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# Tracing the *Hercules* stream with *Gaia* and LAMOST: new evidence for a fast bar in the Milky Way

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## ABSTRACT

The length and pattern speed of the Milky Way bar are still controversial. Photometric and spectroscopic surveys of the inner Galaxy, as well as gas kinematics, favour a long and slowly rotating bar, with corotation around a Galactocentric radius of 6 kpc. On the other hand, the existence of the *Hercules* stream in local velocity space favours a short and fast bar with corotation around 4 kpc. This follows from the fact that the *Hercules* stream looks like a typical signature of the outer Lindblad resonance of the bar. As we showed recently, reconciling this local stream with a slow bar would need to find a yet unknown alternative explanation, based, for instance, on the effect of spiral arms. Here, by combining the TGAS catalogue of the *Gaia* DR1 with LAMOST radial velocities, we show that the position of *Hercules* in velocity space as a function of radius in the outer Galaxy indeed varies exactly as predicted by fast bar models with a pattern speed no less than 1.8 times the circular frequency at the Sun's position.

**Key words:** Galaxy: disc – Galaxy: kinematics and dynamics – Galaxy: structure.

## 1 INTRODUCTION

The distribution and kinematics of stars in the Milky Way disc can be described, to a first approximation, by axisymmetric models (e.g. Binney & Piffl 2015). However, it is well known that the Galactic disc contains non-axisymmetric features. These appear in the distribution of stars in configuration space: For example, the Milky Way disc contains a prominent bar (e.g. Binney, Gerhard & Spergel 1997) and a still relatively poorly known spiral-arm structure in both gas and stars (e.g. Benjamin, Churchwell & GLIMPSE Team 2005; Vallée 2008). However, non-axisymmetries also reveal themselves in the velocity distribution of stars, which can help learning about their dynamical properties such as their pattern speed. In particular, the velocity distribution of stars in the solar neighbourhood would appear as an homogeneous ellipsoid, with a tail for low tangential velocities, if the Galaxy were a pure axisymmetric disc in differential rotation. However, a number of substructures in local velocity space have been known for a very long time (Proctor 1869; Kapteyn 1905; Eggen 1958). The astrometric and spectroscopic surveys of the late twentieth and early twenty-first centuries (Perryman et al. 1997; Nordström et al. 2004; Famaey et al. 2005) have clearly confirmed that the kinematics of disc stars in the solar neighbourhood are rather clumpy, and that the most prominent of these clumps (known as ‘moving groups’) are not disrupted open

clusters keeping coherence in velocity space. An alternative mechanism to create moving groups (or dynamical streams) in local velocity space is the resonant interaction between the stars and the main non-axisymmetric patterns of the Milky Way, namely the bar and the spiral arms. In particular, Dehnen (1999, 2000) pointed out that the *Hercules* moving group, or *Hercules* stream, could be a direct consequence of the Sun being located just outside of the bar's outer Lindblad resonance (OLR). This resonance occurs at the radius  $R_{\text{OLR}}$  where stars make two epicyclic oscillations while making one retrograde rotation in the frame of the bar; hence,

$$\kappa + 2(\Omega - \Omega_b) = 0, \quad (1)$$

where  $\Omega(R)$ ,  $\kappa(R)$  and  $\Omega_b$  are the Galaxy's circular frequency, epicyclic frequency and the bar's pattern speed, respectively (see Binney & Tremaine 2008).

However, careful studies of the photometry of the inner Galaxy (Wegg & Gerhard 2013; Wegg, Gerhard & Portail 2015) recently revealed a long and thin extension of the bar, oriented at an angle of  $\phi_b \sim 27^\circ$  from the Galactic Centre–Sun direction, and reaching a Galactocentric radius  $R \sim 5$  kpc. If this structure is not a loosely wound spiral arm connected to the bar but an extension of the bar itself, this limits its pattern speed since it cannot extend beyond corotation. Such a long and slow bar has subsequently been vindicated by recent dynamical modelling of stellar and gas kinematics in the inner Galaxy (Portail et al. 2015, 2017; Sormani, Binney & Magorrian 2015; Li et al. 2016), converging to a pattern speed of the order of  $40 \text{ km s}^{-1} \text{ kpc}^{-1}$  placing the bar corotation at about

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6 kpc from the Galactic Centre (Portail et al. 2017), and the OLR way beyond the solar neighbourhood.

This is clearly at odds with the structure of local velocity space. Indeed, as we showed recently (Monari et al. 2017), a slow bar with a pattern speed as above does not produce a bimodality in local velocity space. Hence, to reconcile this feature with a slow bar, an alternative explanation, based, e.g. on spiral arms, should be found. For instance, Grand, Kawata & Cropper (2014) showed that the outward radial migrators behind their corotating spiral arms display lower tangential velocities and an outward velocity.

As pointed out by, e.g. Bovy (2010), a way to check whether the OLR explanation holds is to investigate how the *Hercules* feature in velocity space varies with the position in the Galaxy. This was already shown to hold well in the relatively nearby inner Galactic disc by Antoja et al. (2014), using the RAVE survey (Steinmetz et al. 2006). However, the advent of *Gaia* (Gaia Collaboration 2016) provides a unique opportunity to test this in the outer Galactic disc. In particular, cross-matching the recent TGAS catalogue with existing spectroscopic surveys could allow us to trace the position of *Hercules* in velocity space as a function of Galactocentric radius. In this Letter, we show that this can be achieved through the combination of TGAS with the Large Sky Area Multi-Object Fibre Spectroscopic Telescope (LAMOST) DR2 catalogue (Liu et al. 2014).

## 2 CROSS-MATCHING TGAS WITH LAMOST

We cross-match the TGAS catalogue (from which we obtain the right ascension  $\alpha$ , the declination  $\delta$ , the parallax  $\pi$ , and the proper motions  $\mu_\alpha$  and  $\mu_\delta$ ) with the LAMOST DR2 A-, F-, G- and K-type stars catalogue (Liu et al. 2014, from which we obtain the line-of-sight velocity  $v_{\text{los}}$ ) using *GAIA* tools written by Jo Bovy.<sup>1</sup> The LAMOST A-, F-, G- and K-type stars catalogue selects only the spectra whose signal-to-noise ratio (S/N) in the *g* band is greater than 6 in dark time and 20 in bright time. We obtain in this way 108 910 stars. We then select stars with a fractional parallax error<sup>2</sup>  $\sqrt{\sigma_\pi^2 + (0.3 \text{ mas})^2}/\pi < 0.2$ , which provides us a total of 49 075 stars, mostly distributed towards the anti-centre of our Galaxy (while, RAVE, for example, focuses on the central parts). These parallax errors ensure a very limited bias on distances when inverting them. In the following analysis, we used all the stars satisfying the above criteria to maximize our sample. The cross-matched TGAS and LAMOST data sets have a complicated selection function. However, because we focus on the velocity field of relatively nearby stars (distances  $\lesssim 0.7$  kpc), it is unlikely that our conclusions are affected by the observational selection function.

## 3 ANALYSIS

To study the sample, we transform the observables ( $\alpha$ ,  $\delta$ ,  $\pi$ ,  $\mu_\alpha$ ,  $\mu_\delta$ ,  $v_{\text{los}}$ ) to the cylindrical Galactocentric coordinate system ( $R$ ,  $\phi$ ,  $z$ ,  $v_R$ ,  $v_\phi$ ,  $v_z$ ). These transformations can be found in several textbooks (e.g. Binney & Merrifield 1998), and require us to know three fundamental parameters of our Galaxy: the Galactocentric radius of the Sun  $R_0$ , the circular speed of the Galaxy at the Sun  $v_0$  and the motion of the Sun with respect to the local standard of rest (LSR). The latter has a rather uncertain tangential component  $V_\odot$  (see Binney

& Tremaine 2008).<sup>3</sup> We use the values found in Reid et al. (2014), i.e.  $R_0 = 8.34$  kpc,  $v_0 = 240 \text{ km s}^{-1}$  and  $(v_0 + V_\odot) = 254.6 \text{ km s}^{-1}$ . To determine  $v_R$  and  $v_\phi$ , we also use  $U_\odot = 10 \text{ km s}^{-1}$  (e.g. Bovy et al. 2015), the radial motion of the Sun with respect to the LSR.<sup>4</sup>

We focus in particular on the distribution of stars in the  $(R, v_\phi)$  plane, shown in Fig. 1 (left-hand panel). In this space, *Hercules* is a clump of stars localized between  $v_\phi \sim 190$  and  $\sim 200 \text{ km s}^{-1}$ , slightly detached from the main velocity mode at higher  $v_\phi$ . Notice the underdensity of stars between *Hercules* and the main velocity mode (the ‘gap’), which appears slightly inclined in the  $(R, v_\phi)$  space. *Hercules* and the gap disappear when we select only stars moving inwards with  $v_R < 0$  (Fig. 1, central panel), while they are more evident when we select stars with  $v_R > 0$  (right-hand panel). This is due to the fact that *Hercules* is composed of stars moving outwards in the Galaxy (e.g. Dehnen 1998; Famaey et al. 2005).

If *Hercules* is due to the effect of the bar’s OLR, the gap’s location in the  $(R, v_\phi)$  plane corresponds to stars having their guiding radii<sup>5</sup> at  $R_{\text{OLR}}$ . At a radius  $R$ , the  $v_\phi$  of these orbits is given by

$$v_{\text{OLR}} = \frac{R_{\text{OLR}} v_c(R_{\text{OLR}})}{R}. \quad (2)$$

This means that the angular momentum corresponding to the gap in velocity space at radius  $R$  corresponds to the angular momentum of circular orbits at the OLR. As we move outwards in  $R$ ,  $v_{\text{OLR}}$  becomes lower, and the number of stars that is affected by the OLR becomes smaller, which eventually leads to the disappearance of the *Hercules* moving group. We show this in Fig. 2, where, using the formalism of Monari et al. (2016, 2017), we compute the position of the *Hercules* gap for perturbed phase-space distribution functions in the presence of a quadrupole bar perturbation. We show the theoretical shape of velocity space as a function of radius, for a population of intrinsic radial velocity dispersion  $\sim 35 \text{ km s}^{-1}$ , in the presence of a quadrupole bar<sup>6</sup> (see equation (9) of Monari et al. 2017) with  $\Omega_b = 1.89\Omega_0$  (with  $\Omega_0 \equiv v_0/R_0$ ) and the maximum radial force being 1 per cent of the axisymmetric background, here an axisymmetric potential corresponding to a flat rotation curve  $v_c(R) = v_0$ . We also plot in Fig. 2 a red dashed horizontal line indicating  $v_{\text{OLR}}$  computed as in equation (2). In Fig. 1 (right-hand panel), *Hercules* seems to disappear at  $R \sim 8.6$  kpc. We can, however, still trace it at larger  $R$  and lower  $v_{\text{OLR}}$ , using the  $v_\phi$  distribution in Fig. 3.

If we describe the circular velocity curve as a power law  $v_c(R) = v_0(R/R_0)^\beta$  in the proximity of the Sun, then

$$R_{\text{OLR}}(\Omega_b, \beta) = R_0 \left[ \frac{\Omega_0}{\Omega_b} \left( 1 + \sqrt{\frac{1+\beta}{2}} \right) \right]^{1/(1-\beta)} \quad (3)$$

(Dehnen 2000). In the right-hand panel of Fig. 1, we plot the curve  $v_{\text{OLR}}(R)$  for three values of  $R_{\text{OLR}}$ , using  $\beta = -0.3, 0$  and  $0.3$ , and

<sup>3</sup> Each of the fundamental parameters of our Galaxy is still rather uncertain, but their combination  $(v_0 + V_\odot)/R_0$  (i.e. the angular speed of the Sun around the Galactic Centre) is well constrained to  $30.57 \pm 0.43 \text{ km s}^{-1} \text{ kpc}^{-1}$  (Reid et al. 2014).

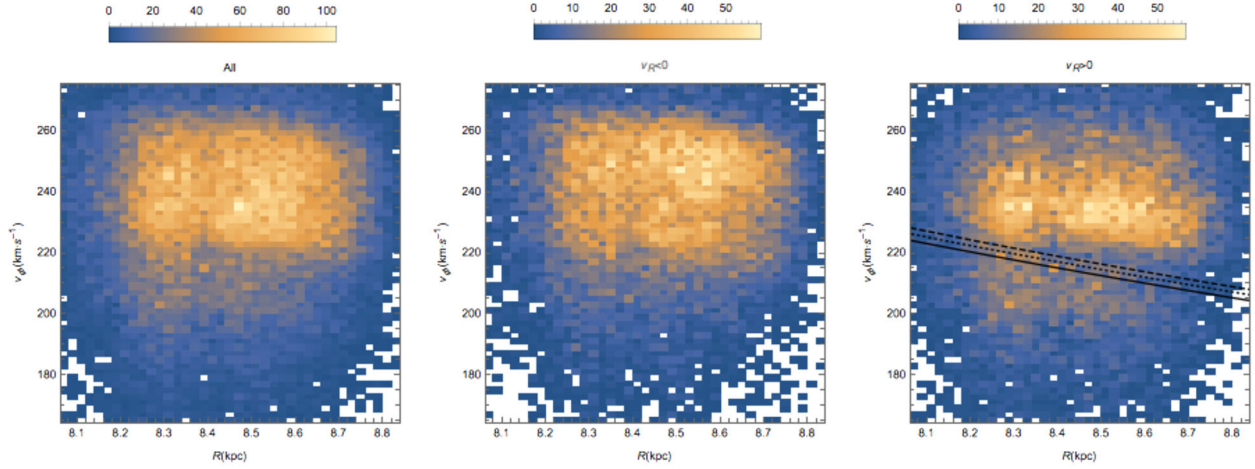
<sup>4</sup> Note that the  $U_\odot$  value is almost uninfluential on the distribution of stars in  $v_\phi$  that we find in this work.

<sup>5</sup> This is, strictly speaking, true only when a star’s orbital eccentricity is null, but still a good approximation for the range of radial energies that we consider in our samples.

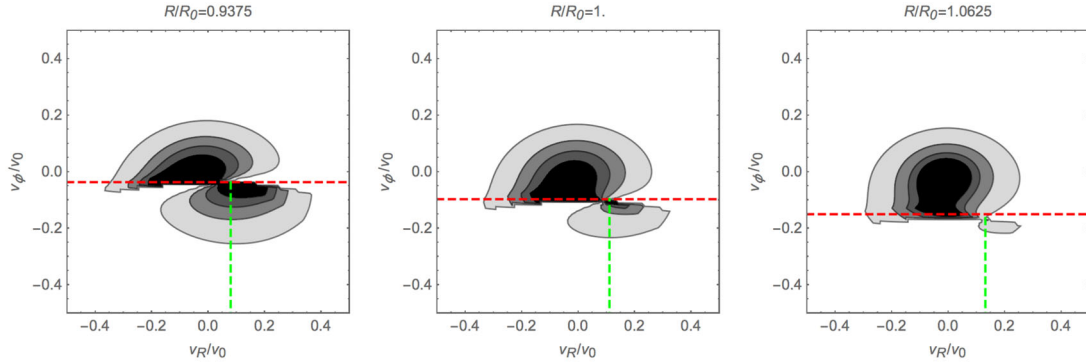
<sup>6</sup> Higher order terms with  $m > 2$  would, in principle, also appear in the expansion of the bar potential for more complex bar shapes than a pure quadrupole. The response to these additional terms in the potential will, however, be of second or higher order compared to the quadrupole amplitude, and will not affect the  $v_{\text{OLR}}$  of the gap.

<sup>1</sup> <http://github.com/jobovy/gaiatools>

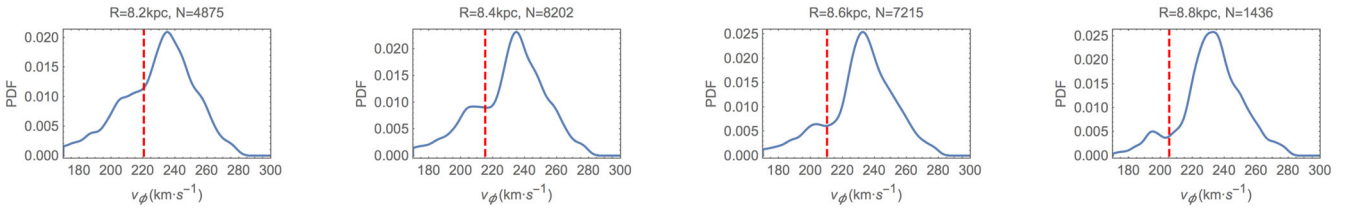
<sup>2</sup> We take into account the recommended value for the systematic error in the parallax of 0.3 mas (see Gaia Collaboration 2016).



**Figure 1.** Distribution of stars in TGAS+LAMOST in the  $(R, v_\phi)$  plane. We select stars with parallax accuracy  $\sqrt{\sigma_\pi^2 + (0.3 \text{ mas})^2}/\pi < 0.2$ . The bin size is 20 pc in  $R$ , and  $2 \text{ km s}^{-1}$  in  $v_\phi$ , and the units of the colour bar indicate the number of stars per bin (the white bins are empty). The left-hand panel represents the whole sample, the central panel stars with  $v_R < 0$  and the right-hand panel stars with  $v_R > 0$ . The different curves in the right-hand panel correspond to different models of  $v_{\text{OLR}}$  with  $\Omega_b = 1.89\Omega_0$ : The solid curve has a flat  $v_c(R)$  ( $\beta = 0$ ), the dashed curve has an increasing  $v_c(R)$  ( $\beta = 0.3$ ) and the dotted curve a decreasing  $v_c(R)$  ( $\beta = -0.3$ ). Note how *Hercules* fades away beyond  $R = 8.6 \text{ kpc}$ , in accordance with Fig. 2.



**Figure 2.** Theoretical shape of velocity space as a function of radius for a population of intrinsic radial velocity dispersion  $\sim 35 \text{ km s}^{-1}$  in the presence of a bar with  $\Omega_b = 1.89\Omega_0$ , angle from the long axis of the bar  $\phi = 25^\circ$  and the maximum radial force being 1 per cent of the axisymmetric background, of a potential corresponding to a flat rotation curve  $v_c(R) = v_0$ , computed using the formalism of Monari, Famaey & Siebert (2016) and Monari et al. (2017), together with a red dashed horizontal line indicating  $v_{\text{OLR}}$  computed as in equation (2) and a green dashed vertical line representing the peak of the low-velocity mode with  $v_\phi < v_{\text{OLR}}(R)$ . The contours include (from the inner to the outer) 34, 50, 68 and 90 per cent of the stars.

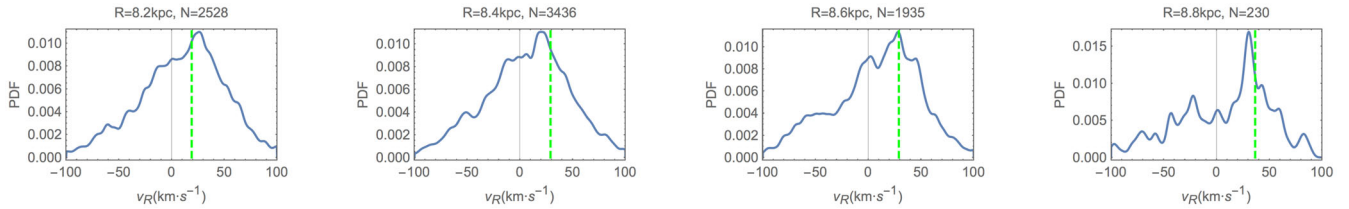


**Figure 3.** Distribution of stars in  $v_\phi$ , for stars with  $v_R > 0$  and  $R_i - \Delta R < R < R_i + \Delta R$ , where  $R_i = 8.2, 8.4, 8.6$  and  $8.8 \text{ kpc}$ , and  $\Delta R = 0.1 \text{ kpc}$ . The PDFs are obtained using Gaussian kernels of bandwidth  $3 \text{ km s}^{-1}$ . The red dashed line corresponds to  $v_{\text{OLR}}(R_i)$  for  $\Omega_b = 1.89\Omega_0$ , and  $\beta = 0$ , indicating the theoretical gap between the high- and low-velocity modes in  $v_\phi$ .

$\Omega_b = 1.89\Omega_0$ . The pattern speed  $\Omega_b$  was found by Antoja et al. (2014) using the RAVE catalogue, which mostly probes regions with  $R < R_0$ . We find that the three curves nicely follow the shape of the gap even for  $R > R_0$ , without *any* tuning of the parameters to obtain a good fit. This appears even more evident when looking at Fig. 3, where the  $v_{\text{OLR}}$  for the flat rotation curve model (vertical red dashed line) perfectly matches, at every  $R$ , the saddle point in the  $v_\phi$  distribution, corresponding to the *Hercules* gap. To check the

consequence of our choice of fundamental parameters for the Milky Way, we also studied the  $(R, v_\phi)$  distribution for stars with  $v_R > 0$ , but for the parameters of Bovy et al. (2015), namely  $R_0 = 8 \text{ kpc}$ ,  $v_0 = 218 \text{ km s}^{-1}$  and  $V_\odot = 24 \text{ km s}^{-1}$ . In this case, a slightly lower pattern speed of the bar relative to the local circular frequency is preferred ( $\Omega_b \approx 1.85\Omega_0$ ).

Ideally, one should compare not only the theoretical position of the gap in  $v_\phi$  with the data, but also the detailed shape of velocity



**Figure 4.** Distribution of stars in  $v_R$ , for stars with  $v_\phi < v_{\text{OLR}}(R)$  and  $R_i - \Delta R < R < R_i + \Delta R$ , where  $R_i = 8.2, 8.4, 8.6$  and  $8.8$  kpc, and  $\Delta R = 0.1$  kpc. The PDFs are obtained using Gaussian kernels of bandwidth  $3 \text{ km s}^{-1}$ . The green dashed lines corresponds to the peak of the low-velocity mode at each  $R_i$  from the theoretical model of Fig. 2.

space. Indeed, the main conclusion from the present analysis is that the *Hercules* moving group is generated by a perturbation with a single resonance radius at  $R \sim 7$  kpc. This readily excludes corotating spiral arms with a varying pattern speed as a possible explanation. However, one should also compare the  $v_R$  distribution of the analytical models with the data. It is not straightforward to make a direct quantitative comparison of the amplitude of the overdensity in the data and models because while the models are reliably predicting the location of the overdensity in velocity space, they do not predict its amplitude due to the need for a refined treatment of the analysis at the resonance itself (Monari et al., in preparation). However, we can indeed compare the location of the overdensity in  $v_R$  for stars in the low-velocity mode in the models and data. This is done in Fig. 4, where the  $v_R$  distribution as a function of radius is compared with the peak of the low-velocity mode from the theoretical model of Fig. 2: The agreement is good, given that the analytical model is precisely predicting the location of the gap in  $v_\phi$  but is not a perfect representation of the actual distribution at the resonance.

#### 4 CONCLUSIONS

The *Hercules* moving group has traditionally been interpreted as a direct signature in local stellar kinematics of the Galactic bar’s OLR (Dehnen 1999, 2000). This explanation is, however, at odds with the slowly rotating long bar favoured by stellar and gas kinematics in the inner Galaxy. This would mean that an alternative explanation, e.g. based on spiral arms, should be found to reproduce the bimodality in local velocity space (Monari et al. 2017). Such an alternative explanation has not yet been found, but Grand et al. (2014) showed that the outward radial migrators behind their corotating spiral arms display lower tangential velocities and an outward velocity, which could help explaining the moving group. However, a consequence of such a model would be that there is no unique resonant radius associated with the perturbation.

One way to test whether *Hercules* is indeed linked to the OLR of the bar is to trace its position in velocity space as a function of position in the Galaxy. Here, we showed that this is already possible today through the combination of the TGAS and LAMOST DR2 catalogues. We found out that the *Hercules* moving group is indeed closely following the prediction of models placing the Sun just outside the OLR of the bar. In these models, the corotation of the bar is close to  $R \sim 4$  kpc, and its OLR is at  $R \sim 7$  kpc. This would mean that alternative explanations, necessary to account for a slowly rotating bar with corotation around  $R \sim 6$  kpc, would have to reproduce not only the position of *Hercules* in local velocity space at the Sun’s position, but also its variation with radius precisely as predicted by fast bar models. As the observed variation with radius of the gap in  $v_\phi$  indicates, the *Hercules* moving group is linked to a single resonance radius, hence excluding a varying pattern speed with radius. This excludes a corotating spiral origin for the group.

What is more, its observed variation in  $v_R$  as a function of radius also closely follows the prediction of the bar model, rendering an explanation based on a spiral density wave unlikely.

The deprojected 3D density of red clump giants in the inner Galaxy nevertheless points to the existence of a long, flat structure, in the direct prolongation of the bar, reaching out to  $R \sim 5$  kpc. If the bar indeed has a high pattern speed  $> 1.8\Omega_0$ , then this long thin structure cannot be rotating with the bar, but could rather be a loosely wound spiral coupled to the end of the bar. It will thus be of extreme importance to model precisely the effect of spiral arms in the inner Galaxy, and to explore their whole possible parameter space, to check whether spirals could help reconcile existing data on gas and stellar kinematics in the inner Galaxy with the fast bar revealed by the stellar kinematics in the outer Galaxy. A second possibility would be that the long extension of the bar and the smaller bar causing the OLR feature in the solar vicinity actually constitute a double-bar feature, the smaller bar having a larger pattern speed (e.g. Wozniak et al. 1995). A last intriguing possibility would be that the pattern speed of the bar has dramatically decreased in the recent past, and that the outer Galactic disc is still affected by its ancient pattern speed.

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