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Optimal modal basis approaches fundamental limits to free-space optical system capacity imposed by atmospheric turbulence

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ABSTRACT

We have considered the problem of finding optimal transmission beams, or eigenmodes, for propagation through the random atmosphere with minimum distortion. We have found, making a realistic assumption that a transmitter knows atmospheric turbulence statistics, but not the instantaneous state of the atmosphere, that these stochastic eigenmodes are Laguerre-Gauss (LG) modes with beam waist chosen adequately relative to the field coherence length in the receiver plane. We have shown in our analysis that these eigenmodes have the remarkable capability of enduring coherence during propagation in turbulence. The ability to order the strength of individual stochastic eigenmodes notwithstanding turbulence is fundamental. It allows using of optimal transmission strategies that only consider the small number of most robust modes and significantly outperform conventional fixed mode sets under weak or strong turbulence conditions. Using these eigenmodes as transmit and receive bases minimizes signal degradation by turbulence, and minimizes the complexity of any signal processing for compensating the impact of turbulence. Adaptive optics can be replaced by adaptive multi-input multi-output signal processing, enabling compensation of fast fluctuations of both phase and amplitude.

Keywords: Free-space communications, space multiplexing, atmospheric eigenmodes, capacity limits.

1. INTRODUCTION

The propagation of beams through random media is of fundamental importance in applications such as optical communications, remote sensing, and imaging. However, coherent fields that propagate through random media such as atmospheric turbulence are subject to distortion and scintillation that can cause considerable degradation in system performance. In a modal communication system [1][2], which exploits spatial degrees of freedom by launching modulated data signals onto different orthogonal spatial optical modes, aberrations induced by turbulence on the transmitted modes result in signal degradation from mode coupling and mode cross-talk, thus reducing the channel capacity of the system.

Here, we consider the problem of finding optimal transmission beams [3][4] for propagation through the random atmosphere based on the knowledge of the second-order moment of the propagated field. The covariance function $C(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2)$ is the crucial ingredient in the coherent mode decomposition analysis, as it encodes the notion of similarity between the fields at points $\boldsymbol{\rho}_1, \boldsymbol{\rho}_2$.¹ The statistics of a field that has propagated through atmospheric turbulence can be characterized by the covariance or, similarly, by its associated normal-mode decomposition, a statistical procedure that uses an orthogonal transformation to convert a set of observations of correlated fields into a set of amplitudes of linearly uncorrelated fields called eigenmodes.

Assuming that the covariance function C of the atmospheric fields is integrable over the region \mathbf{R}_1 occupied by the fields in the transmitter plane, it represents a Hilbert–Schmidt kernel and, by *Mercer’s theorem*, it may be expanded in a series of orthogonal functions of the form²

$$C(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \sum_n \lambda_n \psi_n^*(\boldsymbol{\rho}_1) \psi_n(\boldsymbol{\rho}_2), \quad (1)$$

¹ It is a basic assumption that points with inputs $\boldsymbol{\rho}_1, \boldsymbol{\rho}_2$ that are close in the receiver aperture are likely to have similar field values, and thus, under a Gaussian process view, it is the covariance function that defines nearness or similarity.

² Mercer’s theorem and Hilbert–Schmidt kernels are introduced in the theory of integral equations. (See, for instance, [5].)

where the eigenvalues λ_n and the eigenmodes $\psi_n(\boldsymbol{\rho})$ satisfy the Fredholm integral equation

$$\int_{R_1} C(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) \psi_n(\boldsymbol{\rho}_1) d\boldsymbol{\rho}_1 = \lambda_n \psi_n(\boldsymbol{\rho}_2). \quad (2)$$

The summation in Eq. (1), in general, may be a finite or infinite sum. The eigenvalues are non-negative and, loosely speaking, describe the amount of power allocated on average to the eigenmode ψ_n . Indeed, for $\boldsymbol{\rho}_1 = \boldsymbol{\rho}_2 = \boldsymbol{\rho}$, we have $C(\boldsymbol{\rho}, \boldsymbol{\rho}) = \sum_n \lambda_n |\psi_n(\boldsymbol{\rho})|^2$. The eigenmodes are orthogonal and are typically taken to be orthonormal. The orthonormality implies that $\int_{R_1} \psi_n^*(\boldsymbol{\rho}) \psi_m(\boldsymbol{\rho}) d\boldsymbol{\rho} = \delta_{nm}$, with δ_{nm} the Kronecker symbol. Equation (1) is often called the coherent mode representation of the covariance function.

2. STATISTICAL EIGENMODES OF LASER BEAMS IN THE ATMOSPHERE

The eigenvalues λ_n and the eigenmodes $\psi_n(\boldsymbol{\rho})$ of the coherent mode representation are obtained as the solution of the homogeneous Fredholm integral equation Eq. (2). For coherent beams in Kolmogorov atmospheric turbulence, a propagation model using the extended Huygens-Fresnel principle expresses the covariance function C at the receiver aperture as³

$$C(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \exp\left(-\frac{|\boldsymbol{\rho}_1|^2 + |\boldsymbol{\rho}_2|^2}{2\omega_0^2}\right) \exp\left(-\frac{|\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2|^{5/3}}{2\delta^{5/3}}\right), \quad (3)$$

where ω_0 is the beam intensity radius, and $\delta = r_0/6.88^{3/5}$ is proportional to the coherence diameter r_0 describing the spatial correlation of field fluctuations in the receiver plane. Unfortunately, as happens frequently in the study of propagation effects through the turbulent atmosphere, the 5/3-power law makes the analytic solution to Eq. (2) intractable.

In order to overcome the difficulties with the 5/3-power law, when the coherence width δ is small ($\omega_0 > \delta$), we can consider a statistical model in which the random fields are expressed as finite sums over statistically independent cells in the receiver aperture. To a good approximation, the field can be considered to consist of $(\omega_0/\delta)^2$ independent speckle cells, each of radius δ . As the spatial variation of the random field in any of these independent cells can be described as a quadratic power law, we can approximate the total field covariance as a linear superposition of a number of Gaussian-shaped basis functions. We have shown that this expansion leads to a squared exponential covariance function:

$$C(\boldsymbol{\rho}_1, \boldsymbol{\rho}_2) = \frac{1}{2\pi\omega_0^2} \exp\left(-\frac{|\boldsymbol{\rho}_1|^2 + |\boldsymbol{\rho}_2|^2}{2\sigma_g^2}\right) \exp\left(-\frac{|\boldsymbol{\rho}_1 - \boldsymbol{\rho}_2|^2}{2\sigma_s^2}\right), \quad (4)$$

with σ_g^2 and σ_s^2 described in terms of ω_0^2 and δ^2 as $\sigma_g^2 = (\omega_0^2 + \delta_0^2)$ and $\sigma_s^2 = (\omega_0^2 + \delta_0^2) \delta_0^2 / \omega_0^2$. We have verified numerically that this approach is very accurate, even when the number of speckle cells $(\omega_0/\delta)^2$ within the receiver area is very small.

The advantage of considering the modulated squared exponential covariance function (4) is that its coherent mode decomposition can be developed analytically. We have shown, as can be verified by direct substitution, that its stochastic eigenmodes $\psi_n(\boldsymbol{\rho})$ are given by the normalized Laguerre-Gauss (LG) modes

$$\psi_n(\rho, \phi) = \frac{1}{\omega} \sqrt{\frac{2p!}{\pi(p+|l|)!}} \left(\sqrt{2} \frac{\rho}{\omega}\right)^{|l|} L_p^{|l|} \left(2 \frac{\rho^2}{\omega^2}\right) \exp\left(-\frac{\rho^2}{\omega^2}\right) \exp(jl\phi), \quad (5)$$

where $L_p^{|l|}$ are the normalized Laguerre polynomials and $l = 0, \pm 1, \pm 2, \dots$ and $p = 0, 1, 2, \dots$ are arranged in a double-indexed sequence. The ψ_n are sorted in ascending order $n = 1, 2, \dots$ by decreasing eigenvalues λ_n :

³ Atmospheric turbulence has been studied extensively, and various theoretical models has been proposed to describe turbulence-induced phase degradation and amplitude fluctuations. The extended Huygens Fresnel principle is often used to formulate the field correlations at the receiver aperture. (See, for instance, [6].)

$$\lambda_n = \frac{\pi}{2} \omega^2 (1 - \xi) \xi^{\frac{|l|}{2} + p}. \quad (6)$$

Here,

$$\xi = \frac{1 + 2 \frac{\sigma_g^2}{\sigma_s^2} - \sqrt{1 + 2 \frac{\sigma_g^2}{\sigma_s^2}}}{1 + 2 \frac{\sigma_g^2}{\sigma_s^2} + \sqrt{1 + 2 \frac{\sigma_g^2}{\sigma_s^2}}}$$

and

$$\frac{1}{\omega^2} = \frac{1}{\omega_0^2} + \frac{1}{2\sigma_g^2} \sqrt{1 + 2 \frac{\sigma_g^2}{\sigma_s^2}},$$

where ω is the effective beam width of the modes, which, as expected, depends on the coherence width δ (see Fig. 1).

3. SPACE-MULTIPLEXED LASER COMMUNICATION

In this section, we investigate transmission using orthogonal spatial modes over a general atmospheric channel, applying mode-division multiplexing to near-field line-of-sight (LOS) free-space optical communication. Here, the term ‘‘near-field’’ refers to link geometries in which, on average, the minimum spot size on the receiver plane is smaller than the receiver aperture, yielding near-perfect power coupling when the beam is focused on the receiver. We consider a modal multiplexing system that transmits a complete set of spatial modes, such as all the LG, with a common central rotation axis. We assume that all modes are efficiently multiplexed at the transmitter, and spatially co-propagate through the atmospheric channel. We consider that at the receiver, the modes are demultiplexed without loss or crosstalk (see, e.g., [7][8]).

Given N transmit and receive modes, and frequency-flat fading, the baseband complex model we consider is

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n},$$

where \mathbf{x} and \mathbf{y} are the $N \times 1$ input and output vectors, respectively, while $\mathbf{n} \sim \mathcal{N}(\mathbf{0}, \sigma^2)$ is additive circularly symmetric complex Gaussian noise. The input vector component x_j represents the signal amplitude transmitted in the j th mode, while the output vector component y_i represents the signal amplitude received in the i th mode. The channel is characterized by the $N \times N$ random matrix \mathbf{H} , with entries H_{ij} representing the scattering gain from transmit mode j to receive mode i .

We define a turbulence-free signal-to-noise ratio (SNR) $\gamma = P/\sigma^2$, where P is the total transmitted power in all modes and σ^2 is the receiver noise power per mode. We assume received signals are detected coherently [9][10] and the dominant noise source is local oscillator shot noise, so γ equals the number of received signal photons. As we are considering near-field links, where the receiver collects virtually all the signal power leaving the transmitter, γ also equals the number of transmitted signal photons. Consequently, given transmit and receive bases of N spatial modes, the ergodic average spectral efficiency (in bit/s/Hz) per polarization is [11][12]

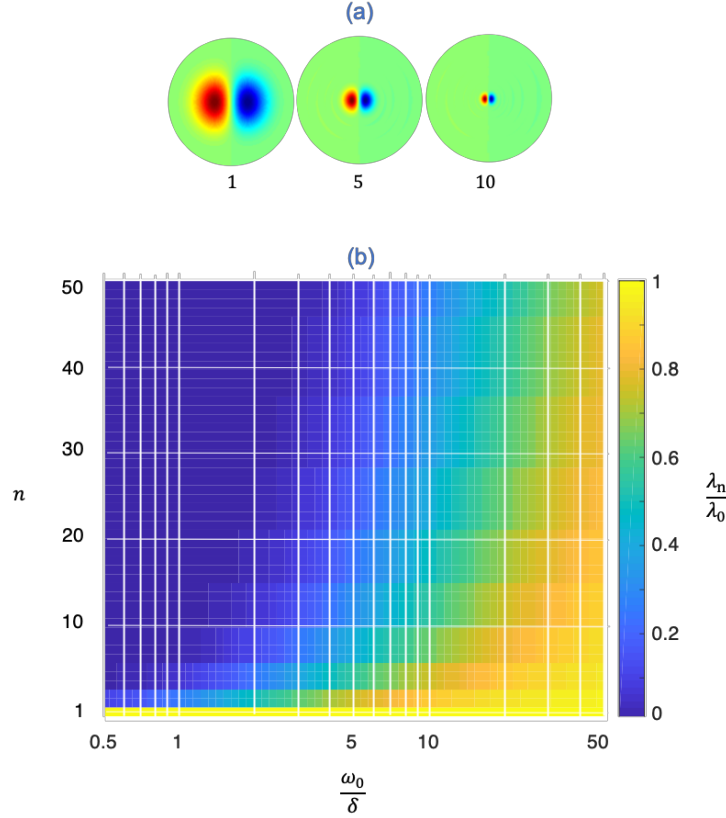


Fig. 1. (a) The effective beam width ω of the modes depends on the coherence width δ . As an example, the plot shows the evolution of the third eigenmode ($n = 3$) with the coherence parameter (ω_0/δ). Modes under weak, medium, and strong turbulence conditions, with (ω_0/δ) equal to 1, 5, and 10, respectively, are considered. (b) Color map plot of the eigenvalues λ_n of the atmospheric covariance function. They have been normalized to the largest eigenvalue λ_1 . Eigenvalues are sorted in ascending order, as given by the mode index n , as a function of coherence parameter (ω_0/δ).

$$S = \max_{\mathbf{Q}} E_{\mathbf{H}} \{ \log_2 |\mathbf{I} + \gamma \mathbf{H} \mathbf{Q} \mathbf{H}^*| \}, \quad (7)$$

where the $N \times N$ matrix $\mathbf{Q} = E[\mathbf{x} \mathbf{x}^*]$ denotes the input covariance matrix \mathbf{Q} whose trace is normalized to unity.⁴ The expectation $E_{\mathbf{H}}\{\}$ is an ensemble average over realizations of the channel matrix \mathbf{H} , \mathbf{I} is an $N \times N$ identity matrix, and $|\cdot|$ represents a matrix determinant.

If we consider channels that either vary slowly over time and/or are reciprocal, information on the instantaneous channel state, i.e., the realization of \mathbf{H} can be assumed known at the transmitter. In this case, the channel can be partitioned by eigenbeam-forming into orthogonal sub-channels that are determined by the instantaneous channel state. Due to the non-uniform gains of these orthogonal sub-channels, or instantaneous eigenmodes, channel capacity can be maximized by

⁴ A given transmission strategy is completely characterized by its covariance matrix \mathbf{Q} . Let us decompose the normalized input covariance as $\mathbf{Q} = \mathbf{V} \mathbf{P} \mathbf{V}^*$ identifying the eigenvectors of \mathbf{Q} with the columns of the unitary matrix \mathbf{V} and its eigenvalues with the diagonal entries of \mathbf{P} . Both the eigenvectors and the eigenvalues have immediate engineering significance. The strategy consists of transmitting independent symbols along the eigenvectors of \mathbf{Q} , with the corresponding eigenvalues specifying the powers allocated to each eigenvector. It is worth considering the capacity-achieving forms of \mathbf{Q} for two relevant regimes, i.e., when instantaneous information is available for tracking the states of the atmospheric channel \mathbf{H} , and when limited statistical information about \mathbf{H} is used.

allocating transmit power control to each sub-channel based on the water-pouring theorem. In this case, the ergodic-average capacity S given by Eq. (7) simplifies to the sum

$$S = E_{\mathbf{H}} \left\{ \sum_{n=1}^N \log_2(1 + \gamma \alpha_n \kappa_n) \right\}, \quad (8)$$

where κ_n is the n th eigenvalue of $\mathbf{H}^* \mathbf{H}$ representing the gain of the n th instantaneous eigenmode, and α_n is the corresponding optimal transmitted power allocation resulting from a power-constrained $\sum_{n=1}^{N_{LG}} \alpha_n = 1$ water-filling optimization. Instantaneous eigenmodes corresponding to larger eigenvalues receive more power and the power allocated to some of the eigenmodes may be zero. As can be seen from Eq. (8), the spectral efficiency is determined by the number N of independently addressable spatially multiplexed sub-channels and by their corresponding λ_n .

If the transmitter has access to the statistical distribution of \mathbf{H} , but not to \mathbf{H} itself, the transmit sub-channel directions and power allocations cannot be based on knowledge of the instantaneous channel state, but can be optimized based on statistics of the channel states that are fed back from the receiver to the transmitter.⁵ Now, we consider that the eigenvectors of the input covariance \mathbf{Q} are equal to the stochastic LG eigenmodes (5) of the channel covariance \mathbf{C} . The elements H_{ij} of the channel matrix \mathbf{H} are now the complex coupling coefficient between transmit and receive stochastic LG eigenmodes. We address the optimization of the eigenvalues of the transmit covariance matrix, i.e., the power allocation policy, with a water-pouring algorithm. The fading channel cannot be partitioned into orthogonal independent channels and the ergodic-average spectral efficiency S is computed as

$$S = E_{\mathbf{H}} \{ \log_2 | \mathbf{I} + \gamma \mathbf{H} \mathbf{D} \mathbf{H}^* | \}. \quad (9)$$

The entries d_n of the diagonal matrix \mathbf{D} are the optimal power allocation resulting from a power-constrained $\sum_{n=1}^{N_{LG}} d_n = 1$ water-filling solution applied on the eigenvalues λ_n (6) of the stochastic LG eigenmodes.

In order to understand the benefit offered by the transmission strategies described above, we also consider the extreme case in which the transmitter has access to neither instantaneous nor statistical channel state information. When the transmitter has statistical knowledge of the channel state, it can adjust its transmission strategy to the correlation properties of the channel and adapt the beam width ω of the set of stochastic eigenmodes to the spatial statistics of the received fields, as described by δ . When no channel information is available at the transmitter, however, the transmitter cannot adapt to potential correlation in the channel. In this scenario, the only possible strategy is to use a family of LG modes with fixed beam width ω_0 and split the transmit power equally over all transmit modes, even though some of these beams will not propagate effectively through the channel. The elements H_{ij} of the channel matrix \mathbf{H} are now the complex coupling coefficient between transmit and receive fixed LG modes. In this extreme case, the fading channel is completely unknown at the transmitter and the ergodic-average spectral efficiency S is computed as

$$S = E_{\mathbf{H}} \left\{ \log_2 \left| \mathbf{I} + \frac{\gamma}{N_{LG}} \mathbf{H} \mathbf{H}^* \right| \right\}. \quad (10)$$

We use Monte Carlo simulations to assess the performance obtained using different SMM FSO transmit and receive modal bases. We approximate the field disturbance caused by turbulence using a phase screen model. The phase-screen turbulence model is valid for the near-field links considered in this analysis, in which the phase aberration is the most significant turbulence distortion, and power scintillation is small. Intuitively, in the presence of turbulence, intense scintillation occurs when a beam passing through random phase distortions interferes with a spatially shifted version of itself as it propagates over a long atmospheric path. We introduce the influence of turbulence in the transmitting modes and consider an overlap integral to estimate the complex coupling coefficient between pairs of transmit and receive modes characterizing the random channel matrix \mathbf{H} . In the Monte Carlo experiments, we generate 10^4 instances of the channel

⁵ If the transmitter receives the channel covariance matrix \mathbf{C} only instead of the concrete channel realization \mathbf{H} , it does not have any information about the actual fading of each transmit-receive pair but possesses directional information regarding the signal subspaces that can be used for beamforming. The motivation for this approach stems from the fact that the channel statistics –i.e., the covariance \mathbf{C} of the atmospheric channel– vary over much larger time scales than the instantaneous channel \mathbf{H} . Therefore, the statistical information can be easily obtained by exploiting reciprocity, or by employing feedback channels with significantly lower bandwidth compared with instantaneous \mathbf{H} feedback systems.

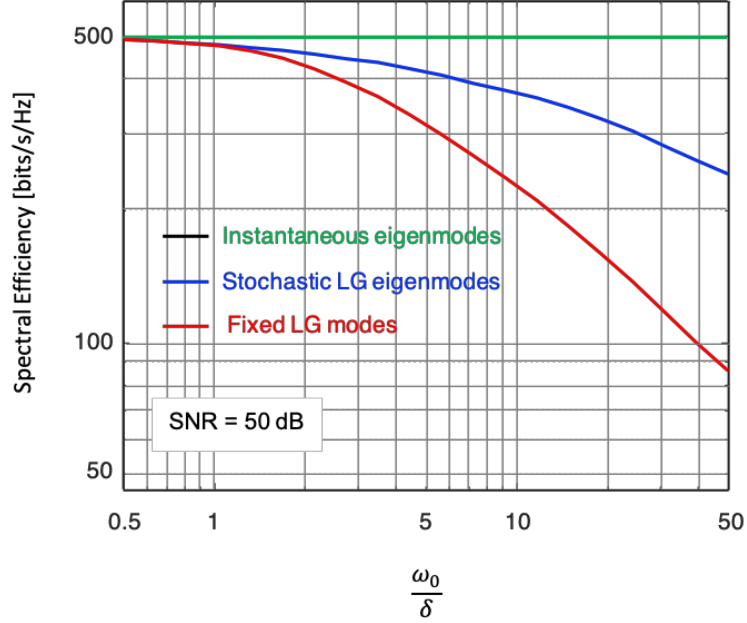


Fig. 2. Spectral efficiency is shown as a function of coherence parameter (ω_0/δ) and considering a 50-dB SNR γ . The analysis assumes several transmission strategies \mathbf{Q} with $N = 45$ modes: Instantaneous eigenmodes (green line) consider instantaneous channel information at the transmitter to partition the channel into orthogonal sub-channels and allocate transmit power with a water-pouring algorithm; Stochastic eigenmodes (blue line) contemplate statistical channel information at the transmitter to find the eigenvectors of the input covariance and use a water-pouring algorithm to address power allocation. Fixed LG modes (red line) assume that no channel information is available at the transmitter and split the transmit power equally over all transmit modes.

matrix \mathbf{H} for the different transmit and receive basis sets and use them to collect the statistics of capacity given by Eqs. (8)-(10).

Figure (2) show plots of the spectral efficiency with $N = 45$ modes as a function of coherence parameter (ω_0/δ) and a signal level described by SNR $\gamma = 50$ dB. It shows that better performance is obtained when statistical information about the channel is used. The capacity improvement obtained by exploiting even partial channel knowledge is substantial. It is interesting to observe the anticipated weak-turbulence behavior at small coherence parameter (ω_0/δ), where the entire transmit power is allocated to a small number of individual eigenvectors. Only when $\omega \sim \omega_0$, for values of the coherence parameter (ω_0/δ) ~ 1 , do the performance of the LG modes and the stochastic eigenmodes become comparable.

4. SUMMARY

We need to understand fundamental limits to FSO system capacity caused by atmospheric turbulence and to recognize transmission schemes that can approach these limits in practice. This necessity is particularly real in SMM FSO systems. By considering separate spatial dimensions linked to different light modes, SMM is an anticipated transmission strategy to increase data capacity in near-field FSO communication. However, light modes affected by distortion and scintillation after propagation through atmospheric turbulence can degrade SMM performance.

Here we have considered the problem of finding optimal transmission beams for propagation through the random atmosphere with minimum distortion and found that these stochastic eigenmodes are LG modes with beam waist chosen adequately relative to the field coherence length in the receiver plane. Using these eigenmodes as transmit and receive bases minimizes signal degradation by turbulence, and minimizes the complexity of any signal processing for compensating the impact of turbulence. We have discussed how transmission of this mode family over a general

atmospheric channel is likely to function in FSO, achieving high capacity transmission through increased spatial multiplicity. In our analysis, we have considered an SMM system that transmits the complete set of spatial light eigenmodes, with a shared central rotation axis, that spatially co-propagate through the atmospheric channel. Technology developed for (de)multiplexing and multi-input multi-output signal processing in fiber SMM systems can be leveraged to help make FSO SMM systems practical and vastly increase transmission capacity. Fundamentally lossless modal multiplexers and demultiplexers can map the stochastic eigenmodes to/from single-mode waveguides, making the implementation of modal transmission systems straightforward.

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