choose the vector $\hat{\gamma}$ that minimizes the sum of the squared errors, we implicitly consider *PTR* models as candidates. Indeed, these two classes of models are nested. Therefore, minimizing the sum of squares allows us to implicitly identify whether it is better to choose our buffered model or the classical threshold model.

We can go further in the interpretation of our results. The minimization of $S(\gamma)$ indicates that the optimal transition mechanism is the one given by buffered models (see Tables 4 and 6). Moreover, we can strengthen our strategy for choosing between the two classes of models by calculating certain selection criteria that are very often used in the literature, such as the *AIC* and *BIC* that select models via the optimization of a penalized objective function. It turns out that based on the minimum *AIC* and *BIC* and for a fixed K , the buffered models are preferred to the classical threshold models.

As we have already mentioned, another approach to tackling sudden jumps is *PSTR* modelling (Gonzales et al., 2017). We thus propose a second alternative model for our data using this approach. More precisely, we consider the following model:

$$y_{i,t} = \mu_i + X_{i,t}\beta_0 + \sum_{j=1}^r X_{i,t}\beta_j (1 + \exp(-\gamma_j (q_{i,t} - c_j)))^{-1} + u_{i,t}.$$

Table 8 provides the results of our tests for the different numbers of regimes. In Model (I), in which the quality of institutions is the thresh-

	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Algeria	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Australia	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
Bolivia	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Brunei Darussalam	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
Cameroon	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Canada	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
Colombia	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Cote d'Ivoire	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Ecuador	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Egypt	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Gabon	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Indonesia	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Kuwait	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	2
Nigeria	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Qatar	1	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
Norway	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
Oman	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
Saudi Arabia	3	2	1	1	1	1	1	3	1	1	1	2	1	1	3	1	3	3	3	3	3	2
Vietnam	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Figure 4: GDP growth and the quality of institutions (as the threshold variable): regime indicator ($R_{i,t}$) values obtained from the estimated PTR model.

	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017
Algeria	1	1	1	1	1	1	1	1	1	3	3	2	3	1	1	1	1	1	1	1	1	1
Australia	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Bolivia	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Brunei Darussalam	1	1	1	1	1	1	1	1	1	3	3	1	1	1	1	1	1	1	1	1	1	1
Cameroon	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Canada	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Colombia	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Cote d'Ivoire	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Ecuador	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Egypt	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Gabon	3	1	1	1	3	1	1	1	1	3	3	3	3	1	3	3	3	3	1	1	1	1
Indonesia	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Kuwait	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
Nigeria	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Qatar	3	2	1	3	3	3	2	3	3	3	3	3	1	2	3	3	1	1	1	1	1	1
Norway	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
Oman	3	3	1	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	1	1
Saudi Arabia	3	3	1	2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	1	1
Vietnam	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1

Figure 5: GDP growth and oil dependence (as the threshold variable): regime indicator ($R_{i,t}$) values obtained from the estimated PTR model.

Linearity against a 2-regime <i>PSTR</i> model.		
	Model (I)	Model (II)
$\hat{\gamma}$	70.2885	2.0617
\hat{c}	0.3944	37.6029
SSR_2	3938.0969	3987.9135
LM _{1,2}	5.5343	26.3876
<i>p</i> -value	0.3542	0.0001
2-regime <i>PSTR</i> model against a 3-regime <i>PSTR</i> model.		
	Model (I)	Model (II)
$\hat{\gamma} = (\hat{\gamma}_1, \hat{\gamma}_2)$	–	(25.8914, 2.1034)
$\hat{c} = (\hat{c}_1, \hat{c}_2)$	–	(3.9882, 37.5683)
SSR_3	–	3463.8230
LM _{2,3}	–	42.2983
<i>p</i> -value	–	0.0000

Table 8: Results of the tests of the *PSTR* models. The $LM_{k,k+1}$ statistic is defined as $LM_{k,k+1} = TN(SSR_k - SSR_{k+1})/SSR_k$, where SSR_k is the panel sum of squared residuals under H_0 (*PSTR* model with k regimes) and SSR_{k+1} is the panel sum of squared residuals under H_1 (*PSTR* model with $(k + 1)$ regimes).

Model	Model (I)			Model (II)		
	GDPG					
Endogenous variable						
Threshold variable	QINST			DEP		
	β_0	β_1	β_2	β_0	β_1	β_2
QINST		–	–	–0.7567 (–0.7515)	–0.7750 (–1.5072)	8.6892*** (3.6096)
DEP	0.1639*** (5.2576)	–	–	–	–	–
OPEN	–2.7269* (–1.9166)	–	–	–0.8193 (–0.7292)	1.2486 (1.1822)	3.5827 (1.0384)
INF	–0.0509** (–2.4788)	–	–	–0.2536*** (–3.7430)	0.2402*** (3.3965)	–0.0595 (–0.2948)
INVEST	0.0908*** (2.9265)	–	–	0.0882** (2.1498)	–0.0747* (–1.6931)	0.1304 (0.9377)
POPG	0.4829*** (3.9210)	–	–	1.1144** (2.5139)	–0.2132 (–0.4643)	–1.6709*** (–4.0201)
$\hat{\gamma} = (\hat{\gamma}_1, \hat{\gamma}_2)$	–			(25.8914, 2.1034)		
$\hat{c} = (\hat{c}_1, \hat{c}_2)$	–			(3.9882, 37.5683)		
AIC	2.2922			2.2797		
BIC	2.3404			2.4631		

Table 9: GDP growth, the quality of institutions and oil dependence: estimated *PSTR* Model.

old variable, we do not reject linearity. Therefore, *PSTR* modelling is not suitable for explaining the observed heterogeneity in our data. Indeed, as our *BTPD* (and even *PTR*) models reject linearity, we can say that our approach fits our data better than a *PSTR* approach. For Model (II), in which the threshold variable is dependence on natural resources, we accept a three-regime *PSTR* model. Let us now compare the two models: the three-regime *BTPD* and the three-regime *PSTR*. The estimation results for the latter are provided in Table 9. In this model, we have two location parameters, $c_1 = 3.98$ and $c_2 = 37.56$. They represent the threshold values that indicate breaks in the relationship within the different regimes. However, the interpretation of the parameters is less easy than in the *BTPD* case. Indeed, in the latter, the beta coefficients correspond to the effect of the exogenous variables in each regime (lower, middle and upper) identified by the threshold values. In the *PSTR* case, the beta coefficients should be combined with the threshold functions to determine the effect of the exogenous variables on the dependent variable. Roughly speaking, we can determine two extreme regimes with certainty: for very low natural resource dependence, the effect of the exogenous variables is β_0 ; for very high dependence rates, this effect is given by $\beta_0 + \beta_1 + \beta_2$. In between, the impact of the exogenous variables varies continuously. In practice, the interpretation of the marginal effect is more interesting (e.g., Belarbi et al., 2016). *BTPD* modelling thus offers a more precise classification while giving, at the same time, a smooth transition between the different regimes. The effect of the quality of institutions is highly statistically significant in the three regimes, while only β_2 is significant in the *PSTR* model. The *BTPD* approach thus better captures the heterogeneity in *GDP* growth among the countries in our sample.

4. Conclusion

In this article, we revisit the question of the relationships among growth, oil dependence and institutions by providing a new approach to address nonlinearities in the panel data framework. Our model is suitable for accounting for the problem of sudden jumps in *PTR* models. The results show that it is a very useful tool for studying smooth transitions between different regimes. It can also be considered a promising alternative to *PSTR* modelling and provides results that are more easily interpretable. Indeed, our application shows that, compared with *PTR* and *PSTR* models, *BTPD* model can provide a richer and more precise description of the evolution of economic

growth in rentier states while taking into account the interactions between oil dependence and the quality of institutions. In our sample, it is clear that the relationships between growth and dependence on oil rents are not linear: there is a gradual positive impact as the quality of institutions increases. Only oil-dependent countries with high-quality institutions are very stable. All the other countries in our sample have experienced a transition into a buffer zone. They are potentially in a transition between different regimes. Moreover, when considering dependence on oil rents as a threshold variable, it appears that the quality of institutions has a positive and significant effect on growth when oil dependence is low or high. More interestingly, the quality of institutions negatively impacts growth in intermediate oil-dependent countries. Some of these countries experience something of an oil-dependence trap. These results are relatively new in the literature dealing with growth in rentier states. Our *BTPD* formulation better highlights the impacts of oil dependence and the quality of institutions on economic growth. Indeed, the *BTPD* model allows us to clearly classify countries into three categories, while this is not the case with classical *PTR* and *PSTR* models. Finally, our model offers a wide perspective in terms of applications to different frameworks and could enhance our understanding of the dynamics of the evolutions of various economic phenomena.

Acknowledgements

We would like to thank the Co-Editor Angus Chu, an Associate Editor and two anonymous referees for their very helpful comments and suggestions that highly improved the quality of the paper.

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5. Appendix

N	T	10		20		50		100		
		True	Mean	Std	Mean	Std	Mean	Std	Mean	Std
10	$\beta_{1,1}$	-0.5	-0.5158	0.1554	-0.5053	0.0932	-0.4994	0.0583	-0.4975	0.0405
	$\beta_{1,2}$	0.6	0.6244	0.1512	0.6027	0.0962	0.6004	0.0571	0.6017	0.0396
	$\beta_{2,1}$	0.3	0.3444	0.2263	0.3177	0.1370	0.2987	0.0798	0.2980	0.0541
	$\beta_{2,2}$	-0.2	-0.2157	0.2276	-0.2130	0.1387	-0.1980	0.0785	-0.1971	0.0543
	$r_{L,1}$	0.25	0.1334	0.4536	0.2312	0.2417	0.2464	0.1105	0.2554	0.0583
	$r_{U,1}$	0.75	0.7656	0.3432	0.7551	0.1700	0.7483	0.0688	0.7494	0.0339
	σ^2	1	0.9183	0.1377	0.9680	0.1038	0.9937	0.0628	0.9968	0.0438
	20	$\beta_{1,1}$	-0.5	-0.5072	0.0981	-0.4971	0.0636	-0.4994	0.0406	-0.4987
$\beta_{1,2}$		0.6	0.6085	0.1002	0.5999	0.0659	0.5996	0.0408	0.5999	0.0280
$\beta_{2,1}$		0.3	0.3073	0.1450	0.3002	0.0880	0.3015	0.0568	0.2964	0.0390
$\beta_{2,2}$		-0.2	-0.2214	0.1467	-0.2013	0.0943	-0.1972	0.0573	-0.1987	0.0380
$r_{L,1}$		0.25	0.2460	0.2664	0.2493	0.1504	0.2513	0.0598	0.2506	0.0287
$r_{U,1}$		0.75	0.7571	0.2040	0.7499	0.0832	0.7486	0.0357	0.7498	0.0213
σ^2		1	0.9650	0.1036	0.9842	0.0732	0.9970	0.0453	1.0003	0.0320
50		$\beta_{1,1}$	-0.5	-0.4996	0.0613	-0.4997	0.0417	-0.4981	0.0257	-0.4977
	$\beta_{1,2}$	0.6	0.5988	0.0584	0.6004	0.0419	0.5987	0.0252	0.5981	0.0175
	$\beta_{2,1}$	0.3	0.2938	0.0862	0.2916	0.0591	0.2961	0.0351	0.2952	0.0257
	$\beta_{2,2}$	-0.2	-0.2016	0.0880	-0.1960	0.0572	-0.1963	0.0345	-0.1969	0.0249
	$r_{L,1}$	0.25	0.2690	0.1415	0.2524	0.0751	0.2505	0.0249	0.2500	0.0158
	$r_{U,1}$	0.75	0.7534	0.0895	0.0645	0.0422	0.7497	0.0195	0.7499	0.0138
	σ^2	1	0.9900	0.0678	0.9988	0.0449	1.0021	0.0278	1.0028	0.0199
	100	$\beta_{1,1}$	-0.5	-0.4965	0.0411	-0.4970	0.0284	-0.4980	0.0185	-0.4976
$\beta_{1,2}$		0.6	0.5994	0.0421	0.5967	0.0291	0.5975	0.0174	0.5981	0.0127
$\beta_{2,1}$		0.3	0.2887	0.0603	0.2959	0.0394	0.2950	0.0245	0.2961	0.0181
$\beta_{2,2}$		-0.2	-0.1931	0.0612	-0.1945	0.0376	-0.1953	0.0253	-0.1957	0.0183
$r_{L,1}$		0.25	0.2620	0.0914	0.2525	0.0350	0.2495	0.0158	0.2499	0.0129
$r_{U,1}$		0.75	0.7505	0.0419	0.7501	0.0240	0.7498	0.0150	0.7497	0.0133
σ^2		1	1.0017	0.0471	1.0047	0.0330	1.0040	0.0204	1.0034	0.0142

Table 10: Results of a simulation study for a two-regime *BTPD* model with $\varepsilon_{i,t} \sim \mathcal{N}(0, 1)$, $X_{i,t} \sim \mathcal{N}(0, I_2)$ and $q_{i,t} \sim \mathcal{N}(0, 2)$.

N	T	10		20		50		100		
		True	Mean	Std	Mean	Std	Mean	Std	Mean	Std
10	$\beta_{1,1}$	0.3	0.2987	0.1620	0.3039	0.1064	0.3011	0.0636	0.3000	0.0441
	$\beta_{1,2}$	-0.2	-0.2061	0.1632	-0.1991	0.1066	-0.1994	0.0619	-0.1996	0.0425
	$\beta_{1,3}$	1.5	1.4981	0.1602	1.5020	0.1090	1.4943	0.0608	1.5017	0.0425
	$\beta_{1,4}$	1.3	1.3092	0.1659	1.3000	0.1108	1.3005	0.0630	1.2985	0.0442
	$\beta_{1,5}$	-0.2	-0.1907	0.1582	-0.1999	0.1126	-0.2063	0.0608	-0.1988	0.0465
	$\beta_{2,1}$	0.5	0.4927	0.3071	0.5033	0.1650	0.5040	0.1023	0.4964	0.0715
	$\beta_{2,2}$	0.2	0.2244	0.3066	0.1985	0.1635	0.2004	0.1020	0.1981	0.0711
	$\beta_{2,3}$	0.9	0.9115	0.3241	0.9073	0.1570	0.9023	0.0997	0.9036	0.0719
	$\beta_{2,4}$	-0.8	-0.7849	0.3284	-0.7999	0.1590	-0.7912	0.1053	-0.7887	0.0711
	$\beta_{2,5}$	-0.5	-0.5081	0.3167	-0.5070	0.1518	-0.5029	0.1058	-0.4980	0.0715
	$r_{L,1}$	0.25	0.2120	0.2550	0.2432	0.1525	0.2487	0.0575	0.2514	0.0285
	$r_{U,1}$	0.75	0.7500	0.1396	0.7414	0.0971	0.7506	0.0263	0.7495	0.0168
	σ^2	1.33	1.1623	0.1356	1.3147	0.0708	1.3180	0.0607	1.3301	0.0407
20	$\beta_{1,1}$	0.3	0.3065	0.1078	0.3061	0.0671	0.3006	0.0435	0.3008	0.0310
	$\beta_{1,2}$	-0.2	-0.1980	0.1085	-0.1970	0.0726	-0.1989	0.0427	-0.1976	0.0309
	$\beta_{1,3}$	1.5	1.4997	0.1056	1.4970	0.0708	1.4982	0.0449	1.4983	0.0298
	$\beta_{1,4}$	1.3	1.2981	0.1083	1.2942	0.0744	1.2995	0.0441	1.2950	0.0313
	$\beta_{1,5}$	-0.2	-0.2005	0.1064	-0.2032	0.0712	-0.2002	0.0428	-0.2024	0.0311
	$\beta_{2,1}$	0.5	0.5026	0.1887	0.4998	0.1231	0.4991	0.0711	0.5003	0.0508
	$\beta_{2,2}$	0.2	0.1903	0.1841	0.1930	0.1275	0.1981	0.0752	0.2013	0.0500
	$\beta_{2,3}$	0.9	0.8912	0.1860	0.9080	0.1203	0.9073	0.0738	0.9059	0.0485
	$\beta_{2,4}$	-0.8	-0.7838	0.1888	-0.7819	0.1216	-0.7788	0.0723	-0.7866	0.0522
	$\beta_{2,5}$	-0.5	-0.4958	0.1926	-0.4988	0.1187	-0.4949	0.0718	-0.4969	0.0510
	$r_{L,1}$	0.25	0.2618	0.1600	0.2585	0.0936	0.2501	0.0286	0.2510	0.0155
	$r_{U,1}$	0.75	0.7513	0.0749	0.7506	0.0364	0.7503	0.0167	0.7501	0.0131
	σ^2	1.33	1.2754	0.1050	1.3174	0.0742	1.3390	0.0438	1.3413	0.0298
50	$\beta_{1,1}$	0.3	0.3035	0.0655	0.2991	0.0440	0.2998	0.0280	0.2998	0.0188
	$\beta_{1,2}$	-0.2	-0.2000	0.0664	-0.1966	0.0439	-0.1985	0.0279	-0.1994	0.0194
	$\beta_{1,3}$	1.5	1.4927	0.0648	1.4962	0.0450	1.4983	0.0279	1.4978	0.0194
	$\beta_{1,4}$	1.3	1.2870	0.0661	1.2917	0.0444	1.2938	0.0278	1.2954	0.0208
	$\beta_{1,5}$	-0.2	-0.2000	0.0666	-0.1999	0.0448	-0.1993	0.0272	-0.2010	0.0188
	$\beta_{2,1}$	0.5	0.4998	0.1135	0.4955	0.0726	0.4982	0.0455	0.4985	0.0310
	$\beta_{2,2}$	0.2	0.1940	0.1109	0.1876	0.0748	0.1967	0.0465	0.1957	0.0326
	$\beta_{2,3}$	0.9	0.9119	0.1129	0.9065	0.0763	0.9047	0.0431	0.9057	0.0319
	$\beta_{2,4}$	-0.8	-0.7661	0.1182	-0.7733	0.0803	-0.7812	0.0472	-0.7845	0.0346
	$\beta_{2,5}$	-0.5	-0.4979	0.1133	-0.4946	0.0769	-0.4978	0.0453	-0.4987	0.0323
	$r_{L,1}$	0.25	0.2663	0.0902	0.2547	0.0369	0.2508	0.0135	0.2500	0.0104
	$r_{U,1}$	0.75	0.7519	0.0355	0.7509	0.0178	0.7510	0.0130	0.7498	0.0111
	σ^2	1.33	1.3465	0.0718	1.3502	0.0448	1.3490	0.0275	1.3489	0.0198

Table 11: Results of a simulation study for a two-regime *BTPD* model with $\varepsilon_{i,t} \sim \mathcal{U}[-2, 2]$, $X_{i,t} \sim \mathcal{N}(0, I_5)$ and $q_{i,t} \sim \mathcal{N}(0, 2)$.

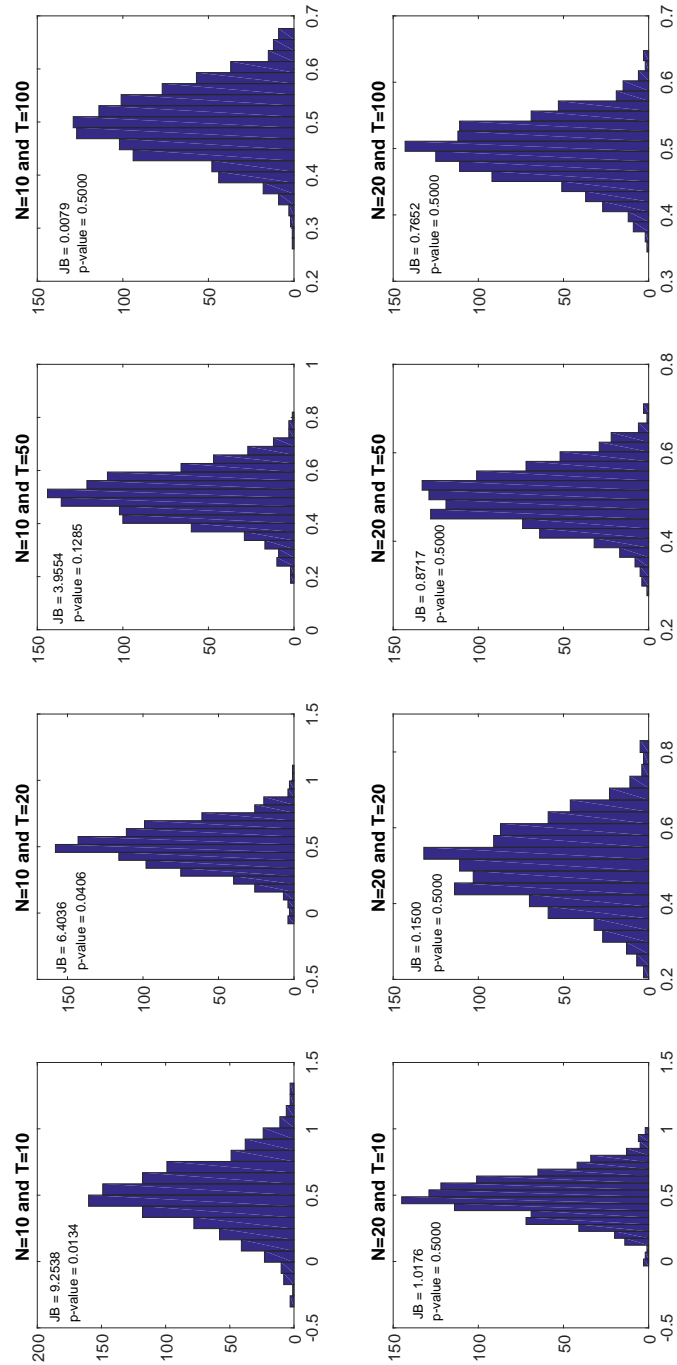


Figure 6: Empirical distributions of the estimator when $\beta_{2,1} = 0.5$. The samples are generated from a three-regime *BTPD*, as defined in Table 11.

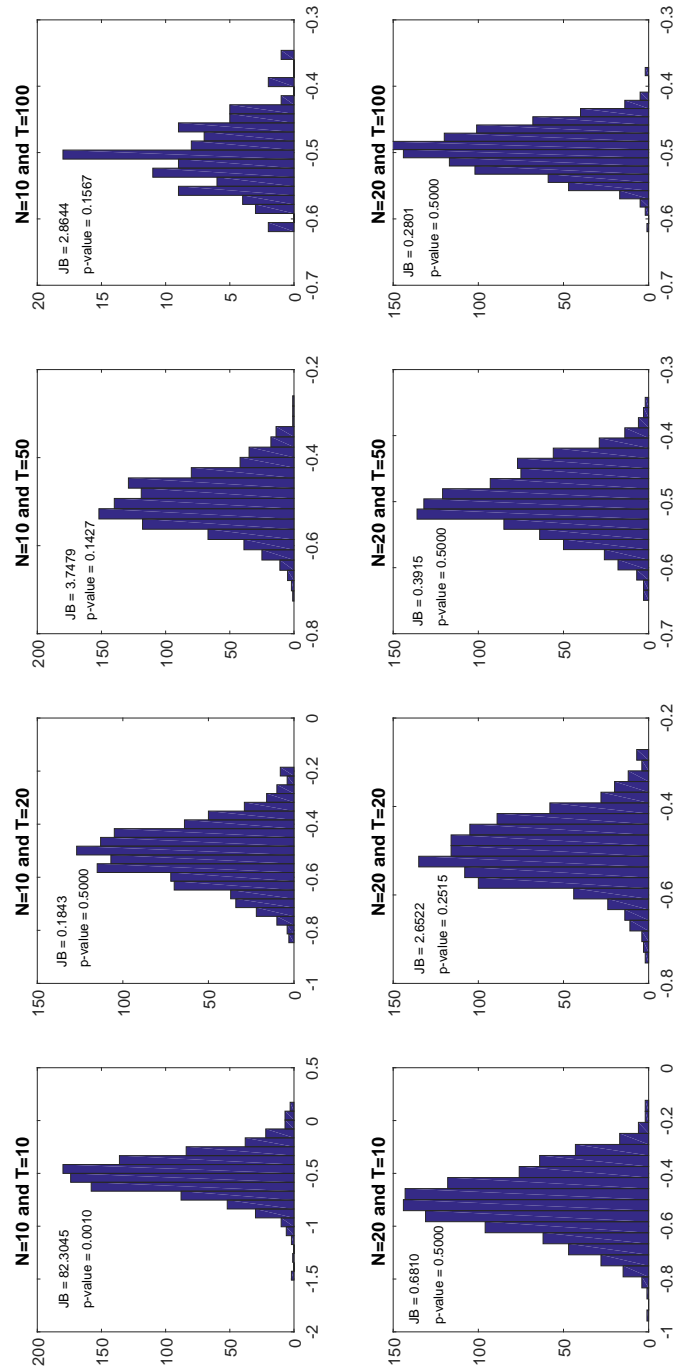


Figure 7: Empirical distributions of the estimator when $\beta_{1,1} = -0.5$. The samples are generated from a three-regime *BTPD*, as defined in Table 12.

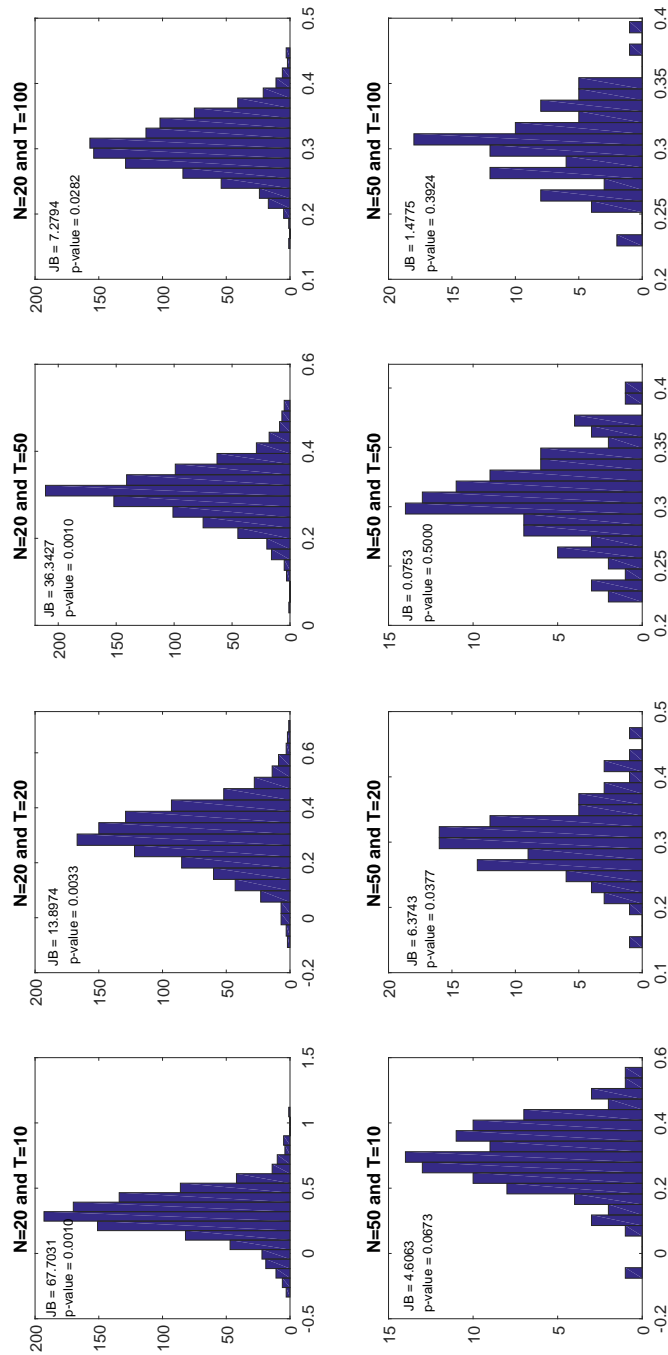


Figure 8: Empirical distributions of the estimator when $\beta_{1,1} = 0.3$. The samples are generated from a three-regime *BTPD*, as defined in Table 13.

N	T	10		20		50		100		
		True	Mean	Std	Mean	Std	Mean	Std	Mean	Std
100	$\beta_{1,1}$	0.3	0.3021	0.0453	0.3008	0.0320	0.2991	0.0203	0.3001	0.0140
	$\beta_{1,2}$	-0.2	-0.2003	0.0445	-0.1980	0.0316	-0.1989	0.0192	-0.1999	0.0134
	$\beta_{1,3}$	1.5	1.4976	0.0472	1.4977	0.0320	1.4977	0.0195	1.4989	0.0137
	$\beta_{1,4}$	1.3	1.2883	0.0501	1.2923	0.0317	1.2946	0.0204	1.2954	0.0137
	$\beta_{1,5}$	-0.2	-0.2004	0.0475	-0.2008	0.0313	-0.1999	0.0191	-0.2007	0.0139
	$\beta_{2,1}$	0.5	0.4960	0.0798	0.4962	0.0540	0.4966	0.0317	0.4981	0.0217
	$\beta_{2,2}$	0.2	0.1887	0.0774	0.1941	0.0521	0.1967	0.0306	0.1963	0.0226
	$\beta_{2,3}$	0.9	0.9183	0.0785	0.9089	0.0521	0.9054	0.0314	0.9038	0.0222
	$\beta_{2,4}$	-0.8	-0.7502	0.0829	-0.7666	0.0569	-0.7783	0.0363	-0.7825	0.0252
	$\beta_{2,5}$	-0.5	-0.4907	0.0773	-0.4964	0.0539	-0.4964	0.0318	-0.4976	0.0233
	$r_{L,1}$	0.25	0.2624	0.0512	0.2529	0.0224	0.2503	0.0107	0.2498	0.0087
	$r_{U,1}$	0.75	0.7505	0.0223	0.7506	0.0137	0.7498	0.0114	0.7493	0.0103
	σ^2	1.33	1.3711	0.0537	1.3620	0.0349	1.3545	0.0198	1.3510	0.0151

Table 11: Continued.

N	T	10			20		50		100	
		True	Mean	Std	Mean	Std	Mean	Std	Mean	Std
10	$\beta_{1,1}$	0.3	0.3351	0.2801	0.3101	0.1814	0.3067	0.0916	0.3031	0.0591
	$\beta_{1,2}$	-0.2	-0.1205	0.3100	-0.1743	0.1895	-0.1937	0.0971	-0.1932	0.0616
	$\beta_{2,1}$	0.5	0.4980	0.6182	0.5102	0.1978	0.5005	0.0617	0.4965	0.0380
	$\beta_{2,2}$	0.2	0.0744	0.6573	0.1836	0.2236	0.2023	0.0697	0.1970	0.0371
	$\beta_{3,1}$	0.3	0.2909	0.2510	0.2991	0.0936	0.3009	0.0423	0.3012	0.0279
	$\beta_{3,2}$	-0.4	-0.4013	0.2327	-0.4049	0.0876	-0.4019	0.0428	-0.3976	0.0288
	$r_{L,1}$	0.3	0.4915	0.2479	0.3717	0.1539	0.3113	0.0473	0.3021	0.0200
	$r_{U,1}$	0.5	0.6898	0.3283	0.5591	0.1950	0.5112	0.1015	0.5037	0.0708
	$r_{L,2}$	0.8	0.9627	0.4588	0.8361	0.2258	0.8029	0.0740	0.8013	0.0264
	$r_{U,2}$	1	1.3702	0.7407	1.0975	0.3838	1.0079	0.1130	0.9989	0.0338
σ^2	2	1.7494	0.2777	1.9023	0.1952	1.9825	0.1291	2.0028	0.0941	
20	$\beta_{1,1}$	0.3	0.3062	0.1772	0.2978	0.1130	0.3058	0.0622	0.3056	0.0415
	$\beta_{1,2}$	-0.2	-0.1582	0.2048	-0.1851	0.1171	-0.1949	0.0616	-0.1937	0.0414
	$\beta_{2,1}$	0.5	0.5055	0.2046	0.5053	0.0725	0.4996	0.0391	0.4973	0.0254
	$\beta_{2,2}$	0.2	0.1654	0.2611	0.2014	0.0812	0.1958	0.0404	0.1933	0.0263
	$\beta_{3,1}$	0.3	0.2979	0.0868	0.3007	0.0485	0.3002	0.0275	0.3012	0.0191
	$\beta_{3,2}$	-0.4	-0.4000	0.1132	-0.4008	0.0493	-0.3991	0.0280	-0.3970	0.0204
	$r_{L,1}$	0.3	0.3907	0.1623	0.3188	0.0593	0.3036	0.0249	0.3021	0.0156
	$r_{U,1}$	0.5	0.5842	0.2107	0.5097	0.1122	0.5001	0.0739	0.4962	0.0452
	$r_{L,2}$	0.8	0.8492	0.2269	0.8014	0.0616	0.8042	0.0387	0.8001	0.0205
	$r_{U,2}$	1	1.1171	0.4104	1.0073	0.1334	1.0007	0.0565	1.0002	0.0249
σ^2	2	1.8779	0.1899	1.9556	0.1500	2.0205	0.0631	2.0056	0.0611	
50	$\beta_{1,1}$	0.3	0.3001	0.1040	0.3038	0.0529	0.3075	0.0363	0.3032	0.0293
	$\beta_{1,2}$	-0.2	-0.2095	0.0946	-0.2022	0.0587	-0.1980	0.0359	-0.1874	0.0239
	$\beta_{2,1}$	0.5	0.4972	0.0599	0.4975	0.0381	0.4973	0.0252	0.4953	0.0174
	$\beta_{2,2}$	0.2	0.2042	0.0575	0.1977	0.0391	0.1914	0.0267	0.1898	0.0181
	$\beta_{3,1}$	0.3	0.3004	0.0418	0.3081	0.0273	0.3011	0.0171	0.3005	0.0116
	$\beta_{3,2}$	-0.4	-0.3953	0.0492	-0.3974	0.0298	-0.4000	0.0177	-0.3980	0.0116
	$r_{L,1}$	0.3	0.3114	0.0445	0.2985	0.0180	0.2998	0.0153	0.3027	0.0161
	$r_{U,1}$	0.5	0.4856	0.0812	0.4962	0.0692	0.4991	0.0387	0.4961	0.0267
	$r_{L,2}$	0.8	0.7951	0.0394	0.8004	0.0292	0.7998	0.0199	0.8000	0.0189
	$r_{U,2}$	1	0.9911	0.0577	0.9976	0.0347	0.9993	0.0212	1.0005	0.0196
σ^2	2	1.9701	0.1322	1.9811	0.1026	2.0239	0.0608	2.0303	0.0394	
100	$\beta_{1,1}$	0.3	0.2999	0.0539	0.3050	0.0493	0.3098	0.0272	0.3097	0.0217
	$\beta_{1,2}$	-0.2	-0.1907	0.0639	-0.1931	0.0487	-0.1881	0.0292	-0.1831	0.0203
	$\beta_{2,1}$	0.5	0.4992	0.0357	0.4929	0.0251	0.4927	0.0155	0.4954	0.0114
	$\beta_{2,2}$	0.2	0.1993	0.0385	0.1979	0.0259	0.1924	0.0144	0.1912	0.0127
	$\beta_{3,1}$	0.3	0.2984	0.0292	0.3000	0.0202	0.3023	0.0117	0.3008	0.0073
	$\beta_{3,2}$	-0.4	-0.3969	0.0268	-0.3973	0.0228	-0.3986	0.0140	-0.3967	0.0096
	$r_{L,1}$	0.3	0.3014	0.0211	0.3016	0.0195	0.3031	0.0159	0.3060	0.0158
	$r_{U,1}$	0.5	0.5052	0.0737	0.5012	0.0353	0.4984	0.0222	0.5022	0.0164
	$r_{L,2}$	0.8	0.8011	0.0260	0.8030	0.0200	0.7987	0.0212	0.8011	0.0200
	$r_{U,2}$	1	1.0082	0.0394	1.0018	0.0252	0.9986	0.0205	1.0002	0.0166
σ^2	2	2.0036	0.0904	2.0251	0.0692	2.0331	0.0399	2.0378	0.0298	

Table 12: Results of a simulation study for a three-regime *BTPD* model with $\varepsilon_{i,t} \sim \mathcal{N}(0, 2)$, $\log(X_{i,t}) \sim \mathcal{N}(0, I_2)$ and $\log(q_{i,t}) \sim \mathcal{N}(0, 2)$.

N	T	10		20		50		100		
		True	Mean	Std	Mean	Std	Mean	Std	Mean	Std
10	$\beta_{1,1}$	-0.5	-0.5145	0.1988	-0.5122	0.1108	-0.4998	0.0646	-0.5044	0.0478
	$\beta_{1,2}$	0.6	0.6179	0.1899	0.6062	0.1145	0.6012	0.0673	0.5963	0.0450
	$\beta_{1,3}$	1.2	1.2051	0.1992	1.1917	0.1074	1.2026	0.0663	1.2036	0.0434
	$\beta_{1,4}$	1.5	1.5021	0.1869	1.5027	0.1119	1.5011	0.0701	1.5032	0.0480
	$\beta_{1,5}$	0.2	0.2114	0.1945	0.2004	0.1150	0.1988	0.0648	0.1922	0.0459
	$\beta_{2,1}$	0.3	0.2764	0.5906	0.3053	0.1967	0.2904	0.1077	0.2932	0.0723
	$\beta_{2,2}$	-0.2	-0.1877	0.5329	-0.2029	0.1941	-0.1946	0.1078	-0.1890	0.0636
	$\beta_{2,3}$	1.5	1.5075	0.8154	1.4991	0.1918	1.4905	0.1075	1.4853	0.0645
	$\beta_{2,4}$	0.9	0.9473	0.5113	0.8804	0.1985	0.9000	0.1130	0.8963	0.0663
	$\beta_{2,5}$	-0.7	-0.6928	0.6169	-0.7005	0.2058	-0.6988	0.1081	-0.6954	0.0752
	$\beta_{3,1}$	-0.4	-0.4005	0.2695	-0.4002	0.1427	-0.3967	0.0852	-0.3798	0.0597
	$\beta_{3,2}$	-0.3	-0.3103	0.2699	-0.2951	0.1493	-0.2982	0.0868	-0.3004	0.0703
	$\beta_{3,3}$	0.9	0.9090	0.2625	0.9052	0.1529	0.8989	0.0850	0.9111	0.0623
	$\beta_{3,4}$	-0.5	-0.5024	0.2800	-0.5035	0.1482	-0.4922	0.0871	-0.4864	0.0588
	$\beta_{3,5}$	-0.8	-0.8176	0.2673	-0.7993	0.1499	-0.7996	0.0875	-0.8070	0.0591
	$r_{L,1}$	-0.2	-0.3261	0.3386	-0.2104	0.1183	-0.2023	0.0575	-0.1920	0.0365
	$r_{U,1}$	0.1	0.0700	0.2503	0.1021	0.1049	0.0965	0.0571	0.0953	0.0370
	$r_{L,2}$	0.6	0.4744	0.2554	0.5554	0.1360	0.5914	0.0711	0.5952	0.0509
	$r_{U,2}$	0.8	0.8441	0.2732	0.8194	0.1099	0.7993	0.0518	0.8022	0.0440
σ^2	1	0.7856	0.1295	0.9113	0.0982	0.9857	0.0639	1.0107	0.0457	
20	$\beta_{1,1}$	-0.5	-0.5085	0.1174	-0.4987	0.0746	-0.4966	0.0488	-0.4958	0.0325
	$\beta_{1,2}$	0.6	0.5997	0.1196	0.5974	0.0735	0.5969	0.0462	0.5946	0.0326
	$\beta_{1,3}$	1.2	1.2010	0.1130	1.1998	0.0795	1.2010	0.0464	1.2011	0.0330
	$\beta_{1,4}$	1.5	1.4981	0.1145	1.4983	0.0747	1.4969	0.0455	1.4971	0.0325
	$\beta_{1,5}$	0.2	0.1999	0.1116	0.1980	0.0772	0.1941	0.0483	0.1955	0.0334
	$\beta_{2,1}$	0.3	0.3175	0.2106	0.2955	0.1271	0.2822	0.0737	0.2784	0.0548
	$\beta_{2,2}$	-0.2	-0.2036	0.2019	-0.1986	0.1207	-0.1912	0.0735	-0.1881	0.0517
	$\beta_{2,3}$	1.5	1.4954	0.2039	1.4944	0.1194	1.4888	0.0749	1.4885	0.0512
	$\beta_{2,4}$	0.9	0.9015	0.2133	0.8987	0.1282	0.8912	0.0759	0.8920	0.0559
	$\beta_{2,5}$	-0.7	-0.7032	0.2040	-0.6916	0.1199	-0.6912	0.0761	-0.6893	0.0519
	$\beta_{3,1}$	-0.4	-0.4027	0.1621	-0.3989	0.0990	-0.3948	0.0589	-0.3952	0.0418
	$\beta_{3,2}$	-0.3	-0.3020	0.1498	-0.3001	0.1000	-0.2983	0.0609	-0.2987	0.0426
	$\beta_{3,3}$	0.9	0.8952	0.1543	0.8997	0.1017	0.9013	0.0613	0.9070	0.0424
	$\beta_{3,4}$	-0.5	-0.4977	0.1511	-0.4932	0.0980	-0.4893	0.0634	-0.4888	0.0455
	$\beta_{3,5}$	-0.8	-0.7923	0.1426	-0.7987	0.0976	-0.8000	0.0577	-0.7985	0.0427
	$r_{L,1}$	-0.2	-0.2199	0.1308	-0.2028	0.0702	-0.1994	0.0413	-0.1999	0.0333
	$r_{U,1}$	0.1	0.0975	0.1201	0.0991	0.0648	0.0995	0.0378	0.0965	0.0308
	$r_{L,2}$	0.6	0.5516	0.1420	0.5883	0.0824	0.6000	0.0518	0.6014	0.0396
	$r_{U,2}$	0.8	0.8152	0.1166	0.8062	0.0602	0.8026	0.0409	0.7989	0.0367
σ^2	1	0.9103	0.1024	0.9736	0.0720	1.0117	0.0488	1.0220	0.0350	

Table 13: Results of a simulation study for a three-regime *BTPD* model with $\varepsilon_{i,t} \sim \mathcal{N}(0, 1)$, $X_{i,t} \sim \mathcal{N}(0, I_5)$ and $q_{i,t} \sim \mathcal{N}(0, 2)$.

N	T	10			20		50		100	
		True	Mean	Std	Mean	Std	Mean	Std	Mean	Std
50	$\beta_{1,1}$	-0.5	-0.5187	0.0661	-0.4942	0.0477	-0.4940	0.0293	-0.4982	0.0189
	$\beta_{1,2}$	0.6	0.6032	0.0705	0.5993	0.0446	0.5949	0.0315	0.5954	0.0195
	$\beta_{1,3}$	1.2	1.1992	0.0686	1.1982	0.0483	1.2048	0.0290	1.2040	0.0232
	$\beta_{1,4}$	1.5	1.4961	0.0658	1.4964	0.0523	1.4937	0.0324	1.5012	0.0182
	$\beta_{1,5}$	0.2	0.2149	0.0740	0.1987	0.0416	0.1956	0.0323	0.1971	0.0212
	$\beta_{2,1}$	0.3	0.2820	0.1035	0.2908	0.0768	0.2779	0.0485	0.2723	0.0367
	$\beta_{2,2}$	-0.2	-0.1894	0.1131	-0.1877	0.0785	-0.1954	0.0507	-0.1926	0.0351
	$\beta_{2,3}$	1.5	1.4794	0.1140	1.4852	0.0707	1.4835	0.0466	1.4829	0.0316
	$\beta_{2,4}$	0.9	0.9172	0.1141	0.8904	0.0779	0.8851	0.0515	0.8791	0.0427
	$\beta_{2,5}$	-0.7	-0.6903	0.1113	-0.6899	0.0843	-0.6814	0.0489	-0.6852	0.0311
	$\beta_{3,1}$	-0.4	-0.4019	0.0869	-0.4007	0.0612	-0.3995	0.0394	-0.3950	0.0249
	$\beta_{3,2}$	-0.3	-0.2962	0.0893	-0.2922	0.0561	-0.2961	0.0397	-0.3010	0.0297
	$\beta_{3,3}$	0.9	0.8996	0.0911	0.8911	0.0631	0.9055	0.0366	0.9033	0.0243
	$\beta_{3,4}$	-0.5	-0.4858	0.0862	-0.4896	0.0676	-0.4864	0.0438	-0.4903	0.0309
	$\beta_{3,5}$	-0.8	-0.8103	0.0844	-0.7983	0.0578	-0.8072	0.0402	-0.7995	0.0260
	$r_{L,1}$	-0.2	-0.2127	0.0746	-0.1955	0.0383	-0.1959	0.0290	-0.2016	0.0288
	$r_{U,1}$	0.1	0.0833	0.0595	0.0947	0.0369	0.0998	0.0301	0.0939	0.0270
	$r_{L,2}$	0.6	0.5825	0.0672	0.6019	0.0554	0.5975	0.0383	0.6067	0.0357
	$r_{U,2}$	0.8	0.7958	0.0590	0.8021	0.0413	0.8049	0.0388	0.8053	0.0362
	σ^2	1	0.9924	0.0679	1.0151	0.0532	1.0249	0.0283	1.0350	0.0241
100	$\beta_{1,1}$	-0.5	-0.4995	0.0476	-0.4956	0.0365	-0.4974	0.0196	-0.4966	0.0152
	$\beta_{1,2}$	0.6	0.6047	0.0536	0.5988	0.0366	0.5959	0.0213	0.5952	0.0164
	$\beta_{1,3}$	1.2	1.2025	0.0461	1.2089	0.0348	1.2000	0.0196	1.2004	0.0148
	$\beta_{1,4}$	1.5	1.5037	0.0440	1.4923	0.0312	1.5009	0.0202	1.4969	0.0160
	$\beta_{1,5}$	0.2	0.2027	0.0478	0.1903	0.0308	0.1960	0.0197	0.1935	0.0141
	$\beta_{2,1}$	0.3	0.2765	0.0786	0.2823	0.0478	0.2786	0.0351	0.2685	0.0274
	$\beta_{2,2}$	-0.2	-0.2053	0.0771	-0.1985	0.0521	-0.1957	0.0341	-0.1926	0.0252
	$\beta_{2,3}$	1.5	1.4926	0.0757	1.4927	0.0516	1.4847	0.0385	1.4808	0.0221
	$\beta_{2,4}$	0.9	0.8907	0.0840	0.8781	0.0576	0.8897	0.0372	0.8757	0.0358
	$\beta_{2,5}$	-0.7	-0.6883	0.0839	-0.7034	0.0521	-0.6877	0.0356	-0.6934	0.0235
	$\beta_{3,1}$	-0.4	-0.3979	0.0669	-0.3938	0.0387	-0.3928	0.0270	-0.3882	0.0201
	$\beta_{3,2}$	-0.3	-0.3031	0.0699	-0.3013	0.0421	-0.2987	0.0243	-0.3010	0.0183
	$\beta_{3,3}$	0.9	0.9037	0.0656	0.9089	0.0419	0.9066	0.0284	0.9066	0.0215
	$\beta_{3,4}$	-0.5	-0.4908	0.0610	-0.4918	0.0441	-0.4826	0.0341	-0.4845	0.0263
	$\beta_{3,5}$	-0.8	-0.7883	0.0626	-0.7939	0.0436	-0.7995	0.0233	-0.8005	0.0195
	$r_{L,1}$	-0.2	-0.1938	0.0404	-0.1978	0.0321	-0.1997	0.0289	-0.1899	0.0250
	$r_{U,1}$	0.1	0.0949	0.0392	0.0990	0.0353	0.0989	0.0261	0.0910	0.0192
	$r_{L,2}$	0.6	0.6041	0.0486	0.6012	0.0392	0.6031	0.0371	0.6112	0.0346
	$r_{U,2}$	0.8	0.8049	0.0422	0.8067	0.0359	0.8034	0.0348	0.8056	0.0332
	σ^2	1	1.0047	0.0479	1.0211	0.0307	1.0331	0.0249	1.0365	0.0179

Table 13: Continued.

Empirical size							
True model and H_0	H_1	N	α	$T = 10$	$T = 20$	$T = 50$	$T = 100$
Linear	Two-regime <i>BTPD</i>	10	0.01	6	9	8	9
			0.05	53	44	56	50
			0.10	103	94	98	98
		20	0.01	9	17	14	10
			0.05	54	54	53	47
			0.10	101	89	96	95
			0.01	13	12	12	10
		50	0.05	49	51	44	49
			0.10	104	98	87	90
		Empirical power					
True model and H_1	H_0	N	α	$T = 10$	$T = 20$	$T = 50$	$T = 100$
Two-regime <i>BTPD</i>	Linear	10	0.01	829	1000	1000	1000
			0.05	935	1000	1000	1000
			0.10	968	1000	1000	1000
		20	0.01	1000	1000	1000	1000
			0.05	1000	1000	1000	1000
			0.10	1000	1000	1000	1000
			0.01	1000	1000	1000	1000
		50	0.05	1000	1000	1000	1000
			0.10	1000	1000	1000	1000

Table 14: Rejection frequencies from the bootstrap-based test for linearity.