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Transformation of the Peregrine breather into grey solitons on a vertically sheared current

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Author contribution statement

H. Hsu, Conceptualization of the project and formal analysis checking.

M. Abid, Formal analysis checking; Software design; Numerical simulations; Validation; Investigation; Visualization.

Y. Y. Chen, Conceptualization of the project and formal analysis checking.

C. Kharif, Conceptualization; Methodology; Formal analysis; Investigation; Writing.

Keywords

Nonlinear schrödinger equation, Water Waves, solitons, Breather, Vorticity

Abstract

Word count: 40

In this Brief Research Report, we have shown, within the framework of the vor-NLS equation in deep water, that the Peregrine breather travelling at the free surface of a shear current of slowly varying vorticity may transform into grey solitons.

Contribution to the field

Based on numerical simulations of the nonlinear Schrödinger equation in infinite depth we conjecture the existence of grey solitons at the sea surface of slowly varying vortical current. This occurrence of dark solitons is due to the transformation of the Peregrine breather traveling at the free surface of the varying vortical current.

Ethics statements

Studies involving animal subjects

Generated Statement: No animal studies are presented in this manuscript.

Studies involving human subjects

Generated Statement: No human studies are presented in this manuscript.

Inclusion of identifiable human data

Generated Statement: No potentially identifiable human images or data is presented in this study.

Data availability statement

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2 ABSTRACT

3 In this Brief Research Report, we show, within the framework of the nonlinear Schrödinger
4 equation in deep water and in the presence of vorticity (vor-NLS), that the Peregrine breather
5 travelling at the free surface of a shear current of slowly varying vorticity may transform into grey
6 solitons.

7 **Keywords:** Peregrine breather, Grey soliton, vorticity, water waves

1 INTRODUCTION

8 Within the framework of a fully nonlinear two-dimensional potential solver, [4] computed the temporal
9 evolution of a Stokes wavetrain with a small modulation. They found that the energy becomes focused, at
10 the maximum of modulation, into a short wave packet of large amplitude they called a steep wave event
11 (SWE). They showed that the Peregrine breather, which is an exact solution of the self-focusing nonlinear
12 Schrödinger equation, was the most convenient approximation of the envelope of the SWE. Note that the
13 Peregrine breather can be derived from the Kuznetsov-Ma breather and the Akhmediev breather in the
14 limit of infinite temporal and spatial period (see [7]). [3] suggested that the Peregrine breather may provide
15 a useful and simple model for rogue wave events. [1] presented the first experimental observation of the
16 Peregrine breather in a water wave tank.

17 More recently, [2] presented the first ever observation in a wave tank of dark solitons on the surface of water,
18 so demonstrating the probable existence at the sea surface of dark solitons in finite depth for $kh < 1.363$
19 where k is the carrier wavenumber and h the water depth. They found a good agreement between the
20 experimental soliton and the dark soliton solution of the defocusing nonlinear Schrödinger equation. Dark
21 solitons occur as envelope holes. Rogue waves are large-amplitude waves which occurs at the sea surface
22 suddenly without warning. Such waves are accompanied by deep holes before and/or after the largest crest.
23 Another possible mechanism of these holes in the ocean could be dark or grey soliton generation.

24 There is an abundant literature on the interaction between surface water waves and spatially uniform
25 currents. In the real ocean, currents are never uniform. Spatially varying currents may affect strongly the
26 water wave behaviour. Herein, we paid attention to the evolution of a Peregrine breather propagating at the
27 surface of a vertically sheared current. In this Brief Research Report we propose, based on the NLS equation
28 in infinite depth, a physical mechanism of grey soliton generation from a Peregrine breather evolving

29 on slowly varying underlying water vorticity. Section 2 is devoted to the presentation of the vor-NLS
 30 equation in the presence of constant vorticity derived by [8]. The vor-NLS equation is self-focusing or
 31 defocusing according to the magnitude of the vorticity. Vorticity effect on the soliton solutions of the
 32 vor-NLS of self-focusing and defocusing types is displayed. A numerical simulation of the transformation
 33 of the Peregrine breather propagating at the free surface of a water flow of slowly varying vorticity is
 34 presented in section 3. A conclusion is given in section 4.

2 THE VOR-NLS

35 We choose an Eulerian frame ($Oxyz$) with unit vectors ($\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z$). The vector \mathbf{e}_z is oriented upwards so
 36 that the acceleration due to gravity is $\mathbf{g} = -g \mathbf{e}_z$ with $g > 0$. The equation of the undisturbed free surface is
 37 $z = 0$. The water waves are travelling at the surface of an underlying vertically sheared current of constant
 38 vorticity given by $U(z) = \Omega z$. The study is restricted to modulated wave trains propagating with positive
 39 phase velocities so long as both negative and positive values of Ω are considered.

40 [8], have shown that the spatio-temporal evolution of the complex envelope $a(\xi, \tau)$ of the surface elevation
 41 of a two-dimensional weakly nonlinear modulated wave train propagating in the presence of constant water
 42 vorticity is governed by the nonlinear Schrödinger equation (NLS equation)

$$ia_\tau + \alpha a_{\xi\xi} + \gamma |a|^2 a = 0. \quad (1)$$

43 In the case of a wave train propagating at the surface of a deep water flow of constant vorticity $\zeta = -\Omega$,
 44 the dispersive and nonlinear coefficients of the vor-NLS equation (1) are

$$\alpha = -\frac{\omega(1 + \bar{\Omega})^2}{k^2(2 + \bar{\Omega})^3}, \quad \gamma = -\frac{\omega k^2 (\bar{\Omega} + 2/3)(3\bar{\Omega}^2 + 6\bar{\Omega} + 6)}{8(1 + \bar{\Omega})},$$

45 with $\bar{\Omega} = \Omega/\omega$.

46 We consider a carrier wave travelling from left to right in deep water whose intrinsic frequency, intrinsic
 47 phase velocity and intrinsic group velocity are

$$\omega = -\frac{\Omega}{2} + \sqrt{\frac{\Omega^2}{4} + gk},$$

48

$$c_p = -\frac{\Omega}{2k} + \sqrt{\frac{\Omega^2}{4k^2} + \frac{g}{k}},$$

49

$$c_g = \frac{\Omega + \sqrt{\Omega^2 + 4gk}}{2\sqrt{\Omega^2 + 4gk}} c_p,$$

50 with k the carrier wavenumber and g the gravitational acceleration. [5] have compared both the linear
 51 intrinsic phase velocities and total energies of gravity waves in the presence of constant vorticity in finite
 52 depth and deep water and came to the conclusion that linear gravity waves in finite depth propagating at the
 53 surface of a water flow of constant vorticity behave like waves in infinite depth if $kh > \pi$. We can conclude
 54 that the unbounded water flow at $z = -\infty$ does not influence the kinematics and dynamics of the surface
 55 waves. In addition to the results of [5], figure 1 shows the dimensionless intrinsic group velocity deviation
 56 between finite and infinite depths for different values of the vorticity. As we can see the difference between

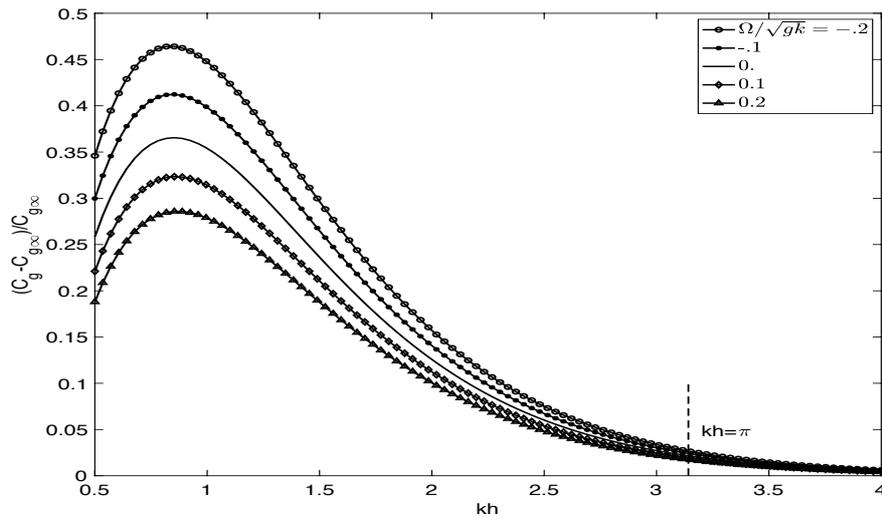


Figure 1. Dimensionless group velocity deviation between finite and infinite depths as a function of the dispersive parameter for several values of the dimensionless vorticity. In dimensionless terms, the units of acceleration and length are the acceleration of gravity g and 2π ($g = 1$ and $k = 1$).

57 the group velocities in finite and infinite depth becomes very weak as kh increases beyond the value π .

58 Equation (1) is focusing for $\bar{\Omega} > -2/3$ and defocusing for $\bar{\Omega} < -2/3$. Note that $\bar{\Omega} > -1, \forall \Omega$.

59 **2.1 Effect of vorticity on the Peregrine breather and and grey soliton**

60 The focusing vor-NLS equation admits the Peregrine breather as solution

$$a_P(\xi, \tau) = a_0 k \sqrt{-\frac{\omega}{2\gamma}} \left(1 - \frac{4(1 - ia_0^2 k^2 \omega \tau)}{1 - \omega a_0^2 k^2 \xi^2 / \alpha + a_0^4 k^4 \omega^2 \tau^2} \right) \exp(-ia_0^2 k^2 \omega \tau / 2). \tag{2}$$

61 Note that $\alpha < 0$ and $\gamma < 0, \forall \bar{\Omega} > -2/3$.

62 The defocusing vor-NLS equation admits the dark, a_D , and the grey, a_G , solitons as solutions.

$$a_D = a_0 \tanh\left(\sqrt{-\frac{\gamma}{2\alpha}} a_0 \xi\right) \exp(i\gamma a_0^2 \tau), \tag{3}$$

63

$$a_G = a_0 \frac{\exp(2im) + \exp\left(2a_0 \sin(m) \left(\sqrt{-\frac{\gamma}{2\alpha}} \xi + a_0 \gamma \tau \cos(m)\right)\right)}{1 + \exp\left(2a_0 \sin(m) \left(\sqrt{-\frac{\gamma}{2\alpha}} \xi + a_0 \gamma \tau \cos(m)\right)\right)} e^{ia_0^2 \gamma \tau}, \tag{4}$$

64 where a_0 is the envelope amplitude of the background carrier wave.

65 The parameter m fixes the minimum of amplitude at the center of the soliton. For $m = \pi/2$ this minimum is
 66 zero, with a phase shift, that corresponds to the dark soliton. Note that $\alpha < 0$ and $\gamma > 0, \forall -1 < \bar{\Omega} < -2/3$
 67 ($\bar{\Omega} > -1$ whatever the value of Ω).

68 Equation (3) can be derived from equation (4) when $m = \pi/2$.

69 In figure 2 are plotted several dimensionless profiles of the Peregrine breather and the grey soliton for
 70 different values of the vorticity $\zeta = -\Omega$. The vorticity does not modify the amplification factor $|a_P|/|a_{P\infty}|$.
 71 The envelope amplitude of the background carrier wave is $|a_{P\infty}| = a_0 k \sqrt{-\omega/(2\gamma)}$ whereas the maximum

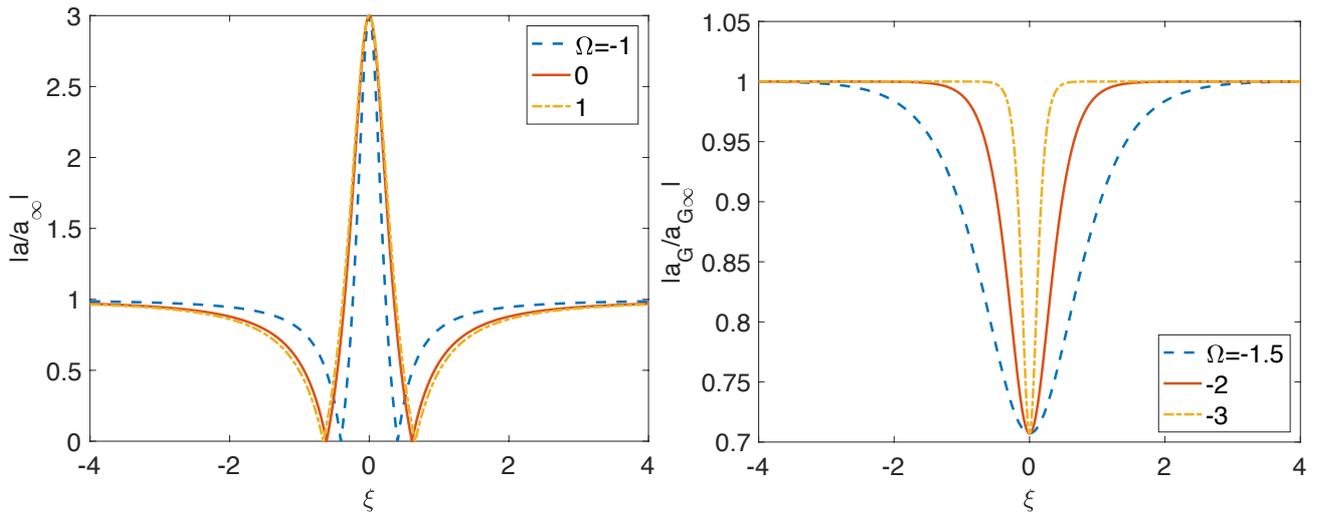


Figure 2. Dimensionless profiles of the Peregrine breather (left) and the grey soliton (right) for several values of the vorticity with $g = 1$ and $k = 1$.

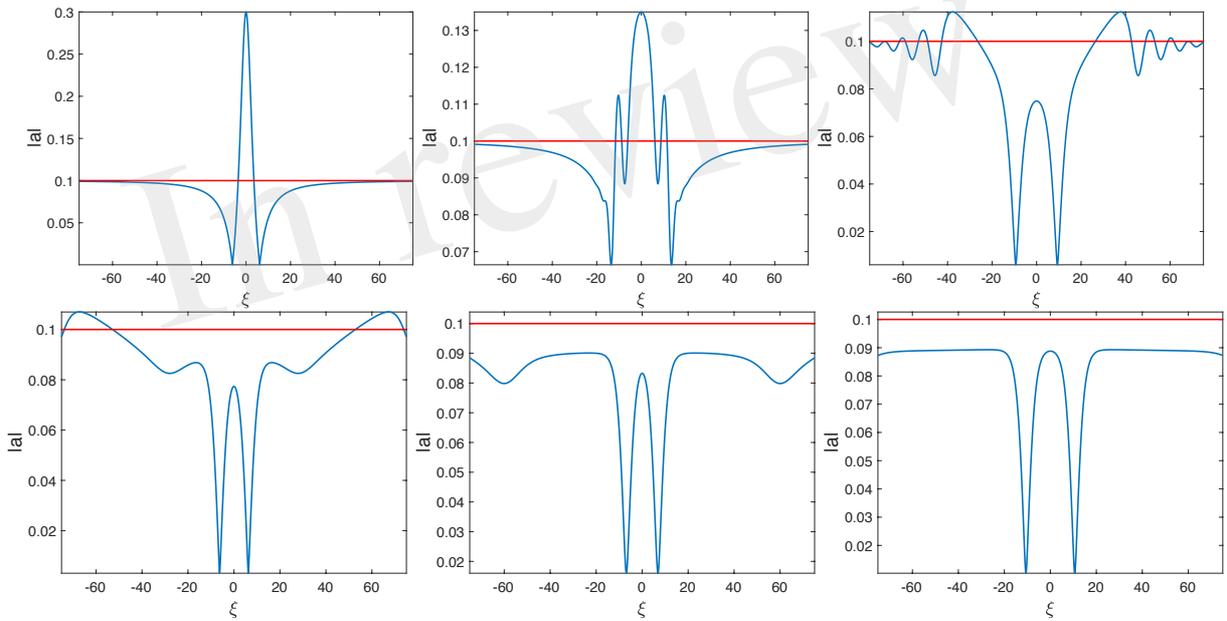


Figure 3. Time evolution of the transformation of the Peregrine breather into a two-grey soliton with $g = 1$ and $k = 1$. Top panels from left to right: $\tau = 0, 23T, 145T$. Bottom panels from left to right: $\tau = 298T, 596T, 901T$. T is the period of the Peregrine breather background.

72 amplitude of the envelope is $3 a_0 k \sqrt{-\omega/(2\gamma)}$. Consequently, the normalized maximum of the modulation
 73 envelope does not depend on the vorticity. The Peregrine breather is narrower (wider) for positive (negative)
 74 vorticity. The width of the breather decreases as the vorticity increases. The minimal amplitude at the
 75 center of the grey soliton is not modified by the presence of the vortical flow. The grey solitons becomes
 76 narrower as the positive vorticity increases.

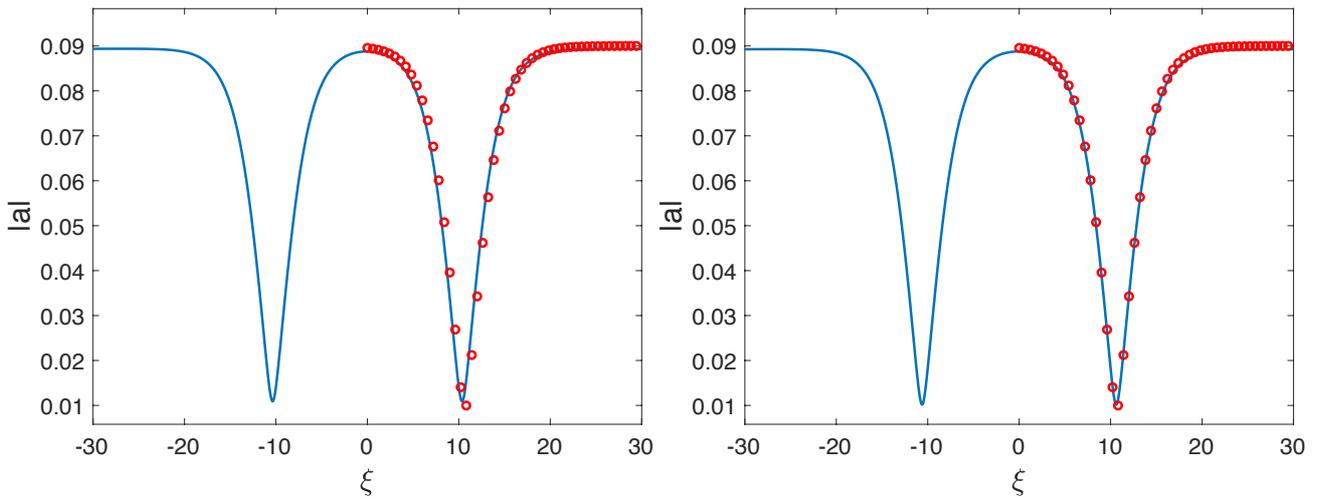


Figure 4. Grey soliton profiles at $t = 867T$ (left) and $t = 900T$ (right). The solid lines correspond to the numerical simulation whereas the circles correspond the analytical solution of equation (4) for $a_0 = 0.09$ and $m = \pi/2 - 1/10$.

3 EVOLUTION OF THE PEREGRINE BREATHER ON SLOWLY VARYING VORTICAL FLOW

77 Within the framework of equation (1), we have performed a numerical simulation of the transformation
 78 of a Peregrine breather travelling at the free surface of a vortical water flow whose vorticity varies very
 79 slowly. $\bar{\Omega}$ varies from zero to a value less than $-2/3$. For $\bar{\Omega} = 0$ equation (1) is focusing whereas for
 80 $\bar{\Omega} < -2/3$ this equation is defocusing. Consequently, the slow variation of the vorticity transforms the
 81 focusing vor-NLS equation into defocusing. We consider a Peregrine breather that meets progressively a
 82 vortical water flow whose temporal variation is

$$\begin{aligned} \Omega(\tau) &= \sin(\sigma\tau + \pi/2) - 1, & 0 \leq \sigma\tau \leq \pi \\ \Omega(\tau) &= -2, & \sigma\tau > \pi. \end{aligned}$$

83 The parameter σ is chosen such that the average temporal variation of the vorticity along the ramp is of
 84 $\mathcal{O}(a_0^2 k^2)$. The vorticity remains constant when $\sigma\tau > \pi$. Note that $\bar{\Omega}(0) = 0$ and $\bar{\Omega}(\sigma\tau = \pi) \approx -0.828$.
 85 The vor-NLS equation is solved numerically using a pseudo-spectral Fourier method. The periodicity
 86 length of the spatial domain is $L = 160\pi$. The number of grid points is $N = 6000$. Dealiasing is used.
 87 Time integration is carried out with a second order time-splitting method and a fourth order Runge Kutta
 88 scheme as well. The numerical code is checked using comparisons with the exact breather solutions and by
 89 the self consistency of the results of the two time integration schemes also. We have used a spatial filter
 90 similar to that of [6] to avoid numerical instabilities. The filter does not modify the envelope evolution
 91 because it cancels the highest modes that do not participate in the global dynamic of the envelope. The
 92 transfer function of the filter is $\exp(-\nu\kappa^2)$, where κ is a wavenumber in the Fourier space and $\nu = 10^{-5}$ is
 93 used. With this value of ν the loss of the energy of the carrier wave is of the order of 1% over approximately
 94 1000 periods of time evolution of the carrier.

95 The numerical simulation has been run in dimensionless units with $k = 1$ and $g = 1$. The initial condition at
 96 $\tau = 0$ is the Peregrine breather with $a_0 = 0.10$ and $\Omega = 0$. Figure 3 shows the evolution of the envelope at
 97 several times. The vorticity $\zeta = -\Omega$ varies slowly along the ramp from $\zeta(\tau = 0) = 0$ to $\zeta(\tau \approx 10T) = 2$.

98 At $\tau \approx 6T$ the vor-NLS equation becomes defocusing. T is the period of the background of the Peregrine
99 breather. During its propagation along the ramp the width of the breather increases whereas its amplitude
100 decreases. At $\tau \approx 20T$, on both sides of the crest of the envelope two small local troughs occur that then
101 deepen during the propagation of the envelope to give rise to the formation of two grey solitons whose
102 profiles are close to that of a dark soliton. Figure 4 shows the profiles of the two grey solitons at two
103 different times. An excellent agreement is obtained between the numerical and analytical profiles.

4 CONCLUSION

104 Within the framework of the vor-NLS equation in infinite depth we have shown numerically that a Peregrine
105 breather propagating at the free surface of a slowly varying vortical flow may generate grey solitons. The
106 present simulation confirms in a different context the result of [2] on the existence of dark solitons on the
107 surface of shallow water ($kh < 1.363$) in the absence of vorticity.
108 Our approach presents two limitations. The vor-NLS equation which was derived for constant vorticity has
109 been used with a slowly varying vorticity to transform the focusing NLS equation into defocusing during
110 a limited time. Nevertheless, the very slow variation of the vorticity on a small number of periods lead
111 us to believe that our results are physically relevant. The time to obtain the formation of the grey solitons
112 is beyond the time range of validity of the NLS equation. However, it is well known that approximate
113 models can be valid beyond their validity domain. [4] found, from their numerical simulations of the NLS
114 equation over several thousands of periods, that their computations were in qualitative good agreement over
115 a much longer time scale than expected, namely when the wave steepness of the carrier wave is not too
116 steep. Despite these limitations we conjecture the existence of grey solitons on deep water in the presence
117 of varying water shear flows. Our numerical simulation provides only preliminary results on this transition
118 which has to be confirmed by using models free of these two limitations.

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CONFLICT OF INTEREST STATEMENT

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136 relationships that could be construed as a potential conflict of interest.

AUTHOR CONTRIBUTIONS

137 H. Hsu, Conceptualization of the project and formal analysis checking.
138 M. Abid, Formal analysis checking; Software design; Numerical simulations; Validation; Investigation;
139 Visualization.
140 Y. Y. Chen, Conceptualization of the project and formal analysis checking.
141 C. Kharif, Conceptualization; Methodology; Formal analysis; Investigation; Writing.

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Figure 1.JPEG

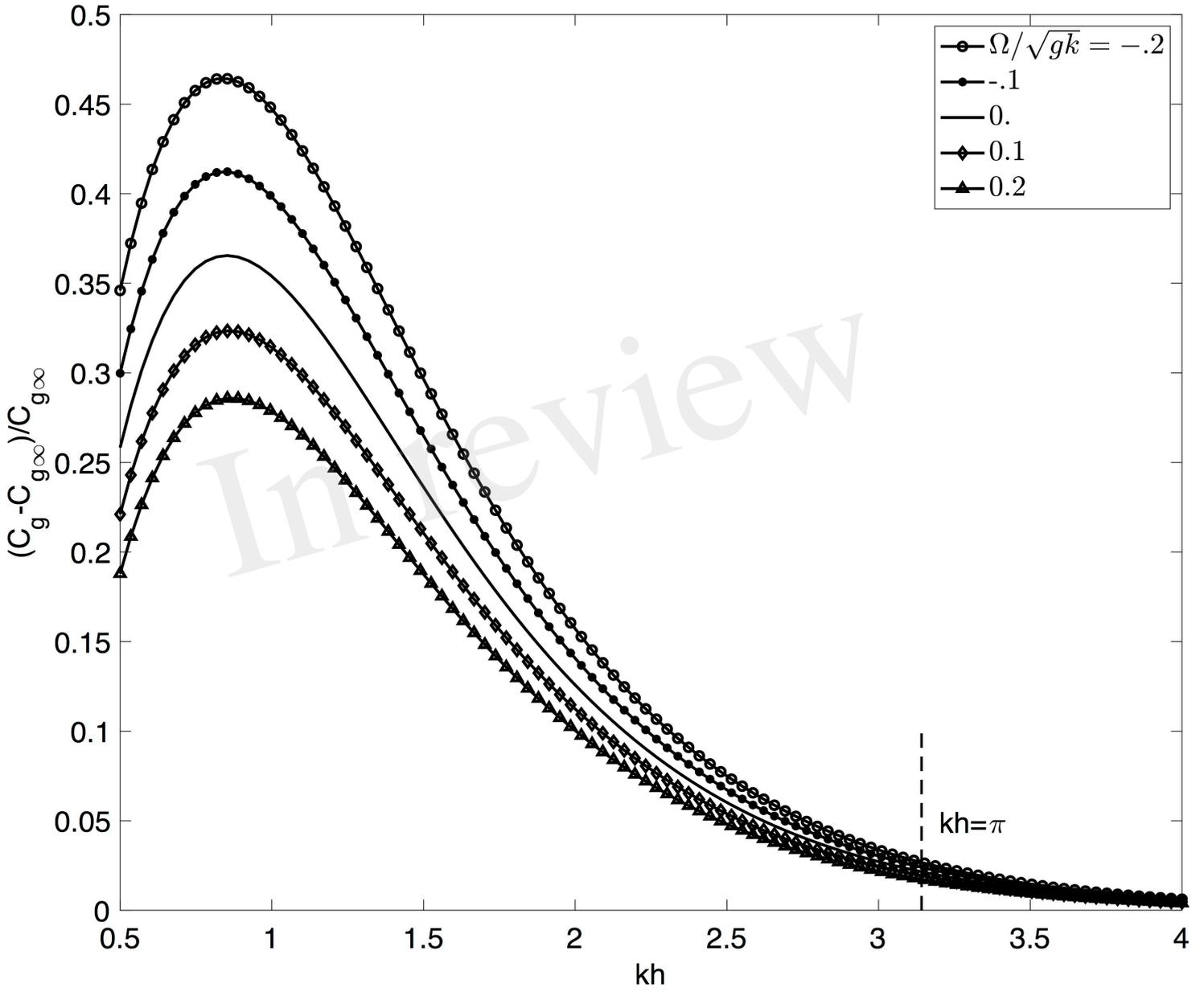


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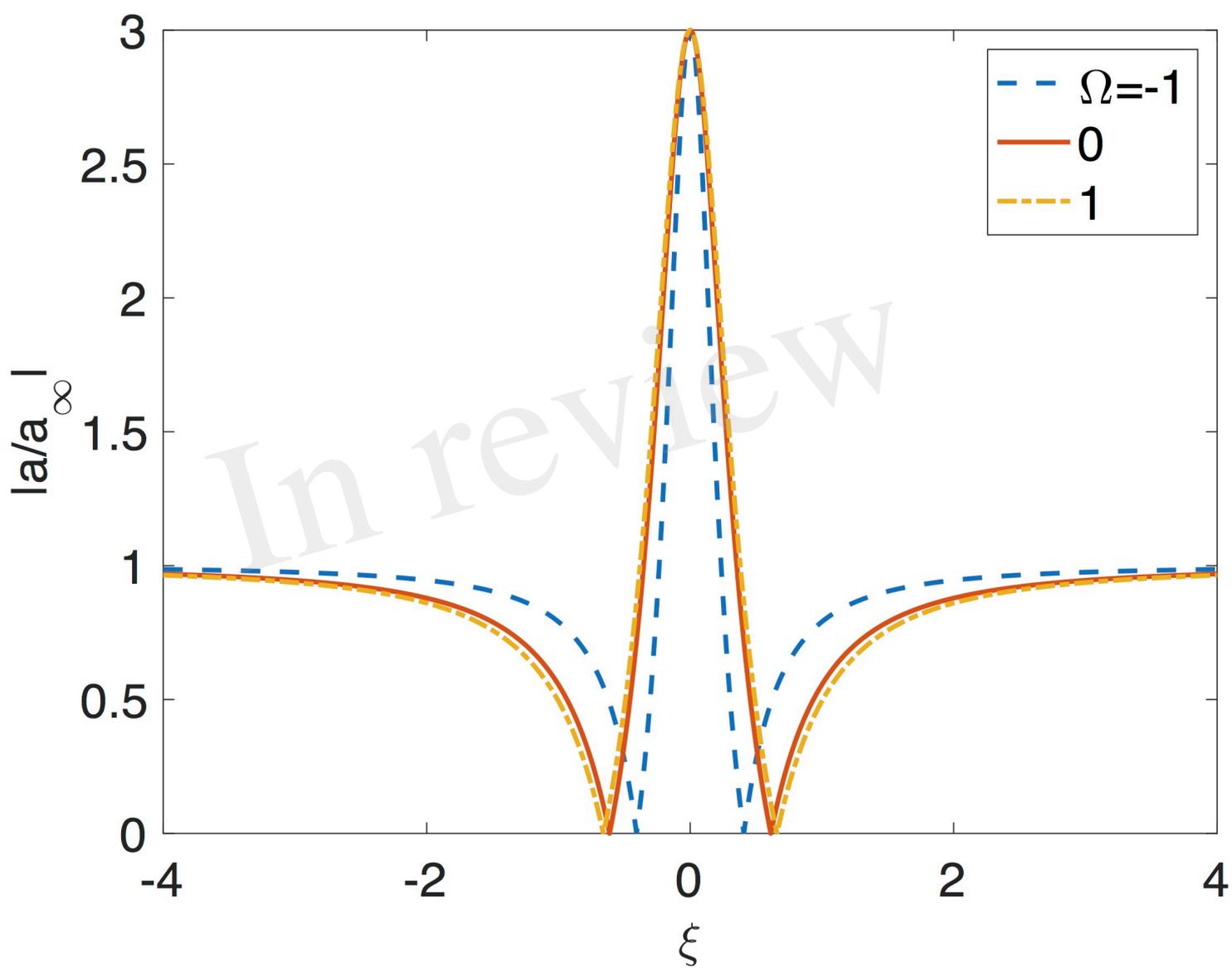


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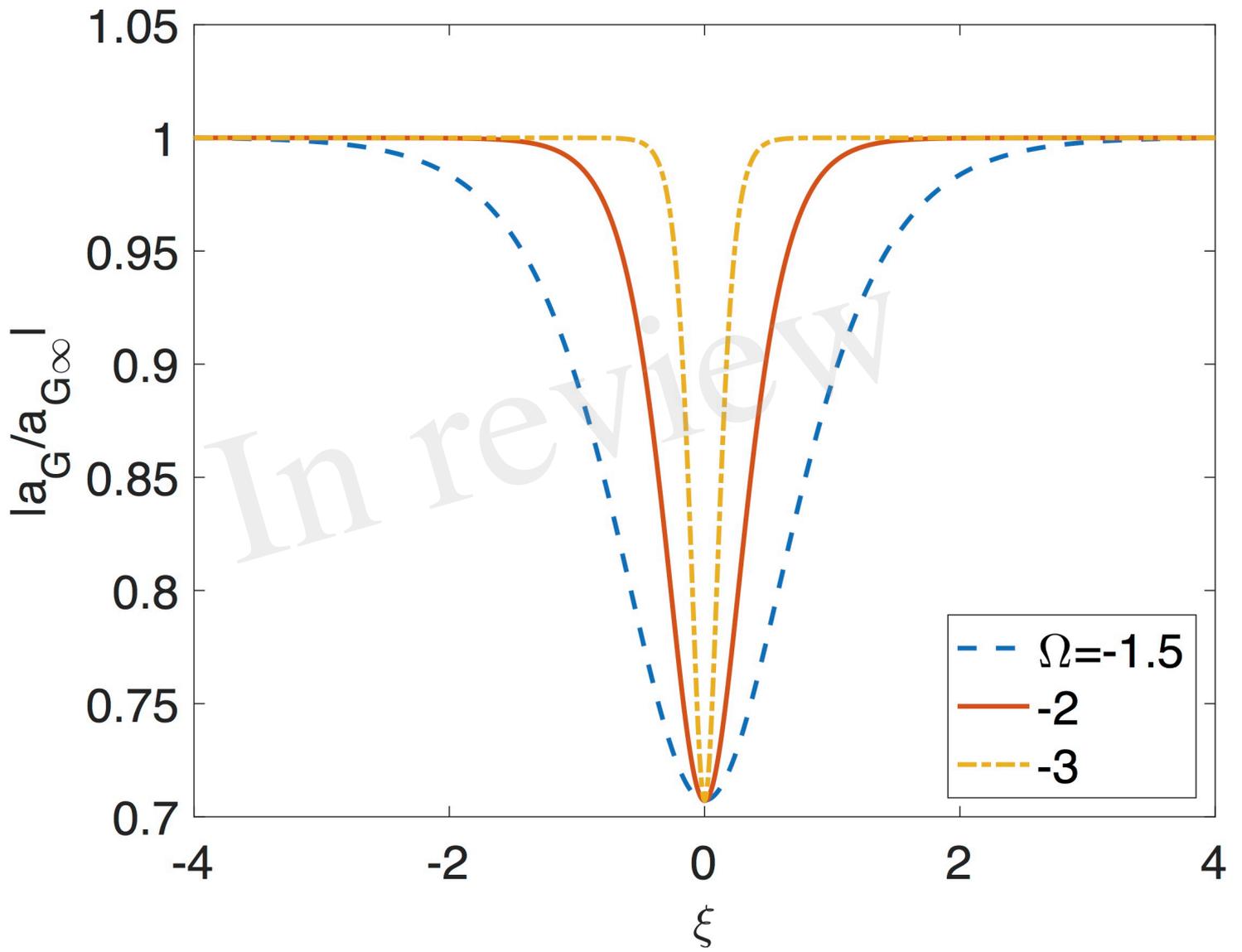


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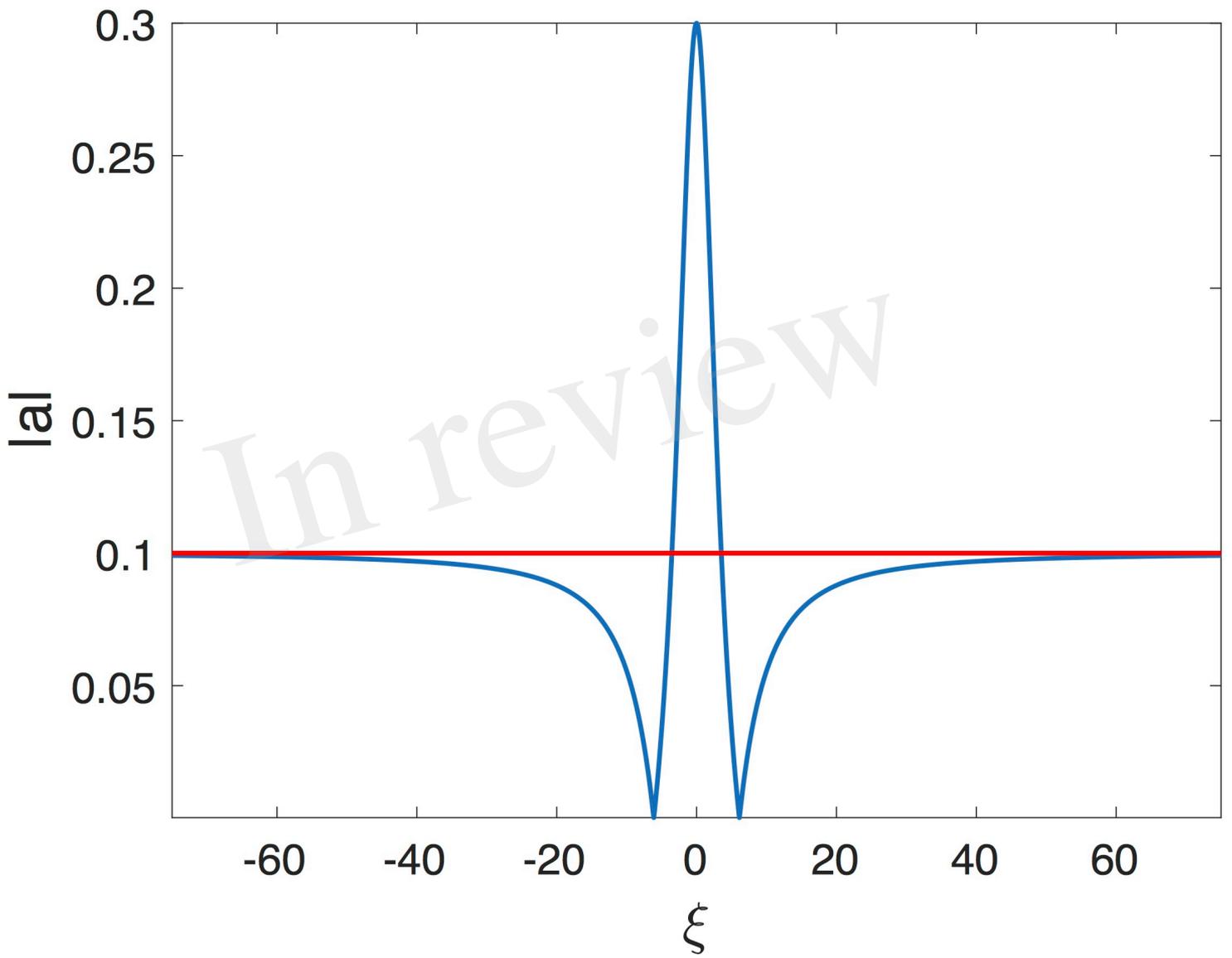


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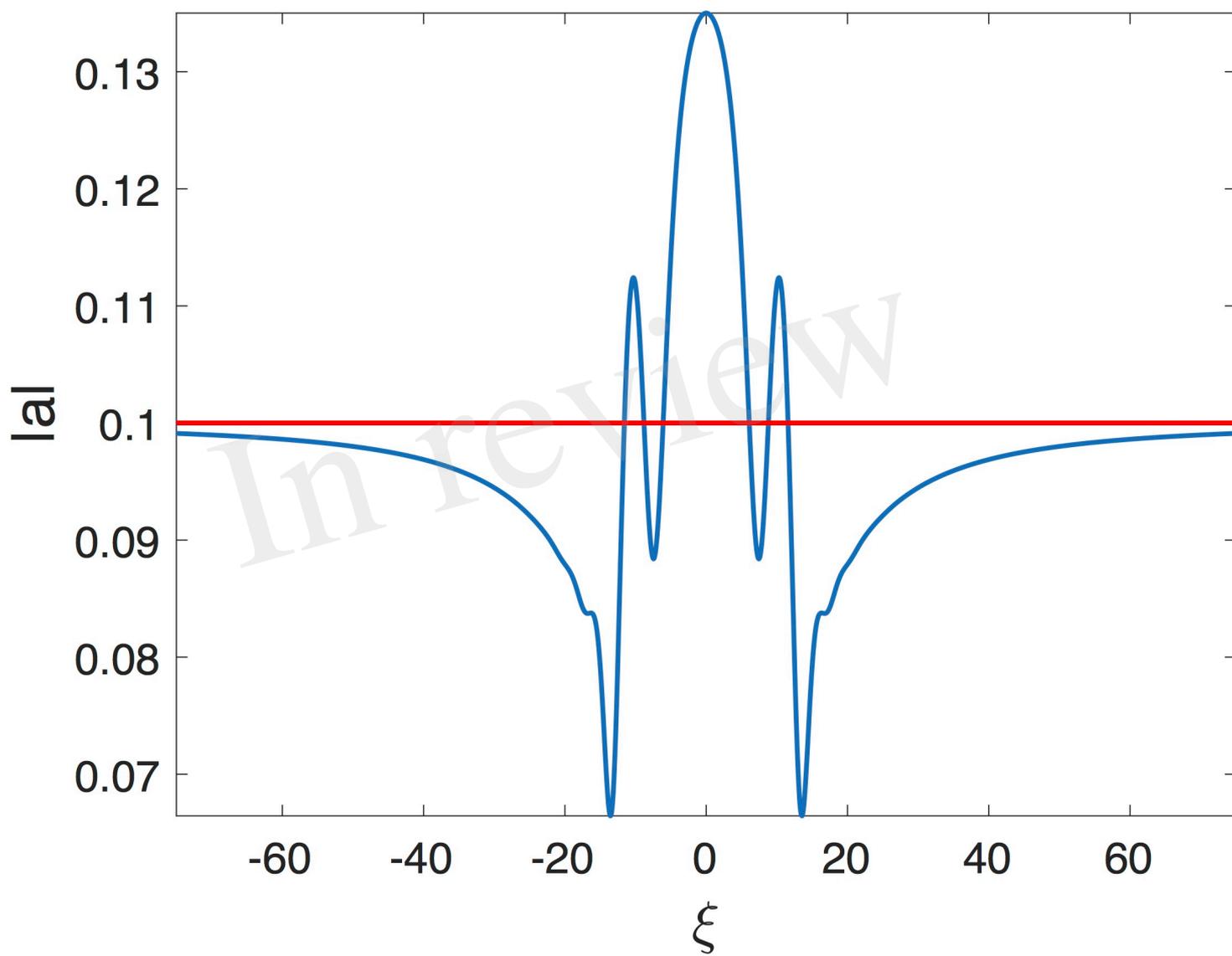


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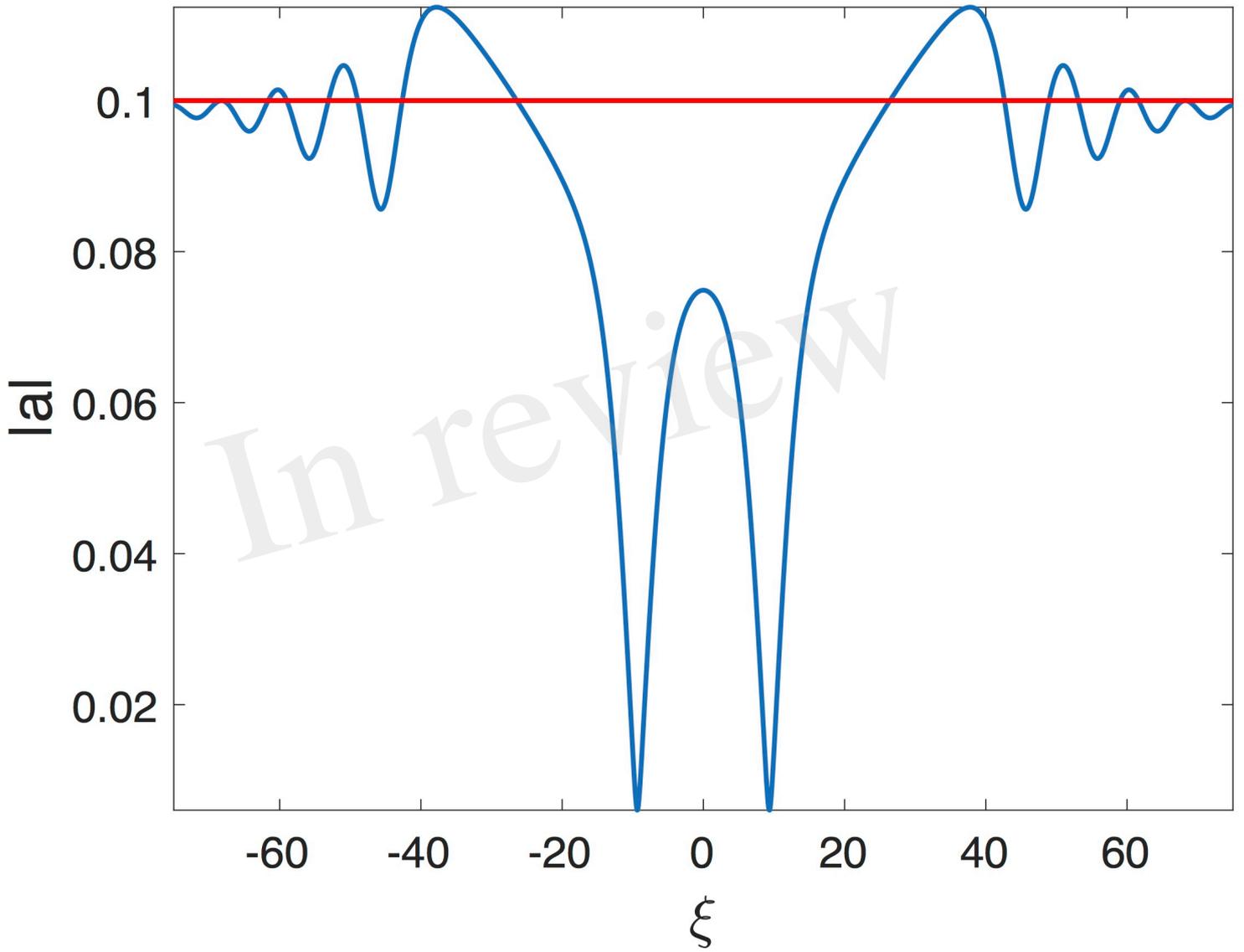


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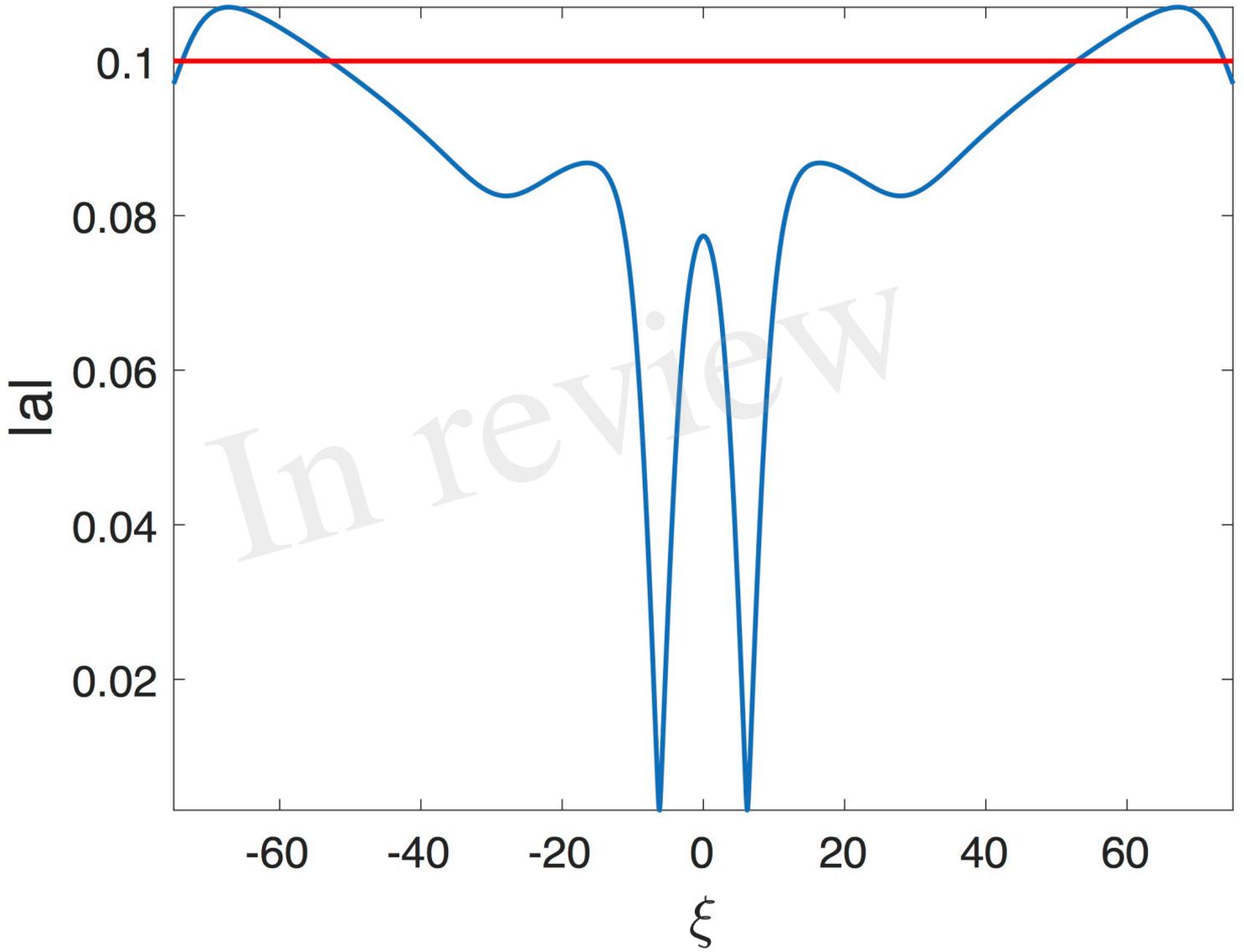


Figure 8.JPEG

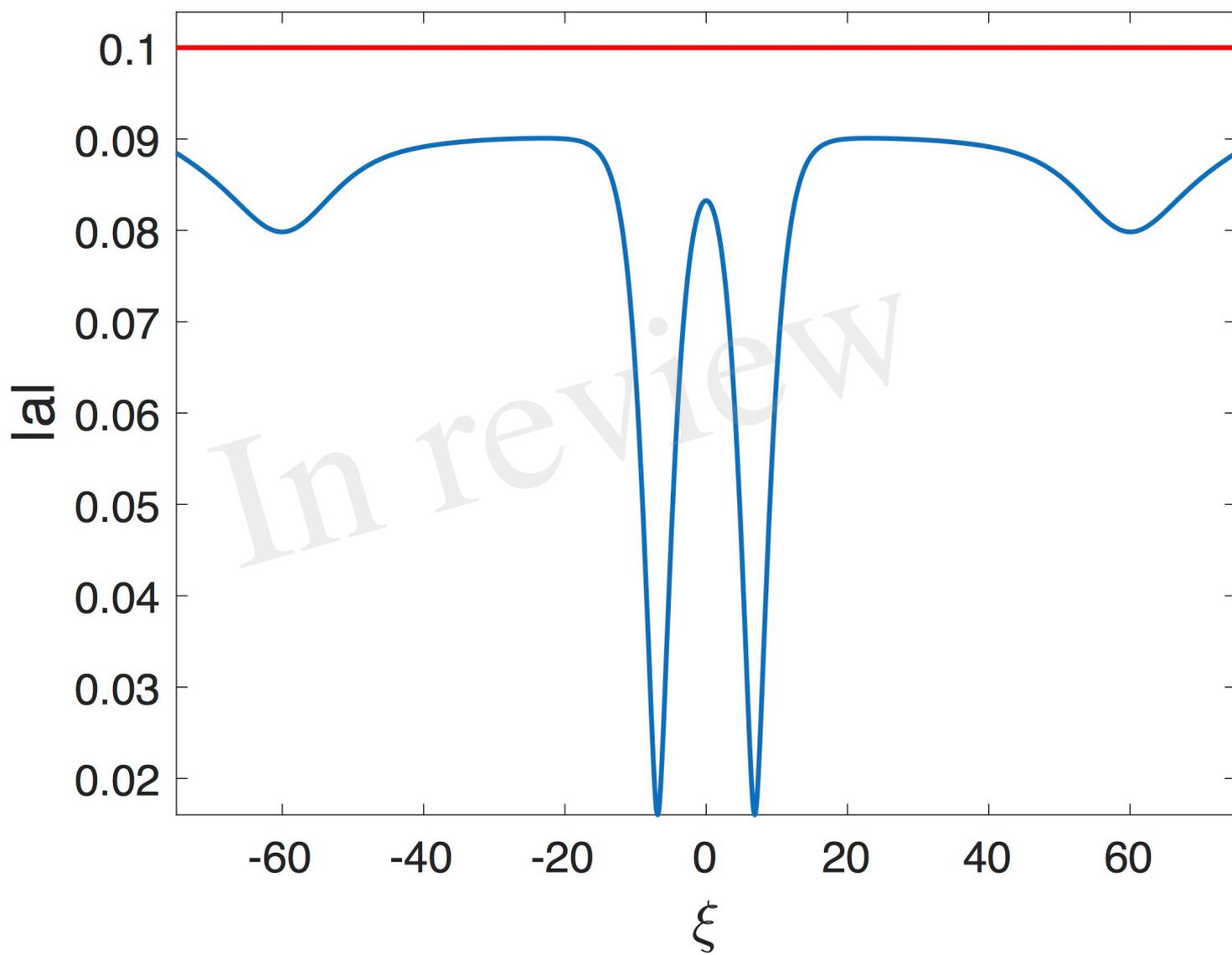


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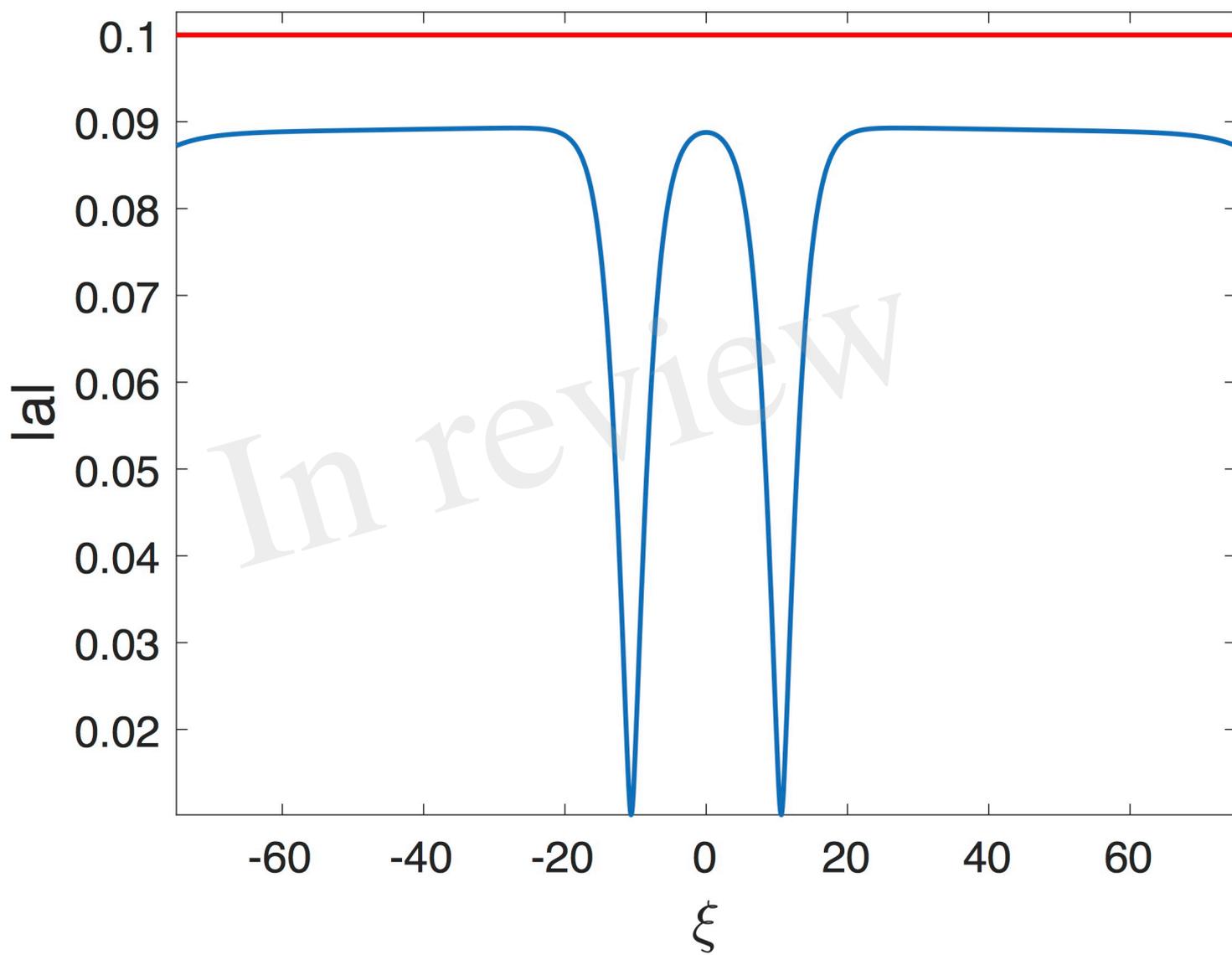


Figure 10.JPEG

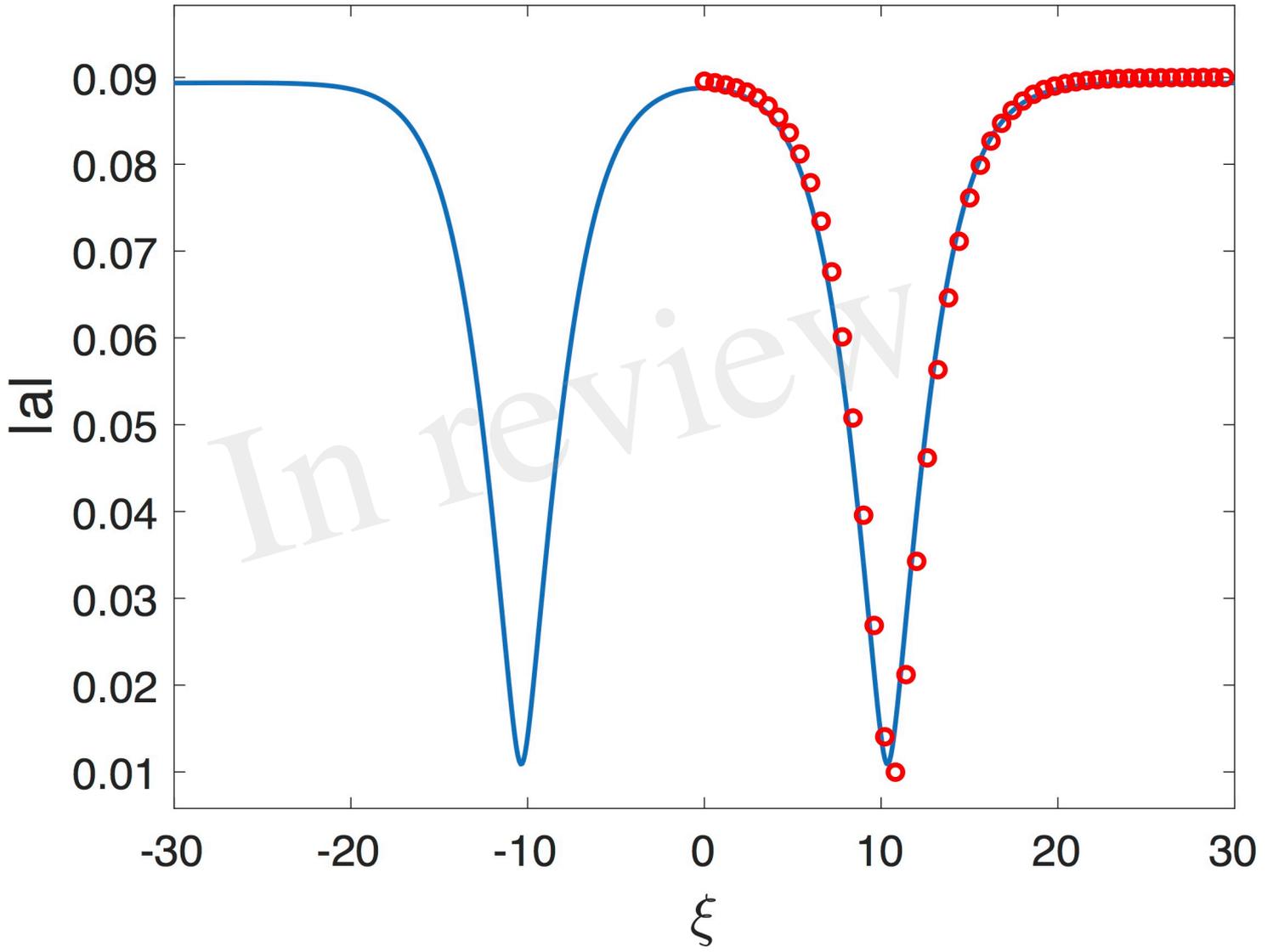


Figure 11.JPEG

