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# The Ground for Negotiation: Zoning for Risk Reduction around Hazardous Plants\*

Céline Grislain-Letrémy<sup>†</sup> Bertrand Villeneuve<sup>‡</sup>

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## Abstract

The industrialists are liable for any damage they cause to neighboring households. Consequently, households do not have to pay for the risk they create by locating in exposed areas. A common and efficient self-insurance strategy for the firm is to freeze land, or to negotiate land-use restrictions. When people understand only simple messages about risk, the boundaries of the building zone are the ground for negotiation with the mayor. Typical scenarios regarding the distribution of bargaining power between the firm and the mayor are examined. In the comparative statics, we show how red zones are revised as technology or demography change. Further, we give the conditions for a purple zone (limit red zone as the population grows) and a green zone (limit inhabitable zone as the risk grows) to exist.

*Keywords:* industrial disasters, self-insurance, externalities, behavioral constraint, labels in risk regulation, land-use negotiation

*JEL classification:* D83; D91; Q53; R52.

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# 1 Introduction

The urbanization in the vicinity of industrial plants increases the magnitude of industrial disasters. The gas leak from the Union Carbide India Limited pesticide plant in Bhopal in 1984 is dramatic evidence of this trend (Ferrante 2011). As is the explosion of the AZF plant in Toulouse (France) in 2001.<sup>1</sup> The number of deaths and casualties due to a fertilizer plant explosion in North Texas in April 2013 is also attributable to the urbanization in the neighborhood around the plant.<sup>2</sup> Overall, the economic costs of industrial disasters has risen dramatically over the last few decades (Figure 4, page 6 in Bevere et al. 2015). Understanding these phenomena requires to explore the interaction between industrialists, mayors, and people.

The firm that causes the damages is liable for them. Indeed, even though the intertwined histories of risk and urban development can be complex, the law is quite simple: there is no right of “initial land use” by which the industrialist can renege on its responsibility for any disaster compensation to the newcomers. Consequently, households and businesses do not have to pay for the risk they create (and expose themselves to) by locating in exposed areas. They do not need to purchase insurance against these risks, except perhaps legal assistance insurance. This protection might explain why there are households and businesses in hazardous areas. Besides this rational carelessness, other causes aggravate the problem: perception biases, rational ignorance, or employment by hazardous facilities. The implementable policies depend on the behavior towards this particular risk in this particular context.

We propose here an analysis of the negative externality that the “curse of unlimited liability” implies: fully compensated households occupy the available land and, in doing so, they increase the cost of compensation for the firm in the case of a disaster. For the sake of simplicity, we do not consider life and health risk and we assume that the firm does not employ any of the households in our area. Thus, households’ losses are only material, direct, and fully compensable. Our simplifying assumptions facilitate comparisons between bargaining scenarios and risk levels. Land use is the ground for negotiation. Indeed, to limit its own exposure, the firm can buy land around its plant to limit its use by households.

We observe red zones in practice. In Louisiana, the Dow Chemical company in 1991 paid for a whole village of 300 inhabitants to move out of the vicinity of one of its chemical plants (Sauvage 1997). Or the state can delimit red zones. European member states

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<sup>1</sup>The AZF website illustrates how the Toulouse agglomeration progressively has encircled the plant. See [https://www.aria.developpement-durable.gouv.fr/wp-content/files\\_mf/A21329\\_ips21329\\_007.pdf](https://www.aria.developpement-durable.gouv.fr/wp-content/files_mf/A21329_ips21329_007.pdf), page 11.

<sup>2</sup>M. Fernandez and S. Greenhouse, “Texas Fertilizer Plant Fell Through Regulatory Cracks.” *New York Times* April 24, 2013 (<http://www.nytimes.com/2013/04/25/us/texas-fertilizer-plant-fell-through-cracks-of-regulatory-oversight.html>).

The New York Times’ website provides a map of the plant and its vicinity (<http://www.nytimes.com/interactive/2013/04/18/us/fertilizer-plant-explosion.html>).

design land use policies such that appropriate distances are maintained between hazardous facilities and residential areas. [Basta \(2009\)](#) compares implemented red zones around hazardous plants between European countries.<sup>3</sup> Red zones result from a negotiation between the households (e.g. through the mayor) and the firm, because extending the region where building is forbidden reduces the total cost of risk but crowds households at the same time.

The motivation for studying red zones can be summarized by three arguments. The first argument is that red zones are a powerful self-insurance strategy of the firm. The firm does not need land *per se*; it only wants to prevent potential victims from occupying the riskiest locations. The second argument is behavioral: people distort the information given by the delimitation of red zone. People understand better simple labels and classification than finely calculated risk indicators. This is a form of narrow bracketing as in [Read et al. \(1999\)](#). A corollary is that people tend to take legal zones, big or small, as homogeneous: any place in the permitted zone is as good as another. This limited understanding of risk exacerbates the externality exerted by the households on the firm: not only they do not receive price signals, but the signals given by zoning are coarse. The third argument is pragmatic: this risk attitude has to be taken as a constraint by the mayors and firms, and to be exploited ([Koszegi 2014](#)). Zoning being a powerful practical solution to many problems, actual zones have to be optimized according to the players objectives. Our study explores risk management by a mayor and a firm under behavioral constraints imposed by limited understanding of risk.

We develop an urban model of a linear city with a significant risk gradient. A good picture is that of a new developing region with a hazardous plant on one side and on the other an already saturated area or a natural barrier like a mountain. Our model is inspired from the classical urban economics literature (e.g., [Fujita and Thisse 2002](#)). We focus the analysis on the mechanisms through which the bargaining power of the industrialist impacts the design of red zones and their evolution with respect to risk reassessment, technological change, and demographic evolution.

We show that the red zone is increasing with the bargaining power of which side ultimately incurs the loss: when the firm is financially independent of the community, a strong firm wants a bigger red zone than a strong mayor, but in the extreme case where the firm entirely belongs to the community, the red zone is even bigger. This result is based on four comparable scenarios: (1) bargaining power lying with the mayor acting in the interest of the citizens, (2) bargaining power lying with the firm, (3) a benchmark case where all players are price takers, and (4) the extreme case where the firm is entirely the property of the households. They are ranked by decreasing bargaining power of the mayor. The first scenario is the most favorable to households since they extract rent from the firm by conceding land. This scenario gives the smallest red zone. In the second scenario, the firm can pay less for the land

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<sup>3</sup>See Article 12 on the Control of Urbanization in the Council Directive 96/82/EC of 9 December 1996 on the control of major accident hazards involving dangerous substances.

it uses and imposes a larger red zone. The third scenario is between the first two scenarios in term of bargaining power and impact. The fourth scenario gives the largest red zone: this is the worst case for households, because they bear the entire cost of risk; this is why they want to contain risk.

Red zones must be revised as risk and urbanization evolve. The comparative statics explain how. The increasing cost of industrial disasters is largely explained by the growing urbanization of risky areas. Industrial hazards can also change because of technological progress or reassessed risk. The natural intuition is that an increase in risk should cause an extension of the red zone, whereas an increase in population should cause a reduction. The analysis shows that there are important exceptions to these intuitions. Indeed, the variations in the red zone depend on the bargaining game played. Besides, we give and explain the conditions for a purple zone (limit red zone as the population grows), and a green zone (limit inhabitable zone as the risk grows).<sup>4</sup>

But the scenario does not alone explain the variations: preferences matter in the trade-off. We propose an analysis based on a decomposition in basic microeconomic effects. In summary, there are two obviously opposite impacts. On the one hand, an increase in risk or in population raises the cost of risk. On the other hand, the demand for land increases as the population increases. A third, and more complex, effect is at play: the variation in the households' willingness to pay for space as their budget varies. The article gives examples where the net effect is clear and illustrates in this way the variety of possibilities under plausible parametric assumptions.

**Literature and discussion.** Our study is related to the self-insurance literature à la Ehrlich and Becker (1972) and the insurance and law economics à la Shavell (1982). We assume perfect compensation to facilitate comparative statics. In reality, the residual risk is not zero and it is internalized in housing prices. Several empirical works that use the hedonic prices method show that the perception of industrial risks can decrease property values to an extent that depends on information release (Boxall et al. 2005; Kiel and McClain 1995; McMillen and Thorsnes 2003) and that varies among sites (Gawande and Jenkins-Smith 2001; Grislain-Letrémy and Katosky 2014; Kiel and Williams 2007). An alternative assumption, namely limited liability of the firm, would produce interesting and realistic effects: more prudent location choices by imperfectly protected households, and excessive risk-taking by the firm. Similarly, if households take into account irreplaceable assets, such as health, they should make less risky location choices. These cases are complex to solve: the equilibrium is a full profile of land occupation, a notion that defies comparative statics, despite remarkable attempts in Pines and Sadka (1986).

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<sup>4</sup>The color purple is used by some regulatory agencies for extreme risks (above red), but this choice must be seen as a mere convention that we adopt. Examples of this convention are <http://aviationweek.com/bca/how-use-nexrad-and-airborne-weather-radar-together> and [https://archive.ipcc.ch/report/ar5/wg2/docs/WGIIAR5\\_SPM\\_Top\\_Level\\_Findings.pdf](https://archive.ipcc.ch/report/ar5/wg2/docs/WGIIAR5_SPM_Top_Level_Findings.pdf)

Risk and space are rarely studied together. The literature on behavioral geography is limited. Even a programmatic paper like [Strauss \(2008\)](#) or a critical one like [Berliant \(2010\)](#) are very classical in their summaries of behavioral economics; they recall the main teaching about the attitude towards risk and information. Mixing industrial risk and location choice within an urban model is almost never done. We are confident that the interest in demographic or regulatory changes will raise the interest for this approach.

Red zones are our main focus. The literature studies the maneuvers by firms to purchase land but not from the angle of risk exposure. Our analysis fills this gap in the study of industrial risks. The analysis of takeover bids as in [Grossman and Hart \(1980\)](#) is a valid source of inspiration for solutions. [Blume et al. \(1984\)](#) and [Nosal \(2001\)](#) study the efficiency of paying compensation; [Miceli and Segerson \(2006\)](#) and [Strange \(1995\)](#) analyze the problem of the holdup a developer faces when individual landowners know that their parcel of land is necessary for project completion and can postpone or even block the overall project.

In terms of basic ingredients and notation, the model is tightly related to [Grislain-Letrémy and Villeneuve \(2019\)](#), a paper dealing with natural disasters. Yet, this paper develops behavioral foundations for our assumptions, and we have a very different legal context and different sets of references. In the case of natural disasters, the firm has no equivalent, and the notion of bargaining power was meaningless. In the case of industrial disasters studied in this paper, the negotiation protocol between parties is key. We study four realistic scenarios and show their contrasted properties.

This paper proceeds as follows: Section 2 sets up the model. Section 3 compares the optimal red zones for the different scenarios of bargaining power between the households and the firm. Section 4 details the impact of technological change and demographic evolution on the size of the red zones, depending on the scenario. Section 5 concludes.

## 2 Model

### 2.1 Households and risk

We model a developing area with a risk gradient. The interval  $[0; X]$  is the linear space of inhabitable locations. The hazardous plant is located at 0. The distance  $x$  to the source determines the risk exposure at a given location. The safest place  $X$  is the outer limit of the territory. In Figure 1, an already dense city creates this limit. In reality, it could be a natural limit (sea, river) or a legal one (national park, private property). The cost of living somewhere comprises the rent and transportation costs. In the tradition of urban economics models, the latter would be related to the distance to the center. Here, the business center could be the plant itself, or the other side of the territory, or anywhere between 0 and  $X$ . Since the impact of transportation costs is well understood, we leave them aside in this study to keep the tractability of the model.

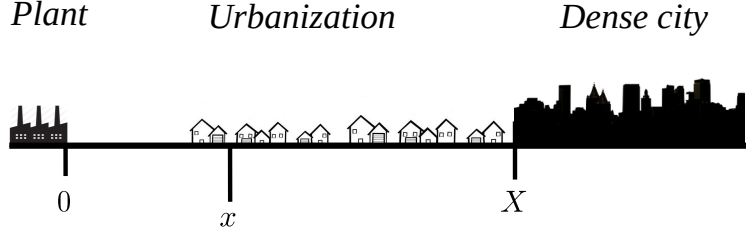


Figure 1: Space and risk. Location  $x$  is generic; 0 and  $X$  are the limits of the area of interest.

Households are identical and have no intrinsic preference for one location over another. Their utility  $U(z, s)$  depends on their consumption  $z$  of the composite good (henceforth money) and on their housing size  $s$ . The function  $U$  is twice differentiable and strictly increases with respect to  $z > 0$  and  $s > 0$ . Indifference curves are strictly convex and do not cut axes. We assume that the Engel curves increase. Technical detail is in Appendix A.1.

Households are price takers; they have an exogenous income  $\omega$ , and they maximize their expected utility under their budget constraint. Households being infinitesimal, we are interested in the population rather than particular households. We search for the distribution  $n(\cdot)$  of households over  $[0, X]$ . Households located at  $x$  consume  $s(x)$ , meaning that they consume collectively  $n(x)s(x)dx$  locally. Local space constraints imply

$$0 \leq n(x)s(x) \leq 1 \text{ for all } x, \quad (1)$$

and the global constraint implies

$$\int_0^X n(x)dx = N, \quad (2)$$

where  $N$  is the total population (for simplicity the number of households).

Location  $x$  will be damaged with probability  $p(x)$  with

$$\forall x, p(x) := \rho f(x), \quad (3)$$

where function  $f(\cdot)$  is positive, decreasing along the space line, and piecewise continuous; and  $\rho > 0$  is a magnitude index for comparative statics. In case of disaster, the damage per dwelling of size  $s$  is  $\lambda_F + \lambda_S s$ ; the first part  $\lambda_F \geq 0$  is fixed and the other part is  $\lambda_S s$  with  $\lambda_S \geq 0$  and is proportional to the house's size. The damage corresponds to the (re)building cost and does not depend on the land's value. There is no damage to empty places.

The expected damage is what will matter in the sequel. For a dwelling of size  $s$  at location  $x$ , it is

$$p(x) (\lambda_F + \lambda_S s).$$

This form is very flexible and allows extensive interpretations. Over a given portion of the territory, the damages increase with the number of dwellings in this portion, and with the occupied surface. The relative strength of these two effects is determined by  $\lambda_F$  and  $\lambda_S$  respectively; these parameters explain the variety of impacts we discuss in Section 4. Function  $p$  is a probability strictly speaking if the disaster can happen only once. If a disaster can happen again,  $p$  is rather an expected number of disasters in the considered period. For example spills or fires can happen several times; in the case of mines and quarries, earthquakes and land collapse can also occur repeatedly. A natural rationale for a decreasing  $f$  is that risk are correlated across locations: if an explosion or contamination reaches location  $x$ , then it has also reached location  $x'$  with  $0 < x' < x$ .

Location-dependent severity  $\rho(x)$  is implicitly comprised in this modeling. Assume the damage is

$$\lambda_F(x) + \lambda_S(x)s,$$

in the case where  $\lambda_F(x)/\lambda_S(x)$  is a constant independent of  $x$ . Therefore the expected damage

$$p(x)(\lambda_F(x) + \lambda_S(x)s)$$

could be rewritten as

$$p(x)\rho(x)(\lambda_F(0) + \lambda_S(0)s),$$

with appropriate normalization of  $\rho(\cdot)$ . Then clearly,  $p(x)\rho(x)$  can be redefined as a new  $\hat{p}(x)$ . The expected local damage has this form

$$\hat{p}(x)(\lambda_F + \lambda_S s).$$

We are back to the previous formulation, which happens to be quite general, especially if  $\hat{p}$  is seen as the expected number of events, as already discussed. Given this, having a scalar  $\rho$  to organize the comparative statics results is very convenient. Indeed, a change in  $\rho$  can alternatively be interpreted as a change of the probability or a change in the intensity.

All households are *de jure* and *de facto* insured by the industrialist. Because the firm is liable for all damages, there is no need for households to take out insurance for the risk associated with the hazardous plant. We assume that after a disaster the industrialist provides complete reimbursement  $\lambda_F + \lambda_S s$ . Limited liability of the firm or incomplete reimbursement are important practical issues. Households would mitigate the risk they would bear by choosing more prudent locations. Yet this approach, though interesting, considerably complicate the analysis, in particular the comparisons between scenarios and the comparative statics of risk. We also assume that the firm itself either is risk-neutral and completely self-insures, or equivalently here, that risk-neutral insurers fully insure the firm.



## 2.2 Red zone

In a perfect world, the Coase theorem says that a first-best could be attained. First-best allocations can be described qualitatively. They are based on a non uniform density of the population: the density increases smoothly as one gets farther away from the plant; the density of population may be zero in a nontrivial zone around the plant. The proof in [Grislain-Letrémy and Villeneuve \(2019\)](#) borrows on the techniques usually employed in urban economics. Understanding the first best is very informative. The main limit of this approach is that in practice, firms are limited in the instruments they use, hence our insistence on red zones. We provide theoretical arguments from behavioral economics justifying zoning. [Spiegler \(2019\)](#) indeed writes that specifications should not be taken for theories, and here are the arguments we put forth.

**Maps are simple.** Precise maps could lead to great legal differences between places, which is generally not understood nor accepted by the public. A stable and frequently observed solution is to establish a very small number of zones. Maps on which land-use regulations and insurance tariffs are based are simplistic, compared to the scientific knowledge regarding risk the society has. The anti-discrimination argument is politically powerful. People would resist being more restricted than their neighbors “on the right” (following the left-right organization in our pictures). Inequity aversion ([Fehr and Schmidt 1999](#)) Il ne manque pas une virgule avant le 1999 ? could be attached to this behavior. Besides, behavioral models sometimes reflect a slightly paranoid attitude towards information. With spatially differentiated risk, this effect is likely to be encountered. Spatially refined regulations, as the first-best choice would request, would be based on the opinion of experts; expertise being highly concentrated by definition, suspicion about the recommendations can easily arise. The beliefs that experts are biased, or even plainly sold, are sometimes strong. In the end, zoning has to be much simpler than it could be.

**Simple maps reinforce distortion in risk perception.** It has been noted at least since [White \(1945\)](#) that the organization of space matters for risk perception. Visibly protective infrastructure has this effect. White’s levee effect has been recalled by Kunreuther and Michel-Kerjan ([2013](#), page 528). La citation apparait bizaremment. A levee causes a false feeling of safety against floods; people may overuse land behind the protection, whereas a levee breach may more disastrous than a flood. Overall, the levee may cause a higher risk. More generally, by organization of space, we mean also legal classification (zoning), labels and safety communication (color codes), differentiated constraints (building codes, insurance tariffs), and any other form of simplified information. Labeling a zone red or blue, which is a purely conventional decision, changes risk perception. First, it creates a strong discontinuity in risk perception between zones. Second, within zones, people get the feeling that the risk, high or low, is homogeneous simply because the color is the same. We consider in this paper



Figure 2: Land use. The red zone is between 0 and  $\bar{x}$ ; the building zone is between  $\bar{x}$  and  $X$ .

what happens when these two effects characterize the behavior of people and when mayors and industrialists must take decisions under this constraint. Nudges take all sorts of forms. People tend to forget about rare risk (Fanta et al. 2019): labeling zone to enforce a building code is of public interest.

Optimal zoning resembles exploitative contracting; see Koszegi (2014), a survey on behavioral contract theory. In this literature, firms or states pursue their objectives by exploiting a behavioral trait in the setting of contractual details. This objective may be profit (for the firm) or social welfare (for the mayor or the state); the tool is the zoning.

In this second-best world, we define the red zone as the area rented by the firm. The firm does not need land *per se*; it only wants to prevent potential victims from occupying the riskiest locations. In the other zone, land use by households is not restricted. The location  $\bar{x}$  denotes both the size of the red zone and the leftmost inhabited location (Figure 2). The red zone is the result of the game between the firm and the mayor, as developed in Section 3.

## 2.3 Equilibrium given the red zone

Because households are fully covered, all of the permitted locations have the same value, the rent is the same everywhere, and no authorized space is lost: the building zone  $[\bar{x}, X]$  is fully and uniformly used. Therefore:

$$\begin{cases} s(x) = \frac{X-\bar{x}}{N} = \frac{1}{n(x)} & \text{if } x \in [\bar{x}, X], \\ s(x) = 0 \text{ and } n(x) = 0 & \text{otherwise.} \end{cases} \quad (4)$$

The rent  $r$  is uniform, as is the occupation, in the authorized area  $[\bar{x}, X]$ . Further, the households have equal shares in a fund that owns all of the land. They receive, in addition

to  $\omega$ , the income  $R$  that they take as given:

$$R := \frac{X - \bar{x}}{N} r. \quad (5)$$

The rents are endogenous and ultimately, functions of  $\bar{x}$ . Households also receive a share  $T/N$  of the net transfer  $T$  from the firm to the community. The value of  $T$  depends on the negotiation. It can be either positive or negative, depending on bargaining powers (Section 3).

Because coverage is complete, the expected utility  $EU$  is no more than the utility  $U$ . Households' choice  $(x, z, s)$  solves

$$\max_{x, z, s} U(z, s) \quad \text{s.t.} \quad \omega + R + \frac{T}{N} \geq z + sr \quad \text{and} \quad x \in [x^*, R], \quad (6)$$

where  $R = sr$  as shown by Equations (4) and (5).

The first definition serves for the backward induction involved in the game played between the firm and the mayor. The equilibrium is conditional on negotiated terms.

**Definition 1** (Conditional equilibrium). *Given the red zone  $\bar{x}$  and the net transfer  $T$ , the conditional equilibrium is summarized by the rent  $r$  such that all the authorized land is uniformly and fully occupied (Equation 4), the collected rents are redistributed (Equation 5), and all authorized locations have the same value to consumers (Equation 6).*

### 3 Bargaining scenarios

The size of the red zone results from a negotiation between the firm and the mayor. The fact that households are landowners makes them likely to benefit from the firm renting their land, but the distribution of the benefits of risk reduction depends on the market's organization. Indeed, the mayor has the formal right to define the red zone but he may not have the real decision power to do so in the best interest of his community. The firm's strength comes essentially from its capacity to relocate or develop elsewhere. Who suffers from the hold-up problem depends on the context.

The red zone and the transfer are here the only variables that the bargaining can change. Increasing  $\bar{x}$  reduces the cost of risk and the available space at the same time, and the firm compensates households for being squeezed by an amount denoted by  $T$ , each of them receive  $T/N$  before they settle in. The red zone partially corrects the imperfect internalization by households of the risk they create by locating in exposed areas.

We study four typical patterns in real estate markets that correspond to different distributions of bargaining power between the firm and the mayor. In three cases, households are neither employees, owners of the firm, nor consumers of its production. In an extreme case (Integrated), we take the assumption that the firm is entirely the property of the households.

**Dominant mayor scenario.** In this scenario, the mayor holds the bargaining power and redistributes to households the benefits of the risk reduction extracted from the firm:

1. The mayor offers two different rents: one for households and one for the firm. The mayor also requires a lump-sum transfer from the firm to be redistributed to the households.
2. The firm accepts or declines the offer. The firm's reference for acceptance is the equilibrium in the absence of a red zone.
3. The conditional equilibrium is attained.

**Dominant firm scenario.** In this scenario, the firm holds the bargaining power and captures all the surplus generated by the transaction:

1. The firm offers a two-part tariff: it chooses the rent per unit of land and the lump-sum transfer to the community.
2. The mayor accepts or declines the offer. Acceptance by the mayor depends on households doing at least as well as without the red zone.
3. The conditional equilibrium is attained.

**Market scenario.** The households and the firm are both rent takers and make their choices simultaneously. The equilibrium in the land market determines the red zone. This scenario is based on a different logic since outside options and the bargaining powers are not explicit. It must be seen as a benchmark for the other bargaining scenarios. The surplus is partly captured by the firm via access to the land and partly recovered by households via rents.

**Integrated scenario.** In this extreme case, the community has to pay for the loss and decides the size of the red zone. This scenario can be seen as the case where the firm is the property of the community, hence the name:

1. The mayor decides the size of the red zone.
2. The conditional equilibrium is attained.

We need a definition of optimality to evaluate and compare the scenarios. A red zone policy combines a net transfer  $\bar{T}$  from the firm to households and an  $\bar{x}$ .

**Definition 2** (Constrained optimality). *A red zone policy  $(\bar{T}, \bar{x})$  is constrained optimal if it is Pareto optimal under the following constraints: (i) all physical constraints (global or local) are satisfied, (ii) households differ only by location but receive the same basket in terms of numéraire and surface.*

A constrained optimum has a continuous red zone: efficient use of the land requires using first the less risky locations. In general, constrained optima are characterized by programs where the utility of the households is maximized given a minimum profit for the firm of  $\bar{\Pi}$ :

$$\begin{cases} \max_{(T,x)} & U\left(\omega + \frac{T}{N}, \frac{X-x}{N}\right), \\ \text{s.t.} & \pi - T - \text{CR}(x) \geq \bar{\Pi}, \\ & 0 \leq x \leq X. \end{cases} \quad (7)$$

where  $\pi$  is the unmodeled profit generated by the firm's primary activity. We assume for simplicity that  $\pi$  depends neither on  $T$  nor on  $x$ . In the general case where  $\pi$  depends on  $T$ , say because the Modigliani-Miller theorem does not hold, the transfer impacts the marginal cost of capital. The dependence on  $x$  would come from the fact that the land has some use to the firm. In such extensions, the firm would not simply try to reduce risk but would preserve profits minus the cost of risk. In a sense, the cost of risk that we use could be redefined to integrate other economic effects.

With uniform land-use over the inhabited area  $[\bar{x}, X]$ , the total expected cost of the risk (CR) amounts to

$$\text{CR}(\bar{x}) := \int_{\bar{x}}^X p(t) (\lambda_F + \lambda_S s(t)) n(t) dt = \left( \frac{N}{X - \bar{x}} \lambda_F + \lambda_S \right) \times \int_{\bar{x}}^X p(t) dt. \quad (8)$$

When we calculate the expected total damages, correlation across places does not matter: the expected total damages are the sum of the expected local damages. To facilitate the comparative statics, we assume that<sup>5</sup>

$$\text{CR}(\cdot) \text{ is convex.} \quad (9)$$

In all the numerical applications in the paper, the cost is convex by linearity of  $p(\cdot)$ .

An equilibrium allocation is generically denoted by  $(\bar{T}_i, \bar{x}_i)$ , where  $i$  indicates the relevant scenario ( $i \in \{\text{Mayor, Market, Firm, Integ}\}$ ).

**Proposition 1.** *In the four scenarios, the equilibrium allocation  $(\bar{T}_i, \bar{x}_i)$  exists and is constrained optimal; for the dominant mayor, dominant firm, and the integrated scenarios, the equilibrium allocation is unique.*

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<sup>5</sup>This assumption is equivalent to having the MRR defined in Appendix A.1 non-increasing. Having  $p(\cdot)$  non-increasing is not a sufficient condition for convexity, yet simple examples with analytic forms are easy to find. [Grislain-Letrémy and Villeneuve \(2019\)](#) proposes conditions for the cost to be convex.

*Proof.* See Appendix [A.2](#). □

The key to the proof is that the players have enough instruments to exploit their positions (first-mover advantage). The two different rents (in the dominant mayor scenario) or the two-part tariff (in the dominant firm scenario) allow efficient extraction of the surplus by the dominant player. The fact that the market scenario is also efficient comes from the internalization by the firm of the damages it could have to pay. Uniqueness of the competitive equilibrium is not established, for reasons well known in mathematical economics.

We can order the sizes of the four red zones. Proposition [1](#) implies that the equilibrium allocations are all located on the constrained-Pareto frontier. Households and the firm rank scenarios in opposite orders because an increase in  $\bar{T}$  benefits the households but hurts the firm, whereas an increase in  $\bar{x}$  benefits the firm but hurts the households. Comparing the transfers from the firm to the households between the scenarios amounts to comparing the bargaining position of the mayor in each scenario. First comes the dominant mayor scenario, and then comes the market scenario and the dominant firm scenario. The integrated scenario is the worst case for households because they bear the entire cost of risk. Thus:

$$\bar{T}_{\text{Mayor}} \geq \bar{T}_{\text{Market}} \geq \bar{T}_{\text{Firm}} \geq \bar{T}_{\text{Integ}}. \quad (10)$$

Proposition [2](#) compares the red zones.

**Proposition 2.** *We order the four red zones as follows:*

$$\bar{x}_{\text{Mayor}} \leq \bar{x}_{\text{Market}} \leq \bar{x}_{\text{Firm}} \leq \bar{x}_{\text{Integ}}. \quad (11)$$

*Proof.* See Appendix [A.3](#). □

Figure [3](#) illustrates the proposition. The proposition shows that the red zone is increasing with the bargaining power of which side ultimately incurs the loss. When the firm is financially independent of the community, a strong firm wants a bigger red zone than a strong mayor. But in the extreme case where the firm entirely belongs to the community, the red zone is even bigger. The dominant mayor scenario is the most favorable to households since they extract rent from the firm by conceding land. This scenario gives the smallest (not necessarily small) red zone. In the dominant firm scenario, the firm can pay less for the land it uses and imposes a larger red zone. The market solution is in between these two scenarios in term of bargaining power and impact. The integrated scenario gives the largest red zone: this is the worst case for households, because they bear the entire cost of risk; this is why they want to contain risk.

## 4 Changing conditions and limits

For given fundamentals, the red zone depends on the bargaining game played. We explore this result further by showing how different societies digest new data. In other terms, we

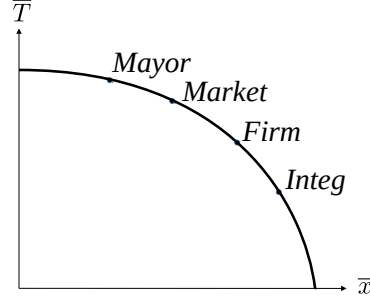


Figure 3: Constrained-efficient frontier.

propose a comparative study of the scenarios when the parameters change.<sup>6</sup>

## 4.1 Causes of change

The conditions surrounding a given community are likely to change. The number of people and businesses located in the exposed areas and the value of their assets strongly determine the economic cost of industrial disasters (Bevere et al. 2011). Technological improvements or, contrariwise, negligence and deterioration of structures happen continually. Better risk regulation and better implementation are obvious causes of a decrease of  $\rho$ .

Further, natural and industrial hazards can also mutually aggravate each other. For example, the increase of landslides and seismic activity due to the Three Gorges Dam in China is now officially recognized by Chinese authorities.<sup>7</sup> Hydraulic fracturing causes earthquakes of low intensity and might also cause moderate earthquakes.<sup>8</sup> In these cases, the causal relationship is statistical, which can lead to an exoneration from liability for the dam owner or the drilling company. More commonly, floods are major issues for many industries: spills for chemical plants, disruption of cooling and other controls for nuclear powerplants, leakages for landfills and waste repositories. Since climate change is likely to increase the intensity and the frequency of such extreme natural hazards (Schneider et al. 2007), it is also a relevant factor to be regularly reassessed. The responsibility of the industrialist can be invoked for lack of due diligence.

<sup>6</sup>Our approach is similar to Pines and Sadka (1986), who establish comparative statics of an empty zone in a city inside which transportation costs vary (but not risk exposure). They show that the empty space can increase or decrease with respect to population size.

<sup>7</sup>M. Hvistendahl, “China’s Three Gorges Dam: An Environmental Catastrophe?”, *Scientific American* March 25, 2008. See [link](#).

<sup>8</sup>Seismologists at Columbia University consider that earthquakes that hit Youngstown (Ohio) in 2011 (including a 4.0 magnitude earthquake) are probably linked to the disposal of the wastewater from hydraulic fracturing. See Lamont-Doherty Earth Observatory, “Ohio Quakes Probably Triggered by Waste Disposal Well, Say Seismologists” (Press release, January 6, 2012). See [link](#).

## 4.2 When the red zone turns purple, or green

The following comparative statics explore the effect of  $\rho$  and  $N$  around the basic scenario:

$$p(x) = \rho \cdot (X - x), \quad (12)$$

which means that  $x = X$  is safe. A large  $\rho$  can indicate a probability of disaster larger than one: this can be seen as multiple occurrences within the same period because we consider only the expected loss. Our specifications lead to closed-form expressions for red zones in all scenarios. These examples have interesting properties concerning the limits of the red zone. A *purple zone* is the limit of the red zone as  $N$  tends to infinity (e.g. if  $\lim_{N \rightarrow +\infty} \bar{x} > 0$ ); in some cases, the demographic pressure is such that it vanishes completely. A *green zone* is a preserved space for households even if  $\rho$  tends to infinity (e.g. if  $\lim_{\rho \rightarrow +\infty} \bar{x} < X$ ); otherwise, households are forced onto the safest place,  $X$ . This term means that it can happen that a zone is always inhabitable given the tradeoff between the cost of risk and the value of land. See Figure 4.

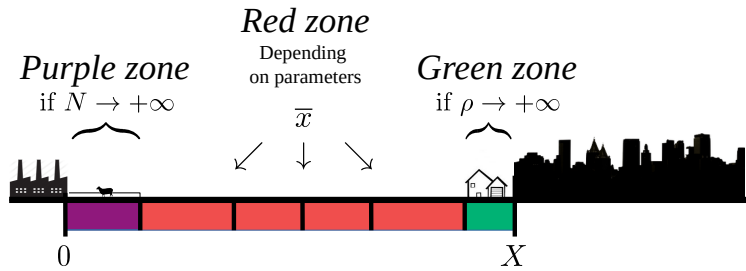


Figure 4: Limits when conditions change.

**Ambiguous in general, predictable sometimes.** One expects an increase in risk to cause an extension of the red zone, whereas an increase in population would cause a reduction in the red zone. In fact, the net impact of a change depends on the bargaining scenario and the preferences. Indeed, an increase in risk leads the firm to bid higher rents to extend the red zone. This increase in rent has a substitution effect on households (because they pay rents as renters) and an income effect (because they receive rents as landowners). The net impact depends on the elasticity of substitution between land and money. The intricacy of effects calls for their systematic separation and quantification and for their illustration by calculable applications.

Three effects can be distinguished: (1) the risk intensification effect that increases the firm's willingness to extend the red zone, (2) the income effect caused by the change in the households' share of wealth, and (3) in the case of the changes in population size, the land



		Risk intens. effect	Income effect	Land sharing effect	Net on $\bar{x}$
$\rho$	Mayor	+	—	0	$\nearrow$ or $\searrow$
	Market	+	+ or —	0	$\nearrow$ or $\searrow$
	Firm	+	0	0	$\nearrow$
	Integ	+	+	0	$\nearrow$
$N$	Mayor	+	+	—	$\nearrow$ or $\searrow$
	Market	+	+ or —	—	$\nearrow$ or $\searrow$
	Firm	+	+	—	$\nearrow$ or $\searrow$
	Integ	+	—	—	$\nearrow$ or $\searrow$

Table 1: Comparative statics of  $\bar{x}$  with respect to  $\rho$  and  $N$  in the general case.

sharing effect, that is the mechanical increase in the marginal bid for land due to demographic expansion. These three effects are mathematically defined in Appendix A.4.

**Proposition 3.** *For a general utility function, the comparative statics with respect to  $\rho$  and  $N$  are given in Table 1. There are only two clear cases: an increase in  $\rho$  always expands the red zone in the dominant firm and integrated scenarios. In the other scenarios, an increase in either  $\rho$  or  $N$  can pull both ways.*

*Proof.* See Appendix A.4. □

The variations of  $\bar{x}$  when  $\rho$  changes depends on the risk intensification effect of  $\rho$  and of its income effect. The land sharing effect mentioned above is zero, by definition. Things are more mixed with the effect of  $N$ . All three effects matter.

The natural reasoning that an increase in risk should cause an extension of the red zone, whereas an increase in population should cause a reduction in the red zone, is thus inexact. For given preferences, the variations in the red zone as the hazard or population size increases depend on the game played. Conversely, in a given scenario, these variations depend on the preferences. A clear lesson can be drawn. In the scenarios where the agents who pay the cost of risk control the red zone, an increase in risk expands the red zone. This situation happens when the firm has the bargaining power and, at the other extreme, when the firm is the property of households and they decide the size of the red zone. In the other scenarios, preferences matter because the marginal rate of substitution of households between money and space, and its variations with wealth are critical. This is the case particularly when the increase in risk makes the firm pay increasingly high rents to circumvent the risk. The following examples illustrate the importance of preferences.

**Log-log utility function.** We take first a Cobb-Douglas utility function:

$$U(z, s) = \log(z) + \alpha \log(s). \quad (13)$$

**Proposition 4.** *For a log-log utility function, the comparative statics with respect to  $\rho$  is given in Table 2, and the comparative statics with respect to  $N$  is given in Table 3. As  $N$  increases, the red zone narrows in the market, dominant firm, and integrated scenarios. In the dominant mayor scenario, the red zone is monotonic with respect to  $N$ , either increasing or decreasing.*

*Proof.* See Appendix A.5. □

As  $\rho$  increases, the red zone expands in all scenarios. Red zones are smaller when the bargaining power is more favorable to households (Proposition 2), and this order between zones is preserved as  $\rho$  tends to infinity. In the dominant mayor and market scenarios, households have more bargaining power and the inhabited zone narrows down to the green zone. In the dominant firm and integrated scenarios, households are forced to the safest place as  $\rho$  tends to infinity. All four limits are never reached for a finite  $\rho$ .

As  $N$  increases, the red zone narrows in the market, dominant firm, and integrated scenarios (Figures 5 and 6). In the dominant mayor scenario, the red zone is monotonic with respect to  $N$ , either increasing or decreasing. The red zone increases with respect to  $N$  if and only if

$$\rho^2 \lambda_F^2 X^2 - 4\alpha(\alpha + 2)\omega\rho\lambda_F X - 4\alpha(\alpha + 2)\omega^2 > 0. \quad (14)$$

In the dominant mayor scenario, the more households value land consumption or the richer they are, (either  $\alpha$  or  $\omega$  large), the more likely the red zone is to decrease. In contrast, the higher the per capita share of expected damage,  $\rho\lambda_F$ , the more the firm is willing to increase its bid on land, which tends to increase the red zone.

In Figure 5, the red zone decreases with respect to  $N$  in all scenarios, whereas the red zone increases for the dominant mayor scenario in Figure 6. Indeed, in all simulations but one, the fact that more people want more space dominates. The important exception happens when  $\lambda_F$  is big and  $\omega$  is small, a case where the revenue households get from the firm's compensation is relatively big compared to the basic income. The net effect is then determined by the income effect. More households mean less money extorted from the firm per household. The willingness to pay for land decreases substantially, given the Cobb-Douglas utility, whose elasticity of substitution is unitary. The decrease of households' bid for land is strong enough to cause a diminution of total inhabited space. The change in the set of parameters between the two figures also leads to contrasted results in terms of purple zones. In Figure 5, the simulations show a completely squeezed purple zone in the dominant mayor, market, and dominant firm scenarios. In contrast, in Figure 6, the red zone tends to a nontrivial purple zone in these scenarios. In the integrated scenario, in both cases, the purple zone is large for almost any  $N$ .

**Log-lin utility function.** We take now a quasi-linear utility function:

$$U(z, s) = \log(z) + \alpha s. \quad (15)$$

The other quasi-linear utility function  $U(z, s) = z + \alpha \log(s)$  does not provide other insights. We take the same loss model as before:  $p(x) = \rho \cdot (X - x)$ .

**Proposition 5.** *For a log-lin utility function, the comparative statics with respect to  $\rho$  is given in Table 4, and the comparative statics with respect to  $N$  is given in Table 5. Contrary to the log-log utility function, this second example illustrates the increase in the red zone with respect to  $N$  in the dominant mayor, market, and dominant firm scenarios.*

*Proof.* See Appendix A.6. □

As with the log-log utility function, the red zone grows with respect to  $\rho$ . In the dominant mayor and market scenarios, some parameters show a nontrivial green zone. In the dominant firm and integrated scenarios,  $X$  is attained for finite values of  $\rho$ .

Contrary to the log-log utility function, this second example illustrates the increase in the red zone with respect to  $N$  in the dominant mayor, market, and dominant firm scenarios (see Figure 7). Remark that with this utility function, the marginal value people give to the surface does not depend on the surface they actually occupy, meaning that they tolerate well being squeezed. The consequence is that, when they are more numerous, they tend to prefer money rather than the conservation of their occupied space. In the integrated scenario, the red zone decreases and then increases with respect to  $N$ . The limit, a completely purple zone, is attained for a finite  $N$  in all scenarios. This case is a paradox that is worth stressing: the demographic pressure can lead to a retreat of the population because the cost of a disaster would be too large. This outcome would be compared in practice with the closure of the firm, a case we cannot evaluate with this model.

**No red zone.** We focused on interior solutions because they are sensitive to parameter changes. Clearly, there is no red zone if the risk is moderate compared to the households' taste for land and their wealth. In the dominant mayor, market, and dominant firm scenarios, the values of  $\rho$  and  $N$  for which there is no red zone are the same. Indeed, in these three scenarios, if there is no red zone, then the transfer from the firm to the households is zero; the other aspects of the game being then identical, the first order conditions are the same. Figures 8 and 9 illustrate this property for the first and second examples, respectively.

In the first example, the two risk factors  $\rho$  and  $N$  are complementary in all scenarios: the higher the severity of the risk  $\rho$ , the higher the demographic pressure  $N$  should be for the red zone to disappear (Figure 8). This case is the most intuitive. In the second example, the two risk factors  $\rho$  and  $N$  are substitutes in all scenarios but the integrated one, where we find the intuitive impact. In the other three scenarios, we retrieve the fact explained in the previous subsection that the red zone increases as population grows.

Variations w.r.t. $\rho$		Green zone ( $\lim \bar{x}$ as $\rho \rightarrow +\infty$ )
$\bar{x}_{\text{Mayor}}$	$\nearrow$	$\leq \frac{1}{1+\alpha} X$ (†)
$\bar{x}_{\text{Market}}$	$\nearrow$	$\frac{1}{1+\alpha} X$
$\bar{x}_{\text{Firm}}$	$\nearrow$	$X$ (None)
$\bar{x}_{\text{Integ}}$	$\nearrow$	$X$ (None)

$$(\dagger) \lim_{\rho \rightarrow +\infty} \bar{x}_{\text{Mayor}} = X - \frac{(1+\alpha)}{2(2+\alpha)} \frac{\lambda_F N}{\lambda_S} \left( \sqrt{1 + 4 \frac{\alpha(2+\alpha)}{(1+\alpha)^2} \frac{\lambda_S X}{\lambda_F N} \left( \frac{\lambda_S X}{\lambda_F N} + 1 \right)} - 1 \right).$$

Table 2: Comparative statics with respect to  $\rho$  in the log-log case.

Variations w.r.t. $N$		Purple zone ( $\lim \bar{x}$ as $N \rightarrow +\infty$ )
$\bar{x}_{\text{Mayor}}$	if (14) is true $\nearrow$ if (14) is false $\searrow$	$\max \left\{ \frac{1}{1+\alpha} X - \frac{2\alpha}{1+\alpha} \frac{\omega}{\rho \lambda_F}; 0 \right\}$
$\bar{x}_{\text{Market}}$	$\searrow$	$\max \left\{ \frac{1}{1+\alpha} X - \frac{2\alpha}{1+\alpha} \frac{\omega}{\rho \lambda_F}; 0 \right\}$
$\bar{x}_{\text{Firm}}$	$\searrow$	$\max \left\{ X - \left( \frac{2\alpha \omega X^\alpha}{\rho \lambda_F} \right)^{\frac{1}{1+\alpha}}; 0 \right\}$
$\bar{x}_{\text{Integ}}$	$\searrow$	$\max \left\{ X - \frac{2\alpha}{1+\alpha} \frac{\omega}{\rho \lambda_F}; 0 \right\}$

Table 3: Comparative statics with respect to  $N$  in the log-log case.

Variations w.r.t. $\rho$		Green zone ( $\lim \bar{x}$ as $\rho \rightarrow +\infty$ )
$\bar{x}_{\text{Mayor}}$	$\nearrow$	$\leq X$
$\bar{x}_{\text{Market}}$	$\nearrow$	$\leq X$
$\bar{x}_{\text{Firm}}$	$\nearrow$	$X$ (None)
$\bar{x}_{\text{Integ}}$	$\nearrow$	$X$ (None)

Table 4: Comparative statics with respect to  $\rho$  in the log-lin case.

Variations w.r.t. $N$		Purple zone ( $\lim \bar{x}$ as $N \rightarrow +\infty$ )
$\bar{x}_{\text{Mayor}}$	$\nearrow$	$X$ (All is freezed)
$\bar{x}_{\text{Market}}$	$\nearrow$	$X$ (All is freezed)
$\bar{x}_{\text{Firm}}$	$\nearrow$	$X$ (All is freezed)
$\bar{x}_{\text{Integ}}$	$\searrow$ then $\nearrow$	$X$ (All is freezed)

Table 5: Comparative statics with respect to  $N$  in the log-lin case.

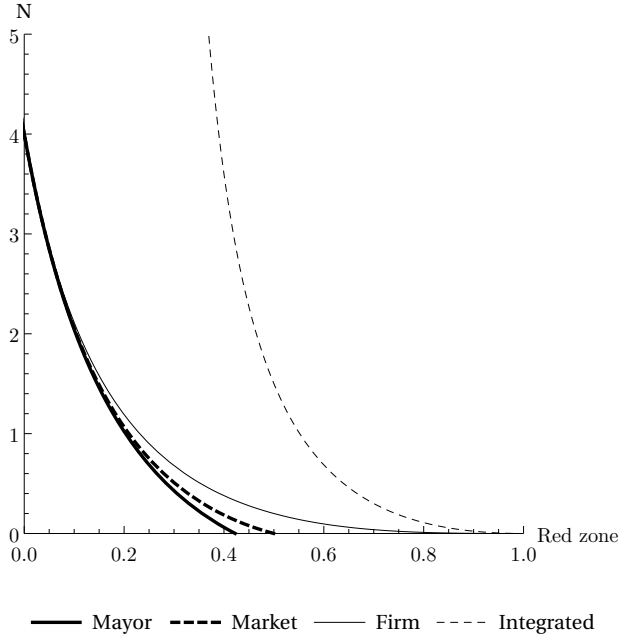


Figure 5: Red zones as a function of  $N$  for log-log utility function and linear loss probability. Parameters:  $X = 1, \lambda_F = 1, \lambda_S = 1, \alpha = 1, \omega = 1.5, \rho = 2$ .

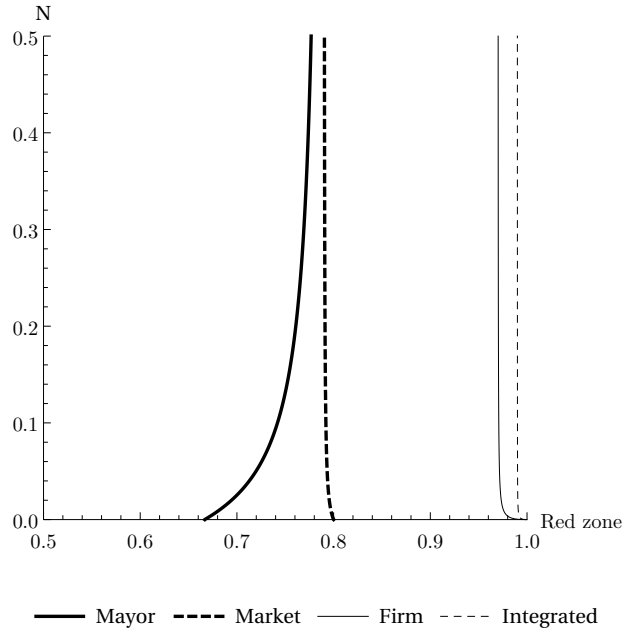


Figure 6: Red zones as a function of  $N$  for log-log utility function and linear loss probability. Parameters:  $X = 1, \lambda_F = 5, \lambda_S = 0.3, \alpha = 0.25, \omega = 0.25, \rho = 1$ .

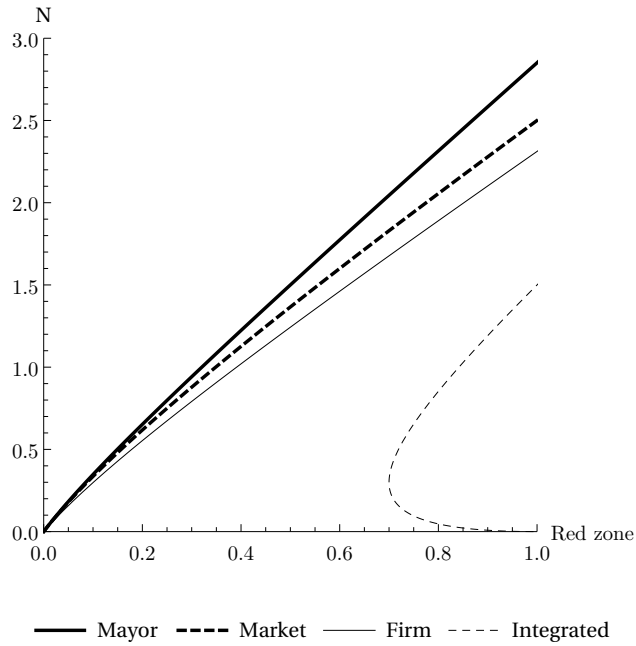


Figure 7: Red zones as a function of  $N$  for log-lin utility function and linear loss probability. Parameters:  $X = 1, \lambda_F = 1, \lambda_S = 1, \alpha = 1, \omega = 1.5, \rho = 2$ .

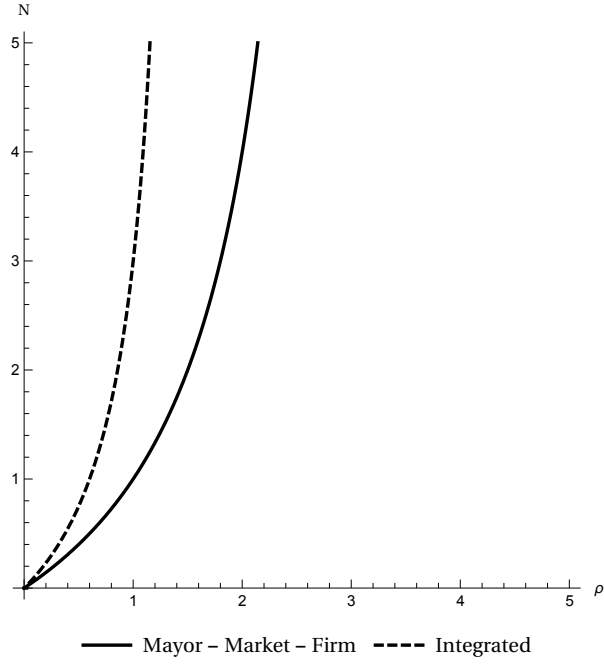


Figure 8: Log-log utility function.

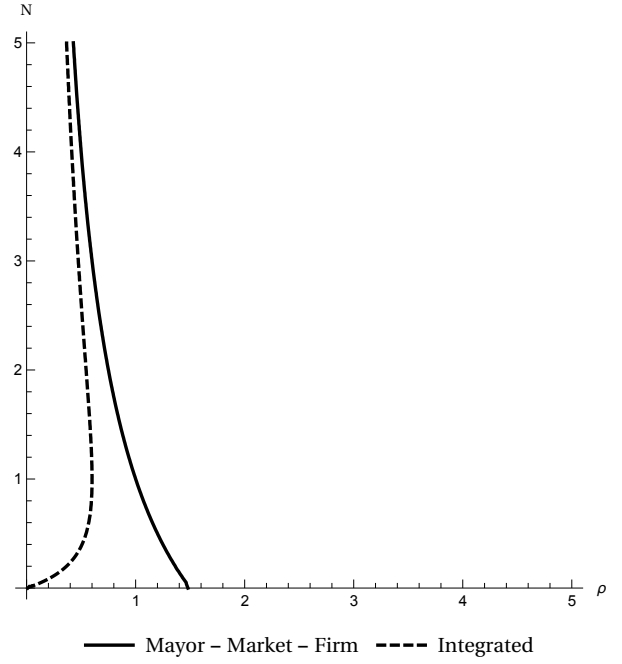


Figure 9: Log-lin utility function.

Note: These pictures tell whether there is a nontrivial red zone for a given pair  $(\rho, N)$ . On the left of the boundaries and for the indicated scenario, there is no red zone. Parameters:  $X = 1, \lambda_F = 1, \lambda_S = 1, \alpha = 1, \omega = 1.5, \rho = 1$

## 5 Conclusion

In a context where price signals towards households are unavailable, zoning and labeling are an effective response to a strong behavioral constraint. For the industrialist, red zones are the best available self-insurance tool. We study the delimitation of red zones around hazardous plants, as well as their limits in extreme cases. The distribution of bargaining power between the mayor and the firm follows several scenarios. We find that the stronger the bargaining power of the industrialist, the larger the red zone. But if the community has to pay for the damages (integrated scenario), the red zone is even bigger. We go further and compare the reactions to exogenous structural changes (demographic and technological) of these different organizations of public affairs. The natural, but inexact, reasoning is that an increase in risk should cause an extension of the red zone, whereas an increase in population should cause a reduction in the red zone. The examples show that there are important exceptions to these intuitions. For given preferences, the variations in the red zone as the hazard or population size increases depend on the game played. Conversely, in a given scenario, these variations depend on the preferences. A clear lesson can be drawn. In the scenarios where the agents who pay the cost of risk control the red zone, an increase in risk expands the red zone. This situation happens when the firm has the bargaining power and, at the other extreme, when the firm is the property of households and they decide the size of the red zone. In the other scenarios, preferences matter because the marginal rate of substitution of households between money and space, and its variations with wealth are critical. This is the case particularly when the increase in risk makes the firm pay increasingly high rents to circumvent the risk. Our examples provide contrasted illustrations in that regard.

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## A Appendix

### A.1 Notation and technical assumptions for proofs

**Preferences.** We denote  $MRS_{sz}$  the marginal rate of substitution of  $s$  for  $z$ , that is,

$$MRS_{sz} := \frac{\partial U / \partial s}{\partial U / \partial z}. \quad (16)$$

$MRS_{zs}$  is simply the reciprocal. We assume that the Engel curves increase:

$$\begin{aligned} \forall(z, s), \quad \frac{\partial MRS_{zs}}{\partial z} &= \frac{\partial}{\partial z} \left( \frac{\partial U / \partial z}{\partial U / \partial s} \right) \leq 0, \\ \forall(z, s), \quad \frac{\partial MRS_{sz}}{\partial s} &= \frac{\partial}{\partial s} \left( \frac{\partial U / \partial s}{\partial U / \partial z} \right) \leq 0. \end{aligned} \quad (17)$$

These two assumptions indicate that the relative value of the commodity that is becoming more abundant decreases; they are ordinal sufficient conditions for increasing Engel curves.

**Cost of risk.** We define the marginal risk reduction (MRR), which is positive:

$$MRR(\bar{x}) := -\frac{dCR}{d\bar{x}} \geq 0. \quad (18)$$

## A.2 Proof of Proposition 1

We show that, for the four scenarios, the equilibrium allocation (corner or interior)  $(\bar{T}_i, \bar{x}_i)$  is constrained optimal and is the unique solution of

$$\begin{cases} \bar{x}_i = 0 & \text{and } \text{MRR}(0) \leq \text{MRS}_{sz} \left( \omega + \frac{\bar{T}_i}{N}, \frac{X}{N} \right) & \text{or} \\ \bar{x}_i \in (0, X) & \text{and } \text{MRR}(\bar{x}_i) = \text{MRS}_{sz} \left( \omega + \frac{\bar{T}_i}{N}, \frac{X - \bar{x}_i}{N} \right) & \text{or} \\ \bar{x}_i = X & \text{and } \text{MRR}(X) \geq \text{MRS}_{sz} \left( \omega + \frac{\bar{T}_i}{N}, 0 \right), \end{cases} \quad (19)$$

where the net transfer  $\bar{T}_i$  from the firm to households is

$$\bar{T}_{\text{Mayor}} = \text{CR}(0) - \text{CR}(\bar{x}_{\text{Mayor}}), \quad (20)$$

$$\bar{T}_{\text{Market}} = r \bar{x}_{\text{Market}} \text{ where } r = \text{MRS}_{sz} \left( \omega + \frac{r \bar{x}_{\text{Market}}}{N}, \frac{X - \bar{x}_{\text{Market}}}{N} \right), \quad (21)$$

$$\bar{T}_{\text{Firm}} \text{ such that } U \left( \omega + \frac{\bar{T}_{\text{Firm}}}{N}, \frac{X - \bar{x}_{\text{Firm}}}{N} \right) = U \left( \omega, \frac{X}{N} \right), \quad (22)$$

$$\bar{T}_{\text{Integ}} = -\text{CR}(\bar{x}_{\text{Integ}}). \quad (23)$$

**Dominant mayor, dominant firm, and integrated scenarios.** For these three scenarios, we first check that the equilibrium of each scenario corresponds to program (7) for a particular value of  $\bar{\Pi}$ . Then we show that the equilibrium allocation  $(\bar{T}_i, \bar{x}_i)$  is constrained optimal and is the solution of (19). We also prove below that the equilibrium allocation is unique.

In the dominant mayor scenario, we recognize that  $\bar{\Pi}_{\text{Mayor}} = \pi - \text{CR}(0)$ . Indeed, the households capture all the benefits from risk reduction and the corresponding transfer from the firm to households is  $\bar{T}_{\text{Mayor}} = \text{CR}(0) - \text{CR}(\bar{x}_{\text{Mayor}})$ .

In the dominant firm scenario, the firm solves a dual program where minimum utility  $U \left( \omega, \frac{X}{N} \right)$  is guaranteed to the households.  $\bar{T}_{\text{Firm}}$  is such that  $U \left( \omega + \frac{\bar{T}_{\text{Firm}}}{N}, \frac{X - \bar{x}_{\text{Firm}}}{N} \right) = U \left( \omega, \frac{X}{N} \right)$ .

In the integrated scenario, we recognize that  $\bar{\Pi}_{\text{Integ}} = -\text{CR}(\bar{x}_{\text{Integ}})$ . Indeed, households own the firm ( $\bar{T}_{\text{Integ}} = \pi$ ) and bear the full cost of risk.

In these three cases, the constraint in (7) is convex in  $T$  and in  $x$  (it is linear in  $T$  and  $\text{CR}(\cdot)$  is convex (9)). As additionally the objective is strictly quasi-concave, the Kuhn-Tucker conditions can be rearranged to give the necessary and sufficient condition that defines the unique constrained optimum. However, the corner solutions are not excluded.

**Market scenario.** For the market scenario, existence of the equilibrium allocation is proved first. Then we show that the equilibrium allocation  $(\bar{T}_{\text{Market}}, \bar{x}_{\text{Market}})$  is constrained optimal and is the solution of (19).

The demand for land of the firm  $x(r)$  only depends on the rent. The demand of a household is  $s_d \left( w + \frac{rx(r)}{N}, r \right)$  where the first argument is the income and the second is the price.

Finding an equilibrium amounts to finding a root  $r$  to the equation

$$N s_d \left( w + \frac{rx(r)}{N}, r \right) + x(r) = X.$$

The LHS will be denoted as  $D(r)$  henceforth.

We show that  $D(\cdot)$  is continuous over  $\mathbb{R}$ :  $x(r)$  is continuous because  $\text{CR}(\cdot)$  is convex (9), which implies in turn that the households experience continuous variations in their incomes and in the prices as  $r$  increases. Their total demand for land is therefore also continuous with respect to  $r$ .

For  $r$  close enough to 0, the households have an unbounded demand for land, meaning that  $D(0^+)$  exceeds  $X$ . For very high  $r$ , the rent paid by the firm is bounded, because it would never pay more than  $\text{CR}(0)$ . This constraint proves that households keep a bounded income when the price of land explodes: their demand goes to 0 and  $D(r)$  is below  $X$ .

We now use the intermediate value theorem: the previous two paragraphs establish that there is a finite  $r > 0$  such that  $D(r) = X$ , which implies in turn that a market equilibrium exists. However, uniqueness is not warranted.

If  $r$  is the equilibrium rent in the market scenario, then we can assume for example that the market allocation is interior. We have

$$\text{MRR}(\bar{x}_{\text{Market}}) = r \text{ firm's optimality,} \quad (24)$$

$$\text{MRS}_{sz} \left( \omega + \frac{r\bar{x}_{\text{Market}}}{N}, \frac{X - \bar{x}_{\text{Market}}}{N} \right) = r \text{ mayor's optimality.} \quad (25)$$

We take  $\bar{\Pi}_{\text{Market}} = \pi - r\bar{x}_{\text{Market}} - \text{CR}(\bar{x}_{\text{Market}})$  in program (7). After eliminating  $T$  by using the binding constraint, the first order condition of program (7) becomes:

$$\text{MRR}(x) = \text{MRS}_{sz} \left( \omega + \frac{r\bar{x}_{\text{Market}} + \text{CR}(\bar{x}_{\text{Market}}) - \text{CR}(x)}{N}, \frac{X - x}{N} \right). \quad (26)$$

By inspection of (24) and (25), we see that  $x = \bar{x}_{\text{Market}}$  is a solution of (26). Because the optimum is unique, we conclude that the market scenario is efficient. This line of reasoning is similar when the market yields a corner solution.

### A.3 Proof of Proposition 2

From (7),

$$\left( \frac{1}{N} \frac{\partial \text{MRS}_{sz}}{\partial s} + \frac{d\text{MRR}}{dx} \right) \frac{d\bar{x}}{dT_i} = \frac{1}{N} \left( \frac{\partial \text{MRS}_{sz}}{\partial z} \right). \quad (27)$$

The conditions (9) and (17) are sufficient for the red zone  $\bar{x}$  to decrease with respect to  $T_i$ . Using (10), we can readily order the sizes of the four red zones.

### A.4 Proof of Table 1

In the equations below,  $a$  stands for  $\rho$  or  $N$ . For a given red zone  $x$ ,  $\hat{s}(x, a) = \frac{X-x}{N}$ , and  $\hat{z}(x, a) = \omega + \frac{T(x, a)}{N}$  where

$$T(x, a) = \begin{cases} \text{CR}(0) - \text{CR}(x) & \text{(Mayor)} & (28) \\ rx \text{ s.t. } r = \text{MRS}_{sz} \left( \omega + \frac{rx}{N}, \frac{X-x}{N} \right) & \text{(Market)} & (29) \\ T \text{ s.t. } U \left( \omega + \frac{T}{N}, \frac{X-x}{N} \right) = U \left( \omega, \frac{X}{N} \right) & \text{(Firm)} & (30) \\ -\text{CR}(x) & \text{(Integ)} & (31) \end{cases}$$

As stated by (19) for interior solutions, the red zones  $\bar{x} \in (0, X)$  are characterized by the equality between the marginal risk reduction (MRR) and the marginal rate of substitution ( $\text{MRS}_{sz}$ ) of households.

$$\hat{\text{MRR}}(\bar{x}, a) = \text{MRS}_{sz}(\hat{z}(\bar{x}, a), \hat{s}(\bar{x}, a)). \quad (32)$$

By derivation of (32) with respect to  $a$ , we get

$$\left( \frac{\partial \hat{\text{MRR}}}{\partial x} \frac{d\bar{x}}{da} + \frac{\partial \hat{\text{MRR}}}{\partial a} \right) = \frac{\partial \text{MRS}_{sz}}{\partial z} \left( \frac{\partial \hat{z}}{\partial a} + \frac{\partial \hat{z}}{\partial x} \frac{d\bar{x}}{da} \right) + \frac{\partial \text{MRS}_{sz}}{\partial s} \left( \frac{\partial \hat{s}}{\partial a} + \frac{\partial \hat{s}}{\partial x} \frac{d\bar{x}}{da} \right). \quad (33)$$

Thus

$$\frac{d\bar{x}}{da} \left( \frac{\partial \hat{\text{MRR}}}{\partial x} - \frac{\partial \text{MRS}_{sz}}{\partial z} \frac{\partial \hat{z}}{\partial x} - \frac{\partial \text{MRS}_{sz}}{\partial s} \frac{\partial \hat{s}}{\partial x} \right) = -\frac{\partial \hat{\text{MRR}}}{\partial a} + \frac{\partial \text{MRS}_{sz}}{\partial z} \frac{\partial \hat{z}}{\partial a} + \frac{\partial \text{MRS}_{sz}}{\partial s} \frac{\partial \hat{s}}{\partial a}. \quad (34)$$

**Dominant mayor, dominant firm, and integrated scenarios.** This decomposition can be used to get the sign of  $d\bar{x}/da$ .

Because  $\partial \hat{z}/\partial x = 1/N \cdot \partial \hat{T}/\partial x > 0$  and  $\partial \hat{s}/\partial x = -1/N < 0$  and thanks to technical assumptions (9) and (17), the factor for  $d\bar{x}/da$  in (34) above is negative.

Therefore the sign of  $d\bar{x}/da$  is the sign of

$$\frac{\partial \hat{\text{MRR}}}{\partial a} - \underbrace{\frac{\partial \text{MRS}_{sz}}{\partial z} \frac{\partial \hat{z}}{\partial a}}_{\geq 0} - \underbrace{\frac{\partial \text{MRS}_{sz}}{\partial s} \frac{\partial \hat{s}}{\partial a}}_{\leq 0}. \quad (35)$$

The signs are derived from (17) and, concerning the first sign, from the fact that  $\text{MRS}_{sz} = (\text{MRS}_{zs})^{-1}$ .

These three terms can be named and interpreted:

$$\text{Risk intensification effect} = \frac{\partial \hat{\text{MRR}}}{\partial a}, \quad (36)$$

$$\text{Income effect} = -\frac{\partial \text{MRS}_{sz}}{\partial z} \frac{\partial \hat{z}}{\partial a}, \quad (37)$$

$$\text{Land sharing effect} = -\frac{\partial \text{MRS}_{sz}}{\partial s} \frac{\partial \hat{s}}{\partial a}. \quad (38)$$

The signs of  $\partial \hat{\text{MRR}}/\partial a$ ,  $\partial \hat{z}/\partial a$  and  $\partial \hat{s}/\partial a$  are given in Table 6. But the  $\partial \hat{z}/\partial a$  depends on the scenario considered. Thus, we can conclude in the dominant mayor, dominant firm, and integrated scenarios.

**Market scenario.** In the market scenario, (34) cannot be directly used: the sign of  $\partial \hat{z}/\partial x$  cannot be straightforwardly computed because the rent  $r$  is endogenous. By derivation of  $\hat{\text{MRR}} = r$  and  $\text{MRS}_{sz} = r$  with respect to  $\rho$  or  $N$ , we get ambiguous expressions (available on request).

All scenarios	$\frac{\partial \hat{\text{MRR}}}{\partial \rho} = \frac{\hat{\text{MRR}}}{\rho}$	$> 0$	$\frac{\partial \hat{\text{MRR}}}{\partial N} = \frac{\lambda_F}{X-\bar{x}} \left( p(\bar{x}) - \frac{\int_{\bar{x}}^X p(t) dt}{X-\bar{x}} \right)$	$\geq 0$
Mayor	$\frac{\partial \hat{z}}{\partial \rho} = \frac{\text{CR}(0) - \text{CR}(\bar{x})}{N\rho}$	$> 0$	$\frac{\partial \hat{z}}{\partial N} = -\frac{\lambda_S}{N^2} \rho \int_0^{\bar{x}} f(t) dt$	$< 0$
Firm	$\frac{\partial \hat{z}}{\partial \rho} = 0$	$= 0$	$\frac{\partial \hat{z}}{\partial N} = -\frac{\bar{x}}{N^2} \frac{U_s}{U_z}$	$< 0$
Integrated	$\frac{\partial \hat{z}}{\partial \rho} = -\frac{\text{CR}(\bar{x})}{N\rho}$	$< 0$	$\frac{\partial \hat{z}}{\partial N} = \frac{\lambda_S}{N^2} \rho \int_{\bar{x}}^X f(t) dt$	$> 0$
All scenarios	$\frac{\partial \hat{s}}{\partial \rho} = 0$	$= 0$	$\frac{\partial \hat{s}}{\partial N} = -\frac{X-\bar{x}}{N^2}$	$< 0$

Table 6: Derivatives of  $\hat{\text{MRR}}$ ,  $\hat{z}$  and  $\hat{s}$  with respect to  $\rho$  and  $N$

## A.5 Comparative statics with log-log utility function and linear loss probability

We compute the comparative statics of  $\bar{x}$ . In the equations below,  $a$  stands for  $\rho$  or  $N$  to economize the exposition.

**Lemma 1** (Comparative statics of the size of the red zone). *Consider the LHS and RHS of an equation defining  $\bar{x}$ . Assume that LHS decreases with respect to  $\bar{x}$  and that RHS increases with respect to  $\bar{x}$ . The LHS and RHS both depend on a parameter  $k$ .*

$$\text{LHS}(\bar{x}, k) = \text{RHS}(\bar{x}, k). \quad (39)$$

- (i) *If the LHS increases or is constant with respect to  $k$  and the RHS decreases or is constant with respect to  $k$ , then  $\bar{x}$  increases with respect to  $k$ .*
- (ii) *If the LHS decreases or is constant with respect to  $k$  and the RHS increases or is constant with respect to  $k$ , then  $\bar{x}$  decreases with respect to  $k$ .*

In the case of a log-log utility function and a linear loss probability, that is,

$$U(z, s) = \log(z) + \alpha \log(s) \quad \text{and} \quad p(x) = \rho \cdot (X - x), \quad (40)$$

we can compute some of the red zones, their variations with respect to  $\rho$  and  $N$ , and their limits (green and purple zones) when these parameters tend to infinity. In the following, we take the interior solution of the equation that defines  $\bar{x}_i$ . Given the spatial constraint, the limits are calculated taking into account the fact that

$$\bar{x}_i = \max \{ \min \{ \bar{x}_i ; X \} ; 0 \}. \quad (41)$$

**Dominant mayor scenario.** The first order condition that characterizes the interior solution

$$\text{MRR}(\bar{x}) = \text{MRS}_{sz} \left( \omega + \frac{\text{CR}(0) - \text{CR}(\bar{x})}{N}, \frac{X - \bar{x}}{N} \right), \quad (42)$$

becomes

$$\frac{\rho \lambda_F N}{2} + \rho \lambda_S (X - \bar{x}_{\text{Mayor}}) = \frac{\alpha N}{X - \bar{x}_{\text{Mayor}}} \left( \omega + \frac{\rho \lambda_F}{2} \bar{x}_{\text{Mayor}} + \frac{\rho \lambda_S}{2N} \bar{x}_{\text{Mayor}} (2X - \bar{x}_{\text{Mayor}}) \right). \quad (43)$$

Dividing (43) by  $\rho$  enables the application of case (i) of Lemma 1 and to conclude that  $\bar{x}_{\text{Mayor}}$  increases with respect to  $\rho$ . The red zone is

$$\bar{x}_{\text{Mayor}} = X - \frac{(1+\alpha)}{2(2+\alpha)} \frac{\lambda_F N}{\lambda_S} \left( \sqrt{1 + 8 \frac{(2+\alpha)}{(1+\alpha)^2} \frac{\lambda_S \alpha}{\lambda_F^2 N \rho} \left( \omega + \frac{\rho \lambda_S}{2N} X^2 + \frac{\rho \lambda_F}{2} X \right)} - 1 \right). \quad (44)$$

$\bar{x}_{\text{Mayor}}$  is always monotonic with respect to  $N$ . The red zone increases with respect to  $N$  if and only if

$$\rho^2 \lambda_F^2 \bar{x}^2 - 4\alpha(\alpha+2)\omega\rho\lambda_F \bar{x} - 4\alpha(\alpha+2)\omega^2 > 0. \quad (45)$$

Green and purple zones exist such that:

$$\lim_{\rho \rightarrow +\infty} \bar{x}_{\text{Mayor}} = X - \frac{(1+\alpha)}{2(2+\alpha)} \frac{\lambda_F N}{\lambda_S} \left( \sqrt{1 + 4 \frac{\alpha(2+\alpha)}{(1+\alpha)^2} \frac{\lambda_S X}{\lambda_F N} \left( \frac{\lambda_S X}{\lambda_F N} + 1 \right)} - 1 \right); \quad (46)$$

$$\lim_{N \rightarrow +\infty} \bar{x}_{\text{Mayor}} = \max \left\{ \frac{1}{1+\alpha} X - \frac{2\alpha}{1+\alpha} \frac{\omega}{\rho \lambda_F}; 0 \right\}. \quad (47)$$

**Market scenario.** The first order condition that characterizes the interior solution

$$\text{MRR}(\bar{x}) = \text{MRS}_{sz} \left( \omega + \frac{r\bar{x}}{N}, \frac{X - \bar{x}}{N} \right), \quad (48)$$

becomes

$$\frac{\rho \lambda_F N}{2} + \rho \lambda_S (X - \bar{x}_{\text{Market}}) = \frac{\alpha \omega N}{X - (1+\alpha)\bar{x}_{\text{Market}}}. \quad (49)$$

Using case (i) of Lemma 1, we conclude that  $\bar{x}_{\text{Market}}$  increases with respect to  $\rho$ . Dividing (49) by  $N$  enables the application of case (ii) of Lemma 1:  $\bar{x}_{\text{Market}}$  decreases so with respect to  $N$ .

The red zone is

$$\begin{aligned} \bar{x}_{\text{Market}} = & \frac{1}{1+\alpha} \frac{X}{2} \left[ 2 + \alpha - \alpha \sqrt{1 + \frac{\alpha}{1+\alpha} \frac{\omega N}{\rho \lambda_S \left( \frac{\alpha}{1+\alpha} \frac{X}{2} + \frac{\lambda_F N}{\lambda_S} \frac{N}{4} \right)^2}} \right] \\ & + \frac{\lambda_F}{\lambda_S} \frac{N}{4} \left[ 1 - \sqrt{1 + \frac{\alpha}{1+\alpha} \frac{\omega N}{\rho \lambda_S \left( \frac{\alpha}{1+\alpha} \frac{X}{2} + \frac{\lambda_F N}{\lambda_S} \frac{N}{4} \right)^2}} \right]. \end{aligned} \quad (50)$$

The green and purple zones are

$$\lim_{\rho \rightarrow +\infty} \bar{x}_{\text{Market}} = \frac{1}{1+\alpha} X; \quad (51)$$

$$\lim_{N \rightarrow +\infty} \bar{x}_{\text{Market}} = \max \left\{ \frac{1}{1+\alpha} X - \frac{2\alpha}{1+\alpha} \frac{\omega}{\rho \lambda_F}; 0 \right\}. \quad (52)$$

**Dominant firm scenario.** The first order condition that characterizes the interior solution

$$\text{MRR}(\bar{x}) = \text{MRS}_{sz} \left( \omega + t, \frac{X - \bar{x}}{N} \right), \quad (53)$$

where  $t$  is defined by

$$\log\left(\omega + t, \frac{X - \bar{x}}{N}\right) = \log\left(\omega, \frac{X}{N}\right), \quad (54)$$

becomes

$$\frac{\rho\lambda_F N}{2} + \rho\lambda_S(X - \bar{x}_{\text{Firm}}) = \frac{\alpha\omega N X^\alpha}{(X - \bar{x}_{\text{Firm}})^{1+\alpha}}. \quad (55)$$

Case (i) of Lemma 1 applies and  $\bar{x}_{\text{Firm}}$  increases with respect to  $\rho$ . Dividing (55) by  $N$  enables the application of case (ii) of Lemma 1 and to conclude that  $\bar{x}_{\text{Firm}}$  decreases with respect to  $N$ . Green and purple zones are completely squeezed.

$$\lim_{\rho \rightarrow +\infty} \bar{x}_{\text{Firm}} = X; \quad (56)$$

$$\lim_{N \rightarrow +\infty} \bar{x}_{\text{Firm}} = \max\left\{X - \left(\frac{2\alpha\omega X^\alpha}{\rho\lambda_F}\right)^{\frac{1}{1+\alpha}}; 0\right\}. \quad (57)$$

**Integrated scenario.** The first order condition that characterizes the interior solution

$$\text{MRR}(\bar{x}) = \text{MRS}_{sz}\left(\omega - \frac{\text{CR}(\bar{x})}{N}, \frac{X - \bar{x}}{N}\right), \quad (58)$$

becomes

$$\frac{\rho\lambda_F N}{2} + \rho\lambda_S(X - \bar{x}_{\text{Integ}}) = \frac{\alpha N}{X - \bar{x}_{\text{Integ}}}\left(\omega - \frac{\rho\lambda_F}{2}(X - \bar{x}_{\text{Integ}}) - \frac{\rho\lambda_S}{2N}(X - \bar{x}_{\text{Integ}})^2\right). \quad (59)$$

Case (i) of Lemma 1 applies and  $\bar{x}_{\text{Integ}}$  increases with respect to  $\rho$ . Dividing (59) by  $N$  enables the application of case (ii) of Lemma 1 and to conclude that  $\bar{x}_{\text{Integ}}$  decreases with respect to  $N$ . The red zone is

$$\bar{x}_{\text{Integ}} = X - \frac{(1+\alpha)}{2(2+\alpha)} \frac{\lambda_F N}{\lambda_S} \left( \sqrt{1 + 8 \frac{(2+\alpha)}{(1+\alpha)^2} \frac{\lambda_S \alpha \omega}{\lambda_F^2 N \rho}} - 1 \right). \quad (60)$$

Green and purple zones are completely squeezed.

$$\lim_{\rho \rightarrow +\infty} \bar{x}_{\text{Integ}} = X; \quad (61)$$

$$\lim_{N \rightarrow +\infty} \bar{x}_{\text{Integ}} = \max\left\{X - \frac{2\alpha}{1+\alpha} \frac{\omega}{\rho\lambda_F}; 0\right\}. \quad (62)$$

## A.6 Comparative statics with log-lin utility function and linear loss probability

We use the same convention as in (41). The proof of monotonicity of the red zone with respect to  $\rho$  relies on the direct calculation of the derivatives.

### Dominant mayor scenario.

$$\bar{x}_{\text{Mayor}} = X + \left( \frac{\lambda_F}{2\lambda_S} + \frac{1}{\alpha} \right) N - \sqrt{X^2 + \frac{\lambda_F^2 N^2}{4\lambda_S^2} + \frac{N^2}{\alpha^2} + \frac{\lambda_F X N}{\lambda_S} + \frac{2\omega N}{\rho\lambda_S}}. \quad (63)$$

$$\lim_{\rho \rightarrow \infty} \bar{x}_{\text{Mayor}} < X \quad \text{iif} \quad N\lambda_F(\alpha X - N) + \alpha\lambda_S X^2 > 0, \quad (64)$$

$$\lim_{N \rightarrow \infty} \bar{x}_{\text{Mayor}} = X \quad \text{and it is attained for finite large enough } N. \quad (65)$$

$$\frac{\partial \bar{x}_{\text{Mayor}}}{\partial \rho} = \frac{2\alpha N \omega}{\rho \sqrt{\rho(\alpha^2 N^2 \rho \lambda_F^2 + 4\alpha^2 N \rho X \lambda_F \lambda_S + 4\lambda_S(\rho \lambda_S(N^2 + \alpha^2 X^2) + 2\alpha^2 N \omega))}} > 0, \quad (66)$$

$$\frac{\partial \bar{x}_{\text{Mayor}}}{\partial N} = \frac{\alpha \rho \lambda_F + 2\rho \lambda_S - \frac{\rho(2\alpha^2 N \rho \lambda_F^2 + 4\alpha^2 \lambda_S(\rho X \lambda_F + 2\omega) + 8N \rho \lambda_S^2)}{2\sqrt{\rho(\alpha^2 N^2 \rho \lambda_F^2 + 4\alpha^2 N \lambda_S(\rho X \lambda_F + 2\omega) + 4\rho \lambda_S^2(N^2 + \alpha^2 X^2))}}}{2\alpha \rho \lambda_S}. \quad (67)$$

The proof of monotonicity with respect to  $N$  is based on three facts: the derivative changes sign only once as  $N$  changes; it is positive for large values of  $N$ ; the explicit expression of the red zone has two zeros only,  $N = 0$  being one of them.

### Market scenario.

$$\bar{x}_{\text{Market}} = \frac{X}{2} + \frac{1}{2} \left( \frac{\lambda_F}{2\lambda_S} + \frac{1}{\alpha} \right) N - \frac{1}{2} \sqrt{X^2 + \frac{\lambda_F^2 N^2}{4\lambda_S^2} + \frac{N^2}{\alpha^2} + \frac{\lambda_F X N}{\lambda_S} + \frac{4\omega N}{\rho\lambda_S} - \frac{\lambda_F N^2}{\alpha\lambda_S} - \frac{2XN}{\alpha}}. \quad (68)$$

$$\lim_{\rho \rightarrow \infty} \bar{x}_{\text{Market}} < X \quad \text{iif} \quad \alpha X - N > 0, \quad (69)$$

$$\lim_{N \rightarrow \infty} \bar{x}_{\text{Market}} = X \quad \text{and it is attained for finite large enough } N. \quad (70)$$

$$\frac{\partial \bar{x}_{\text{Market}}}{\partial \rho} = \frac{2\alpha N \omega}{\rho \sqrt{\rho(\alpha N \lambda_F + 2\lambda_S(N + \alpha X))^2 - 8\alpha N \lambda_S(-2\alpha\omega + N\rho\lambda_F + 2\rho X \lambda_S)}} > 0, \quad (71)$$

$$\frac{\partial \bar{x}_{\text{Market}}}{\partial N} = \frac{\alpha \lambda_F + 2\lambda_S - \frac{\alpha^2 N \rho \lambda_F^2 + 2\alpha \rho \lambda_F \lambda_S(\alpha X - 2N) + 4\lambda_S(2\alpha^2 \omega + \rho \lambda_S(N - \alpha X))}{\sqrt{\rho(\alpha N \lambda_F + 2\lambda_S(N + \alpha X))^2 - 8\alpha N \lambda_S(-2\alpha\omega + N\rho\lambda_F + 2\rho X \lambda_S)}}}{4\alpha \lambda_S}. \quad (72)$$

As for the dominant mayor scenario, the proof of monotonicity with respect to  $N$  is based on three facts: the derivative changes sign only once as  $N$  changes; it is positive for large values of  $N$ ; the explicit expression of the red zone has two zeros only,  $N = 0$  being one of them.



**Dominant firm scenario.** The red zone is not calculated explicitly. A system of equations defines  $\bar{x}_{\text{Firm}}$  and  $\bar{T}_{\text{Firm}}$ , and by the theorem of implicit functions

$$\frac{\partial \bar{x}_{\text{Firm}}}{\partial \rho} = \frac{N^2 (2\lambda_S (X - \bar{x}_{\text{Firm}}) + N\lambda_F)}{2 (\alpha^2 (\bar{T}_{\text{Firm}} + \omega) + N^2 \rho \lambda_S)} > 0, \quad (73)$$

$$\frac{\partial \bar{x}_{\text{Firm}}}{\partial N} = \frac{2\alpha (\bar{T}_{\text{Firm}} (\alpha \bar{x}_{\text{Firm}} + N) + \alpha \omega \bar{x}_{\text{Firm}}) + N^3 \rho \lambda_F}{2N (\alpha^2 (\bar{T}_{\text{Firm}} + \omega) + N^2 \rho \lambda_S)} > 0. \quad (74)$$

When  $N$  is large enough,  $\bar{T} < \rho NT$  where  $T$  is some constant: the firm always pays less than the total loss, hence there is an upper bound proportional to  $\rho$  and  $N$ . Therefore, we find lower bounds of the derivatives:

$$\frac{\partial \bar{x}_{\text{Firm}}}{\partial \rho} > \frac{N^3 \lambda_F}{2 (\alpha^2 (\rho NT + \omega) + N^2 \rho \lambda_S)} > \frac{N^3 \lambda_F}{2 (\alpha^2 (NT + \omega) + N^2 \lambda_S)} \frac{1}{\rho}, \quad (75)$$

$$\frac{\partial \bar{x}_{\text{Firm}}}{\partial N} = \frac{N^3 \rho \lambda_F}{2N (\alpha^2 (\bar{T}_{\text{Firm}} + \omega) + N^2 \rho \lambda_S)} > \frac{N^2 \rho \lambda_F}{2 (\alpha^2 (\rho NT + \omega) + N^2 \rho \lambda_S)} \quad (76)$$

$$> \frac{\lambda_F}{2\lambda_S} - \epsilon \quad \text{for } \epsilon \text{ very small and } N \text{ large enough.} \quad (77)$$

The fact that  $\bar{x}_{\text{Firm}}$  goes to infinity (i.e. reaches  $X$ ) as  $\rho$  or  $N$  goes to infinity can be deduced from the above inequalities by using the Grönwall inequality to conclude. Consequently

$$\lim_{\rho \rightarrow \infty} \bar{x}_{\text{Firm}} = X \quad \text{and it is attained for finite large enough } \rho, \quad (78)$$

$$\lim_{N \rightarrow \infty} \bar{x}_{\text{Firm}} = X \quad \text{and it is attained for finite large enough } N. \quad (79)$$

**Integrated scenario.**

$$\bar{x}_{\text{Integ}} = X + \left( \frac{\lambda_F}{2\lambda_S} + \frac{1}{\alpha} \right) N - \sqrt{\frac{\lambda_F^2 N^2}{4\lambda_S^2} + \frac{N^2}{\alpha^2} + \frac{2\omega N}{\rho \lambda_S}}. \quad (80)$$

$$\lim_{\rho \rightarrow \infty} \bar{x}_{\text{Integ}} = X \quad \text{and it is attained for finite large enough } \rho, \quad (81)$$

$$\lim_{N \rightarrow \infty} \bar{x}_{\text{Integ}} = X \quad \text{and it is attained for finite large enough } N. \quad (82)$$

$$\frac{\partial \bar{x}_{\text{Integ}}}{\partial \rho} = \frac{2\alpha N \omega}{\rho \sqrt{N \rho (\alpha^2 N \rho \lambda_F^2 + 4\lambda_S (2\alpha^2 \omega + N \rho \lambda_S))}} > 0, \quad (83)$$

$$\frac{\partial \bar{x}_{\text{Integ}}}{\partial N} = \frac{\alpha \lambda_F + 2\lambda_S - \frac{\alpha^2 N \rho \lambda_F^2 + 4\lambda_S (\alpha^2 \omega + N \rho \lambda_S)}{\sqrt{N \rho (\alpha^2 N \rho \lambda_F^2 + 4\lambda_S (2\alpha^2 \omega + N \rho \lambda_S))}}}{2\alpha \lambda_S}. \quad (84)$$

Concerning the variations with respect to  $N$ , a case where the red zone is nonmonotonic for positive values of  $N$  has been constructed and simulated (see Figure 7).