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André Chateau Akué Asséko, Benoit Cosson, Salim Chaki, Clément Duborper, Patricia Krawczak, et al.. Coupling inverse fin method with infrared thermography to determine the effective thermal conductivity of extruded thermoplastic foams. *JMST Advances*, 2020, 2 (4), pp.103-110. 10.1007/s42791-020-00036-2. hal-03136293

HAL Id: hal-03136293

<https://hal.science/hal-03136293v1>

Submitted on 24 Jul 2024

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Coupling inverse fin method with infrared thermography to determine the effective thermal conductivity of extruded thermoplastic foams

André Chateau Akué Asséko^{1,2} · Benoît Cosson¹ · Salim Chaki¹ · Clément Duborper^{1,2} ·
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Abstract

An inverse method for determining the in-plane effective thermal conductivity of porous thermoplastics was implemented by coupling infrared thermography experiments and numerical simulation of heat transfer in straight fins having temperature-dependent convective heat transfer coefficient. The microstructure heterogeneity of extruded polyethylene foam, in which pores are filled with air with different levels of open and closed porosity, was taken into account. The obtained effective thermal conductivity values were compared with previous results obtained using a numerical solution based on periodic homogenization techniques (NSHT) and the transient plane source technique (TPS) to verify the accuracy of the proposed method. The results show that the suggested method is in good agreement with both NSHT and TPS. Moreover, it is also appropriate for structural materials such as unidirectional fiber-reinforced plastic composites, where heat transfer is very different according to the fiber direction (parallel or transverse to the fibers).

Keywords Foam · Fin experiment · Infrared thermography · Steady-transient state · Effective thermal conductivity

1 Introduction

Porous materials are used in many applications, e.g., as insulators, filters and separators. Thus, knowledge of their heat transport properties is of great practical interest. The thermal conductivity of a porous material may significantly differ from that of the solid “bulk” material forming its skeleton [1] and strongly depends on the relative density, the cellular characteristics (cell size, cell density, cell anisotropy), and on the

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presence of additional phases and/or fillers (concentration, orientation and dispersion) [2, 3].

The thermal conductivity of polymeric foams varies considerably because of the existence of two phases (gas phase and solid phase) and the presence of pores (cells) causes inherent anisotropy and usually the orthotropy of thermal conductivity tensor. The overall heat transfer through a porous material is due to the interactions among different modes of heat transfer. Indeed, the effective heat flow across a material can be considered as the sum of conduction across the material matrix, convection of the internal gasses in the pores, and radiation among the solid surfaces within the material [4]. Thus, the thermal conductivity of foam can be considered as having four contributions: (1) conduction through the solid (2) conduction through gas (3) convection within the cells (4) radiation through the cell walls and across the cell voids. The influence of convection, which may locally occur in the small air cavities, is often neglected [5–7]. Indeed, convection within the pores is only important when the Grashoff number (which describes the ratio of the buoyant force driving convection to the viscous force opposing it) is greater than ~ 1000 . Using data appropriate to air at 1 atmosphere, this value is achieved when the pore size exceeds 10 mm in diameter [6, 7]. For pore sizes smaller than this, which is the case in our study, thermal transfer through convection, can be ignored. Many experimental methods already exist to measure the thermal conductivity of heterogeneous materials. Most of them are based on the flash method [8–11] and guarded hot plate devices as in [5]. Temperature of the rear side of a planar sample is measured with an infrared camera, while the front side is heated with a flash lamp or a laser pulse. Then, the thermal conductivity in the thickness direction of the sample can be determined from the measured temperature evolution. The flash method can also be applied to determine the in-plane thermal conductivity [12, 13]. However, in that case, because the signal-to-noise ratio must be quite high (at least 10 [13]), a sensitive infrared camera or a high heating temperature is required. However, it should be noted that using high heating temperature leads to complications with modeling the heat-loss effects.

The present paper proposes a new method to overcome the aforementioned drawback. A new design of a simple experimental set-up with an appropriate inverse analysis method are developed to determine the in-plane thermal conductivity in the longitudinal direction of cellular materials.

2 Background for experimental setup design

The experimental device consists of a constant thermal energy source (a heated aluminum plate) into which the end of a straight fin of uniform cross-section is fixed. The experimental procedure consists in heating the sample by

the heated aluminum plate in perfect contact with the sample base, while measuring temperature field with an infrared camera in steady state, instead of the measurement at different points by means of individual temperature sensors [14–16]. Thus, the whole temperature distribution profile in a planar sample can be measured with high accuracy. The fin method is used in a longitudinal direction of heat transfer and considers heat loss on the upper and lower faces. Generally, the convection heat transfer coefficient varies along the fin as well as its circumference, and the usual assumption that this mode of heat transfer is constant is no longer valid [16]. The velocity and thermal boundary bands increase towards the hot end of the sample, and, consequently, the convective heat transfer coefficient also changes. Thus, it is important to take into account the position dependence of this coefficient by introducing a temperature-dependent heat transfer coefficient in the fin model which becomes nonlinear. Exact solutions are often difficult to obtain for nonlinear problems with variable thermal properties [17]. It is, therefore, relevant to use a numerical finite difference solution to easily solve the problem.

A motivation for designing this experimental setup is its simplicity and accuracy. The experimental procedure is also very simple, robust and quick to determine thermal conductivity. In practice, the sample is thin and the temperature gradient in the thickness is negligible [18] (fin method assuming with small Biot number). Thus, the sample geometry allows one-dimensional modeling (assuming that the sample can be considered as isothermal in the thickness direction), and different heat transfer mechanisms can be more easily included in the model. Temperature changes in the longitudinal direction are much larger than those in the transverse direction (one-dimensional conditions are assumed, even though conduction within the material is actually two dimensional) and the propagation in the sample can be easily produced.

3 Fin method analysis

3.1 Method principle

A constant heat energy source is applied at the end of a rectangular cross-section plate (Fig. 1). When the temperature of the sample has reached the steady state (i.e., when the temperature profile is remaining constant), the temperature profiles measurement is performed on one face of the sample using an infrared camera. After that, a post-processing of the obtained thermal maps (thermograms) is achieved and a finite difference solution is applied to identify the thermal conductivity of the material.

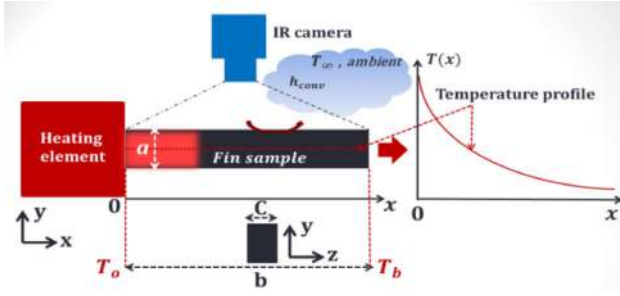


Fig. 1 Fin method principle

3.2 Problem formulation

The models considered in the present work are based on a heat conduction problem resulting in a fin approach [19, 20] in a bar with rectangular cross section and dimensions a , b , and c , associated with y , x , and z directions, respectively (Fig. 1). The analysis for fins of uniform cross-sectional area can be found in any standard heat transfer textbook [18, 19]. It is assumed that the model can be treated as adiabatic at the end ($x = L$), that the convective heat loss from the fin tip is negligible and that the temperature at the base ($x = 0$) is constant, T_0 . To facilitate analysis of the thermal conductivity of the foam glass, the following assumptions are made as in [21, 22]: the medium is an isotropic material, its boundary is adiabatic (meaning that no heat is dissipated at the boundary), thermal conductivity is independent of temperature, the temperature is linearly distributed along the heat flow direction (in the x direction), steady-state conditions are established, no heat generation, heat transfer by both convection and radiation are negligible. Moreover, the fin is assumed to be thin, which implies that the temperature variations in the longitudinal direction are much larger than those in the transverse direction [23, 24]. This assumption can be made when Biot number (Bi) $\ll 1$:

$$Bi = \frac{h_{conv} \times c}{k}, \quad (1)$$

where h_{conv} is the convective heat transfer coefficient, c is the sample thickness and k is its thermal conductivity. This means that the temperature gradients within the solid are small and its initial temperature is that of the ambient air. Indeed, the studied samples are thin (thickness ≈ 2 mm) with a small Biot number ($Bi < 0.1$). Therefore, the internal resistance is negligible and the temperature can be considered as uniform within the body. It is now possible to study the time evolution of the temperature, and therefore, to use the equation of the fins as in [18, 19]. This hypothesis will be verified

at the end of the study. Under these assumptions, the system of energy equation and boundary conditions take the form:

$$\frac{d^2 T(x)}{dx^2} - \frac{ph_{conv}(x)}{kA_s} (T(x) - T_\infty) = 0, \quad (2)$$

where $T = T(x)$ is the temperature of the plate, x is the distance from the heated end of the plate, $\sqrt{\frac{h_{conv}P}{kA_s}}$ is the fin module which characterizes the rapidity with which the heating fades away along the plate for increasing abscissa axis, T_∞ is the ambient air temperature, and $h_{conv}(x)$ is the nonlinear convective heat transfer coefficient (it is position dependent, because of the boundary-layer effects on the air flow across the plate). P is the fin perimeter, A_s the surface area measured from the base to x , and k the thermal conductivity. It is also assumed that the temperature profile $T = T(x)$ is a monotonic function, and thus there is a one-to-one correspondence between x and T , or rather x and $\theta(x) = (T(x) - T_\infty)$. This also means that x can be considered as a function of θ , $x = x(\theta)$. Thus, Eq. (1) can be expressed as:

$$\frac{d^2 \theta(x)}{dx^2} - \frac{ph_{conv}(\theta)}{kA_s} \theta(x) = 0, \quad (3)$$

with an unknown function $h_{conv} = h_{conv}(\theta)$. Also, $h_{conv}(\theta)$ can be replaced by a linear function (as a temperature-dependent heat-loss coefficient). It is assumed that h_{conv} equals to 0 at the ambient temperature (T_∞) (no heat exchange between the environment at the ambient temperature and the sample) and h_{conv} tends towards its maximum value at the sample base (T_0).

$$h_{conv}(\theta) = h_1 \theta, \quad (4)$$

where h_1 is, in principle, a function of T_∞ . However, variation of T_∞ is so small during the experiments that it can be considered as constant. The equation (Eq. 3) is a nonlinear differential heat transfer equation and can easily be solved numerically. Thus, in this equation, the initial and boundary conditions are imposed as follows:

$$\begin{cases} \theta(0) = \theta_0 \\ \frac{\partial \theta}{\partial x} = 0(x = L) \end{cases} \quad (5)$$

3.3 Finite difference solution

The temperature distribution is determined by a numerical scheme using the finite difference method described by Chapra and Canale [25]. In this method, the differential equation of heat conduction is approximated by a set of algebraic equations for temperature at a number of nodal points. The rod is divided into N subregions, each

$\Delta x = b/N$, and the node temperature is denoted by T_n , $n = 0, 1, 2, \dots, N$. The resulting general form of the finite-difference equation for the internal nodes (i.e., $n = 1, 2, \dots, N - 1$) is:

$$\begin{cases} \frac{kA_s}{\Delta x}(T_{n-1} - T_n) + \frac{kA_s}{\Delta x}(T_{n+1} - T_n) - \\ h_{conv}p\Delta x(T_\infty - T_n) = 0 \\ h_{conv} = (T_n - T_\infty)h_1. \end{cases} \quad (6)$$

To determine the thermal conductivity, an inverse problem is considered from the knowledge of stationary temperature profiles measured by infrared thermography. The method consists in minimizing the mean-square deviation between the temperature obtained by the finite-difference solution and the measured temperature (Eq. 7). To minimize this objective function, the “fminsearch” MATLAB function is used, based on the simplex method which is an iterative optimization algorithm for nonlinear minimization without constraints. The experimental data provide θ_0 and h_{conv} and k parameters are determined by minimizing the below integral. In addition, the surrounding air temperature T_∞ is measured using a portable thermocouple.

$$\int_{x_1}^{x_2} (\bar{\theta}(x) - \theta_o(x))^2, \quad (7)$$

where $\bar{\theta}(x)$ is the numerical stationary temperature profile and $\theta_o(x)$ is the measured stationary temperature profile.

3.4 Concept of thermal conductivity measurement

The determination of the effective thermal conductivity of the foamed samples is composed of two steps (Fig. 2). The numerical equation (Eq. 6) contains two terms to be identified (convective heat transfer coefficient and effective thermal conductivity). In the first step, a reference thermoplastic material is used whose thermal conductivity is known so that the nonlinear convective heat transfer coefficient can be experimentally obtained. In the second step, the foamed sample whose effective thermal conductivity is unknown is placed in the same condition as the reference sample. Thus, its effective thermal conductivity can be determined using the heat transfer coefficient previously determined on the reference sample. It is this procedure that was applied in the present study, where the same experimental conditions are imposed to have the same convective heat transfer coefficient. Namely, the temperature gradient or heating rate

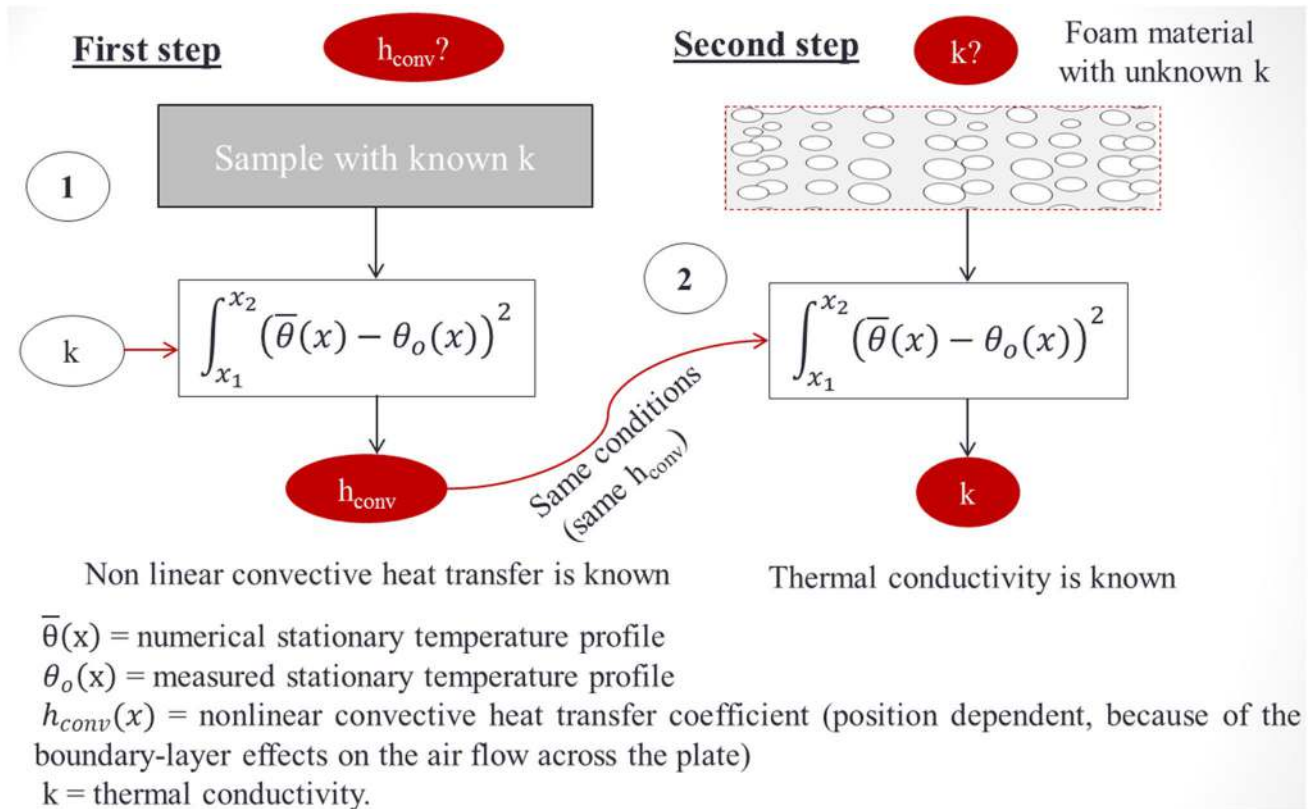


Fig. 2 Identification procedure of foam effective thermal conductivity

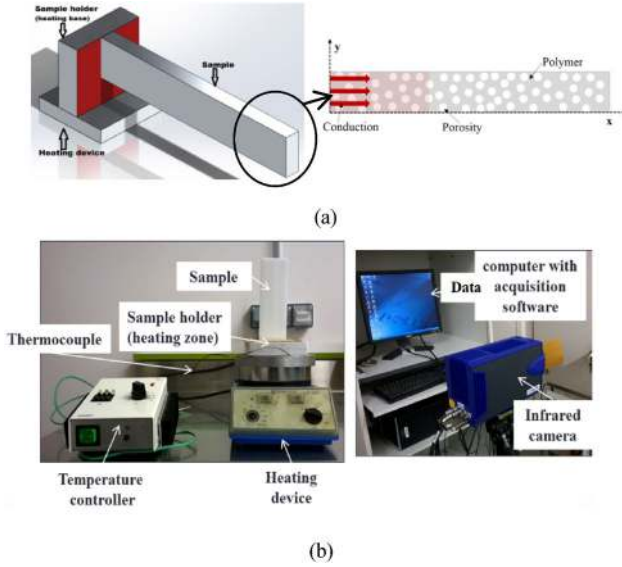


Fig. 3 a Experimental set-up schema. b Experimental devices

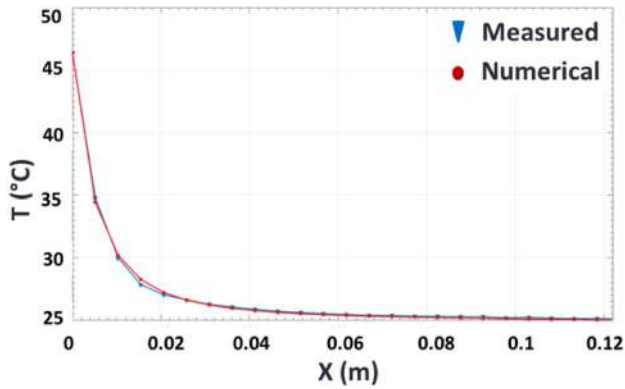


Fig. 4 Comparison of the experimental and computed stationary temperature profiles in the 2 mm thick bulk PE sample

due to the continued convective loss on the surface of the material. Comparison between the temperature profile in the x direction and the numerical finite difference solution based on Eq. 7 shows a very good agreement between numerical and experimental results. The minimization of the difference between the temperature obtained numerically, by the stationary fin model, and the measured temperature gives a value of the parameter $h_1 = 0.44 \text{ W/m}^2\text{K}$. The error between both temperature values is lower than 1%. Figure 5 shows the evolution of convection (h_{conv}) along the x axis in the 2 mm bulk PE sample. The determined value of the constant h_1 is $0.44 \text{ W/m}^2\text{K}$ with an error less than 1%. The convective heat transfer is high at the base of the sample where the flow velocity is the largest. This value of h_{conv} is used in the second inverse

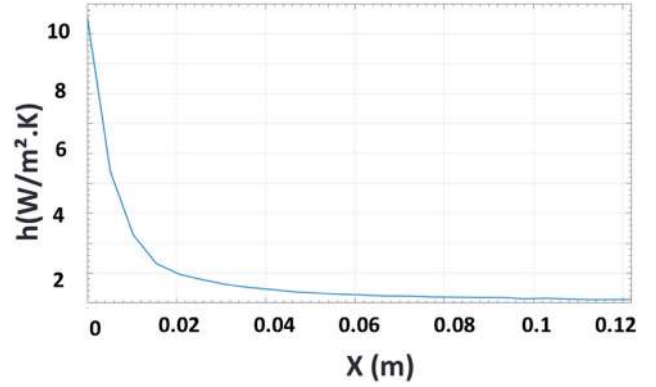


Fig. 5 Temperature dependent convection along x axis in the 2 mm thick bulk PE sample

problem to find out the effective thermal conductivity of the unknown foam materials. The use of this value for all the unknown samples is justified as the surface area in contact with the surrounding and all other experimental conditions, such as heating rate, nature of surrounding medium (air) are the same. The assumption of the small Biot number is well verified ($\text{Bi} = \frac{0.44 \times 0.002}{0.34} = 0.0026 < 0.1$)

5.2 Determination of the effective thermal conductivity of thermoplastic foams

The identification of the effective thermal conductivity of the foam material is performed using the same procedure for identifying the convective heat transfer coefficient (second step on Fig. 1). The experimental and numerical temperature distributions for the base temperature ($T_o = 45 \text{ }^\circ\text{C}$) of PE foam with 45% porosity are presented in Fig. 6. The measured values also match favorably the numerical solution. The relative error between both results remains lower than 1%. The effective thermal conductivity of the last sample, computed by the inverse fin method, is $k = 0.164 \text{ W/(mK)}$. The assumption of the small Biot number is also well verified ($\text{Bi} = \frac{0.44 \times 0.002}{0.164} = 0.0053 < 0.1$). This result is compared with the previous one [21], obtained by a numerical finite element solution and validated with the Transient Plane Source technique (TPS) method (Hot Disk) (Table 2). The experimental values are close to numerical solution. Indeed, the relative error between numerical solution and experimental results, as well as the relative error between numerical solution and TPS (Hot Disk), is lower than 6%. The experimental fin apparatus described in the present paper is consequently successfully validated.

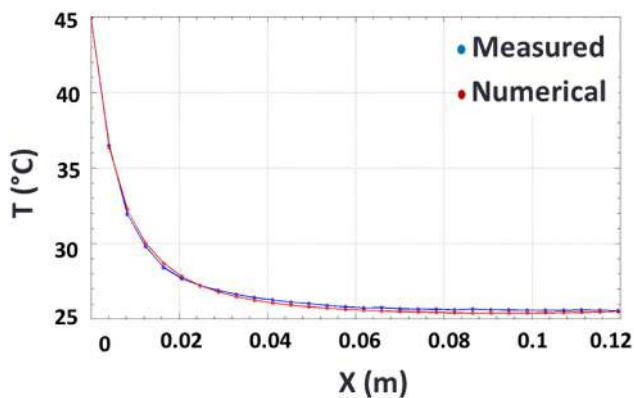


Fig. 6 Temperature dependent convection along x axis in the 2 mm thick bulk PE sample Comparison of the experimental and computed stationary temperature profiles in a 2 mm thick sample of PE foam (45% porosity) (a)

Table 2 Comparison between experimental results from fin method, and previous [21] numerical solution based on periodic homogenization techniques (NSHT) and transient plane source technique (Hot Disk)

Sample	PE foams		
	2	3	4
k (NSHT) [21] (W/(mK))	0.176	0.163	0.042
k (Inverse method) (W/(mK))	0.164	0.154	0.044
k (TPS) [21] (W/(mK))	0.17	0.16	0.041
Deviation (%)	6.8	4.3	4.5

6 Conclusion

An inverse method for determining the effective thermal conductivity of foams was developed. The foamed plate is heated at one end and the steady state. The temperature field of the sample is recorded by an infrared camera. These fields are then fitted by a numerical finite difference solution of the one-dimensional heat equation with temperature-dependent convection heat transfer and effective thermal as two fitting parameters. The relative error between the numerical solution developed in a previous study and experimental results is lower than 6%. Therefore, it is proved that this novel experimental method is well designed and is a suitable alternative for determining thermal conductivity of foamed materials.

The method developed can also be extended to other heterogeneous materials such as structural composites, for example unidirectional fiber-reinforced plastics, where fiber direction should be taken into account because of heat transfer difference parallel or transversally to the fiber direction.

Acknowledgements This project (IFMAS P3A6) has been granted by the French State under the “Programme d’Investissements d’Avenir” Program (contract no. ANR-10-IEED-0004-01) and supported by the French Institute for Biobased Materials (IFMAS, France). The authors thank S. Marcille (Roquette, France) for her support and contribution, and the Hauts-de-France Region (France) for co-funding with IFMAS C. Duborper’s PhD grant (contract no. 15000305).

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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