

Structural damping definitions of multilayered plates

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Abstract

This paper deals with the structural damping of multilayered plates. Three definitions of loss factor are studied. The first one uses an equivalent methodology which assumes that the structure behaves as a Love-Kirchhoff's thin plate. The second approach links the spatial decay rate to the spatial decay rate by means of the group velocity of the wave to account for the dynamic behavior of the structure (shearing effect of the layers). The third approach relies on an energetic method based on the Modal Strain Energy (MSE) method. The loss factors according to these definitions are compared by means of an analytical model of multilayered plates for a typical sandwich structure. Experimental measurements are conducted on constrained-layer damping sandwich plates to validate the definitions. The loss factor is identified using three different protocols based on modal analysis (ESPRIT method), time decay rate estimation and displacement field analysis (CFAT method) and compared to the predictions of the analytical model.

1 Introduction

The reduction of structural vibration level remains a major industrial challenge. Passive damping materials such as viscoelastic layers or damping patches are widely used in multilayered panels to reduce these vibrations. Sandwich composite plates associate these soft materials with stiff skins. This combination provides optimized damping and isolation performances for given frequency bands.

Several experimental techniques exist for damping identification. Commonly known are the half-power band width method (3 dB method) in modal analysis, the decay rate method and the steady-state power input method. Most of these methods meet their limits in terms of precision and resolution in the high-frequency domain. Other approaches are reported in the literature to handle such limitations. For instance, the ESPRIT method [1] extends the high-frequency limit of modal analysis. Approaches based on displacement field analysis can also be cited. Among them, wavenumber fitting methods, such as the IWC [2] (Inhomogeneous Wave Correlation) method or the Hankel fitting approach [3], deals with complex wavenumber identification, and inverse methods, such as the CFAT [4] (Corrected Force Analysis Technique) and the VFM [5] (Virtual Field Method), locally solve the equation of motion of the structure. These field analysis methods can identify the damping of the structure by means of an equivalent methodology, assuming that the multilayer behaves as a thin homogeneous plate under Love-Kirchhoff's theory [6, 7, 8, 9]. Concerning the prediction of damping, the MSE (Modal Strain Energy) method can be used to estimate modal damping levels. This method, largely employed in literature [10, 11, 12], is valid for lightly damped structures. In another approach, Lyon and Dejong [13] defines the structural loss factor as function of the spatial attenuation of a plane wave. This definition relies on the relationship between space and time domain attenuations, which is dependent on the group velocity of the wave.

In this paper, the equivalent methodology, the MSE method and the space domain definition of Lyon and Dejong are used to identify the damping of multilayered panels. Firstly, these three different approaches are detailed in section 2. Then, the definitions are compared in section 3 by means of an analytical model of multilayered plate. Finally, section 4 presents an experimental application conducted on constrained layer sandwich plates. The estimations of the space domain and equivalent definitions are compared to the results of modal analysis, ESPRIT method and decay rate method.

The presented work has already been published in a journal article by the same authors [14]. One may refer to this paper for additional content.

2 Definitions of structural loss factor

2.1 Equivalent definition

This first definition of the structural loss factor relies on an equivalent plate methodology. For a given angular frequency ω , we assume that the multilayer exhibits the same transverse displacement as an equivalent thin plate under Love-Kirchhoff's theory. Based upon this assumption, the flexural wavenumber k_f of the multilayer is solution of the dispersion relation of the equivalent plate:

$$Dk_f^4 = m_S\omega^2, \quad (1)$$

with D the flexural rigidity and m_S the surface mass of the equivalent plate. From this dispersion relation the structural loss factor of the equivalent plate η_{eq} can be defined as function of k_f (using the $e^{j\omega t}$ convention):

$$\eta_{eq} = \frac{\text{Im}(D)}{\text{Re}(D)} = -\frac{\text{Im}(k_f^4)}{\text{Re}(k_f^4)}. \quad (2)$$

This equivalent definition is based on the thin plate theory and is thus valid for thin structures only.

2.2 Space domain definition

The second definition of the structural loss factor is linked to the spatial attenuation of a plane wave which naturally depends on a complex wavenumber k . Using the $e^{j\omega t}$ convention, the spatial decay rate Δx of the wave is defined by:

$$\Delta x = 20\log(e^{\text{Im}(k)}) = 20\text{Im}(k)/\ln(10) \text{ in dB/m}. \quad (3)$$

This definition is not directly related to the structural loss factor η , that governs the decay rate in the time domain:

$$\Delta t = 20\log(e^{-\text{Im}(\omega)}) = 20\log(e^{-\eta\text{Re}(\omega)/2}) = -10\eta\text{Re}(\omega)/\ln(10) \text{ in dB/s}. \quad (4)$$

A link between space and time domain attenuations can however be established by introducing the group velocity of the wave (that represents the velocity at which the energy is conveyed), defined by $C_g = \partial\omega/\partial k$. The group velocity gives a relationship between space and time attenuations: $\Delta t = \Delta x C_g$, which brings [13]:

$$\eta = -2\frac{\text{Im}(k)}{\text{Re}(\omega)}C_g = -2\frac{\text{Im}(k)}{\text{Re}(k)}\frac{C_g}{C_\phi}, \quad (5)$$

where $C_\phi = \text{Re}(\omega)/\text{Re}(k)$ is the phase velocity of the plane wave. This definition of the structural loss factor is valid for all type of structures and is not based on any kind of equivalent model assumptions.

2.3 Energetic definition

The third and last definition corresponds to an energetic approach based on the Modal Strain Energy (MSE) method. This methodology assumes that the equations of free vibration of the structure is described using

the following matrix relation:

$$\mathcal{M}\ddot{X} + (\mathcal{K}_r + j\mathcal{K}_i)X = 0, \quad (6)$$

where \mathcal{M} , \mathcal{K}_r , \mathcal{K}_i and X correspond to the mass, stiffness, loss stiffness matrices and displacement vector of the system. The superscript $\ddot{}$ denotes the second order derivative with respect to time and $j = \sqrt{-1}$. The equation of motion can be solved using an eigenvalue problem:

$$(\mathcal{K}_r + j\mathcal{K}_i)\phi_p^* = \lambda_p^{*2} \mathcal{M}\phi_p^*, \quad (7)$$

where λ_p^* and ϕ_p^* are the p th complex eigenvalue and mode shape, respectively. The complex eigenvalue can be expressed as function of the loss factor of the p th mode:

$$\lambda_p^{*2} = \lambda_p^2(1 + j\eta_p). \quad (8)$$

The complex mode shape ϕ_p^* is approximated by the real mode shape ϕ_p solving the eigenvalue problem with $\mathcal{K}_i = 0$. Then, the loss factor η_p is defined by:

$$\eta_p = \frac{\phi_p^H \mathcal{K}_i \phi_p}{\phi_p^H \mathcal{K}_r \phi_p}, \quad (9)$$

where the superscript H denotes the complex conjugate transpose. Note that, for multilayer systems, Shorter [12] and Ghinet et al. [10] assume that the damping of each layer is uniform, meaning that $\mathcal{K}_i = \sum_{n=1}^N \eta_n \mathcal{K}_r^n$, where \mathcal{K}_r^n and η_n correspond to the stiffness matrix and the loss factor of layer n , respectively. This assumption is not used in this paper.

3 Analytical comparison of the definitions

To compare the definitions of the structural loss factor given in the previous sections, we used an analytical model of multilayered plates which has been developed by Guyader and Lesueur [15] for orthotropic material and has been extended to anisotropic material by Loredo and Castel [16]. The model describes the behaviour of each layer with a Reissner-Mindlin displacement field, considering bending, membrane and shearing effects. The transverse displacement is supposed to be constant for all layers, neglecting the deformation through the thickness. Continuity conditions between layers lead to equations of motion which are independent of the number of layers. In this way, the behaviour of the whole multilayer is defined by the kinematic variables of one reference layer. An energetic aspect governed by Hamilton's principle is used to derive the equations of motion and the dispersion curves of the multilayer.

Table 1: Characteristics of each layer of a sandwich plate composed of aluminium skins and a viscoelastic core.

	Aluminium skins	Viscoelastic core
Thickness (mm)	2	10
Density (kg.m ⁻³)	2700	74
Young's modulus (GPa)	70	0.13
Poisson's ratio (-)	0.3	0.4
Damping (-)	0.001	0.04

A sandwich plate, having the characteristics as given in Table 1, has been simulated using the analytical model. The identified flexural wavenumber has been used to calculate the loss factors according to Eq. (2) and Eq. (5). Figure 1 presents these loss factors and the ratio C_g/C_ϕ on a large frequency band. Whatever the definition used, the frequency evolution of the estimated loss factors looks similar. Such frequency dependence of the loss factor has already been observed by several authors like Millithaler et al. [17], Ege et al. [7, 18], or Butaud et al. [19] on other types of sandwich structures. At the low and high frequency, the dynamic behaviour of the multilayer is essentially governed by the bending of the skins. In

this case, the amount of damping of the skins determines the damping of the structure. In between those limits, the behaviour of the multilayer is dominated by the shearing effect of the core. In this case, the loss factor increases until reaching a maximum at a certain frequency. Differences occur between the definitions concerning this maximum. The space domain definition depends on the velocity ratio C_g/C_ϕ which, as indicated in Figure 1, varies with frequency between values equal to 2 (pure bending motion) and 1 (pure shearing motion). Note that the ratio reaches its minimum when the loss factor is maximum, meaning that the space domain definition takes the shearing effect of the core into consideration. As a consequence, the estimated loss factor is always lower than the damping of the core. On the contrary, the equivalent definition assumes that the velocity ratio is equal to 2 on the whole frequency band since the shearing effects are neglected. Thus, the equivalent definition overestimates the structural loss factor as compared to the space domain definition, since it can reach values up to twice the damping of the core. Finally, the space domain definition is in good agreement with the MSE method on the whole frequency range for the studied sandwich.

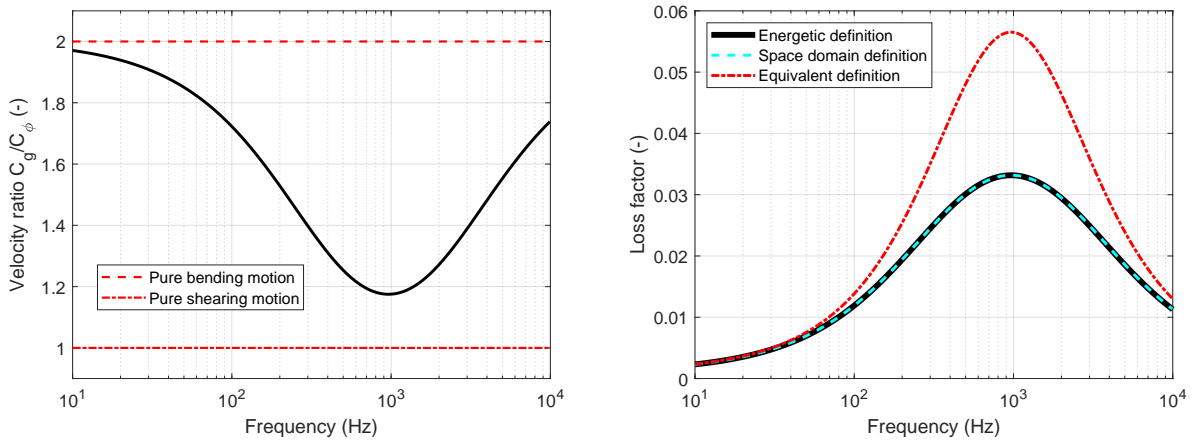
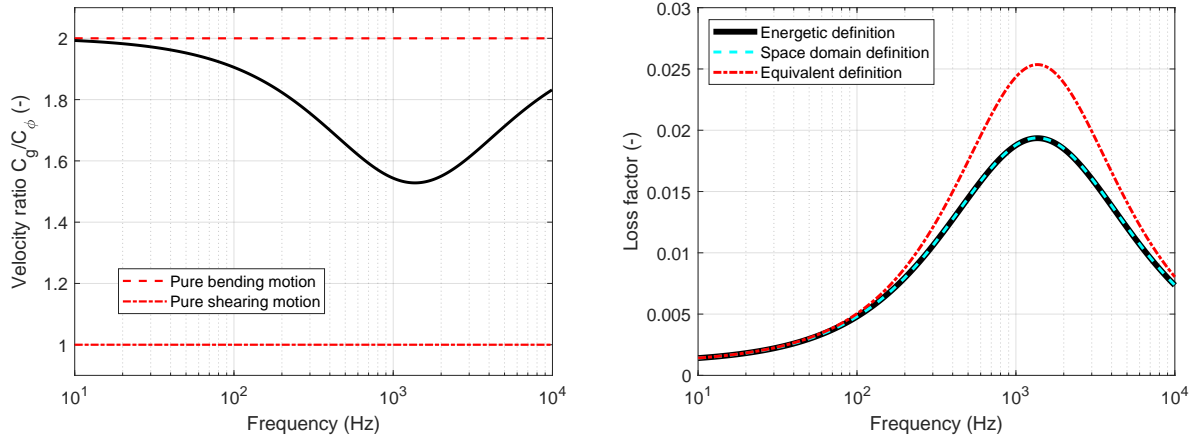
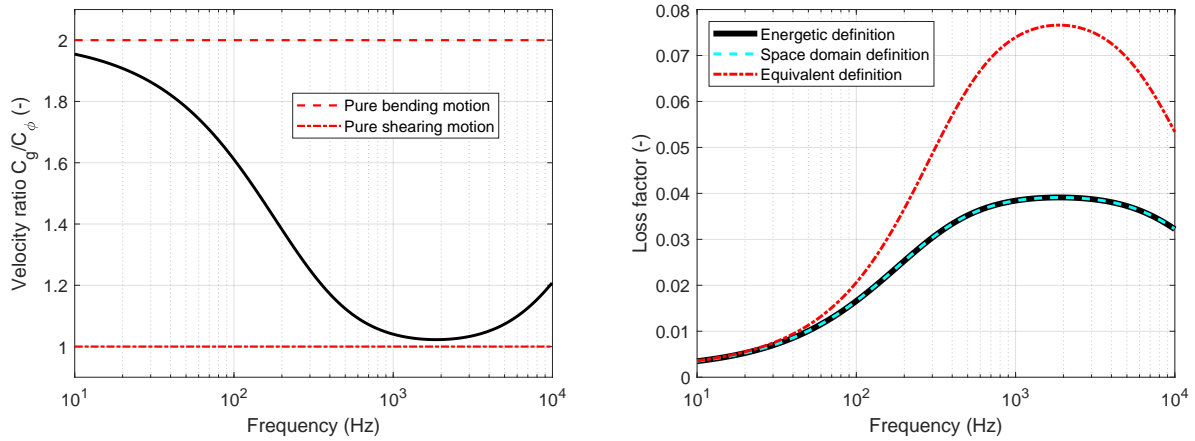


Figure 1: Velocity ratio C_g/C_ϕ (left) and loss factor (right) of a sandwich plate (characteristics of the layer indicated in Table 1) identified with the equivalent, space domain and energetic definitions.

The differences between the definitions depend upon the material parameters of the layers of the sandwich plate. To illustrate this aspect, the loss factor of the studied sandwich has been calculated using the analytical model for different value of the thickness of the core. Figure 2 compares the velocity ratio C_g/C_ϕ and the loss factors η for both configurations. We can observe from these results that the differences are less pronounced if the core is sufficiently thin and stiff as compared to the skins, meaning that the shearing effects can be negligible. On the contrary, when the core is very thick and soft, the minimum of the velocity ratio is close to 1 and the ratio between the maximum of the equivalent and the space domain loss factors tends to 2. We can also notice that the thickness of the core modifies the frequency where the loss factor is maximal. This frequency is also greatly influenced by the shear modulus of the core.



(a) Thickness of the core: 1 mm



(b) Thickness of the core: 100 mm

Figure 2: Influence of the thickness of the core on the velocity ratio C_g/C_ϕ (left) and loss factor (right) of a sandwich plate (characteristics of the layer indicated in Table 1).

4 Experimental estimations of the structural loss factor: validation of the equivalent and space domain definitions

To validate the equivalent and space domain definitions given in section 2, this section presents measurements of the structural loss factor for two constrained-layer damping sandwich plates.

4.1 Three-layer plates

The sandwich plates under study consist of three homogeneous layers: two metal face sheets and a polymer core. Two sets of hybrid composites are studied: symmetrical steel/polymer/steel (called SPS) and non-symmetrical steel/polymer/aluminium (called SPA) rectangular plates of overall dimensions of $0.3 \times 0.4 \text{ m}^2$. Layer thicknesses and material properties of the individual layers are listed in Table 2. The thicknesses correspond to average values determined using optical microscope images of the two plates' cross sections. The Young's modulus and loss factor of the polymer layer are estimated thanks to the extrapolations of DMA (Dynamic Mechanical Analysis) measurements performed on sheets of the polymer layer alone [7]. The density is determined by measuring and weighting large specimens.

Table 2: Dimensions and material properties of the individual layers for the two constrained-layer damping sandwich plates chosen for experimental damping characterisation.

<i>SPS plate</i>	layer 1 (steel)	layer 2 (polymer)	layer 3 (steel)
Thickness (mm)	0.18	0.69	0.18
Density (kg.m ⁻³)	7800	580	7800
Young's modulus (GPa)	210	0.35	210
Poisson's ratio (-)	0.33	0.33	0.33
Loss factor (-)	0.001	0.047	0.001
<i>SPA plate</i>	layer 1 (steel)	layer 2 (polymer)	layer 3 (aluminium)
Thickness (mm)	0.3	0.69	0.13
Density (kg.m ⁻³)	7800	580	2700
Young's modulus (GPa)	210	0.35	69
Poisson's ratio (-)	0.33	0.33	0.33
Loss factor (-)	0.001	0.04	0.001

4.2 Assessment procedures (theories and experimental set-ups)

Figure 3 sums up the experimentally identified loss factors together with analytical predictions for both plates. Before discussing the results of these wide frequency bands damping characterisations, we briefly present the experimental protocols and assessment procedures below.

Modal approaches.

a) Modal analysis (\triangle). To begin, a modal analysis of the plate is performed in order to estimate the first modal frequencies and loss factors. A pseudo-impulse force is applied by means of a small impact hammer (P.C.B. Piezotronics 086E80) on a rectangular mesh of 9×11 points spanning the whole surface of the plate. The mesh is regularly spaced, resulting in spacings between two consecutive points of 37.5 mm along the width x and of 40 mm along the length y . Boundary conditions are kept as close as possible to free-free, by suspending the plate from one of its corner (with rubber bands passing through a tiny hole). The acceleration is measured with a lightweight accelerometer (P.C.B. Piezotronics M353B18) fixed on another corner of the plate. A multi-degree-of-freedom curve fitting method (Rational Fraction Polynomial-Z) is used to estimate modal frequencies and loss factors. For this "low frequency" experimental methodology, modal loss factors have been estimated up to 1 kHz.

b) High resolution modal analysis (ESPRIT methodology) (\bullet). In order to identify the loss factors of the multilayer plate at higher frequencies, a second approach is used. It consists on a high-resolution modal analysis technique [20] based on ESPRIT algorithm [21]. This high-resolution method assumes that the signal $s(t)$ is a sum of complex exponentials $x(t)$ (the modal signal to be determined) and white noise $\beta(t)$. For exponentially damped sinusoids (the signal model considered here), the rotational invariance property of the signal subspace (or *modal* subspace) is used to estimate the modal parameters (see Roy et al. [21] for mathematical developments). Modal frequencies and modal damping factors are derived from the complex poles (eigenvalues of the *spectral matrix* [20]). The experimental protocol is similar to the previous item. The time signal $s(t)$ analysed with ESPRIT algorithm corresponds to an excitation made in the vicinity of the accelerometer near one of the corner of the plate. Modal loss factors have been estimated up to 3 kHz.

Energetic approach.

Time decay rate estimation (\square). Impulse responses used for ESPRIT are also processed following a time decay rate estimation method. Signals are firstly filtered through third octave band filters, then the decay rate of the squared envelope is estimated in dB/s using a linear regression. The loss factor for each frequency band is then assessed using the following relation (see [8]):

$$\eta = \frac{\text{DR}}{27.3f_c}, \quad (10)$$

where DR is the estimated decay rate and f_c the central frequency of the third octave band. Using this third experimental methodology, loss factors have been estimated up to 8 kHz.

Space domain approach.

Displacement field analysis (\times) and (\times). In these measurements, the SPS and SPA plates are freely suspended to a frame, and excited with a shaker fixed at one edge of the plates and driven by a white noise. The transverse displacement fields of the plates are measured by means of a Laser Doppler Vibrometer (PSV 400, Polytec) over an area of $11 \times 11.6 \text{ cm}^2$ far from the excitation point. The scanning mesh is defined by a spatial step of 2.9 mm. In order to identify the complex material parameters of the structure, the inverse method CFAT [4] is applied on the measured displacement fields using the filtering procedure of the FAT [22] (Force Analysis Technique) method to attenuate noise sensibility on the results. The CFAT approach describes the measured structure with a Love-Kirchhoff's thin plate and locally estimates its equation of motion with a finite difference scheme. The complex rigidity and thus the complex flexural wavenumber of the plates are identified with this method up to 20 kHz. The structural loss factor are calculated from the identified complex wavenumber using the equivalent (Eq. (2)) and space domain (Eq. (5)) definitions.

For more details on the assessment procedures presented in this section the reader may refer to Ege et al. [7]. Note that for impulse hammer measurements of the SPA plate, no experimental mesh has been investigated for reason of time-efficiency; the measurement has been done just at one corner of the plate with a relative low sampling frequency, giving results up to 3 kHz with the ESPRIT method only.

4.3 Experimental and analytical comparisons. Discussion

Figure 3 compares the loss factors η_{eq} (Eq.(2)) and η (Eq.(5)) experimentally identified by the CFAT method and analytically predicted by the model with the results of the modal methods (modal analysis and ESPRIT) and energetic approach (reverberation time). Concerning the loss factor identified by the CFAT method using Eq. (5), the ratio C_g/C_ϕ is calculated from the wavenumber given by the analytical model. Another strategy giving similar results would be to use a fitting polynomial curve on the experimental wavenumber results.

We can notice that the predictions of the model are in good agreement with the estimations of the CFAT method for both plates. The loss factors calculated from the model (—) or the CFAT method (\times) using the space domain definition consistently follow the estimations of the modal (\triangle and \bullet) and energetic methods (\square) over a large frequency range. These results demonstrate the validity of the space domain definition and prove that the equivalent definition overestimates the loss factor when the behaviour of the structure is highly modified in the frequency band considered (for instance: shearing effects on the flexural motion). Note that the discrepancies of the modal method in low frequency (for the first modes) could be caused by the losses introduced by the hanging system or accelerometer cable, leading to an overestimation of the loss factor.

Finally, these interesting experimental results, rarely addressed in literature, give a new perspective on damping identification for complex structures. A direct measurement of the complex wavenumber may be sufficient to obtain, using the space domain definition of the loss factor, similar results as energetic or modal methods. This wavenumber can be entirely estimated with spatial experimental methodologies like the CFAT method but also fitting approaches based on Green's functions for example.

5 Conclusion

Three definitions of structural loss factor of multilayer system are compared in this paper using equivalent, space domain and energetic approaches. The equivalent definition, based on Love-Kirchhoff's theory, is valid for thin structures at low frequencies and overestimates the loss factor at high frequencies when the behaviour of the multilayer is dominated by the shearing of the core. The space domain definition describes this effect by considering the ratio of the group and phase velocities. This ratio can be extracted from the (measured or analytically computed) natural wavenumber. The space domain gives consistent results with the MSE method even for thick multilayer systems. Experiments that were conducted on two sandwich plates

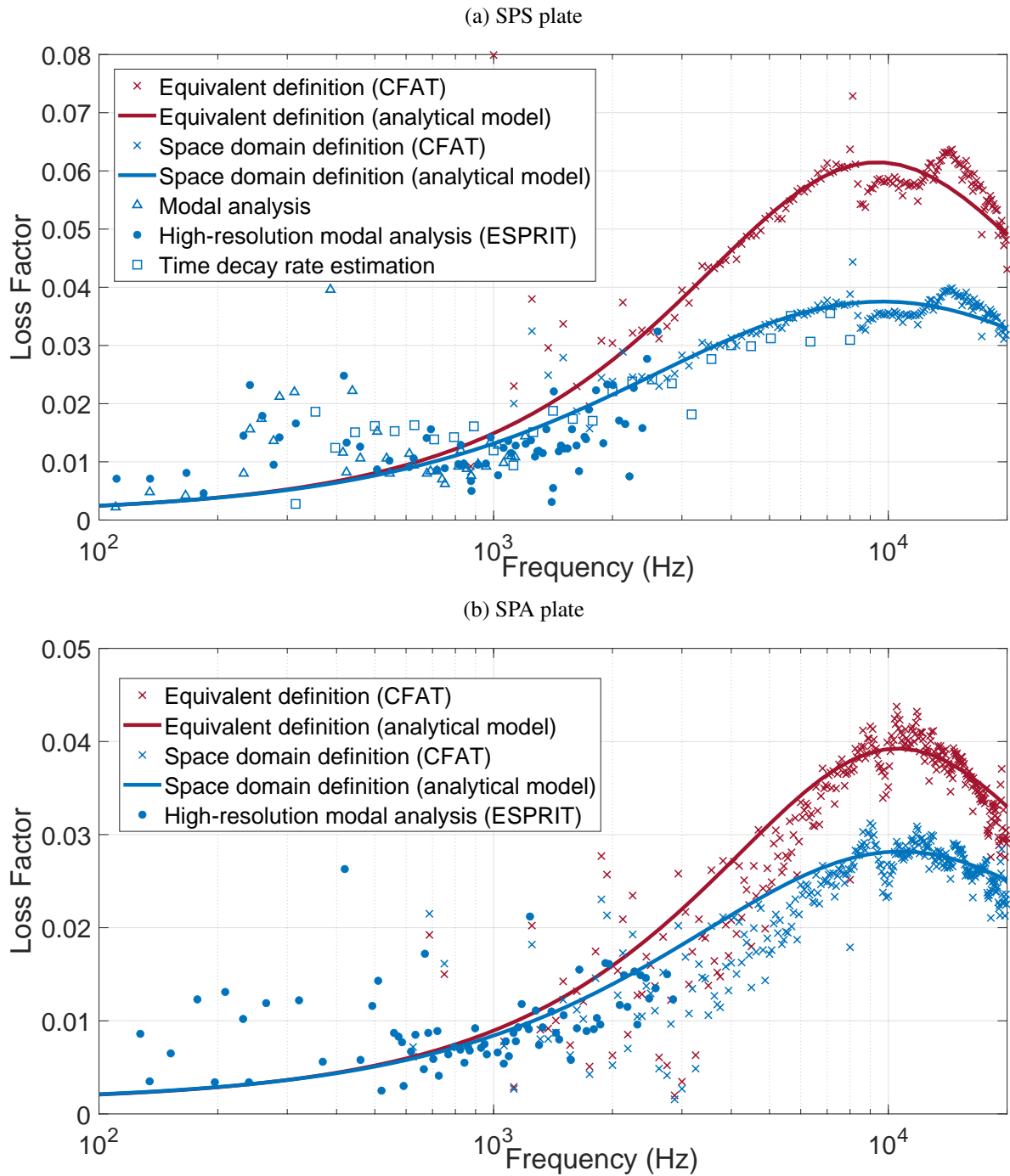


Figure 3: Structural loss factor of the SPS (a) and SPA (b) sandwich plates (characteristics in Table 2). See the legend of the figures for the line types in the graphs and section 4.2 for the experimental procedures.

confirm these statements. The structural loss factors of the plates have been identified from the estimations of the CFAT method and an analytical model using the space domain and equivalent definitions. The results show that the space domain definition is in good agreement with modal and energetic methods. Finally, the space domain definition provides accurate results considering the whole dynamic behaviour of the structure. An interesting application could be to use this definition and damping identification methodologies on other complex structures like ribbed panels [6], periodic structures/meta-materials [23] or porous materials.

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