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# A Lightweight Epistemic Logic and its Application to Planning

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## Abstract

We study multiagent epistemic planning with a simple epistemic logic whose language is a restriction of that of standard epistemic logic. Its formulas are boolean combinations of observability atoms: sequences of ‘knowing whether’ operators followed by propositional variables. This compares favourably with other restricted languages where formulas are boolean combinations of epistemic literals: sequences of ‘knowing that’ epistemic operators and negations followed by propositional variables; or in other terms: epistemic formulas without conjunctions or disjunctions. The reason is that our language enables a richer theory of mind: we can express statements such as “I don’t know whether  $p$ , but I know that you know whether  $p$ ” which are important in communication and more generally in interaction and which cannot be expressed with epistemic literals. Going beyond previous work, we also introduce a ‘common knowledge whether’ operator. We show that satisfiability is nevertheless NP-complete. We then define simple epistemic planning tasks as generalisations of classical planning tasks: action descriptions have sets of observability atoms as add and delete-lists, initial states are sets of observability atoms, and goals are boolean combinations of observability atoms. We show that simple epistemic planning tasks can be polynomially translated into classical planning tasks. It follows that checking solvability of simple epistemic planning

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tasks is PSPACE-complete. We present some application examples such as the gossip problem and some experimental results and clarify the relationship with Dynamic Epistemic Logic-based planning.

*Keywords:* Epistemic planning; multiagent planning; epistemic logic; gossip problem

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## 1. Introduction

Suppose there are  $n$  agents, each of which knows some secret: a piece of information that is not known to the others. They communicate by phone calls, and whenever one person calls another they tell each other all they know at that time. How many calls are required before each item of gossip is known to everyone? This is known as the gossip problem. It is of great interest in the conception of communication networks and in parallel and distributed computing, but there are other less obvious applications like the management of data on storage devices [1], or the computation of the syntenic distance between two genomes (minimum number of fusions, fissions, and translocations required to transform one into the other) [2]. Several variants have been studied in the literature, and a survey of these alternatives and the associated results has been published [3]. The gossip problem can be viewed as a multiagent epistemic planning task where the goal is shared knowledge: everybody knows all secrets. It is a purely epistemic planning task because it is only the agents' knowledge that evolves, while the facts of the world remain unchanged. There are numerous variations of such a planning task; see e.g. [4] for an overview. The goal may in particular be to achieve higher-order shared knowledge: everybody knows that everybody knows all secrets, and so on. We believe that generalised gossiping is a paradigmatic epistemic planning task, much in the same way as the blocksworld is a paradigmatic classical planning task: (1) it is multiagent; (2) it is 'truly epistemic', in the sense that it involves higher-order knowledge; (3) it is as simple as it could be, in the sense that there are no

25 ontic actions; (4) it has many variations.

The aim of the present paper is to introduce a simple logical approach within which we can account for centralised epistemic planning tasks such as the above. We could have chosen Dynamic Epistemic Logic (DEL) for this purpose, as done by [5]: its event models allow us to model telephone calls  
30 as private announcements. However, we take inspiration from a simpler framework: the Dynamic Epistemic Logic of Propositional Assignment and Observation (DEL-PAO), introduced in [6] and further developed in [7, 8, 9]. It is based on observability atoms, that are recursively defined as either propositional variables or atomic pieces of information about whether or not  
35 an agent sees the truth value of a observability atom. DEL-PAO was applied to epistemic planning in [10]. We here simplify this account by disregarding the dynamic aspects and concentrating on the epistemic logic underlying DEL-PAO. That logic is consequently called the Epistemic Logic of Observa-  
40 tion, abbreviated as EL-O. The present paper improves over [10] by simplifying the semantics (models are no longer infinite), by integrating a form of common knowledge into planning, and by a complexity analysis: satisfiability is NP-complete. Our EL-O-based definition of simple epistemic planning tasks parallels that of classical planning tasks, with actions, initial state, and goal described by means of observability atoms. This allows us to poly-  
45 nomially translate the former into the latter. It follows that deciding the solvability of a simple epistemic planning task is PSPACE-complete (just as DEL-PAO satisfiability, validity and model checking are).

The paper is organised as follows. In Section 2 we provide a detailed motivation of our approach and briefly overview related work. In Section 3  
50 we introduce EL-O and in Section 4 we establish that it is a fragment of the standard epistemic logic S5 with an NP-complete satisfiability problem. In Section 5 we give a formal definition of action descriptions and simple epistemic planning tasks within our framework and polynomially translate them into classical planning tasks, from which PSPACE membership  
55 of deciding plan existence follows. In Section 6 we apply our framework

to examples, in particular the gossip problem, and in Section 7 we report on some experiments that were done via an encoding of simple epistemic planning tasks into PDDL. Section 8 discusses related work, where we argue in particular that our action descriptions are a more appropriate way to describe the behaviour of actions than with DEL event models. Section 9 concludes.

## 2. Background and motivation

Reasoning in epistemic logic as introduced by Hintikka [11] and popularised in AI by Halpern and colleagues [12] is strictly more complex than in classical logic: the satisfiability problem is at least in PSPACE, and is EXPTIME-complete if common knowledge is involved [13]. The complexity gap widens for planning tasks: while the solvability problem is PSPACE-complete for classical planning [14], it is undecidable for planning with DEL-based event models [15]. The latter provide rather expressive models for the agents' perception of actions. They parallel the standard epistemic logic modelling of uncertainty by indistinguishability relations between possible states: uncertainty in the perception of actions is modelled by indistinguishability relations between possible events. As explored by several authors in a series of papers, undecidability of DEL planning is already the case under very weak hypotheses about these event models [16, 17, 18, 19]. Basically, DEL planning tasks are only decidable when the event model is a singleton, i.e., when all actions are public. However, actions in the gossip problem are not public, nor are they in many real world applications.

We propose to base epistemic planning on an epistemic logic that is simpler than Hintikka's: epistemic information is restricted to the atomic formulas of the language. We are not the first to do this: several previous approaches have proposed languages where the scope of the epistemic operator  $K_i$  is restricted to literals, or literals that are preceded by a sequence of

85 epistemic operators [20, 21, 22, 23], possibly with negations [24, 25]. Such restrictions however exclude formulas such as  $K_i(K_j p \vee K_j \neg p)$  expressing that agent  $i$  knows that agent  $j$  knows whether the propositional variable  $p$  is true. This is a major drawback because such formulas are fundamental in communication and more generally in any form of interaction: a situation where agent  $i$  does not know whether  $p$  is the case or not ( $\neg K_i p \wedge \neg K_i \neg p$ ) but knows that  $j$  knows ( $K_i(K_j p \vee K_j \neg p)$ ) may lead agent  $i$  to ask  $j$  about  $p$ .  
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We here make a similar restriction, but move from the primitive ‘knowing-that’ operator  $K_i$  to the less standard ‘knowing-whether’ or ‘knowing-if’ operator  $Kif_i$ . In the unrestricted language these two operators are interdefinable: we have  $Kif_i \varphi \leftrightarrow K_i \varphi \vee K_i \neg \varphi$  and  $K_i \varphi \leftrightarrow \varphi \wedge Kif_i \varphi$ , for arbitrary formulas  $\varphi$  [26]. However, the situation changes when we restrict the language to sequences of ‘knowing-whether’ operators followed by propositional variables. On the one hand, any sequence of ‘knowing-that’ operators followed by a literal can be expressed by a sequence of ‘knowing-whether’ operators:  
95

- 100 •  $K_i p$  is equivalent to  $p \wedge Kif_i p$ ;
- $K_i \neg p$  is equivalent to  $\neg p \wedge Kif_i p$ ;
- $K_i K_j p$  is equivalent to  $p \wedge Kif_i p \wedge Kif_j p \wedge Kif_i Kif_j p$ ;
- and so on.

On the other hand and as we will prove in Section 8.2, from  $n \geq 2$  on sequences  $Kif_{i_1} \dots Kif_{i_n} p$  of ‘knowing-whether’ operators followed by propositional variables cannot be expressed by means of boolean combinations of ‘knowing-that’ operators followed by literals. In particular, no such boolean combination is equivalent to  $Kif_i Kif_j p$ . Such increased expressivity makes a difference. For example, consider the above situation where  $i$  does not know whether  $p$  but knows whom to ask, as expressed by the formula  $\neg K_i p \wedge \neg K_i \neg p \wedge K_i(K_j p \vee K_j \neg p)$ : this can be expressed in our language by  $\neg Kif_i p \wedge Kif_j p \wedge Kif_i Kif_j p$ .  
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Instead of  $Kif_i$  we prefer to write  $S_i$ , and we read  $S_i p$  as “agent  $i$  sees whether  $p$  is true or not”. This highlights that when we define  $K_i p$  as  $p \wedge S_i p$ ,  
 115 we consider that  $i$ ’s knowledge comes from what  $i$  sees. More generally, an *observability atom* takes the form  $S_{i_1} \dots S_{i_n} p$  of a sequence of observability operators prefixing a propositional variable. Then propositional variables are the special case where  $n = 0$ . We are going to consider that every such observability atom is an atomic formula and that complex formulas are  
 120 boolean combinations of such atomic formulas. Technically  $S_i$  is therefore not a modal operator, as opposed to  $Kif_i$ , and our move in notation also aims at stressing that. This comes with a move away from Kripke models with accessibility relations: a model is simply a valuation over the set of observability atoms, alias a state. We identify such valuations with sets of  
 125 observability atoms. The only delicate point is that the interpretation of formulas has to be arranged such that introspection is guaranteed.

A semantic counterpart of observability atoms  $S_i p$  of depth 1 were introduced in the model checking literature in order to represent epistemic models in a compact way [27, 28, 29]. As argued there, such compact mod-  
 130 els are more attractive than Kripke models when it comes to model checking. It is shown in [29] how any Kripke model can be captured by means of observability information, possibly by adding new propositional variables. Our observability atoms  $S_i p$  capture their semantic concept in the syntax.

Simplifications of epistemic logic typically lack a modal operator of common knowledge. (It is however present in [29].) We here consider an operator  $JS$ , reading  $JS p$  as “all agents jointly see the value of  $p$ ”, or “all agents jointly see whether  $p$  is true or not”. Metaphorically, joint attention about a propositional variable  $p$  can be understood as eye contact between the agents when observing  $p$ .  $JS$  is a powerful operator that allows us to model  
 135 common knowledge obtained through co-presence. One may think of  $JS p$  as implying the infinite set of propositions of the form  $S_{i_1} \dots S_{i_n} p$  for all  $i_1, \dots, i_n$  and all  $n \geq 1$ . Suppose  $p$  stands for ‘the door is open’. We can imagine that  
 140  $JS p$  is true when all agents are present in the same room and not only

observe the door, but also mutually observe each other. In this concrete  
 145 example, if an agent leaves the room and closes the door behind her,  $JS p$   
 becomes false because the agents no longer mutually observe each other  
 (even if the agent who has left can see the closed door from the outside).  
 In order to guarantee that joint observability of  $p$  implies individual ob-  
 servability of  $p$ , we require in our semantics that states containing  $JS p$  also  
 150 contain every  $S_i p$ , and more generally every  $p$  preceded by any sequence of  
 $S_i$  and  $JS$ . Defining  $CKp$  as  $p \wedge JS p$ , we obtain a common knowledge oper-  
 ator that satisfies all the standard properties except the induction axiom  
 $CK(p \rightarrow \bigwedge_{i \in \mathbb{A}} K_i p) \rightarrow (p \rightarrow CKp)$ . We will discuss this issue in Section 4  
 (Remark 4).

155 It is generally assumed in the AI literature that a reasonable epistemic  
 logic should satisfy introspection: the formulas  $K_i \varphi \rightarrow K_i K_i \varphi$  and  $\neg K_i \varphi \rightarrow$   
 $K_i \neg K_i \varphi$  should both be valid.<sup>1</sup> In terms of observability this means that  
 states should contain  $S_i S_i p$  for every agent  $i$  and propositional variable  $p$ ,  
 and more generally every  $S_i S_i S_j p$ ,  $S_j S_i S_i p$ , etc. Similarly, they should also  
 160 contain  $JS JS p$ . Moreover, whenever a state contains some  $JS p$  then it should  
 also contain  $S_i p$ , for every agent  $i$ . Clearly, these closure conditions make  
 states infinite. We are going to avoid this by integrating closure into the  
 truth condition for observability atoms.

All we have said up to now was about how to describe the agents' static  
 165 knowledge. We complete the picture by adding actions that may modify  
 both the state of the world and the agents' vision: we simply add and  
 delete atoms. This requires some care: we have to guarantee that the  
 above properties of introspection and 'joint sight implies individual sight'  
 are preserved. We do so by requiring action descriptions to be consistent:  
 170 they cannot delete introspective atoms such as  $S_i S_i p$ , they cannot add  $JS p$

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<sup>1</sup>We are aware that negative introspection was criticised in the literature as being too  
 strong [30, 31]. One may however argue with Holliday [32, footnote 4] that these criticisms do  
 not apply to observability-based knowledge.



and delete  $S_i p$  at the same time, etc. A further issue that arises when describing epistemic actions is that the description of effects should cater for conditional effects: whether or not an action changes the knowledge of some agent  $i$  depends on what  $i$  observes when the action is performed. For example, when agent  $i$  switches the light on then the effect of that action on agent  $j$ 's knowledge depends on whether the latter is in the room or not. Another example is a call between two agents in the gossip problem where the agents share all the secrets they know: what is learned is conditional on the agents' prior knowledge. Based on such action descriptions, a simple epistemic planning task is going to be made up of:

- a vocabulary;
- a set of consistent descriptions of actions with conditional effects which add and delete observability atoms;
- an initial state which is an EL-O valuation;
- a goal formula which is an EL-O formula.

Classical planning tasks are particular cases of simple epistemic planning tasks; therefore the existence of a solution to a simple epistemic planning task is PSPACE-hard. It is actually PSPACE-complete: as we will show in Section 5.5, simple epistemic planning tasks can be polynomially reduced to classical planning tasks.

### 3. EL-O: Epistemic Logic of Observation

We introduce the Epistemic Logic of Observation, abbreviated to EL-O. Its language consists of formulas over a vocabulary of observability atoms: propositional variables prefixed by sequences of observability operators  $S_i$  and  $\mathcal{S}$ . Models of EL-O are sets of such observability atoms. The interpretation of formulas is defined in a way such that introspection principles become valid. We show that the EL-O satisfiability problem is NP-complete.

### 3.1. Boolean languages and Classical Propositional Calculus CPC

The language  $\mathcal{L}_{\text{bool}}(\mathbb{V})$  of boolean formulas over some vocabulary  $\mathbb{V}$  combines the elements of  $\mathbb{V}$  by means of the boolean operators. It is defined by the grammar

$$\mathcal{L}_{\text{bool}}(\mathbb{V}) : \varphi ::= p \mid \neg\varphi \mid (\varphi \wedge \varphi)$$

where  $p$  ranges over  $\mathbb{V}$ . The boolean operators  $\top$ ,  $\perp$ ,  $\vee$ ,  $\rightarrow$ , and  $\leftrightarrow$  are defined in the standard way. The *vocabulary of*  $\varphi$  is defined inductively by:  $\text{Voc}(p) = \{p\}$ , for  $p \in \mathbb{V}$ ;  $\text{Voc}(\neg\varphi) = \text{Voc}(\varphi)$ ;  $\text{Voc}(\varphi \wedge \varphi') = \text{Voc}(\varphi) \cup \text{Voc}(\varphi')$ .

The vocabulary  $\mathbb{V}$  is usually supposed to be a countable set of propositional variables  $\mathbb{P}$  that are not analysed further. However, in our lightweight epistemic logic  $\mathbb{V}$  has some structure: its observability atoms involve propositional variables and agents. Related to that, instead of defining the length of an atomic formula to be 1 we suppose in this paper that each  $p \in \mathbb{P}$  has some length  $\ell(p) \geq 1$ . The length of complex formulas is then defined inductively from the length of atomic formulas by:  $\ell(\neg\varphi) = \ell(\varphi) + 1$  and  $\ell(\varphi \wedge \varphi') = \ell(\varphi) + \ell(\varphi') + 1$ . For example, if  $\ell(p) = 1$  then  $\ell(\top) = 5$  because  $\top$  abbreviates  $\neg(p \wedge \neg p)$ .

A *state* is a subset of the vocabulary  $\mathbb{V}$ . We denote states by  $s$ ,  $s'$ , etc. In *Classical Propositional Calculus* CPC the formulas of  $\mathcal{L}_{\text{bool}}(\mathbb{V})$  are interpreted in the set of all states  $2^{\mathbb{V}}$  according to the following truth conditions:  $s \models p$  iff  $p \in s$ , for every  $p \in \mathbb{V}$ ;  $s \models \neg\varphi$  iff  $s \not\models \varphi$ ; and  $s \models \varphi \wedge \varphi'$  iff  $s \models \varphi$  and  $s \models \varphi'$ . A formula  $\varphi$  is *CPC satisfiable* iff  $s \models \varphi$  for every  $s \in 2^{\mathbb{V}}$ , and it is *CPC valid* iff  $s \models \varphi$  for some  $s \in 2^{\mathbb{V}}$ . It is known that CPC-satisfiability of a formula is NP-complete and that CPC-validity of a formula is coNP-complete.

### 3.2. Observability atoms and introspection

We now define the vocabulary we will be using in our epistemic logic. Let  $\mathbb{P}$  be a countable set of *propositional variables* and let  $\mathbb{A}$  be a finite set of *agents*. The set of *observability operators* is  $\text{OBS} = \{S_i : i \in \mathbb{A}\} \cup \{JS\}$ , where  $S_i$  stands for individual observability of agent  $i$  and  $JS$  stands for

joint observability of all agents. The set of all sequences of observability operators is noted  $\text{OBS}^*$  and the set of all non-empty sequences is noted  $\text{OBS}^+$ . We use  $\sigma, \sigma'$ , etc. to denote elements of  $\text{OBS}^*$  and reserve  $\mathbf{nil}$  for the empty sequence. The length of a sequence of observability operators is defined inductively by:  $\ell(\mathbf{nil}) = 0$ ;  $\ell(S_i \sigma) = \ell(\sigma) + 1$ ; and  $\ell(\mathcal{J}S \sigma) = \ell(\sigma) + 1$ .

*Observability atoms*, or atoms for short, are finite sequences of observability operators followed by a propositional variable. The set

$$\text{OA} = \{\sigma p : \sigma \in \text{OBS}^*, p \in \mathbb{P}\}.$$

is the set of all atoms. (It depends on the set of propositional variables  $\mathbb{P}$  and on the set of agents  $\mathbb{A}$ ; we however leave these arguments implicit.) We use the small Greek letters  $\alpha, \alpha', \beta, \dots$  to denote atoms. Here are some examples:  $S_1 p$  reads “1 sees the value of  $p$ ”. Hence 1 knows whether  $p$  is true or false.  $\mathcal{J}S_2 q$  reads “all agents jointly see whether agent 2 sees the value of  $q$ ”. In other words, there is joint attention in the group of all agents concerning 2’s observation of  $q$ : agent 2 may or may not see the value of  $q$ , and in both cases this is jointly observed.  $S_1 S_2 S_3 p$  reads “1 sees whether 2 sees whether 3 sees  $p$ ”. The *length* of an observability atom is  $\ell(\sigma p) = \ell(\sigma) + 1$ . Therefore an atom of length 1 is nothing but a propositional variable. For example,  $\ell(\mathcal{J}S_2 p) = 3$ . As  $\top$  abbreviates  $\neg(p \vee \neg p)$ , it is of length  $\ell(\top) = 2 \times \ell(p) + 3$ .

**Example 1.** In the initial states  $s_0^{G_1}$  of the gossip problem (in which secrets may or may not be true) each agent only knows her own secret. Therefore  $s_0^{G_1} = \{S_i s_i : i \in \mathbb{A}\} \cup S$  where  $S$  is some subset of  $\{s_i : i \in \mathbb{A}\}$ .

As explained in the introduction, individual introspection is expressed with our ‘knowing-whether’ operator as  $S_i S_i \alpha$ . Joint introspection is expressed by  $\mathcal{J}S \alpha$ . The latter implies  $\sigma \mathcal{J}S \alpha$  for every non-empty  $\sigma$  because joint observability implies any nesting of individual observability. We therefore call an atom *introspective* if it contains two consecutive  $S_i$ , or a  $\mathcal{J}S$  that is preceded by a non-empty sequence of observability operators. In other words, an atom is introspective if it is of the form  $\sigma S_i S_i \alpha$  for some  $\sigma \in \text{OBS}^*$ ,

or of the form  $\sigma JS\alpha$  for some  $\sigma \in \text{OBS}^+$ . The set of all introspective atoms is

$$\text{l-OA} = \{\sigma S_i S_i \alpha : \sigma \in \text{OBS}^* \text{ and } \alpha \in \text{OA}\} \cup \{\sigma JS\alpha : \sigma \in \text{OBS}^+ \text{ and } \alpha \in \text{OA}\}.$$

250 The set l-OA is infinite and is a proper subset of OA.

### 3.3. Atomic consequence and introspectively closed states

We define a relation of *atomic consequence* between observability atoms as follows:

$$\alpha \Rightarrow \beta \text{ iff } \alpha = \beta, \text{ or } \alpha = JS\alpha' \text{ and } \beta = \sigma\alpha' \text{ for some } \sigma \in \text{OBS}^+.$$

If this is the case then we say that  $\alpha$  is a *cause* of  $\beta$  and that  $\beta$  is a *consequence* of  $\alpha$ . For example,  $JS p \Rightarrow S_i p$  and  $JS p \Rightarrow JS S_i p$ . The relation  $\Rightarrow$  is clearly reflexive and transitive. Moreover, the set of introspective atoms is  
255 closed under atomic consequence: if  $\alpha \in \text{l-OA}$  and  $\alpha \Rightarrow \beta$  then  $\beta \in \text{l-OA}$ .

We let  $\alpha^\Rightarrow$  denote the set of consequences of  $\alpha$  and  $\alpha^\Leftarrow$  the set of its causes:

$$\alpha^\Rightarrow = \{\beta : \alpha \Rightarrow \beta\}, \quad \alpha^\Leftarrow = \{\beta : \beta \Rightarrow \alpha\}.$$

Here are some examples:

$$\begin{aligned} (p)^\Rightarrow &= \{p\}, & (p)^\Leftarrow &= \{p\}, \\ (S_i p)^\Rightarrow &= \{S_i p\}, & (S_i p)^\Leftarrow &= \{S_i p, JS p\}, \\ (JS p)^\Rightarrow &= \{\sigma p : \sigma \in \text{OBS}^+\}, & (JS p)^\Leftarrow &= \{JS p\}, \\ (S_i S_j p)^\Rightarrow &= \{S_i S_j p\}, & (S_i S_j p)^\Leftarrow &= \{S_i S_j p, JS S_j p, JS p\}. \end{aligned}$$

Observe that  $\alpha^\Leftarrow$  is always finite while  $\alpha^\Rightarrow$  is either infinite (namely when  $\alpha$  starts by  $JS$ ) or the singleton  $\{\alpha\}$  (namely when  $\alpha$  is a propositional variable  
260 or starts by some  $S_i$ ). When  $\beta \Rightarrow \alpha$  then the length of  $\beta$  is less than or equal to the length of  $\alpha$ . Moreover, the set of causes of  $\alpha$  has at most  $\ell(\alpha)$  elements:  $|\alpha^\Leftarrow| \leq \ell(\alpha)$ . It follows that the sum of the lengths of all causes of  $\alpha$  is at most quadratic in the length of  $\alpha$ :

**Proposition 1.** For every  $\alpha$ ,  $\sum_{\beta:\beta\Rightarrow\alpha} \ell(\beta) \leq (\ell(\alpha))^2$ .

We generalize atomic consequence to sets of atoms  $s \in 2^{\text{OA}}$ , alias states:  $s^\Rightarrow = \bigcup_{\alpha \in s} \alpha^\Rightarrow$ . When  $s$  contains all its atomic consequences and all introspective atoms then we say that  $s$  is *introspectively closed* or, for short, *introspective*. The set of all introspective states is

$$I\text{-STATES} = \{s^\Rightarrow \cup \text{I-OA} : s \in 2^{\text{OA}}\}.$$

265 This is an infinite set. Each of its elements is infinite due to infinity of I-OA.

### 3.4. Language and semantics of EL-O

The language of EL-O is the set  $\mathcal{L}_{\text{bool}}(\text{OA})$  of boolean formulas built from the set of observability atoms OA. We recall that the length of formulas is determined by the definition of length of atoms from Section 3.2. For  
 270 example,  $\ell(\mathcal{J}S_1 p \wedge \neg S_2 q) = 7$ . Note that the vocabulary of formulas such as  $\mathcal{J}S_1 p$  does not contain the ‘sub-atoms’  $p$  and  $S_1 p$ : we have  $\text{Voc}(\mathcal{J}S_1 p) = \{\mathcal{J}S_1 p\}$ . Note also that the language  $\mathcal{L}_{\text{bool}}(\mathbb{P})$  is the fragment of  $\mathcal{L}_{\text{bool}}(\text{OA})$  without observability operators.

**Example 2.** The goal of the original gossip problem is  $\text{Goal}^{G_1} = \bigwedge_{i,j \in \mathbb{A}} S_j s_i$ .  
 275 The goal of obtaining shared knowledge of depth 2 is expressed by  $\text{Goal}^{G_2} = \bigwedge_{i,j,r \in \mathbb{A}} (S_j s_i \wedge S_r S_j s_i)$ , i.e., every agent  $r$  knows that every  $S_j s_i$  is true.

We recall that a state is a subset of the vocabulary OA. A way of guaranteeing introspection was proposed in [6] where formulas are interpreted exclusively in the set of introspectively closed states *I-STATES*. As such states are always infinite, it is not immediately clear how to define model checking (which requires finite states). One way out is to work with ‘sufficiently introspective states’, as done in [9, Chapter 2]. Here we work with finite models and instead interpret formulas in such a way that introspection is simulated. The EL-O truth conditions are just as those for CPC of Section 3.1, except that for atomic formulas we stipulate:

$$s \models \alpha \text{ iff } \alpha \in s^\Rightarrow \cup \text{I-OA}.$$

Hence  $\alpha$  is true in  $s$  if and only if  $\alpha$  is introspective or  $\beta \Rightarrow \alpha$  for some  $\beta \in s$ .

The EL-O semantics coincides with the CPC semantics for the fragment  $\mathcal{L}_{\text{bool}}(\mathbb{P})$  of  $\mathcal{L}_{\text{bool}}(\text{OA})$ :

280 **Proposition 2.** *Let  $\varphi \in \mathcal{L}_{\text{bool}}(\mathbb{P})$ . Then  $s \models \varphi$  iff  $s \models^{\text{CPC}} \varphi$ .*

**Example 3.** Let  $s_0^{G_1}$  be the initial state of the gossip problem from Example 1) Then  $s_0^{G_1} \models S_i s_i \wedge \bigwedge_{j \neq i} \neg S_i s_j$  for every  $i \in \mathbb{A}$ .

A formula  $\varphi \in \mathcal{L}_{\text{bool}}(\text{OA})$  is EL-O *satisfiable* iff  $s \models \varphi$  for some  $s \in 2^{\text{OA}}$ ; it is EL-O *valid* iff  $s \models \varphi$  for every  $s \in 2^{\text{OA}}$ . Clearly, an atom  $\alpha$  is EL-O valid if and only if it is introspective. Moreover, for atoms  $\alpha, \beta \in \text{OA}$  we have that  $\alpha \rightarrow \beta$  is EL-O valid iff  $\alpha \Rightarrow \beta$  or  $\beta$  is introspective.

**Proposition 3.** *Let  $A, B \subseteq \text{OA}$  be such that  $(\bigwedge_{\alpha \in A} \alpha) \wedge (\bigwedge_{\beta \in B} \neg \beta)$  is EL-O unsatisfiable. Then  $B$  contains some introspective  $\beta$ , or there are  $\alpha \in A, \beta \in B$  such that  $\alpha \Rightarrow \beta$ .*

290 *Proof.* We prove the contraposition: suppose  $B$  contains no introspective atom and there are no  $\alpha \in A$  and  $\beta \in B$  such that  $\alpha \Rightarrow \beta$ . We show that the state  $s = A$  satisfies  $(\bigwedge_{\alpha \in A} \alpha) \wedge (\bigwedge_{\beta \in B} \neg \beta)$ . First,  $s$  satisfies every element of  $A$ . Second,  $s$  does not satisfy any  $\beta \in B$ : otherwise  $\beta$  would be introspective, or we would have  $\alpha \Rightarrow \beta$  for some  $\alpha \in A$ .  $\square$

295 *Remark 1.* When  $\mathbb{A}$  is a singleton then  $S_i p \wedge \neg \mathcal{J} p$  is satisfiable. While this anomaly could be taken care of by a modification of the semantics, we do not do so for the sake of readability and content ourselves with the observation that the  $\mathcal{J}$  operator is superfluous when there is only one agent.

*Remark 2.* It would be interesting to generalise joint observability from  $\mathbb{A}$  to arbitrary subsets of  $\mathbb{A}$ . For instance, after a gossiping phone call between agents  $i$  and  $j$  all secrets previously known by one of them become jointly observable by the group  $\{i, j\}$ . Such an extension was proposed in [33]; however, the solution turned out to be unsatisfactory. The problem is that joint visibilities of subgroups interact in complex ways, in particular when observability evolves.

In the rest of the section we establish finite model property, given an axiomatisation, and prove NP-completeness of EL-O satisfiability.

### 3.5. Finite model property

A standard property of CPC is that  $s \models^{\text{CPC}} \varphi$  iff  $s \cap \text{Voc}(\varphi) \models^{\text{CPC}} \varphi$ . The  
 310 non-standard truth condition for atoms in EL-O makes that this does not hold in EL-O. (It suffices to consider  $s = \{JS p\}$  and  $\varphi = S_i p$  to see this.) Notwithstanding, we obtain a finite model property by closing the state  $s$ .

First of all, let us say that two states  $s$  and  $s'$  agree on the set of atoms  $A \subseteq \text{OA}$  when they assign the same truth value to every element of  $A$ , i.e.,  
 315 when for every  $\alpha \in A$ ,  $s \models \alpha$  iff  $s' \models \alpha$ . This is not enough to guarantee that  $s \cap A = s' \cap A$ . To witness, consider  $s = \{JS p\}$ ,  $s' = \{S_i p\}$ , and  $A = \{S_i p\}$ :  $s$  and  $s'$  agree on  $A$ , but  $s \cap A = \emptyset$  while  $s' \cap A = \{S_i p\}$ . However by straightforward induction on the structure of formulas we get:

**Proposition 4.** *Let  $\varphi \in \mathcal{L}_{\text{bool}}(\text{OA})$  be a formula and  $s, s' \in 2^{\text{OA}}$  two states  
 320 agreeing on  $\text{Voc}(\varphi)$ . Then  $s \models \varphi$  iff  $s' \models \varphi$ .*

**Proposition 5.** *Let  $\varphi \in \mathcal{L}_{\text{bool}}(\text{OA})$  be a formula and  $s \in 2^{\text{OA}}$  a state. Then  $s \models \varphi$  iff  $(s \Rightarrow \cup \text{I-OA}) \cap \text{Voc}(\varphi) \models \varphi$ .*

*Proof.* The proof is by induction on the structure of  $\varphi$ . The only non-trivial case is the induction base. We have  $s \models \alpha$  iff  $s \Rightarrow \cup \text{I-OA} \models \alpha$  iff  $(s \Rightarrow \cup \text{I-OA}) \cap \text{Voc}(\varphi) \models \alpha$ . The last equivalence holds thanks to Proposition 4.  $\square$   
 325

### 3.6. Complexity of EL-O satisfiability

**Proposition 6.** *Let  $s \in 2^{\text{OA}}$  be a state and  $\varphi \in \mathcal{L}_{\text{bool}}(\text{OA})$  a formula. Then  $s \models \varphi$  iff  $s \Rightarrow \cup \text{I-OA} \models^{\text{CPC}} \varphi$ .*

*Proof.* The proof is by induction on the structure of  $\varphi$ . The only non-trivial case is the induction base. If  $\alpha \in \text{I-OA}$  then both  $s \models \alpha$  and  $\text{I-OA} \models^{\text{CPC}} \alpha$  hold.  
 330 Otherwise, for  $\alpha \notin \text{I-OA}$  we have  $s \models \alpha$  iff  $\alpha \in s \Rightarrow$  iff  $s \Rightarrow \models^{\text{CPC}} \alpha$ .  $\square$

**Proposition 7.** *The problem of deciding satisfiability of  $\mathcal{L}_{\text{bool}}(\text{OA})$  formulas is NP-complete.*

$S_i S_i \alpha$	$(Vis_1)$
$JS JS \alpha$	$(Vis_2)$
$JS S_i S_i \alpha$	$(Vis_3)$
$JS \alpha \rightarrow S_i \alpha$	$(Vis_4)$
$JS \alpha \rightarrow JS S_i \alpha$	$(Vis_5)$

Table 1: Axioms of EL-O

*Proof.* Hardness is the case because EL-O satisfiability and CPC satisfiability coincide for the  $\mathcal{L}_{\text{bool}}(\mathbb{P})$  fragment of  $\mathcal{L}_{\text{bool}}(\text{OA})$  (Proposition 2). For membership, guess some subset  $s \in 2^{\text{Voc}(\varphi)}$  and check in polynomial time:

1. that  $s$  contains all its relevant consequences: for every  $\alpha \in \text{Voc}(\varphi)$  and  $\beta \in s$ , check that if  $\beta \Rightarrow \alpha$  then  $\alpha \in s$ ;
2. that  $s$  contains all introspective atoms from  $\text{Voc}(\varphi)$ : for every  $\alpha \in \text{Voc}(\varphi)$ , check that if  $\alpha \in \text{OA}$  then  $\alpha \in s$ ;
3. that  $s \models^{\text{CPC}} \varphi$ .

Checking that  $s \models^{\text{CPC}} \varphi$  is tantamount to checking that  $s \models \varphi$  because: (a) thanks to the first and second check,  $s$  equals  $(s \Rightarrow \cup \text{I-OA}) \cap \text{Voc}(\varphi)$ , and therefore  $s \models^{\text{CPC}} \varphi$  iff  $(s \Rightarrow \cup \text{I-OA}) \cap \text{Voc}(\varphi) \models^{\text{CPC}} \varphi$ ; (b) in CPC, the latter is the same as  $s \Rightarrow \cup \text{I-OA} \models^{\text{CPC}} \varphi$  (contrarily to EL-O, in CPC irrelevant atoms outside  $\text{Voc}(\varphi)$  can be dropped from valuations); finally, (c) Proposition 6 applies. □

### 3.7. Axiomatisation of EL-O validities

The EL-O validities of our language  $\mathcal{L}_{\text{bool}}(\text{OA})$  are axiomatised by the schemas of Table 1 together with CPC. We prove its completeness via CPC.

**Proposition 8.** *Let  $\varphi \in \mathcal{L}_{\text{bool}}(\text{OA})$  be a formula. Then  $\varphi$  is EL-O valid iff  $\varphi$  is provable in CPC from the axiom schemas  $Vis_1$ – $Vis_5$  of Table 1.*



*Proof.* We again take advantage of Proposition 6 and show that  $Vis_I$ – $Vis_5$  characterise the set of introspectively closed states  $\{s \Rightarrow \cup \text{I-OA} : s \in 2^{\text{OA}}\}$ .

355 The right-to-left direction is clear: each of the five axiom schemas is valid in introspectively closed states. For the left-to-right direction, we show that every  $s$  satisfying  $Vis_I$ – $Vis_5$  is introspectively closed.

For closure under atomic consequence, let  $s \models \alpha$ . The interesting case is when  $\alpha = JS\alpha'$ . Then  $s \models JS_{i_1}\alpha'$  by axiom  $Vis_5$ , and also  $s \models JS_{i_2}S_{i_1}\alpha'$ , and  
 360 so on: we can generate any  $s \models JS_{i_m}\dots S_{i_1}\alpha'$  and then, by  $Vis_4$ , we can obtain  $s \models S_{i_m}\dots S_{i_1}\alpha'$ . Moreover, we have  $s \models JSJS\alpha'$  by  $Vis_2$ , and in the same way, we can generate  $s \models S_{i_m}\dots S_{i_1}JS\alpha'$  for any agents  $i_1, \dots, i_m$  and  $i$ . We therefore obtain that  $s$  satisfies any sequence whose strict postfix is  $\alpha'$ , that is, every  $\sigma\alpha'$  for any  $\sigma \in \text{OBS}^+$ .

365 We use the same technique to show that a state  $s$  satisfying  $Vis_I$ – $Vis_5$  satisfies every  $\sigma S_i S_i \alpha$  for  $\sigma \in \text{OBS}^*$  and every  $\sigma JS\alpha$  for  $\sigma \in \text{OBS}^+$ : we obtain the first with  $Vis_I$  (for  $\sigma$  empty) and  $Vis_3$ ,  $Vis_4$ , and  $Vis_5$  (for  $\sigma$  non-empty), and the second with  $Vis_2$ ,  $Vis_4$ , and  $Vis_5$ .  $\square$

*Remark 3.* We do not require states to satisfy the infinitary constraint “if  
 370  $\sigma\alpha \in s$  for every  $\sigma \in \{S_i : i \in \mathbb{A}\}^+$  then  $JS\alpha \in s$ ”, which is the EL-O counterpart of the transitive closure constraint on Kripke models (see Remark 4 in Section 4.2). The reason is technical: it is not obvious how to extend the axiomatisation of EL-O in a finitary way to account for that constraint. Our recent axiomatisation of ‘common knowledge whether’ of [34] might provide  
 375 a solution; we however do not pursue this further here.

#### 4. EL-O as a fragment of epistemic logic S5

We now situate our lightweight epistemic logic w.r.t. the standard epistemic logic S5 with ‘knowing-that’ operators  $CK$  and  $K_i$ . We are going to exploit that  $S_i\varphi$  can be viewed as an abbreviation of  $K_i\varphi \vee K_i\neg\varphi$  and  $JS\varphi$  as  
 380 an abbreviation of  $CK\varphi \vee CK\neg\varphi$ . Then the language  $\mathcal{L}_{\text{bool}}(\text{OA})$  of EL-O becomes as a fragment of  $\mathcal{L}_{\text{EL}}$ . When we make this identification we are going

to talk about the EL-O *fragment* of  $\mathcal{L}_{\text{EL}}$ .

Our main result is that for the EL-O fragment of  $\mathcal{L}_{\text{EL}}$ , the axioms of Table 1 are sound and complete w.r.t. validity in S5 models (Proposition 11).  
 385 It follows by Proposition 7 that our lightweight epistemic logic is a fragment of standard epistemic logic with an NP-complete satisfiability problem. This is an important result because satisfiability of  $\mathcal{L}_{\text{EL}}$  formulas is PSPACE-complete as soon as there are two agents, and this is the case even without the common knowledge operator [35]. But first of all we recall the  
 390 standard epistemic language  $\mathcal{L}_{\text{EL}}$  and the models in which its formulas are interpreted.

#### 4.1. The standard epistemic language

The grammar of the standard language  $\mathcal{L}_{\text{EL}}$  is:

$$\mathcal{L}_{\text{EL}} : \varphi ::= p \mid \neg\varphi \mid \varphi \wedge \varphi \mid K_i\varphi \mid CK\varphi$$

where  $p$  ranges over the set of propositional variables  $\mathbb{P}$ . The formula  $K_i\varphi$  reads “ $i$  knows that  $\varphi$ ” and  $CK\varphi$  reads “it is common knowledge that  $\varphi$ ”.  
 395 Hence there are no observability atoms: the vocabulary of  $\mathcal{L}_{\text{EL}}$  is just the set of propositional variables  $\mathbb{P}$ , of which we suppose that it is identical to the set  $\mathbb{P}$  from which the observability atoms of Section 3.2 were built.

**Example 4.** The goal of the original gossip problem is to obtain shared knowledge of all secrets. This is expressed in the standard epistemic language  $\mathcal{L}_{\text{EL}}$  by  $\bigwedge_{i,j \in \mathbb{A}} (K_j s_i \vee K_j \neg s_i)$ . If we take  $K_i\varphi$  to be an abbreviation of  $p \wedge S_i p$  (see Section 2) then this goal reduces to the  $\mathcal{L}_{\text{bool}}(\text{OA})$  formula  $Goal^{G_1} = \bigwedge_{i,j \in \mathbb{A}} S_j s_i$ . Shared knowledge up to depth 2 is expressed by  $\bigwedge_{i,j,r \in \mathbb{A}} (K_r K_j s_i \vee K_r K_j \neg s_i)$ ; reducing  $K_j$  and  $K_r$  we get  $\bigwedge_{i,j,r \in \mathbb{A}} (S_j s_i \wedge S_r s_i \wedge S_r S_j s_i)$ , which in CPC is equivalent to  $Goal^{G_2} = \bigwedge_{i,j,r \in \mathbb{A}} (S_j s_i \wedge S_r S_j s_i)$ . We have already seen these  
 405 two  $\mathcal{L}_{\text{bool}}(\text{OA})$  formulas in Example 2. In the generalised gossip problem [6, 36] the goal is shared knowledge up to depth  $k$ , written  $\bigwedge_{i \in \mathbb{A}} \bigwedge_{\vec{K} \in \{K_j : j \in \mathbb{A}\}^k} (\vec{K} s_i \vee \vec{K} \neg s_i)$  in  $\mathcal{L}_{\text{EL}}$ , where  $\{K_j : j \in \mathbb{A}\}^k$  is the set of all sequences of individual ‘knowing-that’ operators of length  $k$ . It reduces to the  $\mathcal{L}_{\text{bool}}(\text{OA})$  formula

$$Goal^{G^k} = \bigwedge_{i \in \mathbb{A}} \bigwedge_{\sigma \in \{S_j : j \in \mathbb{A}\}^{\leq k}} \sigma s_i$$

410 where  $\{S_j : j \in \mathbb{A}\}^{\leq k}$  is the set of all sequences of individual observability operators up to length  $k$ .

#### 4.2. S5 Kripke models and the canonical model for EL-O

We suppose that  $CK$  and the  $K_i$  are S5 modal operators and that common knowledge implies individual knowledge. Semantically, an S5 Kripke model [35] is a tuple  $M = \langle W, \sim_{\mathbb{A}}, \{\sim_i\}_{i \in \mathbb{A}}, V \rangle$  where  $W$  is a nonempty set of possible worlds;  $\sim_{\mathbb{A}}$  and every  $\sim_i$  are equivalence relations on  $W$  such that every  $\sim_i$  is a subset of  $\sim_{\mathbb{A}}$ ; and  $V : W \rightarrow 2^{\mathbb{P}}$  is a valuation. The interpretation of  $\mathcal{L}_{EL}$  formulas in a pointed Kripke model  $(M, w)$  is:

$$M, w \models K_i \varphi \text{ iff } M, w' \models \varphi \text{ for every } w' \text{ such that } w \sim_i w';$$

$$M, w \models CK \varphi \text{ iff } M, w' \models \varphi \text{ for every } w' \text{ such that } w \sim_{\mathbb{A}} w';$$

and as usual for the boolean operators. Formula  $\varphi$  is *valid in  $M$*  if  $M, s \models \varphi$  for every  $s \in W$ ;  $\varphi$  is *valid in S5 Kripke models* if  $\varphi$  is valid in every S5  
415 Kripke model.

*Remark 4.* Every  $\sim_i$  being included in  $\sim_{\mathbb{A}}$ , the accessibility relation for common knowledge contains the transitive closure of the union of the individual accessibility relations  $(\bigcup_{i \in \mathbb{A}} \sim_i)^*$ , but does not necessarily equal it. The former property corresponds to the fact that in introspective states,  $\mathcal{J} \alpha \in s$   
420 implies that  $\sigma \alpha \in s$  for any non-empty sequence  $\sigma$  while the other direction fails to hold, as already discussed in Remark 3. Our notion of common knowledge is therefore weaker than standard common knowledge in that the induction axiom  $CK(\varphi \rightarrow \bigwedge_{i \in \mathbb{A}} K_i \varphi) \rightarrow (\varphi \rightarrow CK \varphi)$  is not valid. A similar notion was called ‘fictitious knower’ or ‘any fool’ by Genesereth and Nils-  
425 son [37] and was used in modal extensions of logic programming in the 90s [38, 39]. We have argued elsewhere that the induction axiom is too strong for a logic of common knowledge [40, 41].

Let us establish that every EL-O valuation  $s$  can be identified with a particular pointed S5 Kripke model  $(M^{\text{EL-O}}, s)$ . The first thing we do is to define

how the states' observability information determines equivalence relations between states. For every  $s, s' \in 2^{\text{OA}}$  we define:

$$sR_i s' \text{ iff } s \text{ and } s' \text{ agree on every } \alpha \text{ such that } s \models S_i \alpha;$$

$$sR_{\mathbb{A}} s' \text{ iff } s \text{ and } s' \text{ agree on every } \alpha \text{ such that } s \models JS \alpha.$$

(Remember that agreement is defined in Section 3.5.) While these relations are clearly reflexive, it is not immediately clear that they are also symmetric and transitive; so let us prove that.

**Proposition 9.** *All relations  $R_i$  and  $R_{\mathbb{A}}$  are equivalence relations.*

*Proof.* We prove symmetry and transitivity of  $R_i$  for an arbitrary  $i$ ; the proof for  $R_{\mathbb{A}}$  is analogous. For symmetry, suppose  $sR_i s'$ . Then for every  $\alpha$ , if  $s \models S_i \alpha$  then  $s$  and  $s'$  agree on  $\alpha$ . Take an arbitrary  $\beta$  such that  $s' \models S_i \beta$ .  $s'$  agrees with  $s$  on  $S_i \beta$  because  $s \models S_i S_i \beta$ . Hence  $s \models S_i \beta$ , and  $s$  and  $s'$  agree on  $\beta$ , and therefore  $s'R_i s$ . For transitivity, suppose  $sR_i s'$  and  $s'R_i s''$  and suppose  $s \models S_i \alpha$ . We show that  $s$  and  $s''$  agree on  $\alpha$ . By hypothesis,  $s$  and  $s'$  agree on  $\alpha$ . As observed above,  $s$  and  $s'$  also agree on  $S_i \alpha$  because  $s \models S_i S_i \alpha$ . Hence  $s' \models S_i \alpha$ , and so  $s'$  and  $s''$  agree on  $\alpha$ . Therefore  $s$  and  $s''$  agree on  $\alpha$ .  $\square$

We are ready to define the *canonical Kripke model* for EL-O as the tuple  $M^{\text{EL-O}} = \langle W^{\text{EL-O}}, \sim_{\mathbb{A}}^{\text{EL-O}}, \{\sim_i^{\text{EL-O}}\}_{i \in \mathbb{A}}, V^{\text{EL-O}} \rangle$  with

$$W^{\text{EL-O}} = I\text{-STATES} = \{s^{\Rightarrow} \cup I\text{-OA} : s \in 2^{\text{OA}}\},$$

$$\sim_{\mathbb{A}}^{\text{EL-O}} = R_{\mathbb{A}} \cap (W^{\text{EL-O}} \times W^{\text{EL-O}}),$$

$$\sim_i^{\text{EL-O}} = R_i \cap (W^{\text{EL-O}} \times W^{\text{EL-O}}),$$

$$V^{\text{EL-O}}(w) = w \cap \mathbb{P} \text{ for every } w \in W^{\text{EL-O}}.$$

Hence the possible worlds of  $M^{\text{EL-O}}$  are the introspectively closed states *I-STATES* that we have defined in Section 3.3.

*Remark 5.* In EL-O, the empty initial state  $s_0 = \emptyset$  models maximal ignorance: the agents do not know anything beyond tautologies. The simplicity of this modelling contrasts with the corresponding pointed Kripke model

445 ( $M^{\text{EL-O}}, s_0$ ): in that model, every agent can access *infinitely many* possible  
 worlds from  $s_0$ . Actually the classical examples in introductory textbooks  
 and articles about epistemic logic (such as the muddy children puzzle) are  
 all modelled by finite Kripke models. As recently observed by Artemov [42],  
 such finite models presuppose more or less tacitly a lot of common knowl-  
 450 edge, which is too strong an assumption in many situations.

**Proposition 10.** *Let  $\varphi \in \mathcal{L}_{\text{bool}}(\text{OA})$  and  $s \in W^{\text{EL-O}}$ . Then  $s \models \varphi$  iff  $M^{\text{EL-O}}, s \models \varphi$ .*

*Proof.* The proof is by induction on the structure of  $\varphi$ . The only interesting  
 case is the base case of atoms  $\alpha \in \text{OA}$ . We use induction on the length of  $\alpha$ .

When  $\alpha = p$  then  $M, s \models p$  iff  $s \models p$  for any  $s \in W^{\text{EL-O}}$  by definition of  $V^{\text{EL-O}}$ .

455 When  $\alpha = S_i\beta$  we prove the two directions of the equivalence.

- If  $s \models S_i\beta$  for some  $\beta \in \text{OA}$  and some  $s \in W^{\text{EL-O}}$ : consider  $s' \in W^{\text{EL-O}}$  such  
 that  $s \sim_i^{\text{EL-O}} s'$ . By definition of  $\sim_i^{\text{EL-O}}$ ,  $s$  and  $s'$  agree on all  $\alpha$  such that  
 $s \models S_i\alpha$ . Therefore if  $s \models \beta$  then  $s' \models \beta$ , and by the induction hypothesis,  
 $M^{\text{EL-O}}, s' \models \beta$ ; hence  $M^{\text{EL-O}}, s \models K_i\beta$ . If  $s \models \neg\beta$ , by the same argument,  
 460 we have that  $M^{\text{EL-O}}, s \models K_i\neg\beta$ . Therefore  $M^{\text{EL-O}}, s \models S_i\beta$ .
- If  $M^{\text{EL-O}}, s \models S_i\beta$  for some  $\beta \in \text{OA}$  and some  $s \in W^{\text{EL-O}}$ , then either  
 $M^{\text{EL-O}}, s \models K_i\beta$  or  $M^{\text{EL-O}}, s \models K_i\neg\beta$ . Suppose that  $s \not\models S_i\beta$ . In particular  
 $\beta \notin \text{I-OA}$ . If  $s \models \beta$ , consider  $s' = s \setminus \beta^{\Leftarrow}$ , and if  $s \not\models \beta$ , consider  $s' = s \cup \beta^{\Rightarrow}$ .  
 In both cases  $s' \in W^{\text{EL-O}}$ , and  $s$  and  $s'$  agree on all  $\alpha$  such that  $s \models S_i\alpha$ .  
 465 Therefore  $s \sim_i^{\text{EL-O}} s'$ , but by the induction hypothesis,  $M^{\text{EL-O}}, s' \models \beta$  iff  
 $s' \models \beta$  iff  $s' \models \neg\beta$  iff  $M^{\text{EL-O}}, s \models \neg\beta$ : this contradicts the hypothesis that  
 $M^{\text{EL-O}}, s \models S_i\beta$ .

For the case  $\alpha = \mathcal{J}\beta$  we proceed similarly. □

### 4.3. Completeness of the EL-O fragment of $\mathcal{L}_{\text{EL}}$ w.r.t. S5 validity

470 We now show that the EL-O semantics in terms of observability and  
 the standard Kripke semantics have the same validities as far as the EL-O  
 fragment is concerned.

**Proposition 11.** *Let  $\varphi \in \mathcal{L}_{\text{bool}}(\text{OA})$  be a formula. Then  $\varphi$  is valid in the S5 Kripke models of Section 4.2 iff  $\varphi$  is provable in CPC from the EL-O axioms of Table 1.*

475

*Proof.* For soundness we show that the schemas  $S_i S_i \alpha$ ,  $JS JS \alpha$ ,  $JS S_i S_i \alpha$ ,  $JS \alpha \rightarrow S_i \alpha$ ,  $JS \alpha \rightarrow JS S_i \alpha$  are valid in S5 Kripke models. For the first it suffices to observe that  $S_i S_i \alpha$  is equivalent to  $K_i(K_i \alpha \vee K_i \neg \alpha) \vee K_i \neg(K_i \alpha \vee K_i \neg \alpha)$ . The latter is equivalent in S5 to the propositionally valid  $K_i \alpha \vee K_i \neg \alpha \vee \neg(K_i \alpha \vee K_i \neg \alpha)$ .

480

Validity of the other axiom schemas can be proved in a similar manner.

For completeness suppose that  $\varphi$  is not provable from the EL-O axioms. By Proposition 8 there exists  $s \subseteq \text{OA}$  such that  $s \not\models \varphi$ . Hence  $s \Rightarrow \cup \text{I-OA} \not\models^{\text{CPC}} \varphi$  by Proposition 6 and, as  $s \Rightarrow \cup \text{I-OA} \in W^{\text{EL-O}}$ ,  $M^{\text{EL-O}}$ ,  $s \Rightarrow \cup \text{I-OA} \not\models \varphi$  by Proposition 10. Hence  $\varphi$  cannot be valid in S5 Kripke models.  $\square$

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#### 4.4. Axiomatisation of the $M^{\text{EL-O}}$ validities in $\mathcal{L}_{\text{EL}}$

It follows from Proposition 11 that in order to check satisfiability in S5 Kripke models of a formula in the EL-O fragment it suffices to take the canonical Kripke model  $M^{\text{EL-O}}$  and go through its states. In other words, there is no formula in the EL-O fragment that is valid in  $M^{\text{EL-O}}$  without being valid in the set of all S5 Kripke models. This fails to hold for the full language  $\mathcal{L}_{\text{EL}}$ . This can be seen from the axiomatisation of the  $\mathcal{L}_{\text{EL}}$  validities in  $M^{\text{EL-O}}$  of [43] and [9, Chapter 2].<sup>2</sup> It is made up of: the EL-O axioms of Table 1; the axioms

$$\begin{array}{ll} K_i \alpha \leftrightarrow \alpha \wedge S_i \alpha, & K_i \neg \alpha \leftrightarrow \neg \alpha \wedge S_i \alpha, \\ CK \alpha \leftrightarrow \alpha \wedge JS \alpha, & CK \neg \alpha \leftrightarrow \neg \alpha \wedge JS \alpha \end{array}$$

---

<sup>2</sup>Our semantics differs from the semantics in these papers in that it is finitary. The only thing that changes is the soundness proof, which is routine.

relating  $K_i$  to  $S_i$  and  $CK$  to  $\mathcal{S}$ ; all S5 principles for  $K_i$  and  $CK$ ; plus two axioms distributing  $K_i$  and  $CK$  over clauses:

$$K_i\left(\bigvee_{\alpha \in A^+} \alpha \vee \bigvee_{\alpha \in A^-} \neg\alpha\right) \leftrightarrow \begin{cases} \left(\bigvee_{\alpha \in A^+} K_i\alpha\right) \vee \left(\bigvee_{\alpha \in A^-} K_i\neg\alpha\right) & \text{if } (A^+)^\Leftarrow \cap A^- = \emptyset, \\ \top & \text{otherwise;} \end{cases}$$

$$CK\left(\bigvee_{\alpha \in A^+} \alpha \vee \bigvee_{\alpha \in A^-} \neg\alpha\right) \leftrightarrow \begin{cases} \left(\bigvee_{\alpha \in A^+} CK\alpha\right) \vee \left(\bigvee_{\alpha \in A^-} CK\neg\alpha\right) & \text{if } (A^+)^\Leftarrow \cap A^- = \emptyset, \\ \top & \text{otherwise.} \end{cases}$$

The last two axioms resort to a generalisation of the ‘caused-by’ function from atoms to sets of atoms  $A \subseteq \text{OA}$ , in the obvious way:  $A^\Leftarrow = \bigcup_{\alpha \in A} \alpha^\Leftarrow$ . They are specific to observability-based knowledge and are typically invalid in normal modal logics. An instance of the first one is  $K_i(p \vee q) \leftrightarrow (K_i p \vee K_i q)$ ,  
490 for different  $p$  and  $q$ . This is a strong principle: to give an example, if it is known that the butler or the gardener was the murderer then it is also known who of them it was. The reason is that our logic is built on atomic observability information: what is modelled in  $M^{\text{EL-O}}$  are forms of individual and common knowledge that are respectively obtained via individual ob-  
495 servation and joint observation of facts. This differs conceptually from the classical operators of individual and common knowledge as studied in epistemic logic [12]. The property of distribution over falsifiable disjunctions of literals is common in epistemic logics that are based on observability of propositional variables. It is shown in [29] that it can be avoided if one  
500 introduces new propositional variables.

The axiomatics allows us to reduce every  $\mathcal{L}_{\text{EL}}$  formula to an equivalent  $\mathcal{L}_{\text{bool}}(\text{OA})$  formula. For example, positive introspection  $K_i p \rightarrow K_i K_i p$  reduces to  $(p \wedge S_i p) \rightarrow (p \wedge S_i p \wedge S_i S_i p)$ . Observe that the latter is EL-O valid.

## 5. Epistemic planning with conditional effects

505 We now define actions and planning tasks within our framework. We assume deterministic actions with conditional effects whose preconditions

are described by EL-O formulas and whose positive and negative effects are sets of observability atoms. We have already argued in Section 2 that conditional effects are important in epistemic planning: the effects of an  
 510 action on agent  $i$ 's epistemic state typically depend on whether  $i$  sees the variables that are modified by the action or not.

We start by defining classical planning tasks [44] and then introduce our simple epistemic planning tasks. The name ‘simple’ will be justified by our translation to classical planning of Section 5.5.

### 515 5.1. Action descriptions and planning tasks

Just as for the definition of boolean formulas in Section 3.1, we consider an abstract vocabulary  $\mathbb{V}$  with which actions are described. An *action description over  $\mathbb{V}$*  (or *action* for short) is a pair  $\mathbf{a} = \langle pre(\mathbf{a}), eff(\mathbf{a}) \rangle$  where  $pre(\mathbf{a}) \in \mathcal{L}_{\text{bool}}(\mathbb{V})$  and  $eff(\mathbf{a}) \subseteq \mathcal{L}_{\text{bool}}(\mathbb{V}) \times 2^{\mathbb{V}} \times 2^{\mathbb{V}}$ . The formula  $pre(\mathbf{a})$  is the precondition of  $\mathbf{a}$ , describing when the action may be applied, and  $eff(\mathbf{a})$  are the conditional effects of  $\mathbf{a}$ , describing which atomic formulas the action may add or remove from the current state under additional conditions. For each conditional effect

$$ce = \langle cnd(ce), ceff^+(ce), ceff^-(ce) \rangle$$

in  $eff(\mathbf{a})$ ,  $cnd(ce)$  is the condition of  $ce$ ,  $ceff^+(ce)$  are the added and  $ceff^-(ce)$  are the deleted atomic formulas. The vocabulary and the length of an action description are defined by:

$$\begin{aligned} \text{Voc}(\mathbf{a}) &= \text{Voc}(pre(\mathbf{a})) \cup \bigcup_{ce \in eff(\mathbf{a})} (\text{Voc}(cnd(ce)) \cup ceff^+(ce) \cup ceff^-(ce)); \\ \ell(\mathbf{a}) &= \ell(pre(\mathbf{a})) + \sum_{ce \in eff(\mathbf{a})} \left( \ell(cnd(ce)) + \left( \sum_{p \in ceff^+(ce)} \ell(p) \right) + \left( \sum_{p \in ceff^-(ce)} \ell(p) \right) \right). \end{aligned}$$

Remember that each  $p \in \mathbb{V}$  has its length  $\ell(p) \geq 1$ , for some integer.

A *planning task over  $\mathbb{V}$*  is a tuple  $\mathcal{P} = \langle \mathbb{V}, Act, s_0, Goal \rangle$  where  $\mathbb{V}$  is some vocabulary,  $Act$  is a set of action descriptions,  $s_0 \in 2^{\mathbb{V}}$  is the initial state, and



$Goal \in \mathcal{L}_{\text{bool}}(\mathbb{V})$  is the goal formula. Its relevant vocabulary and length are:

$$\begin{aligned} \text{Voc}(\mathcal{P}) &= \left( \bigcup_{a \in \text{Act}} \text{Voc}(a) \right) \cup s_0 \cup \text{Voc}(Goal); \\ \ell(\mathcal{P}) &= |s_0| + \ell(Goal) + \sum_{a \in \text{Act}} \ell(a). \end{aligned}$$

The solutions of a planning task depend on the interpretation of formulas and actions. We have already seen the classical EL-O interpretation of formulas (Section 3.1) and the EL-O interpretation of formulas (Section 3.4). As to actions, we are going to define their classical and their EL-O interpretation. Both are relations on the set of states  $2^{\mathbb{V}}$ . Given some set of action-interpreting relations  $\mathcal{R}$ , we say that the state  $s$  is *reachable from the state*  $s_0$  *via*  $\mathcal{R}$  if there exists an integer  $m \geq 0$ , a sequence of states  $t_0, \dots, t_m$  from  $2^{\mathbb{V}}$ , and a sequence of relations  $R_1, \dots, R_m$  from  $\mathcal{R}$  such that  $s_0 = t_0$ ,  $s = t_m$ , and  $t_{k-1}R_k t_k$  for every  $k$  such that  $1 \leq k \leq m$ . Solvability of a planning task amounts to reachability of a state satisfying the goal from the initial state  $s_0$  via the relations interpreting the actions.

## 5.2. Classical planning

When an action description  $a$  has a precondition and two (not necessarily different) conditional effects whose preconditions are jointly satisfiable with  $pre(a)$  then there is a conflict. The other way round, we say that  $a$  is *classically consistent* if and only if for every  $ce_1, ce_2 \in eff(a)$ , if  $pre(a) \wedge cnd(ce_1) \wedge cnd(ce_2)$  is CPC satisfiable then  $ceff^+(ce_1)$  and  $ceff^-(ce_2)$  are disjoint.

**Example 5.** For an example of inconsistency consider the following description of the action  $swap_{p,q}$  swapping the truth values of  $p$  and  $q$ . Suppose this action can always be executed, so its precondition is  $pre(swaps_{p,q}) = \top$ , and suppose its conditional effects are naively described by  $eff(swaps_{p,q}) = \{\langle p, \{q\}, \{p\} \rangle, \langle q, \{p\}, \{q\} \rangle\}$ . These two conditional effects conflict because  $\top \wedge p \wedge q$  is CPC-satisfiable. Observe that we can make the description classically

consistent by replacing the precondition by  $\neg(p \leftrightarrow q)$ : the action is executable only if the truth values of  $p$  and  $q$  differ.

A classically consistent action description determines the following relation on the set of states  $2^{\mathbb{V}}$ :

$$sR_a^{\text{CPC}}s' \text{ iff } s \models^{\text{CPC}} \text{pre}(\mathbf{a}) \text{ and } s' = \left( s \setminus \bigcup_{\substack{ce \in \text{eff}^-(\mathbf{a}), \\ s \models^{\text{CPC}} \text{cnd}(ce)}} \text{ceff}^-(ce) \right) \cup \bigcup_{\substack{ce \in \text{eff}^+(\mathbf{a}), \\ s \models^{\text{CPC}} \text{cnd}(ce)}} \text{ceff}^+(ce).$$

A planning task is *classically solvable* if a state satisfying the goal formula is reachable from the initial state via  $\{R_a^{\text{CPC}} : \mathbf{a} \in \text{Act}\}$ , i.e., via the  
545 CPC-interpretation of the actions in  $\text{Act}$ . It is known that classical solvability of a planning task is a PSPACE-complete reasoning problem [14].

### 5.3. EL-O consistent action descriptions

In order to describe epistemic preconditions and effects we instantiate the vocabulary  $\mathbb{V}$  by the set of observability atoms  $\text{OA}$ . An action description  
560  $\mathbf{a} = \langle \text{pre}(\mathbf{a}), \text{eff}(\mathbf{a}) \rangle$  over  $\text{OA}$  is *EL-O consistent* if and only if

1. for every  $ce \in \text{eff}(\mathbf{a})$ ,  $\text{ceff}^-(ce)$  contains no introspective atoms;
2. for every  $ce_1, ce_2 \in \text{eff}(\mathbf{a})$ , if  $\text{pre}(\mathbf{a}) \wedge \text{cnd}(ce_1) \wedge \text{cnd}(ce_2)$  is satisfiable in EL-O then  $\text{ceff}^+(ce_1)$  and  $(\text{ceff}^-(ce_2))^{\Leftarrow}$  are disjoint.

Proposition 3 provides formal support to these two conditions. Let us explain the intuitions behind them. The first is clear: it makes no sense to  
555 delete introspective atoms. The second condition requires that two conditional effects of an action cannot conflict when  $\text{pre}(\mathbf{a})$  and their triggering conditions are jointly satisfiable. Hence when an action  $\mathbf{a}$  has conditional effects  $ce_1, ce_2 \in \text{eff}(\mathbf{a})$  and there are  $\alpha_1 \in \text{ceff}^+(ce_1)$  and  $\alpha_2 \in \text{ceff}^-(ce_2)$  such  
560 that  $\alpha_1 \Rightarrow \alpha_2$  then  $\text{pre}(\mathbf{a}) \wedge \text{cnd}(ce_1) \wedge \text{cnd}(ce_2)$  must be inconsistent. Note that it follows from the transitivity of introspective consequence  $\Rightarrow$  that  $\text{ceff}^+(ce_1)$  and  $(\text{ceff}^-(ce_2))^{\Leftarrow}$  are disjoint iff  $\text{ceff}^+(ce_1)^{\Rightarrow}$  and  $(\text{ceff}^-(ce_2))^{\Leftarrow}$  are.

**Example 6.** The action flip of flipping a light switch is described by  $\text{pre}(\text{flip}) = \top$  and  $\text{eff}(\text{flip}) = \{\langle \text{On}, \emptyset, \{\text{On}\} \rangle, \langle \neg\text{On}, \{\text{On}\}, \emptyset \rangle\}$ . It is EL-O consistent: first, none

565 of the negative effects is an introspective atom, and second, although there are two conditional effects with contradictory add- and delete-lists, there can be no conflict because the conjunction  $\top \wedge \text{On} \wedge \neg\text{On}$  of the precondition and the conditions of the two conditional effects is unsatisfiable.

**Example 7.** Let us describe calls in the original gossip problem where the goal is to obtain shared knowledge of depth  $k = 1$ . Suppose OA is built from  $\mathbb{A} = \{1, \dots, n\}$  and  $\mathbb{P} = \{s_i : i \in \mathbb{A}\}$ . During the action  $\text{call}_j^i$ , in which a call is made between agents  $i$  and  $j$  (this action being symmetrical), the two agents tell each other every secret they know among all  $n$  secrets. We have  $\text{call}_j^i = \langle \text{pre}(\text{call}_j^i), \text{eff}(\text{call}_j^i) \rangle$  with  $\text{pre}(\text{call}_j^i) = \top$  and:

$$\begin{aligned} \text{eff}(\text{call}_j^i) = & \{ \langle S_i s_1 \vee S_j s_1, \{S_i s_1, S_j s_1\}, \emptyset \rangle, \\ & \dots, \\ & \langle S_i s_n \vee S_j s_n, \{S_i s_n, S_j s_n\}, \emptyset \rangle \}. \end{aligned}$$

Intuitively, a secret becomes observable for both agents if at least one of  
570 them observes it. Each  $\text{call}_j^i$  is EL-O consistent because it has no negative effects. The length of a call is  $\ell(\text{call}_j^i) = 5 + (|\mathbb{A}| \times (8 + 4 + 0)) = 12n + 5$ . (Remember that disjunctions  $S_i s_k \vee S_j s_k$  abbreviate  $\neg(\neg S_i s_1 \wedge \neg S_j s_1)$ .)

Here is a more complex example of a consistent action.

**Example 8.** Consider the action pour of pouring a bottle of liquid on a healthy lawn [45]. Factually, its effect is that if the liquid is poisonous the lawn becomes dead. Epistemically, pour has several conditional effects: (1) if  $i$  knows that Poisonous then  $i$  will know that the lawn is dead; (2) if  $i$  does not know whether Poisonous and if the lawn is known to be healthy<sup>3</sup> then  $i$  will no longer know whether the lawn is healthy or dead. (There is a third possible epistemic situation where  $i$  knows that  $\neg$ Poisonous, but we

---

<sup>3</sup>In [45], the condition of the last effect is only that it is not known *that*  $\neg$ Poisonous, and the effect is that it is no longer known *that*  $\neg$ Dead. This however produces an unintuitive result in states where the liquid is known to be poisonous.

need not say anything here because there is no effect as far as the status of Dead is concerned.) Consider the description of pour with precondition  $pre(\text{pour}) = \top$  and effect

$$\begin{aligned} eff(\text{pour}) = & \{ \langle \text{Poisonous}, \{\text{Dead}\}, \emptyset \rangle, \\ & \langle \text{Poisonous} \wedge S_i \text{ Poisonous}, \{S_i \text{ Dead}\}, \emptyset \rangle, \\ & \langle \neg S_i \text{ Poisonous} \wedge \neg \text{Dead} \wedge S_i \text{ Dead}, \emptyset, \{S_i \text{ Dead}\} \rangle \}. \end{aligned}$$

The first conditional effect is factual and the last two are epistemic. The second represents (1):  $\text{Poisonous} \wedge S_i \text{ Poisonous}$  expresses that  $i$  knows that the liquid is poisonous; and the third represents (2):  $\neg \text{Dead} \wedge S_i \text{ Dead}$  expresses that  $i$  knows that the lawn is healthy. This action description is EL-O consistent in particular because the conjunction of the two last conditions  $\text{Poisonous} \wedge S_i \text{ Poisonous}$  and  $\neg S_i \text{ Poisonous} \wedge \neg \text{Dead} \wedge S_i \text{ Dead}$  is inconsistent.

#### 5.4. Simple epistemic planning tasks

Every EL-O consistent action  $a$  determines a relation  $R_a^{\text{EL-O}}$  between states that is a partial function: for every  $s, s' \in 2^{\text{OA}}$ ,

$$sR_a^{\text{EL-O}} s' \text{ iff } s \models pre(a) \text{ and } s' = \left( s \setminus \bigcup_{\substack{ce \in eff^-(a), \\ s \models cnd(ce)}} (ceff^-(ce)) \right) \cup \bigcup_{\substack{ce \in eff^+(a), \\ s \models cnd(ce)}} (ceff^+(ce)).$$

That is, if its precondition is satisfied then an action

- removes negative effects of firing conditional effects, plus their causes;
- adds positive effects of firing conditional effects, plus their consequences.

Thanks to EL-O consistency the order of removals and additions does not matter. Moreover, if we start from an introspectively closed state in  $I\text{-STATES}$  then the resulting state is also introspectively closed.

A *simple epistemic planning task* is a planning task over some subset of the set of observability atoms OA. It is EL-O *solvable* iff there is a state  $s$  such that  $s \models Goal$  that is reachable from  $s_0^{\rightarrow} \cup I\text{-OA}$  via the EL-O interpretation of the actions in  $Act$ , i.e., via  $\{R_a^{\text{EL-O}} : a \in Act\}$ .

**Example 9.** The planning task corresponding to the original gossip problem is  $G_1 = \langle \text{OA}, \text{Act}^{G_1}, s_0^{G_1}, \text{Goal}^{G_1} \rangle$  with

$$\begin{aligned} \text{Act}^{G_1} &= \{\text{call}_j^i : i, j \in \mathbb{A} \text{ and } i \neq j\}, \\ s_0^{G_1} &= \{S_i s_i : i \in \mathbb{A}\} \cup S \text{ for some } S \subseteq \{s_i : i \in \mathbb{A}\}, \\ \text{Goal}^{G_1} &= \bigwedge_{i, j \in \mathbb{A}} S_i s_j. \end{aligned}$$

It can be solved via  $2n - 4$  calls; when  $n \geq 4$  then there is no EL-O solution that takes less calls [46, 47, 48]. For instance, for  $n = 4$  the sequence  $\text{call}_2^1; \text{call}_4^3; \text{call}_3^1; \text{call}_4^2$  is an EL-O solution with 4 calls.

If the vocabulary is  $\mathbb{P}$  then classical planning tasks as defined in Section 5.2 are particular simple epistemic planning tasks: when action descriptions contain no observability operators then classical consistency and EL-O consistency of actions coincide; moreover,  $\text{ceff}^-(ce) = (\text{ceff}^-(ce))^{\Leftarrow}$  and  $\text{ceff}^+(ce) = (\text{ceff}^+(ce))^{\Rightarrow}$  for all conditional effects, and therefore  $R_a^{\text{EL-O}} = R_a^{\text{CPC}}$  for every action  $a$ . It follows from PSPACE hardness of classical solvability that EL-O solvability of simple epistemic planning tasks is PSPACE-hard, too.

### 5.5. Translation into classical planning

The *expansion* of a simple epistemic planning task  $\mathcal{P} = \langle \text{OA}, \text{Act}, s_0, \text{Goal} \rangle$  adds ‘enough’ consequences of atoms, viz. the relevant ones occurring in  $\mathcal{P}$ :

$$\text{Exp}(\mathcal{P}) = \langle \text{Voc}(\mathcal{P}), \{\langle \text{pre}(a), \text{Exp}_{\mathcal{P}}(\text{eff}(a)) \rangle : a \in \text{Act}\}, (s_0^{\Rightarrow} \cup \text{I-OA}) \cap \text{Voc}(\mathcal{P}), \text{Goal} \rangle$$

where the expansion of the effects of an action relative to  $\mathcal{P}$  is defined as:

$$\text{Exp}_{\mathcal{P}}(\text{eff}(a)) = \{\langle \text{cnd}(ce), \text{ceff}^+(ce)^{\Rightarrow} \cap \text{Voc}(\mathcal{P}), \text{ceff}^-(ce)^{\Leftarrow} \cap \text{Voc}(\mathcal{P}) \rangle : ce \in \text{eff}(a)\}.$$

After expansion classical solvability and EL-O solvability coincide:

**Proposition 12.** *Let  $\mathcal{P} = \langle \text{OA}, \text{Act}, s_0, \text{Goal} \rangle$  be a simple epistemic planning task. Then  $\mathcal{P}$  is EL-O solvable iff  $\text{Exp}(\mathcal{P})$  is classically solvable.*

*Proof.* We restrict the domain of the interpretation of actions to

$$I\text{-STATES}|_{\text{Voc}(\mathcal{P})} = \{s \cap \text{Voc}(\mathcal{P}) : s \in I\text{-STATES}\} = \{(s^\Rightarrow \cup I\text{-OA}) \cap \text{Voc}(\mathcal{P}) : s \in 2^{\text{OA}}\}.$$

For each  $a \in \text{Act}$  we write  $sR_a^{\mathcal{P}}s'$  for the restriction of  $sR_a^{\text{EL-O}}s'$  to the set  $I\text{-STATES}|_{\text{Voc}(\mathcal{P})}$ . We therefore have  $sR_a^{\mathcal{P}}s'$  iff  $s, s' \in I\text{-STATES}|_{\text{Voc}(\mathcal{P})}$ ,  $s \models \text{pre}(a)$  and

$$s' = \left( s \setminus \bigcup_{\substack{ce \in \text{eff}^-(a), \\ s \models \text{cnd}(ce)}} (\text{ceff}^-(ce))^{\Leftarrow} \cap \text{Voc}(\mathcal{P}) \right) \cup \bigcup_{\substack{ce \in \text{eff}^+(a), \\ s \models \text{cnd}(ce)}} (\text{ceff}^+(ce))^{\Rightarrow} \cap \text{Voc}(\mathcal{P}).$$

It is easily shown that if  $s \in I\text{-STATES}|_{\text{Voc}(\mathcal{P})}$  then  $s' \in I\text{-STATES}|_{\text{Voc}(\mathcal{P})}$ .

We now show the following result: for any state  $s \in I\text{-STATES}$  and action  $a \in \text{Act}$ , there exists a state  $s' \in I\text{-STATES}$  such that  $sR_a^{\text{EL-O}}s'$  iff there exists a state  $s'' \in I\text{-STATES}|_{\text{Voc}(\mathcal{P})}$  such that  $(s \cap \text{Voc}(\mathcal{P}))R_a^{\mathcal{P}}s''$ , and in that case we have that  $s'' = s' \cap \text{Voc}(\mathcal{P})$ . The first half is straightforward: as  $\text{Voc}(\text{pre}(a)) \subseteq \text{Voc}(\mathcal{P})$ , for any introspectively closed state  $s$ ,  $s$  and  $s \cap \text{Voc}(\mathcal{P})$  agree on  $\text{pre}(a)$  and hence  $s \models \text{pre}(a)$  iff  $s \cap \text{Voc}(\mathcal{P}) \models \text{pre}(a)$  (following Proposition 4). By the same argument, if  $sR_a^{\text{EL-O}}s'$  and if  $(s \cap \text{Voc}(\mathcal{P}))R_a^{\mathcal{P}}s''$ , then

$$s' = \left( s \setminus \bigcup_{\substack{ce \in \text{eff}^-(a), \\ s \models \text{cnd}(ce)}} (\text{ceff}^-(ce))^{\Leftarrow} \right) \cup \bigcup_{\substack{ce \in \text{eff}^+(a), \\ s \models \text{cnd}(ce)}} (\text{ceff}^+(ce))^{\Rightarrow},$$

and therefore

$$s' = \left( s \setminus \bigcup_{\substack{ce \in \text{eff}^-(a), \\ s \cap \text{Voc}(\mathcal{P}) \models \text{cnd}(ce)}} (\text{ceff}^-(ce))^{\Leftarrow} \right) \cup \bigcup_{\substack{ce \in \text{eff}^+(a), \\ s \cap \text{Voc}(\mathcal{P}) \models \text{cnd}(ce)}} (\text{ceff}^+(ce))^{\Rightarrow}.$$

By intersecting this with  $\text{Voc}(\mathcal{P})$  we conclude that  $s' \cap \text{Voc}(\mathcal{P}) = s''$ .

We can extend this result to any sequence of actions: for every  $s \in I\text{-STATES}|_{\text{Voc}(\mathcal{P})}$ , there exists a state  $s'$  that is reachable from  $s$  via  $\{R_a^{\text{EL-O}} : a \in \text{Act}\}$  iff there exists a state  $s''$  that is reachable from  $s \cap \text{Voc}(\mathcal{P})$  via  $\{R_a^{\mathcal{P}} : a \in \text{Act}\}$ , and in that case  $s'' = s' \cap \text{Voc}(\mathcal{P})$ . Moreover,  $s'$  and  $s''$  agree on  $\text{Voc}(\text{Goal})$ , and therefore  $\mathcal{P}$  is EL-O solvable via  $\{R_a^{\text{EL-O}} : a \in \text{Act}\}$

iff the planning task  $\langle \text{Voc}(\mathcal{P}), \text{Act}, (s_0^{\rightarrow} \cup \text{I-OA}) \cap \text{Voc}(\mathcal{P}), \text{Goal} \rangle$  is EL-O solvable via  $\{R_a^{\mathcal{P}} : a \in \text{Act}\}$ . Finally, thanks to Proposition 6 the latter is the case iff  
615 the expansion of  $\mathcal{P}$  is classically solvable via  $\{R_a^{\text{CPC}} : a \in \text{Act}\}$ .  $\square$

### 5.6. Complexity

Our translation from epistemic planning to classical planning increases task length only polynomially:

**Proposition 13.** *For every simple epistemic planning task  $\mathcal{P}$ ,  $\ell(\text{Exp}(\mathcal{P})) \leq$   
620  $(\ell(\mathcal{P}))^2$ .*

*Proof.* This follows from Proposition 1.  $\square$

**Proposition 14.** *Solvability of a simple epistemic planing task is PSPACE-complete.*

*Proof.* We have seen in Section 5.2 that classical planning tasks can be  
625 viewed as simple epistemic planning tasks. The solvability problem is therefore PSPACE-hard just as classical planning [14]. Membership follows from Propositions 13 and 12 and from Bylander’s proof that classical planning is in PSPACE [14].  $\square$

## 6. Applications

630 In this section, we give some examples of epistemic planning tasks. We start with the Byzantine Generals Problem and some toy examples most of which were introduced in [49]. We then focus on the generalised gossip problem.

### 6.1. Two Generals’ Problem

635 The Two Generals’ Problem [50, 51] is about coordination by communicating over an unreliable channel, highlighting the importance of common knowledge. The Byzantine Generals’ Problem [52] is a generalisation of that problem. In short, two generals need to coordinate an attack; they can

win only if they attack at the same time: either both attack in the morning  
640 ( $ta$ ) or both attack in the afternoon ( $\neg ta$ ). To communicate they must send  
a messenger who can be captured. They must decide on a time to attack,  
agree on this time, and each general must know that the other has agreed,  
and so on. To do this, one general sends a time for the attack, but since he  
cannot be sure the message was received, the other general must send an  
645 acknowledgment. Of course, just like the original message, the acknowl-  
edgment can be lost, so the first general must send an acknowledgment,  
and so on. It is impossible to be fully coordinated, i.e., to obtain common  
knowledge of the fact that the second general knows the time of the attack.

Formally, with  $\mathbb{A} = \{1, 2\}$  and  $\mathbb{P} = \{ta\}$ , we define the atom  $\alpha_n$  as:

- 650
- $\alpha_n = \underbrace{S_1 S_2 S_1 \dots S_2}_{n \text{ alternations}} ta$  if  $n$  is even;
  - $\alpha_n = \underbrace{S_2 S_1 S_2 \dots S_2}_{n \text{ alternations}} ta$  if  $n$  is odd.

For  $n \geq 0$  we define the family of actions  $\text{sndMsg}_n$  by stipulating:

$$\begin{aligned} \text{pre}(\text{sndMsg}_n) &= ta \wedge S_1 ta \wedge S_2 ta \wedge \alpha_2 \wedge \alpha_3 \wedge \dots \wedge \alpha_{n-1} \\ \text{eff}(\text{sndMsg}_n) &= \{\langle \top, \{ta, S_1 ta, S_2 ta, \alpha_2, \alpha_3, \dots, \alpha_n\}, \emptyset \rangle\} \end{aligned}$$

whose effect is that the time of the attack is set, that both agents know it,  
that 1 knows that 2 knows it, and so on until  $n$ .

The planning task to be solved is  $\mathcal{P} = \langle \text{OA}, \text{Act}, s_0, \text{Goal} \rangle$  with  $s_0 = \emptyset$ ,  $\text{Goal} =$   
655  $\mathcal{J} S_2 ta$ , and  $\text{Act} = \{\text{sndMsg}_n : n \geq 0\}$ . There is no solution for this task: while  
all sequences  $\langle \text{sndMsg}_0, \dots, \text{sndMsg}_n \rangle$  are executable, none of them reaches  
a state where  $\mathcal{J} S_2 ta$  is true.

*Remark 6.* The initial state  $s_0 = \emptyset$  of the Two Generals' Problem illustrates  
a case of maximal ignorance as discussed in Remark 5. The infiniteness of  
660 the maximal ignorance Kripke model may explain why the DEL literature  
pays only little attention to this and other Byzantine coordination prob-  
lems: most presentations are in semantical terms and use model checking



action	precondition	conditional effects
$\text{enter}_i$	$\neg \text{In}_i$	$\{\langle \top, \{\text{In}_i\}, \emptyset \rangle\}$
$\text{enter}'_i$	$\neg \text{In}_i \wedge \neg \text{In}_j, i \neq j$	$\{\langle \top, \{\text{In}_i\}, \emptyset \rangle\}$
$\text{leave}_i$	$\text{In}_i$	$\{\langle \top, \emptyset, \{\text{In}_i\} \rangle\}$
$\text{reveal}_i$	$\text{In}_i$	$\{\langle \top, \{S_i m\}, \emptyset \rangle\} \cup$ $\{\langle \text{In}_j, \{JS m\}, \emptyset \rangle, j \neq i\}$
$\text{informs}_{1,2} m$	$(\text{In}_1 \leftrightarrow \text{In}_2) \wedge S_1 m \wedge \neg(S_2 m \wedge S_1 S_2 m)$	$\{\langle \top, \{JS m\}, \emptyset \rangle\}$
$\text{informs}'_{1,2} m$	$(\text{In}_1 \leftrightarrow \text{In}_2) \wedge S_1 m \wedge \neg S_2 m \wedge S_1 S_2 m$	$\{\langle \top, \{JS m\}, \emptyset \rangle\}$

Table 2: Action descriptions for the message task

in finite Kripke models. (A finite modelling was however proposed recently in [19].)

## 665 6.2. Learning a message

Suppose two agents 1 and 2 are outside a room ( $\neg \text{In}_1 \wedge \neg \text{In}_2$ ). The room contains a message that we suppose for simplicity to be the value of the propositional variable  $m$ . Each of the agents can enter and leave the room and can (temporarily) reveal the message when she is in the room. In case  
670 the other agent is also in the room the message is jointly seen. (So this is a conditional effect.) We define a series of planning tasks where the initial state is  $s_0 = \{m\}$  and where we vary the action descriptions and the goal. For convenience we list all the action descriptions in Table 2.

For a start, consider  $Act = \{\text{enter}_i, \text{leave}_i, \text{reveal}_i : i \in \{1, 2\}\}$  where  $\text{enter}_i$  and  $\text{leave}_i$  respectively have preconditions  $\neg \text{In}_i$  and  $\text{In}_i$  and the obvious effects and where  $\text{reveal}_i$  requires to be in the room and has one unconditional effect ( $i$  knows the message) and one conditional effect: if the other agent is in the room then the message becomes common knowledge. We suppose, for simplicity, that there are no epistemic consequences when an agent enters or leaves the room because we are not interested in the epistemic status of

$\ln_1$  and  $\ln_2$ . Consider  $Goal = \neg \ln_1 \wedge \neg \ln_2 \wedge S_1 m \wedge S_2 m$ . The sequences

$$sol_1 = \langle \text{enter}_1, \text{reveal}_1, \text{leave}_1, \text{enter}_2, \text{reveal}_2, \text{leave}_2 \rangle,$$

$$sol_2 = \langle \text{enter}_1, \text{enter}_2, \text{reveal}_1, \text{leave}_1, \text{leave}_2 \rangle$$

are both solutions of  $\langle OA, Act, s_0, Goal \rangle$ .

675 If we replace  $Goal$  by  $Goal' = \neg \ln_1 \wedge \neg \ln_2 \wedge JS m$  then  $sol_2$  still solves  $\langle OA, Act, s_0, Goal' \rangle$ , but  $sol_1$  no longer does so.

Next, consider a variant of  $Act$  where the room is so small that it does not fit two persons: we have to replace  $\text{enter}_i$  by  $\text{enter}'_i$  with precondition  $pre(\text{enter}'_i) = \neg \ln_i \wedge \neg \ln_j$ , for  $i \neq j$ . Let the resulting action description be  $Act'$ .

680 While there is still a solution of  $\langle OA, Act', s_0, Goal \rangle$ , there is no solution of  $\langle OA, Act', s_0, Goal' \rangle$ .

Let us augment the set of actions by adding a further action  $\text{informs}_{1,2}m$  whose precondition is that both agents are in the same place, that the speaker 1 knows whether  $m$  and that it is possible for her that  $m$  is informative for the hearer 2. Let the resulting set of actions be  $Act''$ . Then the sequence

$$sol_3 = \langle \text{enter}_1, \text{reveal}_1, \text{leave}_1, \text{informs}_{1,2}m \rangle$$

becomes a further solution of  $\langle OA, Act'', s_0, Goal \rangle$  as well as of  $\langle OA, Act'', s_0, Goal' \rangle$ . Observe that  $sol_3$  is shorter than the other solutions.

Let us finally modify  $Act''$  into  $Act'''$  by a more demanding precondition of informing, namely by requiring that the speaker knows that her  
685 utterance is relevant for the hearer. Then  $sol_3$  is no longer a solution of  $\langle OA, Act'', s_0, Goal \rangle$  nor of  $\langle OA, Act'', s_0, Goal' \rangle$ : the speaker lacks knowledge about the hearer. It becomes a solution again if we make the hypothesis that there is initial common knowledge of ignorance about  $m$ , i.e., if we set  
690  $s'_0 = \{m, JS S_1 m, JS S_2 m\}$ .

### 6.3. Selective communication

The following example is from Kominis and Geffner:

action	precondition	conditional effects
goright <sub>a</sub>	$\top$	$\{\langle p_{a,k}, \{p_{a,k+1}\}, \{p_{a,k}\} \rangle : 1 \leq k \leq 3\}$
goleft <sub>a</sub>	$\top$	$\{\langle p_{a,k}, \{p_{a,k-1}\}, \{p_{a,k}\} \rangle : 2 \leq k \leq 4\}$
sense <sub>a,q</sub>	$p_{a,2}$	$\{\langle \top, \{S_a q\}, \emptyset \rangle\}$
tell <sub>a,q</sub>	$S_a q$	$\{\langle \varphi_b, S_b q, \emptyset \rangle, \langle \varphi_c, S_c q, \emptyset \rangle\}$

Table 3: Action descriptions for the selective communication task

Let  $a$ ,  $b$ , and  $c$  be three agents in a corridor of four rooms ( $p_1$ ,  $p_2$ ,  $p_3$  and  $p_4$  from left to right). The agents can move from a room to a contiguous room, and when agent  $i$  communicates (tells) some information, all the agents that are in the same room or in a contiguous room, will hear what was communicated. For example, if agent  $i$  expresses in  $p_3$  his knowledge about  $q$ , all agents in rooms  $p_2$ ,  $p_3$  and  $p_4$  will come to know it. We consider the problem where agent  $a$  is initially in room  $p_1$ ,  $b$  in  $p_2$ ,  $c$  in  $p_3$ , and  $a$  has to find out the truth value of a proposition  $q$  and let  $c$  know without agent  $b$  learning it [53].

Let  $p_{i,k}$  mean that agent  $i$  is in room  $p_k$ , for  $i \in \{a, b, c\}$  and  $1 \leq k \leq 4$ . Then the formula  $\varphi_b = \bigvee_{1 \leq k, \ell \leq 4, |k-\ell| \leq 1} (p_{b,k} \wedge p_{b,\ell})$  expresses that  $b$  is close to  $a$ , and likewise for  $\varphi_c$ . We model this as a simple epistemic planning task  $\mathcal{P} = \langle \text{OA}, \text{Act}, s_0, \text{Goal} \rangle$  with  $\text{Act} = \{\text{goleft}_a, \text{goright}_a, \text{sense}_{a,q}, \text{tell}_{a,q}\}$ ,  $s_0 = \{p_{a,1}, p_{b,2}, p_{c,3}\}$ ,  $\text{Goal} = S_c q \wedge \neg S_b q$ .<sup>4</sup> The descriptions of the actions in  $\text{Act}$  are listed in Table 3. Then the sequence

$$\langle \text{goleft}_a, \text{sense}_{a,q}, \text{goright}_a, \text{goright}_a, \text{goright}_a, \text{tell}_{a,q} \rangle$$

is a solution of  $\mathcal{P}$ .

<sup>4</sup>Kominis and Geffner suppose that the possible initial states are common knowledge. We could express this by adding to  $s_0$  the sets  $\{JS_i p_{a,1} : i \in \{a, b, c\}\}$ ,  $\{JS_i p_{b,2} : i \in \{a, b, c\}\}$ , and  $\{JS_i p_{c,3} : i \in \{a, b, c\}\}$ . We however disregard these hypotheses because our approach does not require it.

#### 6.4. Pedestrian crossing

In this 2-agent coordination problem that is derived from Lewis’s [54], a pedestrian wants to cross a road at the same time as a car wants to reach its destination. The driver can either continue or stop to let the pedestrian cross. In the latter case the driver signals to the pedestrian, for example by flashing the car’s headlights, that it is safe for the pedestrian to cross. We assume that the intentions of the pedestrian are obvious to the driver while the car is approaching (but not after it has driven away since the driver will not be looking in the rear-view mirror), but the pedestrian does not know this. Thus, before the pedestrian crosses the road, common knowledge must be established between the two agents (pedestrian and driver) of the pedestrian’s intention to cross.

We can model this problem using the following propositional variables:  $d_c$  is true if the car is driving forward,  $r_c$  is true if the car has reached its destination;  $r_p$  is true if the pedestrian has reached the other side of the road,  $wr_p$  is true if the pedestrian wants to reach the other side of the road.

Then the planning task can be described as follows. The initial state is  $s_0 = \{d_c, wr_p, S_c wr_p\}$ , the goal is  $Goal = r_p \wedge r_c$ , and the set of actions  $Act$  has four elements, three concerning the car and one the pedestrian, whose preconditions and effects are listed in Table 4. The action  $drivesOn_c$  has a precondition  $d_c \wedge \neg r_c$  and (unique) conditional effect that the car reaches its destination, stops driving and no longer sees the value of  $wr_p$  (whether the pedestrian wants to cross or not). Note that the fact that  $S_c wr_p$  becomes false implies that  $JS wr_p$  becomes false because the latter is an atomic (introspective) cause of the former, i.e., because  $JS wr_p \in (S_c wr_p)^\Leftarrow$ . The action  $stopsAndSignals_c$  has a precondition  $d_c \wedge wr_p \wedge S_c wr_p$  and (unique) conditional effect that the car stops and signals (by flashing its lights) that the driver knows that the pedestrian wants to cross which leads immediately to common knowledge of  $wr_p$ . The third possible action of the car is to restart (after a stop). Finally, the only action of the pedestrian is  $crosses_p$ .

action	precondition	conditional effects
$\text{drivesOn}_c$	$d_c \wedge \neg r_c$	$\{\langle \top, \{r_c\}, \{d_c, S_c wr_p\} \rangle\}$
$\text{stopsAndSignals}_c$	$d_c \wedge wr_p \wedge S_c wr_p$	$\{\langle \top, \{JS wr_p\}, \{d_c\} \rangle\}$
$\text{restarts}_c$	$\neg d_c \wedge r_p$	$\{\langle \top, d_c, \emptyset \rangle\}$
$\text{crosses}_p$	$wr_p \wedge ((\neg d_c \wedge JS wr_p) \vee r_c)$	$\{\langle \top, \{r_p\}, \{wr_p\} \rangle\}$

Table 4: Action descriptions for the pedestrian crossing task

Then the two sequences of actions

$\langle \text{drivesOn}_c, \text{crosses}_p \rangle,$

$\langle \text{stopsAndSignals}_c, \text{crosses}_p, \text{restarts}_c, \text{drivesOn}_c \rangle$

both solve our simple epistemic planning task  $\langle \text{OA}, Act, s_0, Goal \rangle$ .

### 6.5. Authorisation via plenary meetings

Here is a further example of actions with common knowledge preconditions. Consider a planning task involving cooperation between different agents that can be divided into  $m$  different stages with tasks to be performed at each stage by each agent. Agents are only authorised to start stage  $k + 1$  if *all* tasks of stage  $k$  have been completed and all agents have common knowledge of this. The only way this can be achieved is by having a plenary meeting at the end of each stage during which each agent announces that her stage- $k$  task has been completed.<sup>5</sup>

Each agent  $i$  requires an authorisation  $a_{ik}$  to start her stage- $k$  task. Initially,  $a_{i1}$  is true for all agents  $i$ , i.e.,  $s_0 = \{a_{i1} : i \in \mathbb{A}\}$ . Let  $t_{ik}$  represent

---

<sup>5</sup>The need for common knowledge can be motivated in the following manner: in order to go back to work, agents must know that other agents have completed their stage- $k$  task. Therefore agents need an acknowledgement of their announcing that they have completed this task in order to know that they are not preventing anybody else from going on with their work. But just like they need an acknowledgement, others need an acknowledgement of their acknowledgement, and so on, just like in the Byzantine Generals' Problem.

action	precondition	conditional effects
doTask <sub>ik</sub>	$a_{ik}$	$\{\langle \top, t_{ik} \wedge S_i t_{ik}, \emptyset \rangle\}$
authorise <sub>k</sub>	$\bigwedge_{i \in \mathbb{A}} \mathcal{J}S t_{i,k-1}$	$\{\langle \top, \{a_{ik} : i \in \mathbb{A}\}, \{p_i : i \in \mathbb{A}\} \rangle\}$
goMeeting <sub>i</sub>	$\top$	$\{\langle \top, p_i, \emptyset \rangle\}$
announce <sub>i</sub>	$\bigwedge_{k \in \mathbb{A}} p_k$	$\{\langle t_{ik} \wedge S_i t_{ik}, \mathcal{J}S t_{ik}, \emptyset \rangle\}$

Table 5: Action descriptions for the authorisation via plenary meetings task

the fact that agent  $i \in \mathbb{A}$  has completed her stage- $k$  task. The propositional variable  $p_i$  is true if agent  $i$  is present at the meeting.

755 The preconditions and effects of the four action types doTask<sub>ik</sub>, authorise<sub>k</sub>, goMeeting<sub>i</sub>, and announce<sub>i</sub> are described in Table 5. For the action doTask<sub>ik</sub>, when an agent completes a task she sees this, but the other agents don't. The action authorise<sub>k</sub> ( $k = 2, \dots, m$ ) authorises the start of stage  $k$ . Authorisations for stage  $k > 1$  are only issued if there is common knowledge that  
760 all tasks of stage  $k - 1$  have been completed. They are thus issued at the end of the plenary meeting. A side effect of this is that all agents leave the meeting. The action goMeeting<sub>i</sub> allows each agent to go to the meeting room whenever she wants. Once everybody is present, each agent  $i$  can announce that her task has been completed via the action announce<sub>i</sub>.

765 The goal is common knowledge that all tasks have been completed:

$$Goal = \bigwedge_{i \in \mathbb{A}} \bigwedge_{k=1, \dots, m} (t_{ik} \wedge \mathcal{J}S t_{ik}).$$

With 2 agents and 2 stages, the following sequence is a solution:

$\langle \text{doTask}_{11}, \text{doTask}_{21}, \text{goMeeting}_1, \text{goMeeting}_2, \text{announce}_1, \text{announce}_2, \text{authorise}_2, \text{doTask}_{12}, \text{doTask}_{22}, \text{goMeeting}_1, \text{goMeeting}_2, \text{announce}_1, \text{announce}_2 \rangle$ .

### 6.6. The generalised gossip problem

Let the set of agents be  $\mathbb{A} = \{1, \dots, n\}$  and let the set of propositional variables be  $\mathbb{P} = \{s_i : i \in \mathbb{A}\}$ . Let us describe the planning task for the

generalised gossip problem of depth  $k$  as a simple epistemic planning task  $G_k = \langle \text{OA}, \text{Act}^{G_k}, s_0^{G_k}, \text{Goal}^{G_k} \rangle$ . The initial state is  $s_0^{G_k} = \{S_i s_i : i \in \mathbb{A}\} \cup S$  for some  $S \subseteq \{s_i : i \in \mathbb{A}\}$  (cf. Example 3). As we have seen in Example 4, the goal of obtaining shared knowledge of depth  $k$  can be described in EL-O by

$$\text{Goal}^{G_k} = \bigwedge_{i \in \mathbb{A}} \bigwedge_{\sigma \in \text{OBS}^+, \ell(\sigma) \leq k} \sigma s_i.$$

The set of actions is  $\text{Act}^{G_k} = \{\text{call}_j^i : i, j \in \mathbb{A}, i \neq j\}$ , where  $\text{pre}(\text{call}_j^i) = \top$  and where for every  $0 \leq m < k$ , every  $\sigma_m \in \text{OBS}^{\leq m}$  of length at most  $m$  and every  $r \in \mathbb{A}$  there are conditional effects  $ce \in \text{eff}(\text{call}_j^i)$  of the form:

$$\begin{aligned} \text{cnd}(ce) &= S_i \sigma_m s_r \vee S_j \sigma_m s_r; \\ \text{ceff}^+(ce) &= \{\sigma S_i \sigma_m s_r : \sigma \in \{S_i, S_j\}^{\leq k-m-1}\} \cup \{\sigma S_j \sigma_m s_r : \sigma \in \{S_i, S_j\}^{\leq k-m-1}\} \\ &= \{\sigma \sigma_m s_r : \sigma \in \{S_i, S_j\}^{\leq k-m}\}; \\ \text{ceff}^-(ce) &= \emptyset. \end{aligned}$$

Remember that  $\{S_i, S_j\}^{\leq k-m}$  denotes the set all sequences of observability operators  $S_i$  and  $S_j$  of length at most  $k - m$ . For  $k = 1$  we obtain  $\text{cnd}(ce) = S_i s_r \vee S_j s_r$ ,  $\text{ceff}^+(ce) = \{S_i s_r, S_j s_r\}$ , and  $\text{ceff}^-(ce) = \emptyset$ , matching Example 7.

We recall that the original gossip problem with  $n \geq 4$  agents can be solved in  $2(n - 2)$  calls [46, 47, 48] and that there is no shorter solution. It is known that the generalised gossip problem  $G_k$  can be solved in at most  $(k+1)(n-2)$  calls [36] and that there is no shorter solution [4]. For instance, for  $k = 2$  and  $n = 5$  the sequence

$$\text{call}_3^1, \text{call}_4^1, \text{call}_5^2, \text{call}_5^1, \text{call}_3^1, \text{call}_4^2, \text{call}_4^1, \text{call}_5^1, \text{call}_3^2$$

770 is a solution with  $3 \times 3 = 9$  calls, which is therefore optimal.

### 6.7. Generalised gossiping with ignorance goals

We can also easily model ignorance goals. Given a set of atoms  $A$ , let  $G\text{-neg}_{k,A} = \langle \text{OA}, \text{Act}^{G\text{-neg}_{k,A}}, s_0^{G\text{-neg}_{k,A}}, \text{Goal}^{G\text{-neg}_{k,A}} \rangle$  be the planning task for the generalised gossip problem of depth  $k$  with the atoms of  $A$  as the only negative

goals. The initial state and the actions remain the same:  $s_0^{G\text{-neg}_{k,A}} = \{S_i s_i : i \in \mathbb{A}\} \cup S$  for some  $S \subseteq \{s_i : i \in \mathbb{A}\}$  and  $Act^{G\text{-neg}_{k,A}} = \{\text{call}_j^i : i, j \in \mathbb{A}, i \neq j\}$ ; but the goal changes:

$$Goal^{G\text{-neg}_{k,A}} = \left( \bigwedge_{\sigma \in \{S_i : i \in \mathbb{A}\}^{\leq k}} \bigwedge_{j \in \mathbb{A}, \sigma s_j \notin A} \sigma s_j \right) \wedge \left( \bigwedge_{\alpha \in A} \neg \alpha \right).$$

Here are some examples:

- $G\text{-neg}_{1, \{S_1, S_2\}}$  corresponds to the case where we want everyone to know all secrets, except that 1 should not know the secret of 2;
- 775 •  $G\text{-neg}_{1, \{S_1, S_3, S_2, S_4\}}$ , to the case where only 1 should not get to know the secret of 3 and 2, the secret of 4;
- $G\text{-neg}_{2, \{S_1, S_2, S_3\}}$ , to the case of epistemic depth 2 where only 1 should not know whether 2 knows the secret of 3, while 1 and 2 should know the secret of 3;
- 780 •  $G\text{-neg}_{3, \{S_1, S_2, S_3\}}$  is the same as  $G\text{-neg}_{2, \{S_1, S_2, S_3\}}$  but with depth 3.

It was shown in [55] that the existence of a solution for gossiping with ignorance goals is an NP-complete problem.

## 7. Encoding into PDDL and experiments

In this section we report about some experiments with those simple epis-  
 785 temic planning tasks of Section 6 that are parametrised: general gossiping, general gossiping with ignorance goals, and authorisation via plenary meetings.

The experiments were done using three planners of the optimal track of  
 IPC 2018: Planning-PDBs, Complementary1 and Complementary2. They  
 790 all gave similar results for the two problems described below, so we chose to only show those for Planning-PDBs. The results were obtained on a GNU/Linux machine running on a 3,6 to 4,4 GHz processor (Ryzen 7 3700x) with 32 GB of RAM and a 30 minutes time limit.



We first show how to encode simple planning tasks into the Planning  
 795 Domain Description Language PDDL [56].

### 7.1. Encoding into PDDL

Consider a simple epistemic planning task  $\mathcal{P} = \langle \text{OA}, \text{Act}, s_0, \text{Goal} \rangle$ . We  
 show how to encode EL-O formulas and actions from  $\text{Exp}(\mathcal{P})$  into PDDL.  
 Some PDDL requirement flags should be set depending on the form of con-  
 800 ditions  $\text{cnd}(ce)$  of conditional effects  $ce$  of actions as well as on the form of  
 the formula  $\text{Goal}$ :

- the default flag `:strips` for conjunctions;
- the flag `:negative-preconditions` for negations;
- the flag `:disjunctive-preconditions` for negations of conjunc-  
 805 tions, and disjunctions, if used to simplify writing.

Fortunately almost all planners from the 2018 International Planning Com-  
 petition (IPC 2018)<sup>6</sup> handle conditional effects and negative preconditions,  
 and most of them handle disjunctive preconditions.

Given a formula  $\varphi$  without introspective atoms, we define a recursive  
 function  $\text{tr}_{\text{PDDL}}(\varphi)$  which returns the encoding of  $\varphi$  into PDDL:

$$\text{tr}_{\text{PDDL}}(S_{i_1} \dots S_{i_m} p) = \begin{cases} (p) & \text{if } m = 0, \\ (\mathbf{S-m} \ i_1 \ \dots \ i_m \ p) & \text{otherwise;} \end{cases}$$

$$\text{tr}_{\text{PDDL}}(\mathbf{JS} \ S_{i_1} \dots S_{i_m} p) = \begin{cases} (\mathbf{JS} \ p) & \text{if } m = 0, \\ (\mathbf{JS-m} \ i_1 \ \dots \ i_m \ p) & \text{otherwise;} \end{cases}$$

$$\text{tr}_{\text{PDDL}}(\neg\varphi) = (\text{not } \text{tr}_{\text{PDDL}}(\varphi));$$

$$\text{tr}_{\text{PDDL}}(\varphi_1 \wedge \varphi_2) = (\text{and } \text{tr}_{\text{PDDL}}(\varphi_1) \ \text{tr}_{\text{PDDL}}(\varphi_2)).$$

with  $p \in \mathbb{P}$ ,  $m \geq 0$ , and  $i_1, \dots, i_m \in \mathbb{A}$ . In words, observability atoms  $S_{i_1} \dots S_{i_m} p$   
 810 and  $\mathbf{JS} \ S_{i_1} \dots S_{i_m} p$  are respectively encoded by the special fluents  $\mathbf{S-m}$  and

<sup>6</sup><https://ipc2018-classical.bitbucket.io/>

**JS**, both with  $m+1$  parameters. For  $m = 0$ , the propositional variable  $p$  is encoded as a fluent without parameters and  $JS p$  is encoded as a special fluent (**JS**  $p$ ). The initial state  $s_0^{\Rightarrow} \cap \text{Voc}(\mathcal{P})$  of  $\text{Exp}(\mathcal{P})$  is trivially encoded as a set of fluents thanks to  $tr_{PDDL}(\alpha)$ . The formula *Goal* and the preconditions of every action can be encoded using  $tr_{PDDL}(\varphi)$  since they are all EL-O formulas. We note that we give no encoding into PDDL for atoms of the form  $\sigma JS \alpha$  where  $\sigma \in \text{OBS}^+$ : these atoms are introspective, and as the translation of planning tasks is normalised so as to remove any introspective atoms, we do not need to worry about encoding such atoms.

As we consider actions with conditional effects, the requirement flag `:conditional-effects must be set`.

Consider an action  $a \in \text{Act}$ . For every  $ce \in \text{eff}(a)$  with  $\text{ceff}^+(ce)^{\Rightarrow} \cap \text{Voc}(\mathcal{P}) = \{\alpha_1, \dots, \alpha_m\}$  and  $\text{ceff}^-(ce)^{\Leftarrow} \cap \text{Voc}(\mathcal{P}) = \{\beta_1, \dots, \beta_\ell\}$ , we add the conditional effect:

$$\begin{aligned} & (\text{when } tr_{PDDL}(\text{cnd}(ce)) \\ & \quad (\text{and } tr_{PDDL}(\alpha_1) \dots tr_{PDDL}(\alpha_m) \\ & \quad \quad (\text{not } tr_{PDDL}(\beta_1)) \dots (\text{not } tr_{PDDL}(\beta_\ell)))) \end{aligned}$$

**Example 10** (Example 7, ctd.). For the original gossip problem, the action description `call2` of Example 7 is coded in PDDL as follows:

```
(:action call-1-2
825  :effect (and
          (when (or (S-1 1 s1) (S-1 2 s1))
                (and (S-1 1 s1) (S-1 2 s1)))
          ...
          (when (or (S-1 1 sn) (S-1 2 sn))
830          (and (S-1 1 sn) (S-1 2 sn))))))
```

This is a direct encoding of a call into PDDL. Observe that we can generalise it to any  $i$  and  $j$  by writing:

```
(:action call
      :parameters (?i ?j)
835  :effect (and (forall (?s)
```

```

      (and
        (when (or (S-1 ?i ?s) (S-1 ?j ?s))
          (and (S-1 ?i ?s) (S-1 ?j ?s))))))

```

Finally, let us consider the generalised gossip problem of depth  $k$  for the case of two agents 1 and 2. According to Section 6.6, call<sub>2</sub><sup>1</sup> can be modelled as follows:

```

(:action call-1-2
  :effect (and
    (when (or (S-1 1 s1) (S-1 2 s1))
      (and (S-1 1 s1) (S-1 2 s1)
        (S-2 1 2 s1) (S-2 2 1 s1)
        ...
        (S-k 1 2 1 ... s1) (S-k 2 1 2 ... s1)))
    (when (or (S-1 1 s2) (S-1 2 s2))
      (and (S-1 1 s2) (S-1 2 s2)
        (S-2 1 2 s2) (S-2 2 1 s2)
        ...
        (S-k 1 2 1 ... s2) (S-k 2 1 2 ... s2)))) )

```

## 7.2. Experiments

In previous experiments we had used the planner FDSS-2014 [57] that handles conditional effects and negative and disjunctive preconditions for the gossip problem [10]. In more recent experiments we have used Planning-PDBs [58], as reported in the sequel. In comparison, the behaviour on the gossip problem was very similar, which is explained by the fact that in both cases the bottleneck was the grounding phase.

The resources available online<sup>7</sup> also contain an open-source PDDL generator for the gossip problem that was developed in Python<sup>8</sup>. It allowed

<sup>7</sup><http://www.irit.fr/%7EAndreas.Herzig/P/Ecai16.html>

<sup>8</sup><https://github.com/FaustineMaffre/GossipProblem-PDDL-generator>

us to create the domain and problem files for the generalised gossip problem and its variant with ignorance goals. In its basic settings, the program  
 865 needs the depth  $k$  and the number of agents  $n$ . It is also possible to specify sets of negative goals, either directly by giving the index of agents such as  $\{S_1 s_3, S_2 s_4\}$ , or with constraints such as  $\{S_i s_j : i \neq j \text{ and } j < 3\}$  if we want no agent except 1 and 2 to know all secrets. Feeding the resulting files into a PDDL planner returns a solution to the corresponding gossip problem,  
 870 namely a sequence of calls.

**Generalised gossip problem.** We ran FDSS-2014 and Planning-PDBs fixing a number of agents  $n$  and increasing the depth  $k$  until the first  $k$  that leads to a timeout. We recall that the original gossip problem with  $n \geq 4$  agents can be solved in  $2n - 4$  calls and the generalised gossip problem (without negative goals) in at most  $(k+1)(n-2)$  calls. For  $k = 2$  and  $n = 5$  the optimal plan found by both FDSS-2014 and Planning-PDBs was the following with 9 calls:

$$\langle \text{call}_2^1, \text{call}_3^1, \text{call}_5^4, \text{call}_4^1, \text{call}_5^3, \text{call}_5^2, \text{call}_5^1, \text{call}_4^2, \text{call}_4^3 \rangle.$$

More generally, for  $k = 2$  and  $n \leq 5$ , FDSS-2014 was able to find all optimal solutions that are computed by the protocol given in [36] and produced a timeout for  $n = 6$ . For  $n \geq 8$ , FDSS-2014 produced a timeout already for  $k = 1$ . The performance of Planning-PDBs was only slightly better: for  
 875  $k = 1$  the timeout occurred for  $n = 9$  and a solution was found for  $n = 8$  in 848s. Figure 1 gives the execution time of Planning-PDBs parametrised by the depth, while Figure 2 gives the execution time parametrised by the number of agents.

**Generalised gossiping with ignorance goals.** We ran FDSS-2014 and  
 880 Planning-PDBs in the same manner as the generalised gossip problem, fixing a number of agents  $n$  and increasing the depth  $k$  until the first  $k$  leading to a timeout. This time the timeout occurred for exactly the same  $n$  and  $k$ . Table 6 contains the optimal plans that were produced for  $n = 4$  agents. The timeout occurred for  $k = 3$ .

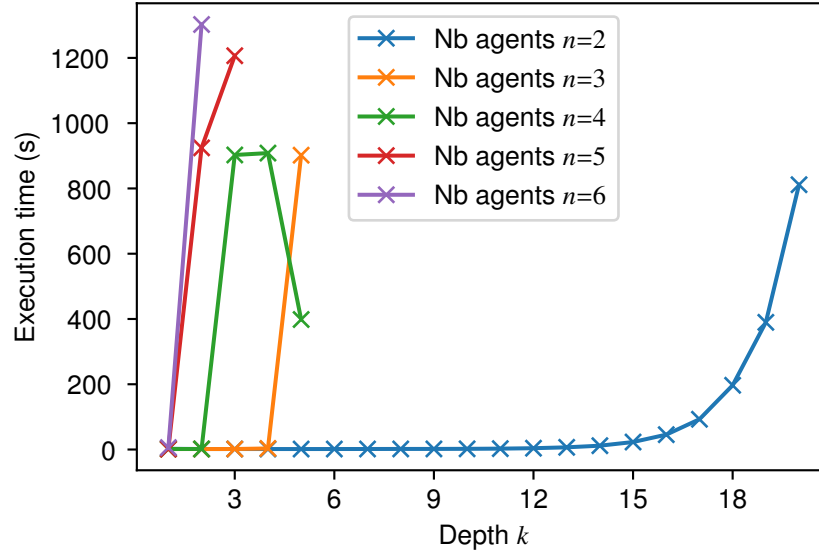


Figure 1: Generalised gossiping: time to find an optimal plan vs. epistemic depth (for different fixed numbers of agents)

$G\text{-neg}_{1,\{S_1, S_2\}}$	$\langle \text{call}_3^1, \text{call}_4^1, \text{call}_4^2, \text{call}_4^3 \rangle$
$G\text{-neg}_{1,\{S_1, S_3, S_2, S_4\}}$	$\langle \text{call}_2^1, \text{call}_4^1, \text{call}_3^2, \text{call}_4^3 \rangle$
$G\text{-neg}_{2,\{S_1, S_2, S_3\}}$	$\langle \text{call}_2^1, \text{call}_4^2, \text{call}_4^3, \text{call}_3^1, \text{call}_3^2, \text{call}_4^2 \rangle$
$G\text{-neg}_{3,\{S_1, S_2, S_3\}}$	(timeout)

Table 6: Generalised gossiping with ignorance goals: optimal plans found by FDSS-2014 and Planning-PDBs, for  $n = 4$  agents

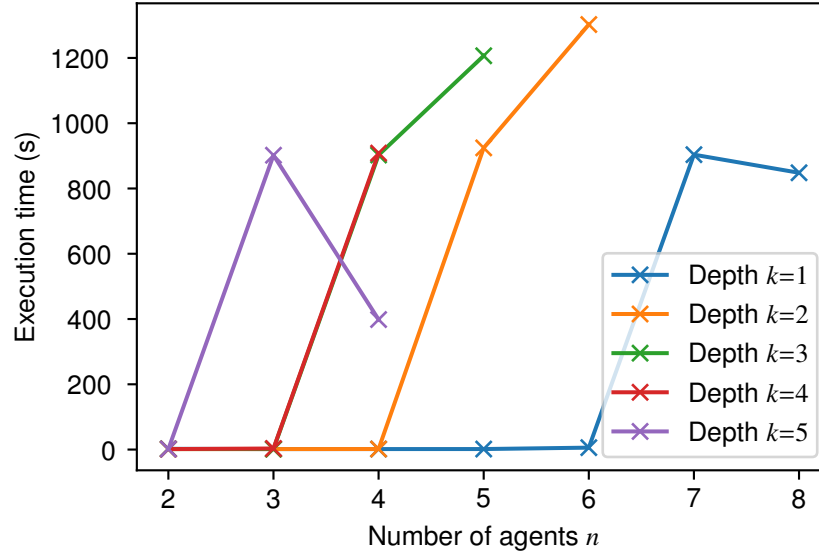


Figure 2: Generalised gossiping: time to find an optimal plan vs. number of agents (for different fixed epistemic depths)

885 **Authorisation via plenary meetings.** We ran Planning-PDBs on the plenary meetings problem for varying numbers of agents  $n$  and numbers of meetings ('steps')  $m$ .

Figure 3 shows the time needed by Planning-PDBs to find an optimal plan, in seconds, relative to the number of agents, where each curve plotted is for a fixed number of meetings. For  $m = 1$  the timeout obtained when  
 890  $n = 7$ ; for  $m = 2$  when  $n = 5$ ; and for  $m = 3$  when  $n = 4$ .

Figure 4 depicts the time to find an optimal plan w.r.t. the number of meetings, each curve being for a fixed number of agents. The curve for two agents provides some evidence for an exponential effect depending on the  
 895 number of meetings.

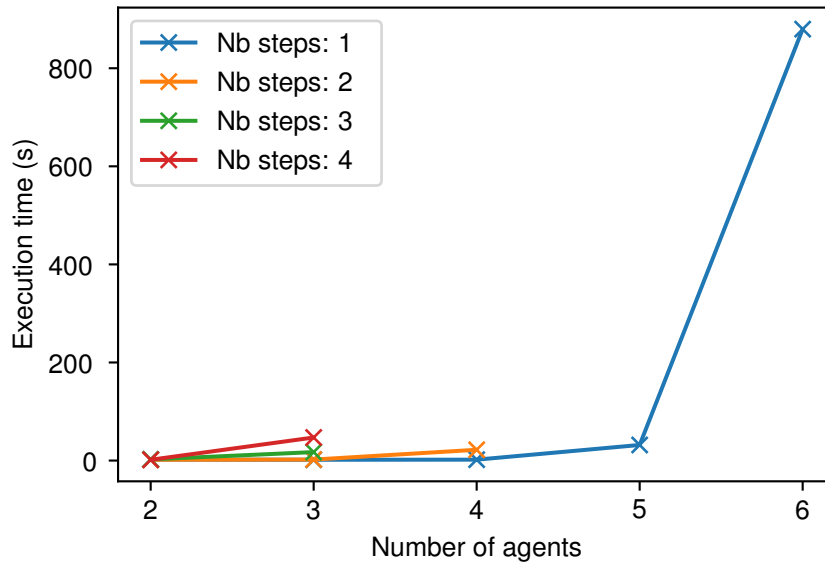


Figure 3: Plenary meetings: time to find an optimal parallel plan vs. number of agents (for different fixed numbers of meetings)

## 8. Related work

We now compare our approach with DEL-based planning as well as with the three existing approaches that can be viewed as particular cases of it. The latter either restrict the language to epistemic literals or restrict DEL's event models in a way such that actions are public or semi-private.

### 8.1. Planning with DEL event models

DEL-based planning was studied extensively in the literature in the last ten years [15, 16, 17, 18, 59, 19]. We show that our action descriptions can capture several important kinds of DEL event models (precisely, public, private and semi-private announcements) and the other way round, we show for some of our action descriptions how they correspond to DEL event models. One might expect that all such descriptions have a corresponding DEL event model; however, the correspondence is not obvious due to fundamen-

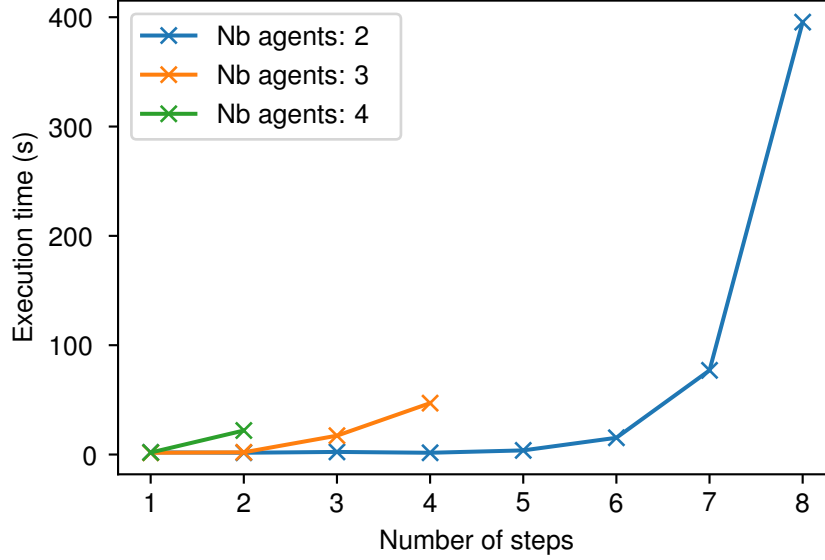


Figure 4: Plenary meetings: time to find an optimal parallel plan vs. number of meetings (for different fixed numbers of agents)

tal differences in the way epistemic effects are modelled. We explain this in  
 910 detail in Section 8.1.5.

### 8.1.1. Event models and product updates

A DEL *event model* is a tuple  $\text{Evt} = \langle W^{\text{Evt}}, \sim_{\mathbb{A}}^{\text{Evt}}, \{\sim_i^{\text{Evt}}\}_{i \in \mathbb{A}}, \text{pre}^{\text{Evt}}, \text{post}^{\text{Evt}} \rangle$  where  
 $W^{\text{Evt}}$  is a finite set of events;  $\sim_{\mathbb{A}}^{\text{Evt}}$  and the  $\sim_i^{\text{Evt}}$  are equivalence relations on  
 $W^{\text{Evt}}$  such that every  $\sim_i^{\text{Evt}}$  is a subset of  $\sim_{\mathbb{A}}^{\text{Evt}}$ ;  $\text{pre}^{\text{Evt}} : W^{\text{Evt}} \rightarrow \mathcal{L}_{\text{EL}}$  maps events  
 915 to their preconditions; and  $\text{post}^{\text{Evt}} : W^{\text{Evt}} \rightarrow (\mathbb{P} \rightarrow \mathcal{L}_{\text{EL}})$  maps events to  
 partial functions such that for every  $e \in W^{\text{Evt}}$ ,  $\text{post}^{\text{Evt}}(e)$  is undefined almost  
 everywhere. A *pointed event model* is a pair  $(\text{Evt}, e)$  where  $e \in W^{\text{Evt}}$  is the  
 designated event (or the actual event). A *multipointed event model* is a pair  
 $(\text{Evt}, E)$  where  $E \subseteq W^{\text{Evt}}$  is such that for any two distinct events  $e, e'$  in  $E$ ,  
 920 the preconditions of  $e$  and  $e'$  are incompatible, that is,  $\text{pre}^{\text{Evt}}(e) \wedge \text{pre}^{\text{Evt}}(e')$  is  
 unsatisfiable.

Here are some examples and their representations as graphs, in which





Figure 5: DEL event models for the public assignment of  $\varphi$  to  $p$  (on the left) and for the truthful public announcement of  $\varphi$  (on the right).

nodes are events, labelled arrows represent the equivalence relations, the actual event is denoted via a double outline, and pre- and postconditions are given respectively in the top and bottom line in the nodes. An undefined postcondition is denoted by *skip*. We will often omit transitive and reflexive arrows for clarity.

1. The public assignment of  $\varphi$  to  $p$ , i.e., the event where  $p$  publicly gets the truth value of  $\varphi$ , is modelled by an event model  $\text{Assign}(p, \varphi)$  with a single point  $e_1$  whose precondition is  $\top$ , with total relations, i.e., such that  $\sim_{\underset{A}{A}}^{\text{Evt}} = \sim_i^{\text{Evt}} = \{\langle e_1, e_1 \rangle\}$ , and with the postcondition function  $\text{post}^{\text{Evt}}(e_1)$  such that  $\text{post}^{\text{Evt}}(e_1)(p) = \varphi$  and undefined for all  $q \neq p$ . This event model is shown on the left-hand side in Figure 5.
2. The truthful public announcement of a formula  $\varphi$  is modelled by an event model  $\text{PubAnn}(\varphi)$ , represented on the right-hand side of Figure 5, with a single point  $e_2$  whose precondition is  $\varphi$ , with total relations and with an undefined postcondition function.
3. The truthful semi-private announcement whether  $\varphi$  to agent  $i$  is when  $i$  learns whether  $p$  and the other agents only learn that  $i$  learns whether  $p$  without learning whether  $p$  themselves. This is modelled by an event model  $\text{SemiPrivAnn}(\varphi, i)$  with two points  $e^+$  and  $e^-$  where  $\text{pre}^{\text{Evt}}(e^+) = \varphi$  and  $\text{pre}^{\text{Evt}}(e^-) = \neg\varphi$ , with an undefined postcondition function, and with  $\sim_{\underset{A}{A}}^{\text{Evt}} = \sim_j^{\text{Evt}} = \{e^+, e^-\} \times \{e^+, e^-\}$  for every  $j \neq i$  and  $\sim_i^{\text{Evt}} = \{\langle e^+, e^+ \rangle, \langle e^-, e^- \rangle\}$ . When we want to model that  $i$  learns that  $\varphi$  then  $e^+$  is the designated event; otherwise it is  $e^-$ , as is the case in Figure 6.

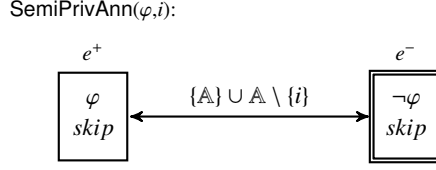


Figure 6: DEL event model for the truthful semi-private announcement of  $\neg\varphi$  to agent  $i$ .

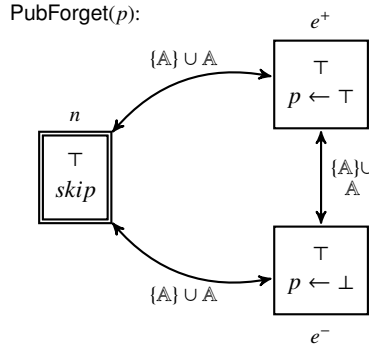


Figure 7: DEL event model for the public forgetting of  $p$ .

4. The public forgetting of a propositional variable  $p$  by all agents [60] can be captured by an event model  $\text{PubForget}(p)$  with three points  $n$ ,  $e^+$  and  $e^-$  where  $\text{pre}^{\text{Evt}}(n) = \text{pre}^{\text{Evt}}(e^+) = \text{pre}^{\text{Evt}}(e^-) = \top$ , where  $\sim_{\mathbb{A}}^{\text{Evt}} = \sim_i^{\text{Evt}} = \{n, e^+, e^-\} \times \{n, e^+, e^-\}$  for every  $i$ , and where  $\text{post}^{\text{Evt}}$  is such that  $\text{post}^{\text{Evt}}(n)(p) = p$ ,  $\text{post}^{\text{Evt}}(e^+)(p) = \top$ ,  $\text{post}^{\text{Evt}}(e^-)(p) = \perp$ , and  $\text{post}^{\text{Evt}}(n) = \text{skip}$ . This event model is represented in Figure 7. Note that a (single-pointed) event model with two points would not do the job: it would force commitment to some ontic event taking place.

Given a Kripke model  $M = \langle W, \sim_{\mathbb{A}}, \{\sim_i\}_{i \in \mathbb{A}}, V \rangle$  as defined in Section 4 and an event model  $\text{Evt}$ , the *product update* of  $M$  by  $\text{Evt}$  is the Kripke model  $M \otimes \text{Evt} = \langle W', \sim'_{\mathbb{A}}, \{\sim'_i\}_{i \in \mathbb{A}}, V' \rangle$  with

- $W' = \{(s, e) : s \in W, e \in W^{\text{Evt}}, \text{ and } (M, s) \models \text{pre}^{\text{Evt}}(e)\}$ ;
- $(s, e) \sim'_{\mathbb{A}} (t, f)$  iff  $s \sim_{\mathbb{A}} t$  and  $e \sim_{\mathbb{A}}^{\text{Evt}} f$ ;

- $(s, e) \sim'_i (t, f)$  iff  $s \sim_i t$  and  $e \sim_i^{\text{Evt}} f$ , for every  $i \in \mathbb{A}$ ;
- 960 •  $V'((s, e)) = \{p : \text{post}^{\text{Evt}}(e)(p) \text{ is undefined and } (M, s) \models p\} \cup$   
 $\{p : \text{post}^{\text{Evt}}(e)(p) \text{ is defined and } (M, s) \models \text{post}^{\text{Evt}}(e)(p)\}.$

The product update of a pointed Kripke model  $(M, s)$  with a pointed event model  $(\text{Evt}, e)$  is the pointed Kripke model  $(M \otimes \text{Evt}, (s, e))$ , defined only when  $(M, s) \models \text{pre}^{\text{Evt}}(e)$ . The product update of a pointed model  $(M, s)$  with a multi-  
 965 pointed event model  $(\text{Evt}, E)$  is the pointed model  $(M \otimes \text{Evt}, (s, e))$  where  $e \in E$  is such that  $(M, s) \models \text{pre}^{\text{Evt}}(e)$ , defined when such an  $e$  exists in  $E$ . In that case the hypothesis in the definition of a multipointed event model that the preconditions of different events are incompatible ensures that it is unique.

### 8.1.2. Bisimulations

970 We recall a standard notion in modal logics that will be instrumental to prove our results: a *bisimulation* [61] between two Kripke models  $M = \langle W, \sim_{\mathbb{A}}, \{\sim_i\}_{i \in \mathbb{A}}, V \rangle$  and  $M' = \langle W', \sim'_{\mathbb{A}}, \{\sim'_i\}_{i \in \mathbb{A}}, V' \rangle$  is a relation  $Z \subseteq W \times W'$  such that for all  $w \in W$  and  $w' \in W'$  such that  $wZw'$ :  $V(w) = V'(w')$  (atomic); if  $w \sim_{\mathbb{A}} u$  then  $uZu'$  and  $w' \sim'_{\mathbb{A}} u'$  for some  $u'$ , and if  $w \sim_i u$  then  $uZu'$  and  
 975  $w' \sim'_i u'$  for some  $u'$  (forth); if  $w' \sim'_{\mathbb{A}} u'$  then  $uZu'$  and  $w \sim_{\mathbb{A}} u$  for some  $u$ , and if  $w' \sim'_i u'$  then  $uZu'$  and  $w \sim_i u$  for some  $u$  (back). Two pointed models  $(M, w)$  and  $(M', w')$  are *bisimilar* if there exists a bisimulation  $Z$  between  $M$  and  $M'$  such that  $wZw'$ . In that case, we have  $M, w \models \varphi$  iff  $M', w' \models \varphi$  for every formula  $\varphi \in \mathcal{L}_{\text{EL}}$  [35].

### 980 8.1.3. From DEL event models to EL-O action descriptions

In what follows we show that we can express several kinds of event models, where we sometimes restrict preconditions to literals and conjunctions thereof. We are going to associate an EL-O action description  $\mathfrak{a}(\text{Evt}, e)$  to a given pointed event model  $(\text{Evt}, e)$ . In all cases, the precondition of  $\mathfrak{a}(\text{Evt}, e)$   
 985 is  $\text{pre}^{\text{Evt}}(e)$ . (More precisely, it is the reduction of  $\text{pre}^{\text{Evt}}(e)$  to an  $\mathcal{L}_{\text{bool}}(\text{OA})$  formula obtained by exhaustively applying the reduction axioms of Section 4.4.) So  $M^{\text{EL-O}}, s \models \text{pre}^{\text{Evt}}(e)$  exactly when  $R_{\mathfrak{a}(\text{Evt}, e)}^{\text{EL-O}}$  is defined at  $s$ . For such

points  $s$ , it can be shown that the pointed product update  $(M^{\text{EL-O}} \otimes \text{Evt}, (s, e))$  is bisimilar to the pointed Kripke model  $(M^{\text{EL-O}}, t)$  for the state  $t$  such that

$$s R_{a(\text{Evt}, e)}^{\text{EL-O}} t.$$

First, let us look at some kinds of public assignments.  $\text{Assign}(p, \top)$  is captured by the action description  $a(\text{Assign}(p, \top), e_1) = \langle \top, \{\langle \top, \{\mathcal{J}S p, p\}, \emptyset\} \rangle$  and  $\text{Assign}(p, \perp)$  is captured by  $a(\text{Assign}(p, \perp), e_1) = \langle \top, \{\langle \top, \{\mathcal{J}S p\}, \{p\}\} \rangle$ . Finally,  $\text{Assign}(p, \neg p)$  is the public toggling of the truth value of  $p$  and is captured by an action description with two conditional effects

$$a(\text{Assign}(p, \neg p), e_1) = \langle \top, \{\langle \neg p, \{p\}, \emptyset \rangle, \langle p, \emptyset, \{p\} \rangle\}.$$

Observe that, contrarily to  $\text{Assign}(p, \top)$  and  $\text{Assign}(p, \perp)$ ,  $\text{Assign}(p, \neg p)$  does not modify  $\mathcal{J}S p$ : if an agent does not know whether  $p$  before the public toggling of  $p$  then she also does not know whether  $p$  afterwards.

Second, let us look at some kinds of public announcements. The truthful public announcement of a propositional variable  $p$  is captured by the action description with precondition  $p$ , unconditional positive effect  $\mathcal{J}S p$ , and without negative effects:  $a(\text{PubAnn}(p), e_2) = \langle p, \{\langle \top, \{\mathcal{J}S p\}, \emptyset\} \rangle$ . Symmetrically, the public announcement of  $\neg p$  is captured by  $a(\text{PubAnn}(\neg p), e_2) = \langle \neg p, \{\langle \top, \{\mathcal{J}S p\}, \emptyset\} \rangle$ . More generally, we can capture the public announcement of conjunctions of atoms and negations of atoms:

$$a(\text{PubAnn}((\bigwedge_{\alpha \in A^+} \alpha) \wedge (\bigwedge_{\alpha \in A^-} \neg \alpha)), e_2) = \langle (\bigwedge_{\alpha \in A^+} \alpha) \wedge (\bigwedge_{\alpha \in A^-} \neg \alpha), \{\langle \top, \{\mathcal{J}S \alpha : \alpha \in A^+ \cup A^-\}, \emptyset\} \rangle.$$

The precondition guarantees that for example the public announcement of  $\mathcal{J}S p \wedge \neg S_i p$  cannot be made. Here are some examples of public announcements of the above conjunctions. We also add the equivalent formulation in the standard epistemic language.

$$\begin{aligned} a(\text{PubAnn}(\neg K_i p \wedge \neg K_i \neg p), e_2) &= a(\text{PubAnn}(\neg S_i p), e_2) = \langle \neg S_i p, \{\langle \top, \{\mathcal{J}S S_i p\}, \emptyset\} \rangle, \\ a(\text{PubAnn}(K_i p), e_2) &= a(\text{PubAnn}(p \wedge S_i p), e_2) = \langle p \wedge S_i p, \{\langle \top, \{\mathcal{J}S p\}, \emptyset\} \rangle, \\ a(\text{PubAnn}(p \wedge \neg K_i p), e_2) &= a(\text{PubAnn}(p \wedge \neg S_i p), e_2) = \langle p \wedge \neg S_i p, \{\langle \top, \{\mathcal{J}S p\}, \emptyset\} \rangle. \end{aligned}$$

In the announcement of  $p \wedge S_i p$  and of the Moore sentence  $p \wedge \neg S_i p$  we have

dropped  $JS_i p$  from the add-list because it already contains  $JS p$  (cf. the EL-O axiom  $Vis_5$ ).

Third, the semi-private announcement to  $i$  that  $p$  is true is captured by the action description

$$1005 \quad a(\text{SemiPrivAnn}(p, i), e^+) = \langle p, \{\langle \top, \{S_i p, JS_i p\}, \emptyset \rangle\} \rangle.$$

So the effect is that  $i$  sees whether  $p$  and that all agents jointly see that; in other words, that it becomes common knowledge that  $i$  sees whether  $p$ .

Fourth, the event model  $\text{PubForget}(p)$  of publicly forgetting  $p$  corresponds to the action description

$$1010 \quad a(\text{PubForget}(p), n) = \langle \top, \{\langle \top, \{JS_i p : i \in \mathbb{A}\}, \{S_i p : i \in \mathbb{A}\} \rangle\} \rangle.$$

That is, all  $S_i p$  become false and this becomes common knowledge.

**Proposition 15.** *For each of the above event models  $\text{Evt}$ :*

1.  $M^{\text{EL-O}}, s \models \text{pre}^{\text{Evt}}(e)$  if and only if  $R_{a(\text{Evt}, e)}^{\text{EL-O}}$  is defined at  $s$ ;
2. If  $sR_{a(\text{Evt}, e)}^{\text{EL-O}} s'$  then  $(M^{\text{EL-O}}, s')$  and  $(M^{\text{EL-O}} \otimes \text{Evt}, (s, e))$  are bisimilar.

1015 *Proof.* The first item is the case because, as already mentioned, the precondition of the action descriptions  $a(\text{Evt}, e)$  are all equivalent to  $\text{pre}^{\text{Evt}}(e)$ .

We sketch the proof of the second item for the public announcement of  $p$ . Let  $M^{\text{EL-O}} \otimes \text{PubAnn}(p) = \langle W, \sim_{\mathbb{A}}, \{\sim_i\}_{i \in \mathbb{A}}, V \rangle$ , and consider  $s \in W^{\text{EL-O}}$  such that  $sR_{a(\text{PubAnn}(p))}^{\text{EL-O}} s'$ , i.e., such that  $s' = s \cup \{JS p\}^{\Rightarrow}$ . We show that the pointed models  $(M^{\text{EL-O}}, s')$  and  $(M^{\text{EL-O}} \otimes \text{PubAnn}(p), (s, e_2))$  are bisimilar, where  $e_2$  is the single event of the event model  $\text{PubAnn}(p)$ . To that end we define the relation  $Z$  between  $W^{\text{EL-O}}$  and  $W$  as:  $s'Z(s, e_2)$  iff  $s' = s \cup \{JS p\}^{\Rightarrow}$ . It can be checked that the three conditions for  $Z$  being a bisimulation are satisfied: atomic, forth, and back.

The other proofs are similar; we only sketch the case of public forgetting of  $p$ ,  $\text{PubForget}(p)$ . Suppose  $sR_{a(\text{PubForget}(p))}^{\text{EL-O}} t$ , i.e.,

$$t = (s \setminus \{S_i p : i \in \mathbb{A}\}^{\Leftarrow}) \cup \{JS_i p : i \in \mathbb{A}\}^{\Rightarrow}.$$

For every  $s$ , this state  $t$  is unique: we denote it by  $R_{a(\text{PubForget}(p))}^{\text{EL-O}}(s)$ . We show that  $(M^{\text{EL-O}}, t)$  and  $(M^{\text{EL-O}} \otimes \text{PubForget}(p), (s, n))$  are bisimilar via the following

relation  $Z$  between  $W$  and  $W^{\text{EL-O}}$ :

$$\begin{aligned} Z = & \{ \langle R_{\text{a(PubForget}(p))}^{\text{EL-O}}(s), (s, n) \rangle \} \cup \\ & \{ \langle R_{\text{a(PubForget}(p))}^{\text{EL-O}}(s) \cup \{p\}, (s, e^+) \rangle \} \cup \\ & \{ \langle R_{\text{a(PubForget}(p))}^{\text{EL-O}}(s) \setminus \{p\}, (s, e^-) \rangle \} \end{aligned}$$

1025 It can be checked that  $Z$  is indeed a bisimulation.  $\square$

Something that cannot be modelled in our framework is the public announcement that  $\neg K_i p$ , or, expressed in the EL-O fragment, that  $\neg(p \wedge S_i p)$ . The reason is that it is not clear how the disjunctive effect of that announcement could be described in terms of an add-list.

#### 1030 8.1.4. From EL-O action descriptions to DEL event models

We now consider the converse direction: given an EL-O action description  $a$ , can we find a DEL event model  $(\text{Evt}, E)$  such that for any state  $s \in W^{\text{EL-O}}$ , the product  $(M^{\text{EL-O}} \otimes \text{Evt}, (s, e))$  of  $(M^{\text{EL-O}}, s)$  and  $(\text{Evt}, E)$  is defined iff  $s \models \text{pre}(a)$ , and such that in that case,  $(M^{\text{EL-O}} \otimes \text{Evt}, (s, e))$  and  $(M^{\text{EL-O}}, s')$  are 1035 bisimilar, where  $s'$  is the state such that  $s R_a^{\text{EL-O}} s'$ ?

Let us here restrict our attention to a few very basic EL-O action descriptions in which there is a single conditional effect either adding or deleting one atom. The idea behind the construction of an equivalent event model is the following: when adding or deleting an atom from a world, only agents 1040 who see this atom know that it has been modified.

The easiest way to translate this into DEL event models is through the use of Bolander's *edge-conditioned event models* [62, 63]. Such models incorporate conditions on accessibility relations, so that a model is a tuple  $\text{Evt} = \langle W^{\text{Evt}}, R_{\mathbb{A}}^{\text{Evt}}, \{R_i^{\text{Evt}}\}_{i \in \mathbb{A}}, \text{pre}^{\text{Evt}}, \text{post}^{\text{Evt}} \rangle$  where  $R_{\mathbb{A}}^{\text{Evt}} : W^{\text{Evt}} \times W^{\text{Evt}} \rightarrow \mathcal{L}_{\text{EL}}$ , 1045  $R_i^{\text{Evt}} : W^{\text{Evt}} \times W^{\text{Evt}} \rightarrow \mathcal{L}_{\text{EL}}$  for every  $i \in \mathbb{A}$ , and the rest is defined as previously. The product of such an edge-conditioned event model with a model  $M = \langle W, \sim_{\mathbb{A}}, \{\sim_i\}_{i \in \mathbb{A}}, V \rangle$  is  $M \otimes \text{Evt} = \langle W', \sim'_{\mathbb{A}}, \{\sim'_i\}_{i \in \mathbb{A}}, V' \rangle$  where

- $(s, e) \sim'_{\mathbb{A}} (t, f)$  iff  $s \sim_{\mathbb{A}} t$  and  $(M, s) \models R_{\mathbb{A}}^{\text{Evt}}(e, f)$  and  $(M, t) \models R_{\mathbb{A}}^{\text{Evt}}(e, f)$ ;

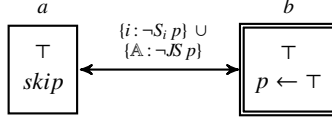


Figure 8: An event model equivalent to  $\langle \top, \{\langle \top, \{p\}, \emptyset \rangle\} \rangle$ .

- $(s, e) \sim'_i (t, f)$  iff  $s \sim_i t$  and  $(M, s) \models R_i^{\text{Evt}}(e, f)$  and  $(M, t) \models R_i^{\text{Evt}}(e, f)$ ,  
 1050 for every  $i \in \mathbb{A}$ ;

and the rest is defined as previously. As mentioned in [63], edge-conditioned event models can be simulated by standard event models, though the former are more succinct.

We now give a few examples of DEL event model translations of such  
 1055 simple action descriptions. We will only give the bisimulation relations without the proofs that they are indeed bisimulations, as those proofs are tedious but uncomplicated.

An equivalent event model to the assignment of  $p$  to true,  $\langle \top, \{\langle \top, \{p\}, \emptyset \rangle\} \rangle$ ,  
 is given in Figure 8. The bisimulation relation  $Z$  is as follows: for every state  
 1060  $s \in W^{\text{EL-O}}$ ,  $sZ(s, a)$  and  $(s \cup \{p\})Z(s, b)$ . If we wish to assign  $p$  to false rather than true (action  $\langle \top, \{\langle \top, \emptyset, \{p\} \rangle\} \rangle$ ), we simply replace the assignment  $p \leftarrow \top$  in the actual event by  $p \leftarrow \perp$ , and change the bisimulation relation so that  $(s \setminus \{p\})Z(s, b)$  for any  $s \in W^{\text{EL-O}}$ . This action can be interpreted as the value of  $p$  (the property of an object) changing, while only agents who are looking  
 1065 at this object can see the change.

For the action  $\langle \top, \{\langle \top, \{S_i \alpha\}, \emptyset \rangle\} \rangle$  making  $S_i \alpha$  true, we need in the general case two actual events, one for when  $\alpha$  is true and one for when it is false. An event model equivalent to this action description is given in Figure 9. The corresponding bisimulation  $Z$  is the following: for every state  $s \in W^{\text{EL-O}}$ ,

- $sZ(s, a)$ ;
- $(s \cup \{S_i \alpha\})Z(s, b)$  if  $s \models \alpha$ ;
- $(s \cup \{S_i \alpha\})Z(s, c)$  if  $s \models \neg \alpha$ .

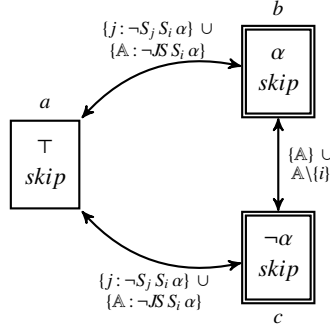


Figure 9: An event model equivalent to  $\langle \top, \{\langle \top, \{S_i \alpha\}, \emptyset \rangle\} \rangle$ .

A more complex example, given in Figure 10, is that of an event model equivalent to an action making the atom  $S_i p$  false. This can be interpreted as agent  $i$  looking away from  $p$ . Agents who saw agent  $i$  looking at  $p$  will see her look away, and the other agents will not be aware of this change. While this is a fairly natural action to consider, it is difficult to generalize to assigning  $S_i \alpha$  to false for any given  $\alpha$ , as only propositional variables can receive assignments in DEL event models. The bisimulation  $Z$  in the case of Figure 10 is as follows: for every state  $s \in W^{\text{EL-O}}$ ,

- $sZ(s, a)$ ;
- $(s \setminus \{JS p, S_i p\})Z(s, b)$ ;
- $(s \setminus \{JS p, S_i p, p\})Z(s, c)$  if  $s \models p$ ;
- $((s \cup \{p\}) \setminus \{JS p, S_i p\})Z(s, c)$  if  $s \models \neg p$ ;
- $(s \setminus \{p\})Z(s, d)$  if  $s \models p$ ;
- $(s \cup \{p\})Z(s, d)$  if  $s \models \neg p$ .

Let us give one more example, this time involving a precondition: private announcements. In an S5 setting, an announcement can only be private if no one is watching (as we cannot represent agents “believing that nothing has happened”). That is, we can only privately announce whether



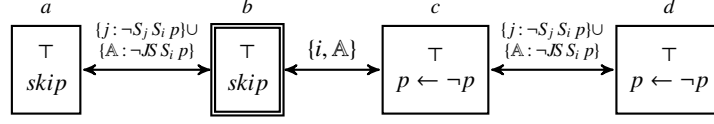


Figure 10: An event model equivalent to  $\langle \top, \{\langle \top, \emptyset, \{S_i p\}\rangle\} \rangle$ .

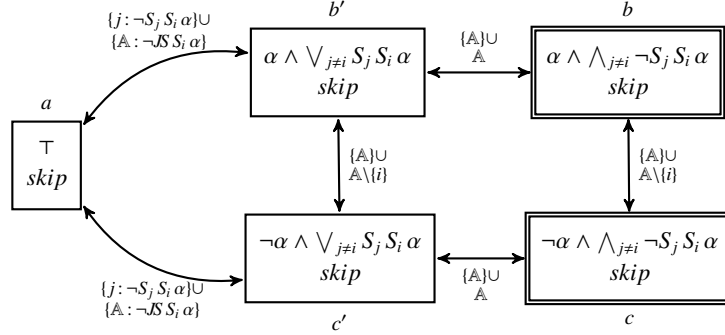


Figure 11: An event model equivalent to the private announcement whether  $\alpha$  to  $i$ .

$\alpha$  is true to an agent  $i$  if for all  $j \neq i$ ,  $\neg S_j S_i \alpha$  holds. The corresponding action description has condition  $\bigwedge_{j \neq i} \neg S_j S_i \alpha$  and a single conditional effect  $\langle \top, \{S_i \alpha\}, \emptyset \rangle$ . The corresponding event model is given in Figure 11. The bisimulation  $Z$  is: for every state  $s \in W^{\text{EL-O}}$ ,  $sZ(s, a)$ , and  $(s \cup S_i \alpha)Z(s, e)$  if  $s \models \text{pre}^{\text{Evt}}(e)$  for  $e \in \{b, b', c, c'\}$ .

We stop here without delving into the territory of deleting longer atoms or combining several effects. It is, we hope, fairly obvious that given an action description, the construction of an equivalent DEL event model is neither straightforward nor systematic. This, we believe, is an argument in favour of our framework, in which many actions can be defined very succinctly and in a natural manner.

### 8.1.5. Discussion: DEL vs. EL-O

The comparison with DEL leads us to a fundamental question: how should an action be described? All the approaches in the literature advo-

1105 cating DEL for epistemic planning presuppose that event models provide an appropriate tool for the description of actions. This, however, seems questionable to us. For example, how should we describe the action of pushing a light button? It is useful for our discussion to recall the fundamental distinction in philosophy of action between *action types* and *action tokens* [64].  
1110 Pushing a light button is an example of the former, while an example of the latter is the pushing of a particular button at a particular point in time by a particular agent. The elements of the set of action descriptions *Act* are clearly action types. In contrast, the elements of the execution of a solution to a planning task are typically action tokens.<sup>9</sup>

1115 Let us suppose that the action type of pushing a light button can be described by means of preconditions and effects. Reasonably, the precondition is that the agent is close to the switch<sup>10</sup> and the effect is that the light is on. However, a DEL event model has to contain more information than that: it also has to say how the button-pushing is perceived (or not) by each of  
1120 the agents. Now remember that we have to describe an action type: we have to account for all possible circumstances of button pushing. For each agent there are at least two cases: the one where the agent is present and observes the light bulb and the one where she is absent. Altogether, this requires an exponential numbers of points in an event model. Worse, we also  
1125 have to take care of the evolution of the agents' higher-order knowledge: for each couple of observing agents *i* and *j* we have to distinguish whether *i* sees that *j* observes the bulb, and so on, the limit case being when there is joint observation of the bulb. So exhaustive action descriptions by means of DEL event models systematically have to have an infinite number of points  
1130 in order to take all the possible effects into account! This can actually be

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<sup>9</sup>In DEL-based planning this distinction is clarified for the first time in [65].

<sup>10</sup>There might be more, such as that the wires leading to the light bulb are working, that the light bulb is not broken etc.: we here neglect the qualification problem [66] which is that the precondition is typically an infinite conjunction.

seen as an epistemic version of the ramification problem [66].

Contrasting with DEL event models, our EL-O-based action descriptions do not require to fully describe all possible ramifications: the direct effect of pushing the light button is simply that the light is on, and the indirect epis-  
1135 temic effects follow from the description of the state, namely who observes the light bulb and who doesn't. This is a fundamentally different answer to the question of how actions should be described. It makes the modeller's task much simpler and more natural. (A similar argument was first put forward in [67].)

1140 To sum it up, we believe that our action descriptions are a more natural way of modelling action types... at least when this can be done. For example, the public announcement of the disjunction  $p \vee q$  (the event model  $\text{PubAnn}(p \vee q)$ ) or the assignment of  $q$  to  $p$  (the event model  $\text{Assign}(p, q)$ ) cannot be captured. This is because all propositional variables are independent  
1145 in our approach, as witnessed by the fact that the knowledge operator distributes over disjunctions of literals: for different  $p$  and  $q$ ,  $K_i(p \vee q)$  is equivalent to  $K_i p \vee K_i q$ , and  $K_i(p \leftrightarrow q)$  is equivalent to  $(K_i p \wedge K_i q) \vee (K_i \neg p \wedge K_i \neg q)$ . This is clearly a limitation of our approach. In particular, we cannot capture the muddy children problem, in which the children learn that one of  
1150 them has a muddy forehead without knowing who. Note that while DEL is more general as far as preconditions are concerned (any complex formula can be announced), its postconditions are restricted: they assign propositional variables and therefore cannot model actions with disjunctive ontic effects such as  $p \vee q$ .

1155 For the same reason we cannot capture the poisonous liquid example from [45]. In this problem there is a bottle of liquid, a healthy lawn, and two actions: pour and senseLawn. The first is the action of pouring some liquid on the lawn that we have already described in Example 8 (Section 5.3). The second senses whether or not the lawn is dead: we have  $\text{pre}(\text{senseLawn}) = \top$   
1160 and  $\text{eff}(\text{senseLawn}) = \{\langle \top, \{S_i \text{Dead}\}, \emptyset \rangle\}$ . Then for the set of actions  $\text{Act} = \{\text{pour}, \text{senseLawn}\}$ , there is no solution of the planning task

$\langle OA, Act, \{\neg Dead, S_i Dead\}, S_i \text{ Poisonous} \rangle$ .

In particular, pouring some liquid on the lawn and checking whether the lawn is dead does not inform the agent whether Poisonous is the case. This  
 1165 is because after pouring the liquid the agent should know that the state of the lawn is tied to the toxicity of the liquid. Such knowledge cannot be captured in our framework.

An interesting avenue for future research are implicitly coordinated plans as introduced by Bolander et al. [59]. For example, suppose agent 2  
 1170 would like to borrow the apartment of his friend 1 while 1 is away on vacation. Agent 1 has an action `putMat` of putting the key under the door mat that is described by  $pre(\text{putMat}) = \top$  and  $eff(\text{putMat}) = \{\langle \top, \{\text{Mat}\}, \emptyset \rangle\}$ ; agent 2 has an action `tryTake` of trying to take the key with  $pre(\text{tryTake}) = \top$  and  $eff(\text{tryTake}) = \{\langle \top, \{S_2 \text{ Mat}\}, \emptyset \rangle, \langle \text{Mat}, \{\text{hasKey}_2\}, \emptyset \rangle\}$ . The initial state is  $s_0 = \emptyset$ .  
 1175 Then 1 putting the key under the mat and 2 taking the key solves the planning task

$\langle OA, Act, s_0, \text{hasKey}_2 \rangle$ .

However, when 2 arrives at the apartment he will not know that the key is under the mat, unless 1 has told him. A better plan, baptised ‘implicitly  
 1180 coordinated’ in [59], involves 1’s action `infoMat` which can be described in EL-O terms by  $pre(\text{infoMat}) = \top$  and  $eff(\text{tryTake}) = \{\langle \top, \{S_2 \text{ Mat}\}, \emptyset \rangle\}$ . The formal characterisation of such plans requires a language with action operators in which one can describe solutions in the language as sequences of actions, prefixing each action of agent  $i$  by  $K_i$ : it is not enough that the action leads  
 1185 to the goal, the acting agent must also know that. This fails to be the case for 2’s action `tryTake`: as 2 does not know whether the key is under the mat, he does not know whether the conditional effect  $\langle \text{Mat}, \{\text{hasKey}_2\}, \emptyset \rangle$  will be triggered. The extension of EL-O with dynamic modal operators, DEL-PAO, has such operators and should therefore enable reasoning about implicitly  
 1190 coordinated plans. The details however remain to be worked out.

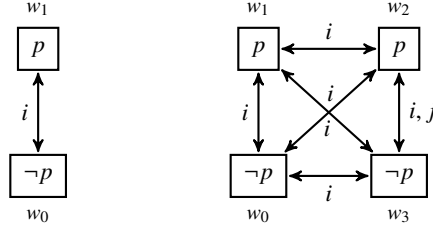


Figure 12: Models  $M$  (left) and  $M'$  (right); reflexive arrows omitted.

## 8.2. Muise et al.

Muise et al. [25] follow a strategy that is very similar to ours where they restrict formulas to boolean combinations of epistemic literals: sequences of belief operators and negations that are followed by a propositional variable; in other terms, formulas of  $\mathcal{L}_{EL}$  without conjunctions and disjunctions. So their grammar of epistemic literals is:

$$\lambda ::= p \mid \neg\lambda \mid K_i\lambda$$

where  $p$  ranges over  $\mathbb{P}$  and  $i$  over  $\mathbb{A}$ . Muise et al.’s proper epistemic knowledge bases (PEKBs) are conjunctions of such epistemic literals [25]. The authors have also started to consider the integration of ‘knowing-whether’ operators in [68], but have not integrated this into the planning formalism of [25].

The major advantage of our EL-O-based approach over the epistemic literals-based approach is that boolean combinations of epistemic literals cannot express  $K_i(K_j p \vee K_j \neg p) \wedge \neg K_i p \wedge \neg K_i \neg p$  is true, that is,  $i$  knows that  $j$  knows whether  $p$  while  $i$  does not know about  $p$  herself. As we had argued in Section 2, such situations are important in interaction and more specifically in communication. They can be expressed in our framework by  $\mathcal{L}_{\text{bool}}(\text{OA})$  formulas of the form  $S_i S_j p \wedge \neg S_i p$ . To prove this formally it suffices to show that  $K_i(K_j p \vee K_j \neg p)$  cannot be expressed by boolean combinations of epistemic literals.

**Proposition 16.** *Let  $i$  and  $j$  be different. Then there is no boolean combination of epistemic literals  $\varphi$  that is equivalent to  $K_i(K_j p \vee K_j \neg p)$  in S5 Kripke models.*

*Proof.* Consider the two S5 Kripke models  $M = \langle W, \sim_{\mathbb{A}}, \{\sim_i\}_{i \in \mathbb{A}}, V \rangle$  and  $M' = \langle W', \sim'_{\mathbb{A}}, \{\sim'_i\}_{i \in \mathbb{A}}, V' \rangle$  with

$$\begin{aligned} W &= \{w_0, w_1\}, & W' &= \{w_0, w_1, w_2, w_3\}, \\ \sim_{\mathbb{A}} &= W \times W = \sim_i, & \sim'_{\mathbb{A}} &= W' \times W' = \sim'_i, \\ \sim_j &= \delta_W, & \sim'_j &= \delta_{W'} \cup \{\langle w_2, w_3 \rangle, \langle w_3, w_2 \rangle\}, \\ V(w_0) &= \emptyset \text{ and } V(w_1) = \{p\}; & V'(w_0) &= V'(w_3) = \emptyset \text{ and } V'(w_1) = V'(w_2) = \{p\}; \end{aligned}$$

where  $\delta_W = \{\langle w_0, w_0 \rangle, \langle w_1, w_1 \rangle\}$  is the identity relation of  $W$  and  $\delta_{W'}$  is the identity relation of  $W'$ . They are depicted in Figure 12. Clearly,  $M, w_0 \models K_i(K_j p \vee K_j \neg p)$  and  $M', w_0 \not\models K_i(K_j p \vee K_j \neg p)$ . We prove that no boolean combinations of epistemic literals can tell the pointed models  $(M, w_0)$  and  $(M', w_0)$  apart. From that it immediately follows that there can be no such boolean combination that is equivalent to  $K_i(K_j p \vee K_j \neg p)$ .

To establish that  $(M, w_0)$  and  $(M', w_0)$  satisfy the same formulas as far as the language of boolean combinations of epistemic literals is concerned it suffices to prove that  $M, w_0 \models \lambda$  iff  $M', w_0 \models \lambda$  for every epistemic literal  $\lambda$ . We use induction on the length of  $\lambda$ . We have to check the following exhaustive list of cases; each of them is straightforward.

**Case 1.**  $\lambda = q$  for some  $q \in \mathbb{P}$ . Obvious. ( $M, w_0 \models \lambda$  and  $M', w_0 \models \lambda$ , regardless whether  $q$  equals  $p$  or not.)

**Case 2.**  $\lambda = \neg\mu$ , for some epistemic literal  $\mu$ . Straightforward application of the induction hypothesis.

**Case 3.**  $\lambda = K_i\mu$ . We have the following subcases.

**Case 3.1.**  $\lambda = K_i q$  for some  $q \in \mathbb{P}$ . Obvious. ( $M, w_0 \models \lambda$  and  $M', w_0 \models \lambda$ .)

**Case 3.2.**  $\lambda = K_i \neg\mu$ , for some epistemic literal  $\mu$ . We have to dive deeper into the subcases here.

**Case 3.2.1.**  $\lambda = K_i \neg q$ . Obvious. ( $M, w_0 \models \lambda$  and  $M', w_0 \models \lambda$ .)

**Case 3.2.2.**  $\lambda = K_i \neg \mu$ . We use that  $\lambda \leftrightarrow K_i \mu$  is valid and apply the induction hypothesis. 1230

**Case 3.2.3.**  $\lambda = K_i \neg K_i \mu$ . We use that  $K_i \neg K_i \mu$  and  $\neg K_i \mu$  are equivalent in S5 and apply the induction hypothesis.

**Case 3.2.4.**  $\lambda = K_i \neg K_j \mu$ . We do yet another case analysis.

**Case 3.2.4.1.**  $\lambda = K_i \neg K_j q$ . Obvious. ( $M, w_0 \not\models \lambda$  and  $M', w_0 \not\models \lambda$ .)

**Case 3.2.4.2.**  $\lambda = K_i \neg K_j \neg \mu$ . This is the last case analysis we have to do: 1235

**Case 3.2.4.2.1.**  $\lambda = K_i \neg K_j \neg q$ . Obvious. ( $M, w_0 \not\models \lambda$  and  $M', w_0 \not\models \lambda$ .)

**Case 3.2.4.2.2.**  $\lambda = K_i \neg K_j \neg \neg \mu$ . We use that  $\lambda \leftrightarrow K_i \neg K_j \mu$  is valid and apply the induction hypothesis.

**Case 3.2.4.2.3.**  $\lambda = K_i \neg K_j \neg K_i \mu$ . We have  $M, w_0 \models \lambda \leftrightarrow K_i \mu$  and  $M', w_0 \models \lambda \leftrightarrow K_i \mu$ , 1240 allowing us to apply the induction hypothesis.

**Case 3.2.4.2.4.**  $\lambda = K_i \neg K_j \neg K_j \mu$ . We use that  $K_i \neg K_j \neg K_j \mu$  and  $K_i K_j \mu$  are equivalent in S5 and apply the induction hypothesis.

**Case 3.2.4.3.**  $\lambda = K_i \neg K_j K_i \mu$ . We have  $M, w_0 \models \lambda \leftrightarrow K_i \mu$  and  $M', w_0 \models \lambda \leftrightarrow K_i \mu$ , allowing us to apply the induction hypothesis.

**Case 3.2.4.4.**  $\lambda = K_i \neg K_j K_j \mu$ . We use that  $K_i \neg K_j K_j \mu$  and  $K_i \neg K_j \mu$  are equivalent in S5 and apply the induction hypothesis. 1245

**Case 3.3.**  $\lambda = K_i K_i \mu$ . We use that  $K_i K_i \mu$  and  $K_i \mu$  are equivalent in S5 and apply the induction hypothesis.

**Case 3.4.**  $\lambda = K_i K_j \mu$ . We have  $M, w_0 \models \lambda \leftrightarrow K_i \mu$  and  $M', w_0 \models \lambda \leftrightarrow K_i \mu$ , 1250 allowing us to apply the induction hypothesis.

**Case 4.**  $\lambda = K_j \mu$ . We have  $M, w_0 \models \lambda \leftrightarrow \mu$  and  $M', w_0 \models \lambda \leftrightarrow \mu$ , allowing us to apply the induction hypothesis. □

### 8.3. Kominis and Geffner

Kominis and Geffner's approach [53, 69] distinguishes three kinds of actions: physical actions modifying the world, public updates (that are nothing but DEL-like public announcements), and sensing actions by means of which an agent learns whether a formula is true or not. 1255

On the one hand, their approach is more general than ours because agents can sense arbitrary formulas. This allows them to model the muddy children problem, which we cannot express as a simple epistemic planning task (see Section 8.1.5). However, if we restrict the formulas describing the set of initially possible states, the formulas that are publicly announced, and the formulas that are sensed to observability atoms (more precisely: to the equivalent  $\mathcal{L}_{EL}$  formulas) then everything that can be modelled in Kominis and Geffner’s approach can also be in ours.

On the other hand, Kominis and Geffner’s approach imposes three severe restrictions:

- all actions can be split up in one of the three categories;
- the set of initially possible states is common knowledge among all agents;
- all physical actions and all public updates are public;
- all sensing actions are semi-private.

The first hypothesis is clearly too strong for many natural everyday situations, as we have already said in Section 4.2 (Remark 5). The second hypothesis is also very strong: it forbids the modelling of private actions modifying the world as required in several of our examples of Section 6. The third hypothesis means that when an agent  $i$  senses the truth value of  $\varphi$  then all other agents see this: they learn that  $i$  knows whether  $\varphi$  is true or not but ignore whether what  $i$  has learned is  $\varphi$  or  $\neg\varphi$ . This means that there can be no private communication; in particular, one cannot model what we take to be a paradigmatic epistemic planning task, viz. the gossip problem.

#### 8.4. *Petrick and Bacchus*

There is also an older approach due to Petrick and Bacchus about planning under incomplete information for a single agent [70, 45] which bears some similarities to ours. Their language is first-order but does not have



epistemic operators. Instead, there are several kinds of knowledge bases by means of which the epistemic status of pieces of information is represented. In particular, there is a ‘knowing-that’ database whose elements are literals and a ‘knowing-whether’ database whose elements are atoms. The point of view is subjective, while ours is objective, i.e., the agent’s knowledge is represented, but not what is true in the world. Therefore the identity of  $K_i p$  and  $p \wedge K_i p$  cannot be expressed in their language. Just as the original proposal in [70], our approach does not account for postdiction (deducing the past from the present), as illustrated by Example 8. This is because in our approach the knowledge operator distributes over disjunctions of literals. This limitation of our approach is also the reason why we cannot account for the muddy children problem: the latter requires actions where the children learn a disjunction, namely that one of them is dirty. A solution to the muddy children problem where public announcements were integrated into DEL-PAO was presented in [7]; however, in that paper only the semantics was designed, without an axiomatisation or complexity result. The study of such mathematical properties will be subject of future work.

## 9. Conclusion

We have made a first step towards realistic multi-agent epistemic planning. Our approach is based on observability atoms, which, we argue, are more appropriate than epistemic literals as previously proposed in the literature. We have also argued that observability atoms provide a simple, finite way to represent situations of maximal ignorance, namely by states that equal the empty set, while such situations require infinite Kripke models in standard epistemic logic (cf. Remark 5 in Section 4.2 and Remark 6 in Section 6.1).

We have shown that goals and conditions of our action descriptions can be reduced to boolean formulas, which has allowed us to encode epistemic planning into classical planning. As our reduction is polynomial, we main-

tain the PSPACE complexity of classical planning. Building on this we have  
1315 encoded epistemic action descriptions into PDDL and performed some ex-  
periments with a PDDL planner in order to find plans efficiently. Moreover,  
our use of a logic of knowledge together with a state of the art automated  
1320 planner—which we assume to be correct in the case of classical planning  
with conditional effects—provides a method for producing plans that are  
guaranteed to be correct. We have illustrated our approach with several  
examples, most importantly the generalised gossip problem, which, we ar-  
gue, is an interesting candidate for a paradigmatic epistemic planning task.

Alternatively we could encode epistemic planning tasks into Answer-  
1325 Set Programming (ASP). An encoding of several variants of the generalised  
gossip problem together with an experimental evaluation that fares better  
than our encoding can be found in [71].

An extension of our approach to parallel planning was recently proposed  
in [49], based on the account of non-epistemic parallel planning of [72].

1330 The decidability of EL-O-based epistemic planning contrasts with the  
undecidability of DEL-based epistemic planning, which is the case even  
for simple fragments. For example, if actions also change the world (and  
not only the agents’ knowledge), then plan existence is undecidable when-  
ever epistemic operators are allowed in preconditions; if actions are purely  
1335 epistemic, then it is undecidable whenever two agents are involved or the  
epistemic depth exceeds 2 [16]. We have nevertheless shown that several  
simple kinds of event models can be captured by our action descriptions  
(Section 8.1). Moreover and as we have explained there, there is a funda-  
mental difference between the modelling of actions by means of DEL event  
1340 models on the one hand and by means of action descriptions on the other:  
in an EL-O-based modelling, the agents’ observational abilities are part of  
the state, while they are part of the action description in a DEL-based mod-  
elling. We have argued that our approach is more natural and that DEL-  
based approaches face an epistemic version of the ramification problem.

1345 As we have said in Remark 2, it is not obvious how EL-O’s operator of

joint observability of the grand coalition  $\mathbb{A}$  should be generalised to joint observability of arbitrary groups. We would like to address this issue in future work.

In this paper we have assumed a centralised approach in which a single  
1350 planner decides the actions of all agents. Other approaches have studied the classical gossip problem from a completely different perspective, assuming that all agents are autonomous [5, 73, 74]. An interesting research avenue is to study generalised gossiping as a distributed planning task.

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1355 We would like to thank the reviewers who did an excellent job in spotting problematic points and suggesting improvements. Thanks are also due to Julien Vianey who ran the experiments on which we report in Section 7.

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### References

- [1] S. Khuller, Y. A. Kim, Y. J. Wan, On generalized gossiping and broadcasting (extended abstract), in: G. Di Battista, U. Zwick (Eds.), Algorithms - ESA 2003, 11th Annual European Symposium, Budapest, Hungary, September 16-19, 2003, Proceedings, Vol. 2832 of Lecture Notes in Computer Science, Springer, 2003, pp. 373–384.  
1365
- [2] D. Liben-Nowell, Gossip is synteny: Incomplete gossip and the syntenic distance between genomes, *J. Algorithms* 43 (2) (2002) 264–283.
- [3] S. M. Hedetniemi, S. T. Hedetniemi, A. L. Liestman, A survey of gossiping and broadcasting in communication networks, *Networks* 18 (4) (1988) 319–349. doi:10.1002/net.3230180406.  
1370

- [4] M. C. Cooper, A. Herzig, F. Maffre, F. Maris, P. Régnier, The epistemic gossip problem, *Discrete Mathematics* 342 (3) (2019) 654–663.  
1375 URL <https://doi.org/10.1016/j.disc.2018.10.041>
- [5] M. Attamah, H. van Ditmarsch, D. Grossi, W. van der Hoek, Knowledge and gossip, in: *Proceedings of the 21st European Conference on Artificial Intelligence (ECAI 2014)*, 2014, pp. 21–26.
- [6] A. Herzig, E. Lorini, F. Maffre, A poor man’s epistemic logic based on propositional assignment and higher-order observation, in: W. van der Hoek, W. H. Holliday, W.-f. Wang (Eds.), *Proceedings of the 5th International Conference on Logic, Rationality and Interaction (LORI 2015)*, Springer Verlag, 2015, pp. 156–168.  
1380 URL <http://www.irit.fr/~Andreas.Herzig/P/Lori15.html>
- [7] T. Charrier, A. Herzig, E. Lorini, F. Maffre, F. Schwarzentruher, Building epistemic logic from observations and public announcements, in: C. Baral, J. P. Delgrande, F. Wolter (Eds.), *Principles of Knowledge Representation and Reasoning: Proceedings of the Fifteenth International Conference, KR 2016*, Cape Town, South Africa, April 25-29, 2016, AAAI Press, 2016, pp. 268–277.  
1385 URL <http://www.aaai.org/ocs/index.php/KR/KR16/paper/view/12899>
- [8] A. Herzig, E. Lorini, F. Maffre, F. Schwarzentruher, Epistemic boolean games based on a logic of visibility and control, in: S. Kambhampati (Ed.), *Proceedings of the Twenty-Fifth International Joint Conference on Artificial Intelligence, IJCAI 2016*, New York, NY, USA, 9-15 July 2016, IJCAI/AAAI Press, 2016, pp. 1116–1122.  
1390 URL <http://www.ijcai.org/Abstract/16/162>
- [9] F. Maffre, Ignorance is bliss: observability-based dynamic epistemic logics and their applications. (le bonheur est dans l’ignorance : logiques épistémiques dynamiques basées sur l’observabilité et leurs  
1400

applications), Ph.D. thesis, Paul Sabatier University, Toulouse, France (2016).

URL <https://tel.archives-ouvertes.fr/tel-01488408>

- 1405 [10] M. C. Cooper, A. Herzig, F. Maffre, F. Maris, P. Régnier, A simple account of multi-agent epistemic planning, in: Proceedings of the 22nd European Conference on Artificial Intelligence (ECAI 2016), 2016, pp. 193–201.
- [11] J. Hintikka, Knowledge and Belief: An Introduction to the Logic of the  
1410 Two Notions, Cornell University Press, 1962.
- [12] R. Fagin, J. Y. Halpern, Y. Moses, M. Y. Vardi, Reasoning about Knowledge, MIT Press, 1995.
- [13] J. Y. Halpern, Y. Moses, A guide to completeness and complexity for modal logics of knowledge and belief, *Artificial Intelligence* 54 (3)  
1415 (1992) 319–379.
- [14] T. Bylander, The computational complexity of propositional STRIPS planning, *Artificial Intelligence* 69 (1994) 165–204. doi:10.1016/0004-3702(94)90081-7.
- [15] T. Bolander, M. B. Andersen, Epistemic planning for single and multi-  
1420 agent systems, *Journal of Applied Non-Classical Logics* 21 (1) (2011) 9–34.  
URL <http://www.tandfonline.com/doi/abs/10.3166/jancl.21.9-34>
- [16] G. Aucher, T. Bolander, Undecidability in epistemic planning, in:  
1425 F. Rossi (Ed.), Proceedings of the 23rd International Joint Conference on Artificial Intelligence (IJCAI 2013), AAAI Press, 2013, pp. 27–33.
- [17] T. Bolander, M. H. Jensen, F. Schwarzentruher, Complexity results in epistemic planning, in: Q. Yang, M. Wooldridge (Eds.), Proceedings of

- 1430 the Twenty-Fourth International Joint Conference on Artificial Intel-  
ligence, IJCAI 2015, Buenos Aires, Argentina, July 25-31, 2015, AAAI  
Press, 2015, pp. 2791–2797.  
URL <http://ijcai.org/Abstract/15/395>
- [18] S. L. Cong, S. Pinchinat, F. Schwarzenruber, Small undecidable prob-  
1435 lems in epistemic planning, in: J. Lang (Ed.), Proceedings of the  
Twenty-Seventh International Joint Conference on Artificial Intelli-  
gence, IJCAI 2018, July 13-19, 2018, Stockholm, Sweden, ijcai.org,  
2018, pp. 4780–4786.  
URL <https://doi.org/10.24963/ijcai.2018/664>
- [19] T. Bolander, T. Charrier, S. Pinchinat, F. Schwarzenruber, Del-based  
1440 epistemic planning: Decidability and complexity, *Artif. Intell.* 287  
(2020) 103304. doi:10.1016/j.artint.2020.103304.  
URL <https://doi.org/10.1016/j.artint.2020.103304>
- [20] R. Demolombe, M. del Pilar Pozos Parra, A simple and tractable exten-  
1445 sion of situation calculus to epistemic logic, in: Z. W. Ras, S. Ohsuga  
(Eds.), Foundations of Intelligent Systems, 12th International Sympo-  
sium, ISMIS 2000, Charlotte, NC, USA, October 11-14, 2000, Proceed-  
ings, Vol. 1932 of Lecture Notes in Computer Science, Springer, 2000,  
pp. 515–524.  
URL [https://doi.org/10.1007/3-540-39963-1\\_54](https://doi.org/10.1007/3-540-39963-1_54)
- 1450 [21] T. C. Son, C. Baral, Formalizing sensing actions A transition function  
based approach, *Artif. Intell.* 125 (1-2) (2001) 19–91.  
URL [https://doi.org/10.1016/S0004-3702\(00\)00080-1](https://doi.org/10.1016/S0004-3702(00)00080-1)
- [22] M. Steedman, R. P. Petrick, Planning dialog actions, in: Proceedings of  
1455 the 8th SIGDIAL Workshop on Discourse and Dialogue (SIGdial 2007),  
2007, pp. 265–272.
- [23] G. Lakemeyer, Y. Lespérance, Efficient reasoning in multiagent epis-  
temic logics, in: L. D. Raedt, C. Bessière, D. Dubois, P. Doherty, P. Fras-

- 1460 coni, F. Heintz, P. J. F. Lucas (Eds.), ECAI 2012 - 20th European Conference on Artificial Intelligence. Including Prestigious Applications of Artificial Intelligence (PAIS-2012) System Demonstrations Track, Montpellier, France, August 27-31 , 2012, Vol. 242 of Frontiers in Artificial Intelligence and Applications, IOS Press, 2012, pp. 498–503.  
URL <https://doi.org/10.3233/978-1-61499-098-7-498>
- 1465 [24] H. Palacios, H. Geffner, Compiling uncertainty away: Solving conformant planning problems using a classical planner (sometimes), in: Proceedings, The Twenty-First National Conference on Artificial Intelligence and the Eighteenth Innovative Applications of Artificial Intelligence Conference, July 16-20, 2006, Boston, Massachusetts, USA, AAAI Press, 2006, pp. 900–905.  
1470 URL <http://www.aaai.org/Library/AAAI/2006/aaai06-142.php>
- [25] C. Muise, V. Belle, P. Felli, S. A. McIlraith, T. Miller, A. R. Pearce, L. Sonenberg, Planning over multi-agent epistemic states: A classical planning approach, in: Proceedings of the 29th AAAI Conference on Artificial Intelligence (AAAI 2015), AAAI Press, 2015, pp. 3327–3334.  
1475
- [26] J. Fan, Y. Wang, H. van Ditmarsch, Contingency and knowing whether, *Rew. Symb. Logic* 8 (1) (2015) 75–107.  
URL <https://doi.org/10.1017/S1755020314000343>
- [27] K. Su, A. Sattar, X. Luo, Model checking temporal logics of knowledge via OBDDs, *The Computer Journal* 50 (4) (2007) 403–420.  
1480
- [28] W. van der Hoek, P. Iliev, M. Wooldridge, A logic of revelation and concealment, in: W. van der Hoek, L. Padgham, V. Conitzer, M. Winikoff (Eds.), Proceedings of the 11th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2012), IFAAMAS, 2012, pp. 1115–1122.  
1485  
URL <http://dl.acm.org/citation.cfm?id=2343856>

- [29] J. van Benthem, J. van Eijck, M. Gattinger, K. Su, Symbolic model checking for dynamic epistemic logic - S5 and beyond, *J. Log. Comput.* 28 (2) (2018) 367–402.  
1490 URL <https://doi.org/10.1093/logcom/exx038>
- [30] W. Lenzen, *Recent work in epistemic logic*, North Holland Publishing Company, Amsterdam, 1978.
- [31] W. Lenzen, On the semantics and pragmatics of epistemic attitudes, in: A. Laux, H. Wansing (Eds.), *Knowledge and belief in philosophy and AI*, Akademie Verlag, Berlin, 1995, pp. 181–197.  
1495
- [32] W. H. Holliday, *Epistemic logic and epistemology*, in: *Introduction to Formal Philosophy*, Springer, 2018, pp. 351–369.
- [33] M. Cooper, A. Herzig, F. Maffre, F. Maris, E. Perrotin, P. Regnier, When ‘knowing whether’ is better than ‘knowing that’, in: *13èmes Journées d’Intelligence Artificielle Fondamentale (JIAF 2019)*, hal-02302984, 2019.  
1500
- [34] A. Herzig, E. Perrotin, *On the axiomatisation of common knowledge*, in: *Proceedings of the 13th conference on Advances in Modal Logic (AiML 2012)*, College Publications, 2020.
- [35] P. Blackburn, M. de Rijke, Y. Venema, *Modal Logic*, Cambridge Tracts in Theoretical Computer Science, Cambridge University Press, 2001.  
1505
- [36] A. Herzig, F. Maffre, *How to share knowledge by gossiping*, in: *Proceedings of the 3rd International Conference on Agreement Technologies (AT 2015)*, Springer-Verlag, 2016, pp. 249–263.
- [37] M. R. Genesereth, N. J. Nilsson, *Logical Foundations of Artificial Intelligence*, Morgan Kaufmann Publishers, 1989.  
1510
- [38] M. Baldoni, L. Giordano, A. Martelli, *A modal extension of logic programming: Modularity, beliefs and hypothetical reasoning*, *J. Log.*



- Comput. 8 (5) (1998) 597–635. doi:10.1093/logcom/8.5.597.  
1515 URL <https://doi.org/10.1093/logcom/8.5.597>
- [39] M. Baldoni, Normal multimodal logics: Automatic deduction and logic programming extension, Ph.D. thesis, Università degli Studi di Torino, Dipartimento di Informatica (1998).
- [40] E. Lorini, A. Herzig, Direct and indirect common belief, in: A. Konzelmann Ziv, H. B. Schmid (Eds.), *Institutions, Emotions, and Group Agents: Contributions to Social Ontology*, Springer, 2014, pp. 355–372.  
1520
- [41] A. Herzig, Logics of knowledge and action: critical analysis and challenges, *Auton. Agents Multi Agent Syst.* 29 (5) (2015) 719–753.  
URL <https://doi.org/10.1007/s10458-014-9267-z>
- 1525 [42] S. N. Artemov, Observable models, in: S. N. Artemov, A. Nerode (Eds.), *Logical Foundations of Computer Science - International Symposium, LFCS 2020, Deerfield Beach, FL, USA, January 4-7, 2020, Proceedings*, Vol. 11972 of *Lecture Notes in Computer Science*, Springer, 2020, pp. 12–26.  
1530 URL [https://doi.org/10.1007/978-3-030-36755-8\\_2](https://doi.org/10.1007/978-3-030-36755-8_2)
- [43] A. Herzig, E. Lorini, F. Maffre, Possible worlds semantics based on observation and communication, in: H. van Ditmarsch, G. Sandu (Eds.), *Jaakko Hintikka on Knowledge and Game-Theoretical Semantics*, Springer International Publishing, Cham, 2018, pp. 339–362.  
1535 URL [https://doi.org/10.1007/978-3-319-62864-6\\_14](https://doi.org/10.1007/978-3-319-62864-6_14)
- [44] H. Geffner, B. Bonet, *A Concise Introduction to Models and Methods for Automated Planning*, *Synthesis Lectures on Artificial Intelligence and Machine Learning*, Morgan & Claypool Publishers, 2013.
- [45] R. P. A. Petrick, F. Bacchus, Extending the knowledge-based approach  
1540 to planning with incomplete information and sensing, in: D. Dubois,

C. A. Welty, M. Williams (Eds.), *Principles of Knowledge Representation and Reasoning: Proceedings of the Ninth International Conference (KR2004)*, Whistler, Canada, June 2-5, 2004, AAAI Press, 2004, pp. 613–622.

1545 URL <http://www.aaai.org/Library/KR/2004/kr04-064.php>

[46] B. Baker, R. Shostak, Gossips and telephones, *Discrete Mathematics* 2 (3) (1972) 191–193. doi:10.1016/0012-365X(72)90001-5.

[47] R. Tijdeman, On a telephone problem, *Nieuw Archief voor Wiskunde* 19 (3) (1971) 188–192.

1550 [48] A. Hajnal, E. C. B. Milner, E. Szemerédi, A cure for the telephone disease, *Canadian Mathematical Bulletin* 15 (3) (1972) 447–450.

[49] M. Cooper, A. Herzig, F. Maris, E. Perrotin, J. Vianey, Lightweight Parallel Multi-Agent Epistemic Planning, in: *Proceedings of the 17th International Conference on Principles of Knowledge Representation and Reasoning*, 2020, pp. 274–283.

1555 URL <https://doi.org/10.24963/kr.2020/28>

[50] E. A. Akkoyunlu, K. Ekanadham, R. V. Hubert, Some constraints and tradeoffs in the design of network communications, in: *Proceedings of the 5th ACM Symposium on Operating Systems Principles (SOSP 1975)*, ACM Press, 1975, pp. 67–74.

1560

[51] J. Gray, Notes on data base operating systems, in: *Operating Systems, An Advanced Course*, Springer-Verlag, 1978, pp. 393–481.

[52] L. Lamport, R. Shostak, M. Pease, The Byzantine generals problem, *ACM Transactions on Programming Languages and Systems* 4 (3) (1982) 382–401.

1565

[53] F. Kominis, H. Geffner, Beliefs in multiagent planning: From one agent to many, in: R. I. Brafman, C. Domshlak, P. Haslum, S. Zilberstein (Eds.), *Proceedings of the Twenty-Fifth International Conference*

- on Automated Planning and Scheduling, ICAPS 2015, Jerusalem,  
1570 Israel, June 7-11, 2015, AAAI Press, 2015, pp. 147–155.  
URL [http://www.aaai.org/ocs/index.php/ICAPS/ICAPS15/  
paper/view/10617](http://www.aaai.org/ocs/index.php/ICAPS/ICAPS15/paper/view/10617)
- [54] D. Lewis (Ed.), *Convention: A Philosophical Study*, Harvard University Press, 1969.
- 1575 [55] M. C. Cooper, A. Herzig, F. Maris, J. Vianey, Temporal epistemic gossip problems, in: M. Slavkovik (Ed.), *Multi-Agent Systems - 16th European Conference, EUMAS 2018, Bergen, Norway, December 6-7, 2018, Revised Selected Papers, Vol. 11450 of Lecture Notes in Computer Science*, Springer, 2018, pp. 1–14. doi:10.1007/  
1580 978-3-030-14174-5\\_1.  
URL [https://doi.org/10.1007/978-3-030-14174-5\\_1](https://doi.org/10.1007/978-3-030-14174-5_1)
- [56] D. McDermott, M. Ghallab, A. Howe, C. Knoblock, A. Ram, M. Veloso, D. Weld, D. Wilkins, *PDDL – The Planning Domain Definition Language*, Tech. rep., Yale Center for Computational Vision and Control (1998).  
1585
- [57] G. Röger, F. Pommerening, J. Seipp, Fast downward stone soup 2014, in: *Eighth International Planning Competition (IPC-8): planner abstracts*, 2014, pp. 28–31.
- [58] M. Martinez, I. Moraru, S. Edelkamp, S. Franco, Planning-PDBs planner in the IPC 2018, *IPC-9 planner abstracts (2018)* 63–66.  
1590
- [59] T. Bolander, T. Engesser, R. Mattmüller, B. Nebel, Better eager than lazy? How agent types impact the successfulness of implicit coordination, in: *Principles of Knowledge Representation and Reasoning: Proceedings of the Sixteenth International Conference, KR 2018, 2018*, pp. 445–453.  
1595  
URL [https://aaai.org/ocs/index.php/KR/KR18/paper/  
view/18070](https://aaai.org/ocs/index.php/KR/KR18/paper/view/18070)

- [60] H. van Ditmarsch, A. Herzig, J. Lang, P. Marquis, Introspective forgetting, *Synthese* 169 (2) (2009) 405–423.  
1600 URL <https://doi.org/10.1007/s11229-009-9554-4>
- [61] R. Milner, An algebraic definition of simulation between programs, in: D. C. Cooper (Ed.), *Proceedings of the 2nd International Joint Conference on Artificial Intelligence*. London, UK, September 1-3, 1971, William Kaufmann, 1971, pp. 481–489.  
1605 URL <http://ijcai.org/Proceedings/71/Papers/044.pdf>
- [62] T. Bolander, Seeing is believing: Formalising false-belief tasks in dynamic epistemic logic, in: A. Herzig, E. Lorini (Eds.), *Proceedings of the European Conference on Social Intelligence (ECSI-2014)*, Barcelona, Spain, November 3-5, 2014, Vol. 1283 of CEUR Workshop Proceedings, CEUR-WS.org, 2014, pp. 87–107.  
1610 URL [http://ceur-ws.org/Vol-1283/paper\\_14.pdf](http://ceur-ws.org/Vol-1283/paper_14.pdf)
- [63] T. Bolander, Seeing is believing: Formalising false-belief tasks in dynamic epistemic logic, in: H. van Ditmarsch, G. Sandu (Eds.), *Jaakko Hintikka on Knowledge and Game-Theoretical Semantics*, Springer International Publishing, Cham, 2018, pp. 207–236.  
1615 URL [https://doi.org/10.1007/978-3-319-62864-6\\_14](https://doi.org/10.1007/978-3-319-62864-6_14)
- [64] R. Audi (Ed.), *The Cambridge Dictionary of Philosophy*, Second Edition, Cambridge University Press, 1999, entry ‘Action’.
- [65] A. O. Liberman, A. Achen, R. K. Rendsvig, Dynamic term-modal logics for first-order epistemic planning, *Artif. Intell.* 286 (2020) 103305.  
1620 URL <https://doi.org/10.1016/j.artint.2020.103305>
- [66] J. McCarthy, Epistemological problems of artificial intelligence, in: B. L. Webber, N. J. Nilsson (Eds.), *Readings in Artificial Intelligence*, Morgan Kaufmann Publishers, 1981, pp. 459 – 465.  
1625 URL <http://www.sciencedirect.com/science/article/pii/B9780934613033500350>

- [67] T. Bolander, H. van Ditmarsch, A. Herzig, E. Lorini, P. Pardo, F. Schwarzentruher, Announcements to attentive agents, *Journal of Logic, Language and Information* 25 (1) (2016) 1–35.  
1630 URL <https://doi.org/10.1007/s10849-015-9234-3>
- [68] T. Miller, P. Felli, C. J. Muise, A. R. Pearce, L. Sonenberg, 'Knowing whether' in proper epistemic knowledge bases, in: D. Schuurmans, M. P. Wellman (Eds.), *Proceedings of the Thirtieth AAAI Conference on Artificial Intelligence*, February 12-17, 2016, Phoenix, Arizona, 1635 USA, AAAI Press, 2016, pp. 1044–1050.  
URL <http://www.aaai.org/ocs/index.php/AAAI/AAAI16/paper/view/12291>
- [69] F. Kominis, H. Geffner, Multiagent online planning with nested beliefs and dialogue, in: L. Barbuiescu, J. Frank, Mausam, S. F. Smith (Eds.), 1640 *Proceedings of the Twenty-Seventh International Conference on Automated Planning and Scheduling, ICAPS 2017*, Pittsburgh, Pennsylvania, USA, June 18-23, 2017, AAAI Press, 2017, pp. 186–194.  
URL <https://aaai.org/ocs/index.php/ICAPS/ICAPS17/paper/view/15748>
- 1645 [70] R. P. A. Petrick, F. Bacchus, A knowledge-based approach to planning with incomplete information and sensing, in: M. Ghallab, J. Hertzberg, P. Traverso (Eds.), *Proceedings of the Sixth International Conference on Artificial Intelligence Planning Systems*, April 23-27, 2002, Toulouse, France, AAAI, 2002, pp. 212–222.  
1650 URL <http://www.aaai.org/Library/AIPS/2002/aips02-022.php>
- [71] E. Erdem, A. Herzig, Solving gossip problems using answer set programming: An epistemic planning approach, *Electronic Proceedings in Theoretical Computer Science* 325 (2020) 52–58, F. Ricca, A. Russo et al. (Eds.): *Proc. 36th International Conference on Logic Program-* 1655

ming (Technical Communications) (ICLP 2020), EPTCS 325. doi:  
10.4204/eptcs.325.11.

URL <http://dx.doi.org/10.4204/EPTCS.325.11>

- 1660 [72] A. Herzig, F. Maris, J. Vianey, Dynamic logic of parallel propositional  
assignments and its applications to planning, in: S. Kraus (Ed.), Pro-  
ceedings of the Twenty-Eighth International Joint Conference on Ar-  
tificial Intelligence, IJCAI 2019, Macao, China, August 10-16, 2019,  
[ijcai.org](http://ijcai.org), 2019, pp. 5576–5582.

URL <https://doi.org/10.24963/ijcai.2019/774>

- 1665 [73] H. v. Ditmarsch, D. Grossi, A. Herzig, W. v. d. Hoek, L. B. Kuijer, Pa-  
rameters for epistemic gossip problems, in: Proc. LOFT 2016, 2016.

- [74] H. van Ditmarsch, J. van Eijck, P. Pardo, R. Ramezani, F. Schwarzentru-  
ber, Epistemic protocols for dynamic gossip, *J. Applied  
Logic* 20 (2017) 1–31.