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Limits to growth and structural change

Marc Germain*

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Abstract

This paper examines the path of an economy constrained by bio-physical limits, due to (i) limited natural capital availability and (ii) compliance with the postulate of strong sustainability. The economy tends towards a stationary state characterized by lower and higher endowments of natural capital and human factors respectively than in the initial state. But this evolution is not monotonous in the sense that GDP and consumption have a path in four phases: growth, reversal, decrease and a quasi-stationary phase leading to steady state. On the contrary, the natural capital stock is declining almost monotonically, involving increasing natural capital operating costs. This results in a structural change by which the human factors share devoted to exploitation increases continuously at the expense of that devoted to final production. Taking pollution into account results in a peak of GDP less pronounced and advanced over time compared to the pollution-free situation.

Keywords : limits to growth, strong sustainability, structural change

JEL: C63, O41, 044, Q20, Q57

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1 Introduction

This article is at the heart of an important and very contemporary issue: the possibility of pursuing economic growth in a finite (bio-physical) world. This question is far from being a new one, since it was already raised by classical authors such as Ricardo or Jevons. Closer to us, it was the subject of the well-known report to the Club of Rome by Meadows et al. (2004)¹. For various reasons (depletion of natural resources (especially land), increased costs of their exploitation, accumulation of pollution), these authors tended to answer the question in the negative in a laissez-faire framework. Indeed, most of the scenarios described in their report anticipate trajectories of the world economy in the 21st century in the form of a growth phase followed by a degrowth phase².

Other contributions from the energy science literature also raise serious doubts about the possibility of further growth. Capellán-Pérez et al. (2014) reach the same conclusion as Meadows et al. (2004) in the more limited context of future energy supply. For other authors (e.g. Hall et al. (2009), Dale et al. (2012), van den Bergh (2013), Hall et al. (2014)), the problem is not the decline in the quantity of available energy resources, but rather the decline in their *quality*. The reasoning starts from the observation that to extract these resources and transform them in order to make them usable by the rest of the economy, energy is needed. The quality of an energy resource is measured through the EROEI (energy returned on energy invested) ratio of the processes that extract and transform it, i.e. the ratio between energy production on the one hand, and the quantity of energy required to obtain this production on the other. For two centuries, the contemporary industrial societies could rely on fossil fuels that offered very high EROEI ratios. Unfortunately, this may not be the case in the future. Indeed, (i) non-renewable resources are characterized by a fall in their EROEI because their extraction gets increasingly energy consuming and (ii) most renewable energies seem characterized by lower EROEI ratios than those observed in the past for fossil fuels. Now, a decline in the EROEIs of energy resources forces the economy to devote ever more human factors (labour and capital) to the exploitation of resources rather than to final production, with deleterious consequences for investment, accumulation and growth.

The "pessimistic" position of the above authors is a matter of debate. Several processes have made it possible in the past and could a priori make it possible in the future to pursue growth. First are the discoveries or exploitation of new resources, as well as resource-saving technological progress (Faber and Proops, 1993). Another explanation is the substitution between natural (resources) and human (labour and capital) factors of production (Solow (1974), Stiglitz (1974)). A fourth process is structural change (induced by shifts in final demand, particularly within household consumption, or by differences of productivity improvements between sectors) which translates into a growing weight of more resource-efficient activities (such as services or research) at the expense of resource-intensive activities (such as industry) (López et al. (2007), Bretschger and Smulders (2012), Campiglio (2014)).

However, even if they are far from complete, the question arises as to whether these processes can be used indefinitely to circumvent the above-mentioned constraints and guarantee growth. Whether their results are based on substitution between human and natural factors, technological progress or structural change, contributions that answer in the affirmative to this question are based on the assumption of *weak sustainability*, in the sense that natural resource productivity is *not* upper bounded (Fagnart-Germain, 2012).

However, other contributions (such as Anderson (1987) and Baumgartner (2004)) have shown that a technology characterized by unbounded resource productivity violates the laws of physics (in particular the conservation laws and the second principle of thermodynamics). As these laws govern all production processes, it follows that the assumption of weak sustainability is physically unfounded (Krysiak, 2006).

Taking into account the argument of the previous paragraph, this article raises the question of the feasibility of long-term growth (i) in an economy subject to limited natural resource availability and (ii) while respecting the assumption of *strong sustainability*, in the sense that natural resource

¹First published in 1972 and updated in 1992 and 2004.

²By contrast, Meadows et al. (2004) show that global sustainable society scenarios are possible if population and production control policies are combined and if technical progress is stimulated in terms of resource and environmental protection.

productivity is upper bounded. It also examines the structural change that may occur during the trajectory of such an economy.

In this sense, it is in line with Meadows et al. (2004), Capellán-Pérez et al. (2014) and Dale et al. (2012). But unlike these contributions, whose approach belongs to systems theory and is based on modeling conventions different from those of economists (in particular, agents' behaviours are not modeled and market mechanisms are implicit), the model developed here belongs to growth theory with environment³.

Among this vast literature, a number of articles are devoted to the energy transition of an economy facing resource and/or climate constraints (e.g. Tahvonen and Salo (2001), Tsur and Zemel (2005), Amigues et al. (2011), Bonneuil and Boucekine (2016)). However, even if energy is fundamental, the limits to growth question is not limited to energy. Indeed the natural environment provides other essential resources such as ecosystem services, water, forests, minerals, etc. In addition, the production of energy (particularly renewable) is inseparable from the mining of metals (common or rare), which in turn requires more and more energy as the concentration of minerals decreases. A vicious circle thus appears in the sense that the constraint of availability of one resource reinforces that of another and inversely (Bihouix (2014), Fizaine and Court (2015)), thus reinforcing the position of the "pessimistic" authors described above.

For the reasons that have just been put forward, the model developed here uses the concept of *natural capital*, which encompasses biological diversity and all natural resources. Natural capital is a stock and to describe its dynamics, the concepts of *ecological footprint* and *biocapacity* developed by the Global Network Footprint will be used together. Biocapacity and ecological footprint refer respectively to the flows that renew and are taken from the natural capital stock. These concepts are also intended to be extensive indicators, if not exhaustive (Zuinen and Gouzée, 2010).

Still from a global perspective and in the manner of Meadows et al. (2004), the model developed is that of a global economy, therefore closed and without Government. The advantage of this assumption is that it forces the economy to fully support the limits of its natural capital, without the possibility of bypassing it through trade with the rest of the world (either by importing natural resources directly or by specializing in low-resource intensive production and exporting it in exchange for resource-rich imports).

While studying the feasibility of growth in a resource-constrained economy, the analysis also aims to highlight the structural change underlying the reasoning of Hall et al. (2009) and the other contributions of the energy science literature mentioned above. Even if they do not explicitly use this notion, the process they describe can be termed structural change when it refers to the shift of human factors from final production to resource exploitation as a result of the decline in energy availability.

In order to study structural change, a two-sector model is developed: a primary sector exploiting natural capital and a final sector producing a general-purpose good for consumption and investment. The question of whether the evolution of structural change is reversible will also be addressed. In other words, if the factor share of the primary sector increases when the economy grows, does it decrease if the economy degrows?

The work of Meadows et al. (2004) shows that growth limits are also likely to result from the negative effects of pollution of all kinds that accompany human activities⁴. Without going so far as to anticipate degrowth scenarios à la Meadows, the NGO DARA study (DARA, 2012) estimates that the overall costs of pollution (including those due to climate change) could exceed 10% of world GDP by the end of the century. Given the above, a second version of the model is developed that postulates a global pollution that negatively affects the economy's production. Comparing the results obtained with the two versions of the model makes it possible to shed light on the impact of pollution on the economy's path and also to see to what extent it modifies structural change

³The World3 model developed by Meadows et al. (2004) is, as its name suggests, a world model that includes several hundred equations (often non-linear). It is therefore much more elaborated than the stylized models usually developed in economics, but also, for the same reason, much less transparent. Although much smaller in size, the models of Capellán-Pérez et al. (2014) and Dale et al. (2012) are also much more detailed in describing the energy sector. With the exception of Dale et al. (2012), the articles related to the decline of the EROEIs mentioned above hold a partial equilibrium reasoning.

⁴These negative effects can even lead to economic collapse in the most severe cases (Germain, 2012).

compared to the pollution-free situation. The second version also makes it possible to analyze the role of pollution persistence in its impacts on the economy.

The structure of this article is as follows. Section 2 describes the model, i.e. the behaviour of households and firms in both sectors, the primary and final sectors. Given the assumption of strong sustainability that underlies the model, a balanced growth path is impossible. However, a stationary state exists under certain conditions and is solved analytically in section 3. It is not possible to do the same at the dynamic level, so the model is solved numerically. Section 4 is devoted to the description of the different simulations carried out with the basic model (without pollution). Section 5 describes the results obtained with the version of the model with pollution. The conclusion summarizes the main results obtained, particularly in relation to the literature.

2 The model

In the tradition of ecological economics, the model distinguishes between human and natural production factors. Thereafter these factors are grouped under the generic terms *human factors* and *natural capital* respectively. These terms must be understood in a very broad sens: (i) human factors (HFs) include all man-made factors (physical capital, human capital and labour force) and (ii) natural capital (NC) includes all living species and natural resources provided by the environment.

Human factors and natural factors play very different roles in the production process. The former extract and transform the latter to produce goods and services. Production is therefore considered not only as a process of value creation, but also as a process of *transformation* of materials and energies, subject to the laws of physics (e.g. the law of matter conservation). As a result, human and natural factors are fundamentally complementary (the possibilities for substitution between the latter are limited).

The model is that of a closed economy without Government. There are three categories of agents: households, primary sector firms that exploit natural capital and final sector firms that produce a final all-purpose good. The absence of Government implies the absence of regulation, particularly in the management of natural capital. To simplify the analysis, it is also assumed that (i) the two sectors are made up of firms in perfect competition (which makes it possible to describe the behaviour of each sector through that of a representative firm) and (ii) NC has free access (which makes it possible to ignore the rents due to the resource owners)⁵.

2.1 The households

We assume long-lived households, who receive the income from production, consume part of it and invest the rest. They rent the HFs they own to firms.

It is assumed that household preferences are represented by homothetic and identical functions. Their propensity to consume is then independent of income, which makes it possible to describe household behaviour through that of a single agent who would receive all the macroeconomic income. At period t , the household budget constraint is written:

$$C_t + I_t = v_t H_t \tag{1}$$

⁵Actually, some natural resources are open access (e.g. oceans), others are not (e.g. fossil resources). In the latter case, a rent due to the owner is added to the operating costs to determine the price of the resource. Neglecting the rent can be justified on the basis of the following arguments:

- Historically, the rent's share of national income has remained relatively low, on average. For the period 1970-2005 and for a large sample of countries, Monge-Naranjo et al. (2019) estimate it at about 6%. This probably explains why the literature on value-added sharing has gradually neglected rent.
- However, the rent's share could increase in the future as a result of the increasing scarcity of resources. In a model similar to the one developed here but with a rent, Germain (2018) studies the polar case where *all* resources are owned. While the rent is likely to take a larger share of national income than historically observed, the author shows that this phenomenon is only transitory and that it is the increase in operating costs that plays the decisive role in the dynamics of the economy. Moreover, he obtains results that are generally similar to those obtained here (although at the cost of significant complications), which suggests that taking into account the rent is not crucial for the study of limits to growth discussed in this paper.

where C_t is consumption, H_t is the HFs stock, I_t is investment in HFs and v_t is their remuneration. Since the HFs stock is understood in a broad sense (as explained above), investment includes expenditure on new productive capital, training and labour force reproduction. The price of final goods (used for consumption and investment) is chosen as the numeraire.

The equation describing the accumulation of HFs is written:

$$H_{t+1} = [1 - \delta] H_t + I_t \quad (2)$$

where δ is the depreciation rate of the HFs stock.

Households choose their consumption C_t and investment I_t in order to maximize their intertemporal utility. Formally, the problem is written:

$$\max_{\{C_t, I_t\}_{t=1, \dots, T}} \sum_{t=1}^T \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$$

under the constraints (1) and (2) to be satisfied at each period with H_1 given. β is the discount factor ($0 < \beta < 1$). σ measures the intertemporal substitution elasticity ($\sigma > 0$). T is the time horizon (very large and possibly infinite). All these parameters are exogenous.

The first-order optimality conditions lead to (cfr. Annex 8.1):

$$\left[\frac{C_{t+1}}{C_t} \right]^\sigma = \beta [v_{t+1} + 1 - \delta] \quad (3)$$

(3) is Ramsey's familiar condition describing the consumption and saving behaviour of households.

2.2 The primary sector

The primary sector exploits natural capital. As it is composed of all living species and natural resources (renewable or not), the primary sector therefore includes all activities directly exploiting Nature (agriculture, forestry, fishing, mining,...).

The stock of NC obeys the following equation:

$$R_{t+1} - R_t = F_t - X_t \quad (4)$$

where :

- X_t refers to the quantities taken on the NC stock by the primary sector;
- F_t includes all the renewal flows of NC (for example, rain that replenishes groundwater, births in the case of an animal species, plant growth due to photosynthesis,...). The previous equation assumes that quantities R, X, F are, despite their heterogeneity, measurable in a common unit. We return to this in section 4.

NC is open access but its exploitation is expensive. To extract a quantity X_t from the NC stock, the primary sector must mobilize the quantity of HFs defined by:

$$H_{pt} = B_t(E_t)X_t \quad (5)$$

wher E_t refers to the exploitation rate of the NC stock:

$$E_t = \frac{X_t}{R_t} \quad (6)$$

The NC exploitation rate is the ratio between the taking by the primary sector and the stock. It goes without saying that $0 \leq E_t \leq 1$.

The function B_t measures an external effect (to the firm) that implies that the amount of HFs required to extract a unit of NC increases with the *global* exploitation rate (E) of the latter. B_t is assumed to be an increasing and convex function of E ($B'_t > 0, B''_t > 0$), reflecting the generally accepted fact that the exploitation of resources constituting NC is characterized by increasing

marginal costs. In addition, the t index indicates that B_t decreases over time as a function of technical progress (see below).

The operating cost of a NC unit is the remuneration of a HF's unit v_t , multiplied by the quantity of HF's per operated NC unit $B_t(E)$. Under perfect competition, the zero profit condition implies that the selling price of the primary sector is equal to :

$$q_t = v_t B_t(E_t) \quad (7)$$

2.3 The final sector

Primary sector production X_t is used as intermediate consumption by the final sector, which transforms it into all-purpose goods used for consumption and investment. The representative firm of the final sector combines HF's and intermediate consumption according to the following constant-substitution elasticity (CES) and constant-return production function:

$$\frac{1}{Y_t^\rho} = \frac{\alpha}{[A_t H_{ft}]^\rho} + \frac{1 - \alpha}{[X_t]^\rho} \quad (8)$$

where $0 < \alpha < 1$ and $0 < \rho$. H_{ft} is the amount of HF's devoted to the manufacture of goods and X_t is intermediate consumption. A_t is the productivity index of HF's likely to change as a result of technological progress. The elasticity of substitution between factors is $\nu = 1/[1 + \rho]$.

The assumption $\rho > 0$ implies $\nu < 1$ and excludes the possibility of producing an infinite quantity of final production with a finite quantity of NC (remember that both production and natural capital must be understood in a very aggregate sense). This assumption (theoretically consistent with Anderson (1987), Baumgartner (2004) and Krysiak (2006)) is reasonable to the extent that, although the share of services was expected to increase over time, a fraction of final production will still have a material and energy content, obtained from NC. Moreover, the production of services depends on factors that require quantities of matter and energy for their manufacture (equipment) or reproduction (work). In other words, services are not produced solely with services.

At each period, the representative firm computes the HF's and intermediate consumption it needs in order to minimize its production costs. Formally, it solves the following problem:

$$\min_{H_{ft}, X_t} v_t H_{ft} + q_t X_t \quad (9)$$

under constraint (8). Under perfect competition, prices q_t and v_t are exogenous at the firm level. The optimality conditions lead to the following relationships:

$$\alpha \left[\frac{Y_t}{A_t H_{ft}} \right]^{1+\rho} A_t = v_t \quad (10)$$

$$[1 - \alpha] \left[\frac{Y_t}{X_t} \right]^{1+\rho} = q_t \quad (11)$$

2.4 Macroeconomic equilibrium

Aggregate production is divided between consumption and investment:

$$Y_t = C_t + I_t \quad (12)$$

The equilibrium on the HF market is written:

$$H_t = H_{ft} + H_{pt} \quad (13)$$

The complete model is described by the equations (2), (3), (4), (5), (6), (7), (8), (10), (11), (12), (13), that is a dynamic system of 11 equations with 11 unknowns

$H_{t+1}, H_{f,t+1}, H_{p,t+1}, X_t, Y_t, C_t, I_t, v_t, q_t, E_t, R_{t+1}$, $t = 1, \dots, T$. Parameters are $\alpha, \beta, \sigma, \delta$. Sequences

$A_t, F_t, t \geq 1$ and the function B_t are known. The initial conditions are H_1 and R_1 , to which must be added the final condition induced by the household's problem.

Particular attention will also be paid to the following two variables:

- the share of HFs used by the final sector:

$$m_t = \frac{H_{ft}}{H_t} \quad (14)$$

m_t determines the distribution of HFs between the primary and final sectors. The variation of this variable over time characterizes the *structural change* at work during the economy's trajectory.

- the ratio between the extraction and renewal flows of NC:

$$o_t = \frac{X_t}{F_t} \quad (15)$$

If $o_t > 1$, the economy is living beyond its (natural) means and the stock of RN is decreasing.

Finally, the following relative variables are introduced:

$$h_t = \frac{A_t H_{ft}}{X_t} \quad (16)$$

$$y_t = \frac{Y_t}{X_t} \quad (17)$$

$$x_t = \frac{Y_t}{A_t H_{ft}} = \frac{y_t}{h_t} \quad (18)$$

h_t is the ratio between HFs (measured in effective terms i.e. multiplied by A_t) and intermediate consumption in the final sector. y_t and x_t are the productivity of HFs and intermediate consumption, respectively. The production function (8) and the optimal conditions of the final sector (10) and (11) can then be rewritten as follows:

$$1 = \alpha x_t^\rho + [1 - \alpha] y_t^\rho \quad (19)$$

$$\alpha x_t^{\rho+1} A_t = v_t \quad (20)$$

$$[1 - \alpha] y_t^{\rho+1} = v_t B_t(E_t) = q_t \quad (21)$$

It follows that⁶:

$$m_t = \alpha x_t^\rho \quad (22)$$

We obtain a simple relationship between the distribution of HFs between sectors and the productivity of HFs in the final sector. The higher the latter, the higher the final sector's share.

2.5 Technical progress

In this model, technical progress occurs at two different levels. It can (i) increase HFs' productivity in the final sector and/or (ii) lower the operating costs of the primary sector.

In case (i), technical progress formally means that $A_{t+1} > A_t > 0, \forall t$, with

$$\lim_{t \rightarrow +\infty} A_t = \bar{A} \quad (23)$$

where \bar{A} is possibly infinite.

In case (ii), technical progress implies a downward shift of function $B_t(E)$ over time. In other words, whatever the exploitation rate $E \in [0,1]$, we have $0 \leq B_{t+1}(E) < B_t(E), \forall t$. At the limit, operating costs can eventually disappear.

⁶Indeed, (1), (12), (5), (13), (14) and (17) $\Rightarrow Y_t = v_t H_t = v_t H_{ft} + v_t B(E_t) X_t \Rightarrow 1 = m_t + \frac{v_t B(E_t) X_t}{Y_t} = m_t + \frac{v_t B(E_t)}{y_t}$, which with (21) $\Rightarrow m_t = 1 - [1 - \alpha] y_t^\rho$. Given (19), (22) follows.

3 Steady state

The fact that ρ is strictly positive implies that the substitution elasticity between factors $\nu = \frac{1}{1+\rho} < 1$. This property implies that the productivity of NC is less than or equal to an upper bound equal to $\frac{1}{[1-\alpha]^{\frac{1}{\rho}}}$ ⁷. As the stock of NC is finite, it is then impossible to produce an infinite quantity of goods (even asymptotically). In this context, a balanced growth path characterized by increasing production at a constant rate is excluded.

On the other hand, the fact that the NC stock benefits from the renewal flow $F_t > 0$ ensures that the economy does not disappear. If certain conditions are met (in particular $\lim_{t \rightarrow +\infty} F_t \rightarrow F$ positive and constant), the economy tends towards a stationary state characterized by the constancy of variables.

This stationary state can be solved analytically (see Appendix 8.2):

- HF's unit income:

$$v = \frac{1}{\beta} - 1 + \delta \quad (24)$$

v is an inverse function of the HF's supply from households. This is all the higher as (i) the discount factor β is high (i.e. household patience is high) and (ii) the depreciation rate of the stock of HF's δ is low.

- price of the primary sector :

$$q = [1 - \alpha] \left[\frac{1 - \alpha x^\rho}{1 - \alpha} \right]^{\frac{1+\rho}{\rho}} \quad (25)$$

where

$$x = \left[\frac{v}{\alpha \bar{A}} \right]^{\frac{1}{1+\rho}} \quad (26)$$

At the steady state, HF's productivity in the final sector x and the selling price of the primary sector q are respectively positively and negatively correlated with HF's unit income. A low unit income implies greater final production, greater demand for NC and therefore a higher price q .

- takings from NC:

$$X = F \quad (27)$$

Constancy of the NC stock necessarily implies equality between taking and renewal flows.

- final production (GDP):

$$Y = \left[\frac{1 - \alpha x^\rho}{1 - \alpha} \right]^{\frac{1}{\rho}} F \quad (28)$$

At the steady state, production is an increasing function of β , ν , \bar{A} (via x and v) and F . It is therefore all the higher as household patience, the elasticity of substitution between human and natural factors, the potential for technological progress and biocapacity are high.

The previous results are based on the assumption that \bar{A} is finite. However, even if the HF's productivity tends to infinity in the *two* sectors⁸, the economy tends towards the steady state characterized by the previous values with $x = 0$. This equilibrium has the following two properties (see Annex 8.2): (i) all HF's are concentrated in the primary sector and (ii) the exploitation rate of NC is unitary (all NC available at the beginning of a period is operated during that period). Even if the productivity of HF's becomes infinite, the fact that the productivity and availability of natural factors remain upper bounded imposes a finite production.

The previous results also assume that the renewal flow of NC F_t tends towards a given and exogenous limit F . In the case where F_t depends on the resource stock, i.e. $F_t = f(R_t)$ where f

⁷This bound is obtained from (8) by making A_t or K_{ft} tend towards infinity. We observe that this bound becomes infinite if $\rho = 0$ (case where the CES reduces to a Cobb-Douglas function), which we exclude here.

⁸I.e. if $A_t \rightarrow \bar{A} = +\infty$ and $B_t(E) \rightarrow B(E)$, where $B(E) = 0$ ($0 \leq E < 1$) and $B(1) = +\infty$

is a known function, then (4) implies at the steady state that $X = F = f(R)$. Combined with the identity $E = X/R$, this equation leads to:

$$ER = f(R) \quad (29)$$

Given that the stationary exploitation rate E is independent of R (whether F is exogenous or not), the previous equation allows to compute R (see Appendix 8.2).

4 Dynamics analysis

Given its complexity and the fact that we are also interested in the transient phase, the dynamic model is solved numerically, which presupposes (i) setting the parameter values and (ii) explicitly defining the exogenous functions. The model is first solved in the framework of a so-called "baseline simulation" (BS). In order to test the robustness of the results, different variants are then carried out by changing one or the other hypothesis on which the BS is based.

In the same way as Laitner (2000) and Fagnart et al. (2016), the depreciation rate of the HFs stock δ is chosen to be equal to 1, which amounts to considering relatively long periods of time (around fifteen years). The advantage of this hypothesis is that it reduces significantly computation costs without fundamentally changing the dynamics of the model.

The model is that of a global economy. In order to quantify the dynamics of the NC stock at a global level, we use the concepts of ecological footprint and biocapacity developed by the Global Footprint Network. Biocapacity F_t refers to "The capacity of ecosystems to produce useful biological materials and to absorb waste materials generated by humans, using current management schemes and extraction technologies", while ecological footprint X_t is "A measure of how much biologically productive land and water an individual, population or activity requires to produce all the resources it consumes and to absorb the waste it generates using prevailing technology and resource management practices" (GFN, 2019). These are physical quantities measured in global hectares (GFN, 2019).

Recall that NC is composed of all living species and natural resources (renewable or not). Biocapacity and ecological footprint are flows that contribute positively and negatively to NC respectively. Under the baseline simulation, biocapacity is considered as exogenous, because at the level of a global economy, biocapacity does not reduce to a particular resource. The method used by the Global Footprint Network to compute it is complex and cannot be described by a "simple" dynamic equation that could be integrated into the stylized model developed here.

Unit operating costs are written:

$$B_t(E) = b_t G(E) = b_t \frac{E}{1 - E} \quad (30)$$

This formula (based on Dale et al., 2012) breaks down operating costs into two factors, one related to technology and the other to NC accessibility:

- sequence b_t is exogenous and decreases over time as a function of technical progress;
- function $G(E)$ is zero at the origin, increasing and tends towards infinity if the exploitation rate $E \rightarrow 1$. This last property reflects the fact that marginal costs become infinite once the physical limits of a resource are reached (devoting additional HFs to exploitation no longer yields anything).

Technical progress is described by the following sequences:

$$A_t = \bar{A} + \frac{A_1 - \bar{A}}{g_A^{t-1}} \quad (31)$$

$$b_t = b_1 \frac{A_1}{A_t} \quad (32)$$

with $t \geq 1$ and $0 < A_1 < \bar{A} < +\infty, g_A > 1, b_1 > 0$. Technical progress is therefore assumed to be exogenous and bounded. b_t being inversely proportional to A_t (cfr. (32)), the primary sector is characterized by the same technical progress in terms of HF usage as the final sector.

Simulations are based on values of exogenous parameters and sequences (i) documented by the literature or (ii) calibrated to respect the existence conditions of the steady state and to ensure that the trajectories generated by the model have certain desirable properties (see Appendix 8.4 for more details).

Before presenting the simulations, it is important to stress that they are not intended to describe a specific reality but rather to highlight the effects on the economy induced by the bio-physical limits to which it is subject. The numerical values are of little interest in themselves and it is mainly the shape of the curves that is important.

4.1 The baseline simulation

Table 1 provides for each variable (i) the stationary value (last column) and when it makes sense, (ii) the maximum or minimum value, (iii) the period during which it is observed and (iv) the ratio between the extreme value and the stationary value⁹. The trajectories of the main variables are illustrated in Figures 1.a-h.

Table 1: The baseline simulation

	max /min	t_{\max} / t_{\min}	Δ	SS
Y	215.3763	6	1.6177	133.1407
C	69.4414	6	1.7385	39.9422
I	146.2319	5	1.5690	93.1985
H	146.2319	6	1.5690	93.1985
H _f	85.3832	6	1.9527	43.7249
H _p	67.3709	7	1.3618	49.4736
X/F	2.5976	4	2.5976	1
R	21.0163 (local min) 21.8655 (local max)	11 14		19.6780
s	0.6772 (min) 0.7032 (local max)	7 12		0.7
m	0.5065 (local min) 0.5263 (local max)	9 12		0.4692
E	0.7087 (local max) 0.6749 (local min)	9 12		0.7664
v	1.2112 (min) 1.4561 (local max)	8 14		1.4286
q	4.0509 (local max) 3.7318 (local min)	9 12		4.6865
k	15.2318 (local max) 14.0685 (local min)	9 12		17.6860
y	8.2079 (local max) 7.8780 (local min)	9 12		8.8284
x	0.5389 (local min) 0.5600 (local max)	9 12		0.4992

⁹In the case of the maximum of a certain variable x_t , the ratio is equal to $\Delta_x = \frac{\max_t x_t}{x_\infty}$, where x_∞ refers to the stationary value.

4.1.1 Analysis

In the very long term, the economy tends towards the steady state characterized in section 3. The variable paths from the initial state to the final state are non-monotonous. In addition, the paths may be at least contrasted from one variable to another.

Absolute quantities (such as final production Y_t , consumption C_t , investment I_t and stocks of HFs H_t, H_{ft}, H_{pt}) have a four-step trajectory: (i) a growth phase that slows down, (ii) a reversal phase, (iii) a decreasing phase, and finally (iv) a quasi-stationary phase leading to the steady-state (Fig. 1.a-b). The same applies for the ecological footprint. These four phases are now analyzed successively.

1st phase: slowdown of growth (periods 1 to 4)

During the first 4 periods, the economy is growing (see Figure 1.a). However, growth slows down in the sense that the growth rate continues to decline. The growth of the economy is reflected in the growth of the ecological footprint X_t . As a result, the ecological overshoot (measured by (15)) that was already greater than 1 at the beginning of the path is worsening (Fig. 1.c)¹⁰. The increase in the ecological overshoot results in a significant decrease in the natural capital stock R_t (Fig. 1.d).

The increase in the ecological footprint and the decrease in the NC stock implies a significant increase of the exploitation rate E_t (Fig. 1.e), which in turn results in a decrease of HFs' productivity in the primary sector $1/B_t(E_t)$ (Fig. 1.h). More and more HFs are required to generate the same extraction of NC, despite technical progress that saves these HFs (Fig. 1.b). This results in a decrease of m_t , the share of HFs allocated to the final sector (Fig. 1.e). This decrease of m_t reflects a *structural change in the form of a diversion of HFs from the final sector to the primary sector*. This structural change implies a lower growth of the HFs stock used by the final sector and consequently a slowdown of the economy's growth.

Figure 1.f illustrates the decline of the HFs unit income v_t . Because $v_t = Y_t/H_t$, this decrease reflects the decrease in the global productivity of HFs (H_t increases faster than GDP Y_t). The selling price of the primary sector $q_t = v_t B_t(E_t)$ is subject to two contradictory influences: (i) on the one hand, the HF intensity within the primary sector $B_t(E_t)$ is increasing (as seen in the previous paragraph); (ii) on the other hand, the HF unit income is decreasing. The first influence is by far the strongest, as shown in Fig. 1.f. The shape of q_t (at the beginning of the trajectory) echoes the trend increase in commodity prices observed since the beginning of the century.

The first phase is also characterized by a slight decrease in the savings rate. This allows consumption growth to be higher than GDP growth.

2nd phase: economic reversal (periods 4 to 7)

The continued increase in the selling price of the primary sector ultimately dampens the growth of the economy. But it is interesting to note that the absolute quantities (production, stocks of HFs, ecological footprint) do not reach their respective maximum at the same period (see Table 1).

The first variable to reach a maximum (in $t = 4$) is the ecological overshoot o_t . As shown in Figure 1.c, the overshoot is then slightly higher than 2.5. In short, this corresponds to the generalization of the current average European overshoot to all humanity within the next fifty years or so¹¹.

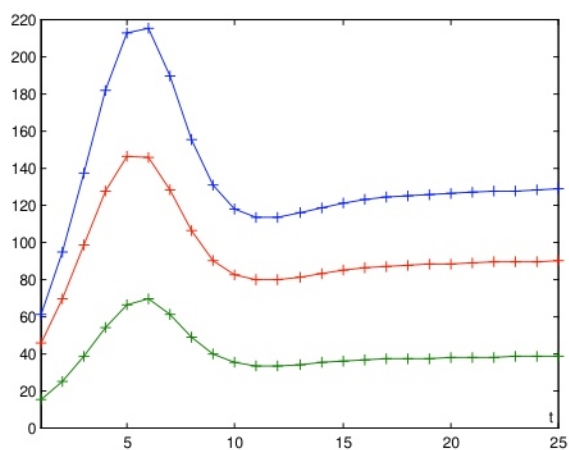
Because biocapacity itself is increasing, the ecological footprint is only at its maximum in the following period. The continuous increase of the ratio q_t/v_t prompts the final sector to substitute HFs for its intermediate consumption, thus reducing its demand to the primary sector. The growth of H_{ft} and technical progress (the growth of A_t) temporarily allow production Y_t to continue to grow while X_t is reversed. This explains the *absolute decoupling* between GDP and ecological footprint observed between periods 5 and 6. But this decoupling is not sustainable given the strong complementarity between human and natural factors, and the decline of X_t ultimately leads to the decline of the economy.

¹⁰In $t = 1$, the ecological overshoot is 1.56, obtained as the ratio between the respective global ecological footprint and biocapacity averages observed over the period 2000-2014.

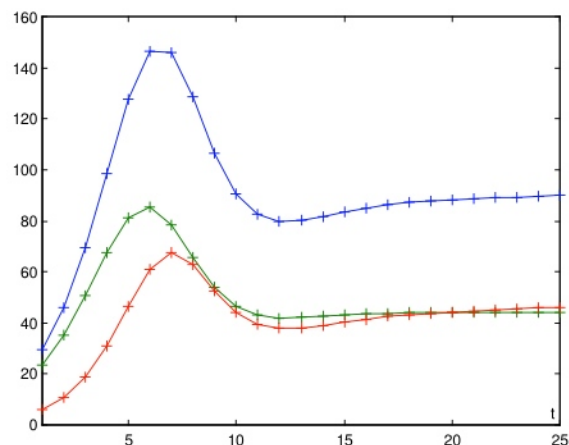
¹¹The EU's ecological overshoot was 2.89 in 2016.

Figure 1: The baseline simulation (b: blue, g: green, r: red, t: turquoise)

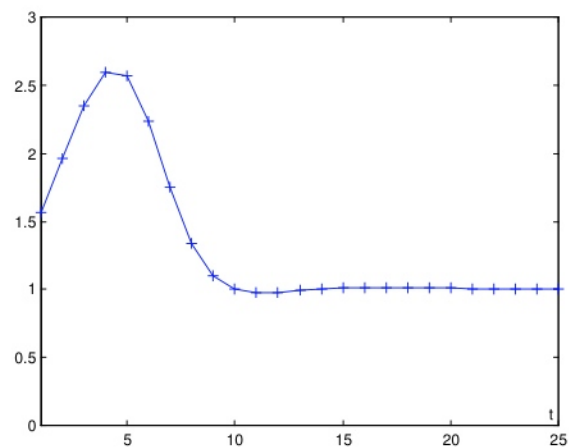
(a) production Y_t (b), consumption C_t (g), investment I_t (r)



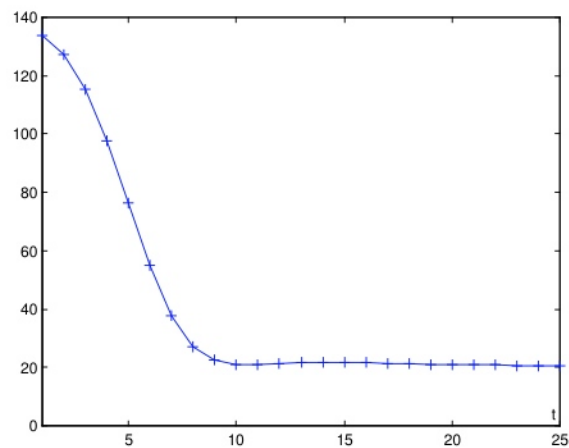
(b) human factors: total H_t (b), final sector H_{ft} (g), primary sector H_{pt} (r)



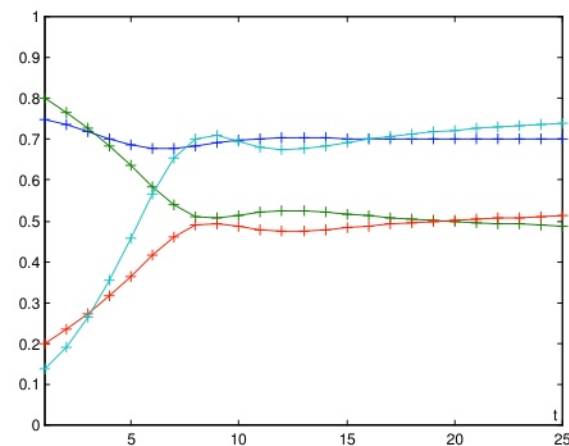
(c) ratio between extraction and renewal flows of NC X_t/F_t



(d) natural capital stock R_t



(e) saving rate s_t (b), HF share of final sector m_t (g), HF share of primary sector $1-m_t$ (r), exploitation rate E_t (t)



(f) prices of human factors v_t (b) and resource q_t (g)

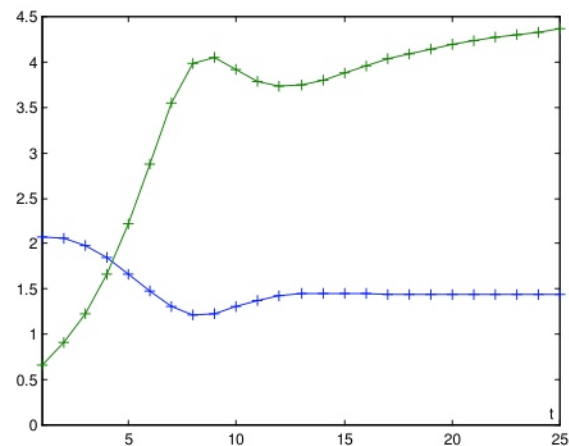
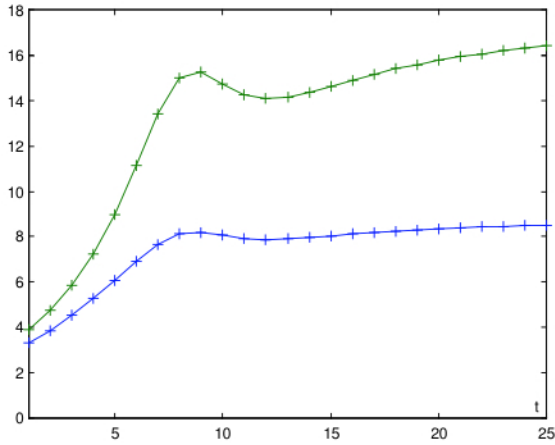
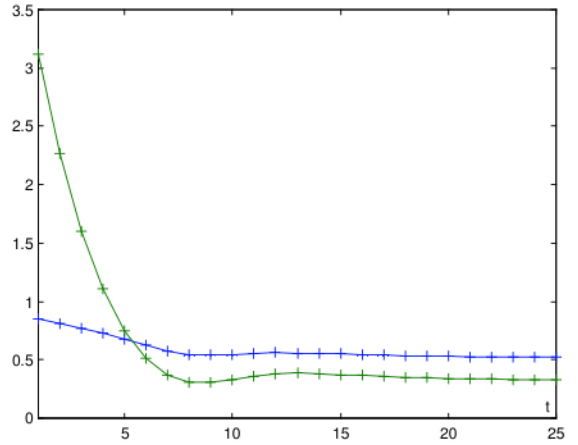


Figure 1: The baseline simulation (cont.)

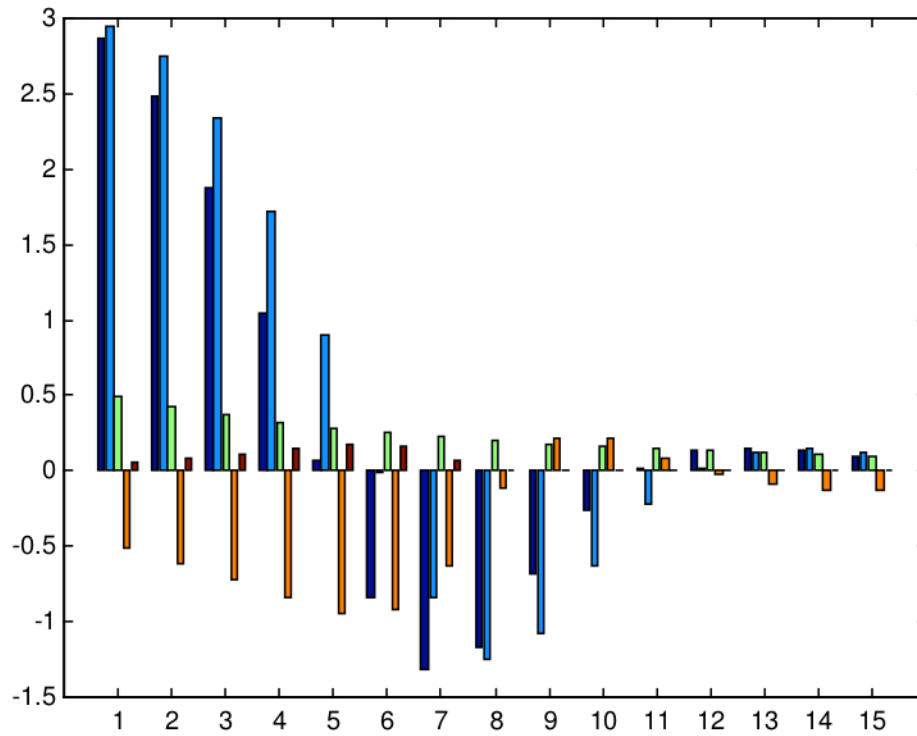
(g) resource productivity y_t (b) and factor ratio h_t (g) in final sector



(h) HF productivity in final sector x_t (b) and primary sector $1/B_t$ (g)



(i) decomposition of g_{Yt} (black); g_{Ht} (blue), g_{At} (green), h_{Et} (orange), error (brown)



The near stagnation of Y_t coupled with the continued (slight) decrease in the savings rate leads to the reversal of investment I_t in $t = 5$ (Fig. 1.a). Given the delay of one period between investment and the total HFs stock, H_t reaches its maximum in $t = 6$ (Fig. 1.b). The reversal of H_t brings the reversal of H_{ft} during the same period as the structural change continues at the expense of the final sector (m_t continues to decline as shown in Figure 1.e). The same applies to GDP Y_t because technical progress (the increase of A_t) is no longer sufficient to compensate for the decrease in factor consumptions (H_{ft} and X_t) by the final sector.

Despite the reversal in activity, the ecological overshoot o_t remains well above 1, resulting in a continuous decline of the NC stock. It follows that even if X_t decreases, the exploitation rate E_t continues to increase (Fig. 1.e). As a result, structural change is continuing in the sense that more and more HFs must be devoted to the primary sector. This explains why the HFs used by the primary sector H_{pt} are still growing while the other absolute quantities (in particular Y_t and H_{ft}) are beginning to decline.

The reversal of H_{pt} in $t = 7$ follows the fact that the decrease in HFs total stock H_t ultimately outweighs the increase of the share in HFs devoted to the primary sector ($1 - m_t$). From period $t = 7$, the reversal phase of the economy is complete in the sense that *all* absolute quantities are falling.

While Phase 2 is clearly at odds with Phase 1 in terms of the behaviour of absolute quantities, this is not the case for prices and relative variables. As shown by the figures, prices q_t and v_t , the exploitation rate E_t , the NC stock R_t , the share of HFs m_t , the ratios $h_t = A_t H_{ft}/X_t$ and $y_t = Y_t/X_t$ characterizing the final sector continue their path from phase 1 and will only turn around during the 3rd phase, when the NC stock begins to stabilize.

3rd phase: degrowth and beginning of stabilization (periods 7 to 10)

Economic growth implies that the NC's exploitation ends up being significantly in excess of what it can sustain in the long term. It is therefore inevitable that the economy will decline and so will ecological footprint X_t . However, the intertemporal smoothing of consumption desired by households does not allow a sudden adjustment towards a sustainable level of activity. As a result of this inertia phenomenon and although the economy has entered a downward trend, the NC stock continues to decline for a few more periods.

After $t = 6$, the prolonged degrowth of activity significantly decreases the ecological footprint, as a result of which the decline in NC stock slows down. The exploitation rate E_t reaches its maximum in $t = 9$ (Fig. 1.e) and the same applies to the selling price of primary sector q_t (Fig. 1.f). Primary production ceases to be more and more expensive in comparison to final production and HFs.

As the gap between factor prices v_t and q_t stops growing (see Fig 1.f), substitution of HFs for intermediate consumption in the final sector stops and the ratio between factors $h_t = H_{ft}/X_t$ stabilizes. As a result, the ratio $y_t = Y_t/X_t$ stabilizes in turn (see Fig 1.g).

The decline of NC in place since the beginning of the path slows down and finally stops. The economy is no longer obliged to divert more and more HFs to the primary sector and the structural change (hitherto reflected in the decrease of m_t) stops.

The decrease in activity and in the ecological footprint ultimately reconciles the latter with biocapacity from $t = 10$, at which point the ecological overshoot o_t constantly becomes close to 1 (Fig. 1.c). The convergence of the economy towards the steady state is on the way.

4th phase: stabilization and convergence towards the steady state ($t \geq 11$)

During periods 11 to 13, the ratio between ecological footprint and biocapacity is reversed, allowing a very slight reconstitution of NC (Fig. 1.d). This factor, coupled with the fact that there is still some technical progress potential at that time, allows for a slight decrease in the exploitation rate E_t and the primary sector selling price q_t (Fig. 1.e-f). However, these variables still remain at much higher levels than in $t = 1$.

This new reversal of q_t in turn allows a reversal of structural change: for a few periods, the share of HFs allocated to the final sector m_t increases (Fig. 1.e). Coupled with the residual technical progress and the fall of q_t , the increase of m_t allows a slight recovery of production Y_t from $t = 12$ (Fig. 1.a). But as E_t and q_t rise again, the recovery process does not deepen. The NC stock R_t

(which stabilizes at a very low level compared to the beginning of the trajectory) does not allow this.

After the slight rebound in activity, all variables converge towards their long-term values. From $t = 13$, structural change is again reflected in a shift of HFs from the final sector to the primary sector. However, this has no deleterious effect on final production because (i) this change is of modest magnitude (see Fig. 1.e) and (ii) it is offset by residual technical progress. In the end, the economy tends towards a steady state which, compared to the initial state, is characterized by a much lower NC stock and a much higher HFs stock. As NC has become significantly rarer, a much higher proportion of HFs must be devoted to operating NC rather than to final production.

4.1.2 Decomposition of growth

The decomposition of the economy' growth rate is based on the following formula (see Annex 8.3):

$$g_{Yt} \approx g_{Ht} + \alpha x_t^\rho g_{At} - [1 - \alpha] y_t^\rho \left[g_{bt} + \frac{g_{Et}}{1 - E_t} \right] \quad (33)$$

$$\approx g_{Ht} + h_{At} + h_{bt} + h_{Et} \quad (34)$$

where (i) $g_{zt} = \ln \left(\frac{z_{t+1}}{z_t} \right)$ refers to the rate of change of variable z_t in period t and (ii) $h_{At} = \alpha x_t^\rho g_{At}$, $h_{bt} = -[1 - \alpha] y_t^\rho g_{bt}$ and $h_{Et} = -\frac{[1 - \alpha] y_t^\rho}{1 - E_t} g_{Et}$.¹²

This equation shows the "contributions" to GDP growth of (i) the accumulation of HFs h_{Ht} , (ii) the technical progress rates h_{At} and h_{bt} (respectively in the final and primary sectors) and (iii) the variation in the exploitation rate h_{Et} . Alongside the two traditional components of growth, namely technological progress and HFs accumulation, another source appears, linked to NC's availability. If E_t increases ($g_{Et} > 0$), operating costs increase and economic growth is negatively impacted. The term $h_{Et} < 0$ can then be understood as *the loss of growth due to the lower availability of NC*.

In the context of the baseline simulation, b_t depends on A_t via (32) (technical progress is uniform across both sectors). Then (34) reduces to:

$$g_{Yt} \approx g_{Ht} + g_{At} + h_{Et} \quad (35)$$

From the numerical simulations, it is possible to quantify the respective weights of the different contributions to economic growth.

The decomposition of growth relative to the baseline simulation is illustrated in Figure 1.i. For each period, the bars represent (as an annual average) (i) the GDP growth rate (black) and the respective contributions of HFs accumulation (blue), technical progress (green) and the variation in the exploitation rate (orange). The last column represents the error, i.e. the difference between the two members of (35). As the figure shows, the approximation is reasonably good (or even very good)¹³.

At the beginning of the trajectory, the growth rate g_{Yt} is mainly explained by the growth rate of HFs. However, g_{Yt} is less than g_{Ht} because the (negative) contribution of the exploitation rate h_{Et} outweighs the (positive) contribution of technical progress¹⁴. We can see that the loss of growth rate induced by the increase of E_t (which forces the economy to divert more and more HFs from the final sector to the primary sector) is not negligible: from .5% in $t = 1$, we approach 1% in $t = 5$. During this last period, h_t offsets the accumulation of HFs and technical progress and there is almost no more growth. The (almost) disappearance of this one in $t = 5$ causes the end of the increase in HFs in $t = 6$, consequently g_{Yt} is almost entirely determined by h_{Et} and becomes negative. From $t = 7$, the decrease of GDP is exacerbated by the decrease in the stock of HFs; g_{Ht} becomes negative and once again becomes the main component of g_{Yt} .

¹²The sign \approx is justified by the fact that the decomposition described by (33) is only exact for infinitesimal variations. However, the model assumes a discrete time framework where periods are long.

¹³From $t = 8$, the error even becomes virtually zero.

¹⁴The technical progress rate being decreasing, its contribution is especially significant at the beginning of the trajectory.

The economy's entry into degrowth (in $t = 6$) coincides with the gradual reduction in the negative contribution of operating costs (orange bars). From $t = 9$, h_{Et} even becomes positive for a few periods as the exploitation rate decreases. After about ten periods, all the components of growth become weak (in absolute terms) as the economy stabilizes.

In summary, the largest component of economic growth is found to be the variation in HFs, except around the peak of GDP. The contribution of the exploitation rate is far from negligible and is even temporarily the most important contribution of growth around the peak of GDP.

4.2 Variants

The baseline BS is based on certain hypotheses (concerning progress and biocapacity in particular) whose influence on the results can be questioned. The following subsection presents a series of comparative dynamics exercises, obtained by alternately changing an underlying assumption of BS.

Before presenting them, it is important to draw the reader's attention to the following points:

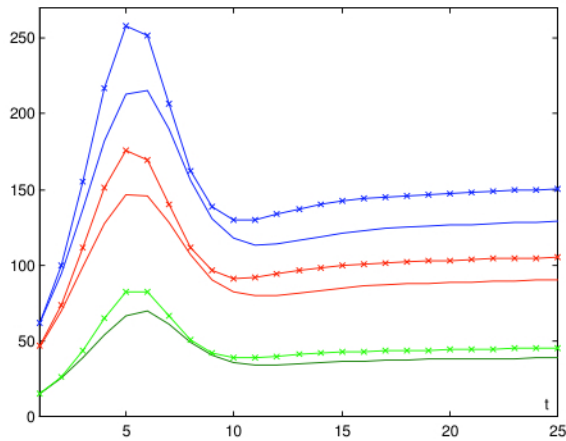
- The following comments focus on the comparison of the variants with BS. Thus, when we say without further precision that a certain variable is increasing, it should be understood that the corresponding curve has moved upwards relative to BS (whatever t);
- Each variable is characterized by the same color in both simulations (e.g. output in blue). The paths characterizing BS are in solid lines.

4.2.1 Change in technical progress potential

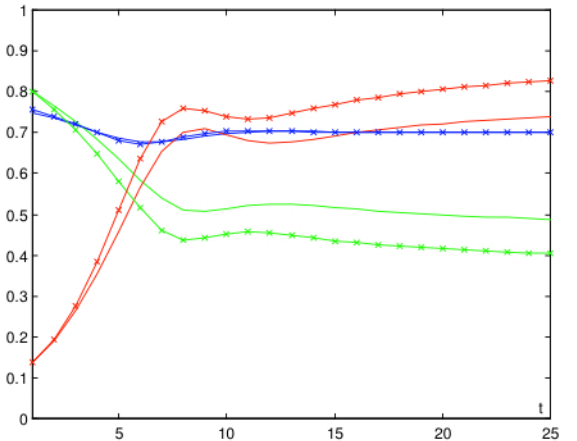
Technical progress is described by the sequences $\{A_t, t \geq 1\}$ (at the final sector level) and $\{b_t, t \geq 1\}$ (at the primary sector level). Variant V1 is obtained by increasing the technical progress potential at the beginning of the path by 100% in both sectors.

Figure 2: Variant V1 - variation of \bar{A} (BS: —, V1: -x- ; b: blue, g: green, r: red)

(a) production Y_t (b), consumption C_t (g), investment I_t (r)



(b) saving rate s_t (b), HF share of final sector m_t (g), exploitation rate E_t (r)



Concerning production, consumption and investment, we observe that (i) the peaks are more pronounced and are advanced over time and (ii) the values at the steady state increase (Fig. 2.a). Concerning HFs stocks, the impact of the technical progress increase is asymmetric across sectors. It is stronger on H_{pt} than on H_{ft} , especially at the steady state. More technical progress results in more intensive exploitation of NC, involving a lower NC stock and a higher exploitation rate

throughout the trajectory (Fig. 2.b). More technical progress therefore results in a more predatory economy and a more pronounced structural change in favour of the primary sector (m_t decrease).

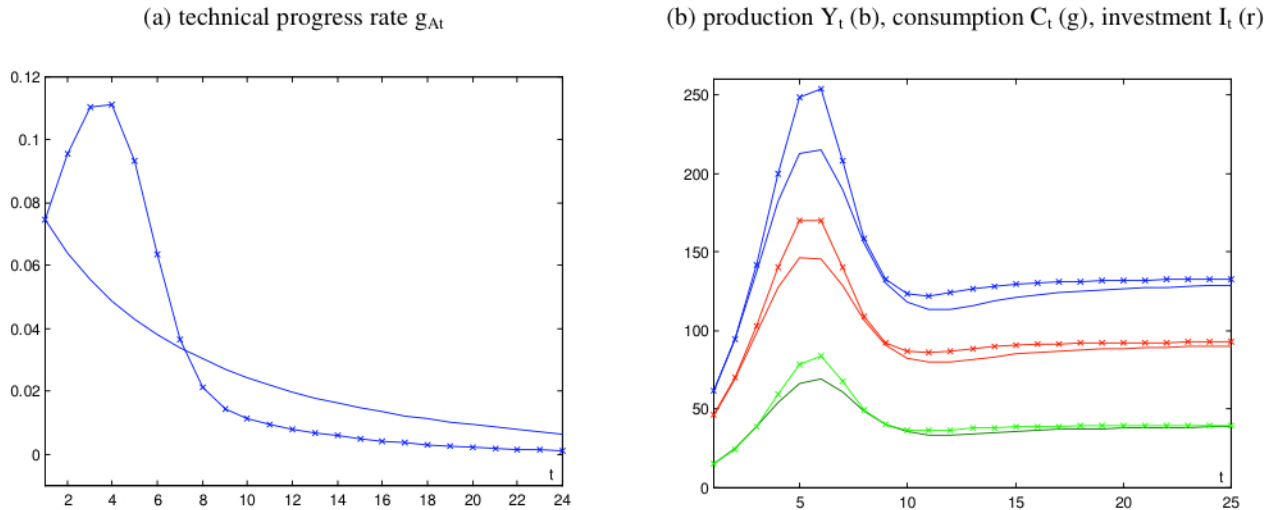
4.2.2 Endogenous technical progress

In the context of BS, technical progress is exogenous. This subsection proposes to endogenize technical progress that governs A_t and b_t and to assess its impact on the results obtained under the baseline. A simple way to endogenize technical progress is to assume a learning by doing process. More specifically, it is assumed that A_t evolves according to the following formula:

$$A_t = \bar{A} + \frac{A_1 - \bar{A}}{\exp_{g_A} \left(\sum_{\tau=1}^{t-1} Y_\tau \right)} \quad (36)$$

Starting from the initial value A_1 , A_t increases as a function of *cumulative* final production and tends towards the final value \bar{A} . W.r.t. BS, the limit value of A_t remains unchanged (the technical progress potential at the beginning of the trajectory is identical). The same is true for the extreme values of the sequence b_t , which remains governed by (32).

Figure 3: Variant V2 - endogenous technical progress (BS: — , V2: -x- ; b: blue, g: green, r: red)



Figures 3.a-b compare variant V2 where A_t obeys (36) with the baseline BS where A_t obeys (31). Compared to BS, the endogenization of technical progress has a significant effect on this variable: the technical progress rate increases during the first periods (Fig. 3.a), which implies that the economy uses its technical progress potential more quickly w.r.t. BS (A_t approaches its final value \bar{A} much faster). This results in a more pronounced peak of output Y_t , though occurring at the same period (Fig. 3.b). The steady state is not affected since the sequence $\{A_t, t \geq 1\}$ evolves between the same boundaries as in BS.

As a whole, results are similar to those obtained under previous variant V1. Endogenizing technical progress has a quantitative effect on the transition phase, but it does not fundamentally change the results: we always observe a trajectory composed of the same successive phases (growth, reversal, degrowth and stabilization) with similar durations.

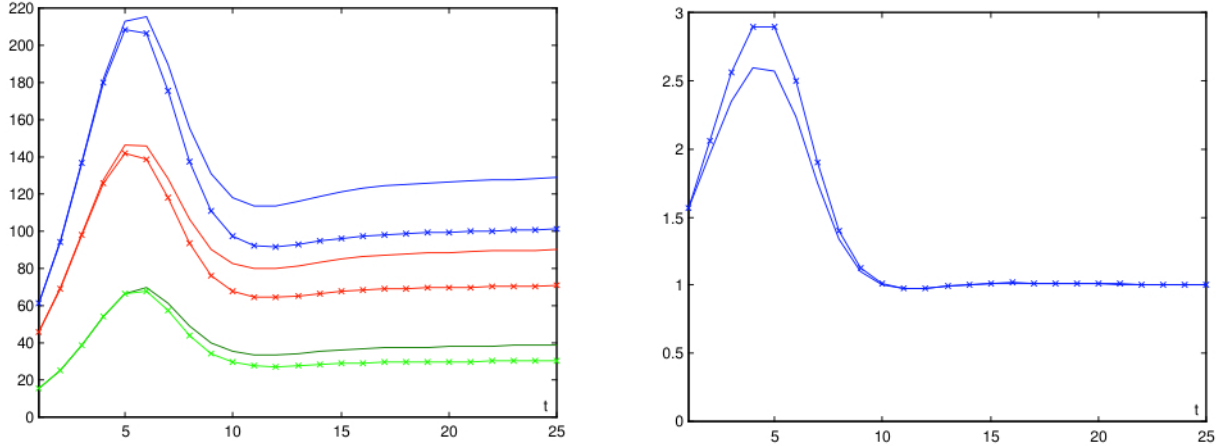
4.2.3 Change in biocapacity

The baseline BS assumes that biocapacity continues to grow (at a rate itself decreasing), with a stationary level F higher by almost 30% compared to its initial level F_1 . Variant V3 takes a

different view to this optimistic assumption by assuming that biocapacity remains constant after $t = 1$.

Figure 4: Variant V3 - variation of F_t (BS: —, V3: -x- ; b: blue, g: green, r: red)

(a) production Y_t (b), consumption C_t (g), investment I_t (r) (b) ratio between extraction and renewal flows of NC X_t/F_t



At the beginning of the trajectory, BS and V3 are practically the same. It takes time for a lower biocapacity to result in a significantly lower level of the natural capital stock. If the economy's growth is not affected during the first periods, V3 is characterized by a peak of GDP lower and reached one period earlier. After the peak, the gap between the two trajectories continues to grow and this observation applies to all absolute quantities (production, consumption and stocks of HFs). At the steady state, all these variables decrease with F in accordance with the results of section 3 (Fig. 4.a).

The difference between the paths of the ecological overshoot X_t/F_t behaves differently. It increases from the beginning (since the ecological footprint grows with the economy while biocapacity is stationary), deepens with economy's growth, and then disappears after the peak (Fig. 4.b).

The fact that the natural capital stock declines after a few periods is accompanied by a temporary increase in the exploitation rate with expected consequences for operating costs and the selling price of the primary sector. On the other hand, none of the relative or price variables (such as s_t, m_t, q_t, E_t) are modified in the long term because, as shown in section 3, the stationary values of these variables do not depend on F .

Whether as part of the BS or the V3 variant, the model deals differently with biocapacity on the one hand and NC and ecological footprint on the other. The latter two variables are endogenous to the model, whereas biocapacity is treated as exogenous. This is because biocapacity does not reduce to a particular natural resource, whose dynamics could be reduced to a "simple" equation that could be incorporated into the stylized model developed here.

However, there is a legitimate suspicion that the significant decline in NC stock observed during the path of the economy (Fig. 1.d) will eventually have an influence on biocapacity¹⁵. To shed light on this point, the argument in the previous paragraph will be overruled by assuming that biocapacity behaves like a homogeneous renewable resource and obeys the following equation:

$$F_t = \lambda \left[1 - \frac{R_t}{\bar{R}} \right] R_t \quad (37)$$

where $\lambda, \bar{R} > 0$. This equation is well known in resource economics (see e.g. Dasgupta and Heal (1979, chapter 5) for a presentation). \bar{R} is the stationary value of the NC stock in the absence

¹⁵I am indebted to a referee from the journal for making this point to me.

of exploitation. Starting from a level below \bar{R} , the stock evolves as a logistic function over time. In the presence of exploitation, the previous equation and the equality between biocapacity and ecological footprint at steady state implies that $X = ER = \lambda [1 - R/\bar{R}] R \Rightarrow R = [1 - E/\lambda] \bar{R}$, where E does not depend on R (see Appendix 8.2). The previous equality and $R > 0$ constrain the choice of λ to be greater than E . Knowing R , we can deduce the other level variables: $X = ER, Y = yX, K = Y/v, I = \delta K$.

The dynamics of the model with (37) leads to the following observations¹⁶:

- Compared to BS, biocapacity begins to grow significantly faster. However, the monotonous decline in NC implies that biocapacity peaks after a few periods and then reverses. Its asymptotic value depends on that of the NC stock, which may be higher or lower than that observed in BS depending on the values of λ and \bar{R} .
- The general shape of the trajectories of the different variables is preserved with respect to BS. There is always convergence towards a steady state even if it is more smooth.
- The stronger growth of biocapacity at the beginning of the trajectory translates into a significantly higher peak in GDP that is slightly delayed in time.

4.2.4 Other variants

The impact on the economy's path of changes in exogenous parameters of the model is now being studied. While the literature allows the identification of variation ranges for these parameters (see Appendix 8.4), there is still some freedom in the choice of their values. In order not to lengthen this paper too much, we will limit ourselves to a summary of the obtained results.

Variante V4 : $\sigma = 1$ Increasing σ means that households want to smooth their consumption more strongly over time. V4 assumes a logarithmic utility function. In comparison with BS (where $\sigma = .75$), the trajectories of absolute quantities (production, consumption, stocks of HFs, ecological footprint) remain very similar to those of BS, except for a slight shift to the right. As σ does not influence the steady state (see section 3), it is not modified.

Variante V5 : $\beta = .9$ Increasing β amounts to giving greater weight to the future, which (ceteris paribus) translates into a greater propensity to save. In comparison with BS (where $\beta = .7$), there is a significant increase in HFs stocks, production and ecological footprint, both at their respective peaks and at the steady state. The higher abundance of HFs results in a higher exploitation of NC and a more pronounced structural change in favour of the primary sector.

Variante V6 : $\nu = .4$ A decrease of the substitution elasticity between production factors induces poorer production possibilities at the final sector level, in the sense that given quantities of factors H_{ft} and X_t imply a lower production Y_t . Given the deterioration of production conditions w.r.t. BS (where $\nu = .5$), absolute quantities (production, consumption, HFs stock) decrease, whether during the transitional phase or at the stationary state. The decrease in production is mainly due to the decrease in HFs because the ecological footprint X_t does not change much. This only decreases significantly around its peak. However, this is sufficient to drive the NC stock R_t (resp. the exploitation rate E_t) to a significantly higher (resp. lower) level at the steady state. Structural change remains but is less pronounced w.r.t. BS.

In summary, the different variants show a high stability of the model's behaviour. While there are of course sometimes significant variations of certain variables (given the magnitude of the shocks), they are more quantitative than qualitative. Indeed, the four-phase trajectory (growth, reversal, degrowth, stabilization) of the absolute quantities (such as output) is systematically found. The same is true with regard to the trend evolution of relative variables (such as the exploitation rate of NC) and structural change.

¹⁶For this simulation, \bar{R} has been calibrated so that the trajectory starts from the same values F_1, X_1, R_1 as that of the BS.

5 The model with pollution

The model developed in Section 2 belongs to the category of growth models with natural resources. It ignores pollution issues, in particular global pollution such as climate change. The resulting trajectories can therefore be considered as "optimistic", insofar as the possible deleterious effects of pollution on the economy are not taken into account.

The purpose of this section is to complement the previous model by taking into account pollution resulting from economic activity. Pollution is extremely diverse and so are its impacts. Greenhouse gases, heavy metals, air and water pollution,... affect human factors, the economy and/or the environment in many ways.

The modeling of pollution and its impacts chosen here has the advantage of being simple and easy to calibrate from the (rare) data available on pollution damage at a global level. We postulate a global pollution that causes a loss of production through the following equation:

$$Z_t = Y_t [1 - d_t] \quad (38)$$

where

$$d_t = D(S_t) \quad (39)$$

Z_t is net production, i.e. after taking damages into account. d_t measures damages in the form of a loss rate function on the pollution stock S_t . Pollutant emissions over a period are assumed to be proportional to the extraction of NC (i.e. to the ecological footprint). The pollution stock S_t results from emissions over θ periods ($\theta \geq 1$ and exogenous):

$$S_t = S(X_\tau, \tau = 1, \dots, \theta) \quad (40)$$

To save a variable, the pollution stock is written directly according to the takings of NC. Through simulations, the parameter θ will shed light on the impact of pollution persistence on the economy's trajectory.

Subsequently, it is assumed that firms of the final sector include the damage function in their calculations. Formally, they solve the problem (9) under constraints (8) and (38), which gives the optimality conditions:

$$\begin{aligned} \alpha \left[\frac{Y_t}{A_t H_{ft}} \right]^{1+\rho} A_t &= \frac{v_t}{1 - d_t} \\ [1 - \alpha] \left[\frac{Y_t}{X_t} \right]^{1+\rho} &= \frac{q_t}{1 - d_t} \end{aligned}$$

Compared to conditions (10) and (11) of the pollution-free model, marginal costs are divided by $1 - d_t$, which implies *ceteris paribus* lower factor demands and final output.

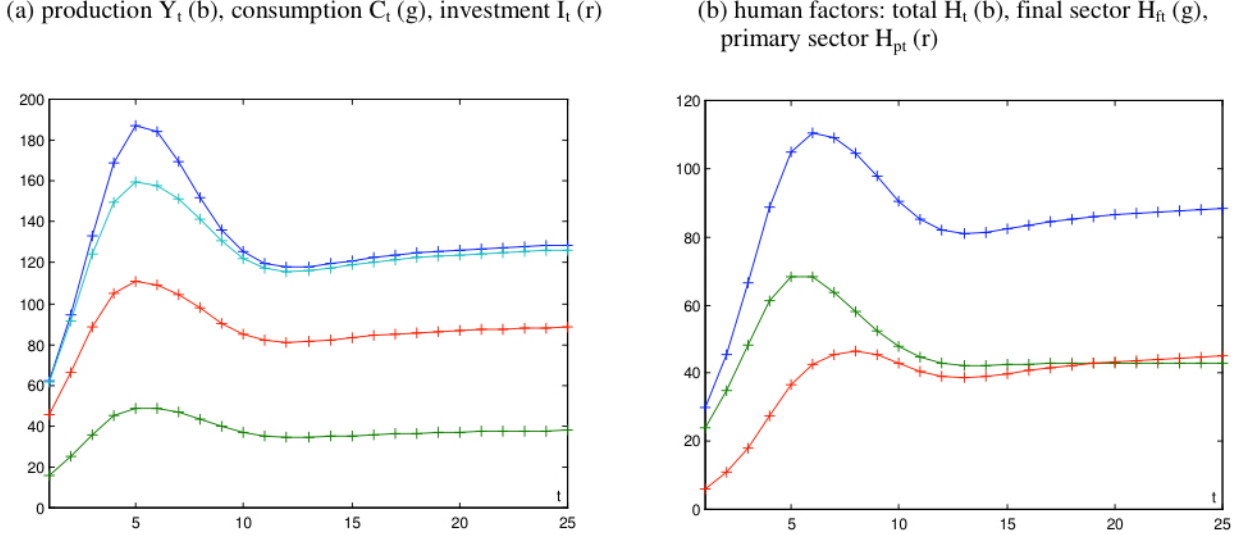
Figures 5.a-b illustrate the baseline simulation obtained with the pollution model (BS2) in the case where $\theta = 1$ (i.e. pollution only accumulates over one period of time)¹⁷. Whatever the absolute quantity considered (gross and net production, consumption, stocks of HFs), we find the growth-reversal-degrowth-stabilization phases observed in the absence of pollution. However, the excess of the peak over the stationary value is less pronounced than in the absence of pollution¹⁸. This is due to the fact that pollution reduces the final sector profitability (*ceteris paribus*), which has a deleterious effect on business. The stationary values are also lower than in the absence of pollution, but the difference in level is minimal.

Comparison of Tables 1 and 2 shows that some variables (Y_t, C_t, H_{ft}) reach their peak one period earlier than in the absence of pollution. For others (H_t, X_t), the peak period is unchanged. Only the peak of H_{pt} is delayed by one period. Taking into account pollution damage results in

¹⁷The parameterization is similar to the one for the pollution-free model. However, the observed output in $t = 1$ corresponds here to Z_1 (i.e. output net of damages), whereas in the pollution-free model, the observed output in $t = 1$ corresponds to Y_1 (since there are no damages).

¹⁸In the case of H_{pt} , we even observe that the peak is a local maximum, since it is lower than the long-term value.

Figure 5: Baseline simulation of model with pollution (b: blue, g: green, r: red, t: turquoise)



an extension of the reversal phase (during which all absolute quantities switch from growth to degrowth), which now extends over periods 4 to 8, or about sixty years.

Table 2 also shows that (i) the relative and price variables (except for HF's price v_t) and (ii) the natural capital stock R_t have a monotonous upward or downward trajectory, which was not the case in the absence of pollution. However, the simulations show that the general shape of their trajectory remains similar when pollution damage is taken into account.

The blue and turquoise curves in Figure 5.a correspond respectively to Y_t and Z_t . The difference between the two curves therefore represents damages in absolute terms. It can be seen that these are growing mainly around the peak of GDP. Figure 6.b confirms this by describing the evolution of the loss rate d_t (solid line curve). d_t reaches a maximum of almost 15% in $t = 5$ (i.e. around 2070), a figure comparable to that of DARA (2012), which estimates that the overall costs of pollution could exceed 10% of world GDP before the end of the century.

Figures 6.a-b shed light in a schematic way on the impact of more or less persistent pollution on the economy's path. Simulations BS2, V7 and V8 are respectively characterized by $\theta = 1, 2, 3$, i.e. the emissions of a given period cause damages that extend over the next 1,2 or 3 periods respectively. In addition, it is assumed that emissions degrade linearly (i.e. totally, by half or a third depending on whether $\theta = 1, 2, 3$).

The following observations can be made:

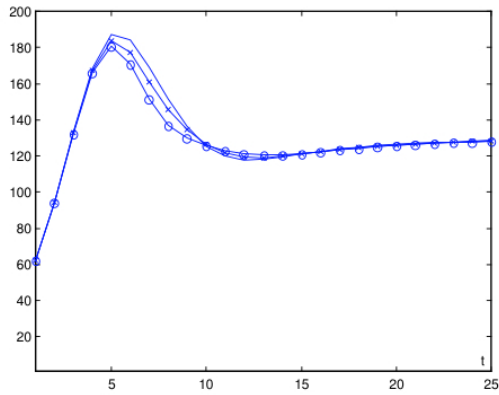
- The influence of pollution persistence only starts to be visible when the economy approaches its peak. It takes some time for the damages due to the pollution stock S_t to begin to emerge.
- The longer the pollution persistence, the greater the negative impact of pollution on gross output (i.e. damages non deducted), whether around the peak of production Y_t or in the long term (Figure 6.a). On the other hand, the timing of the peak is not changed.
- The longer the pollution persistence, the higher the loss rate d_t (the relative difference between gross output Y_t and net output Z_t) (Figure 6.b). Whatever the simulation, this loss rate is highest around the peak of production.

Table 2: Baseline simulation of model with pollution

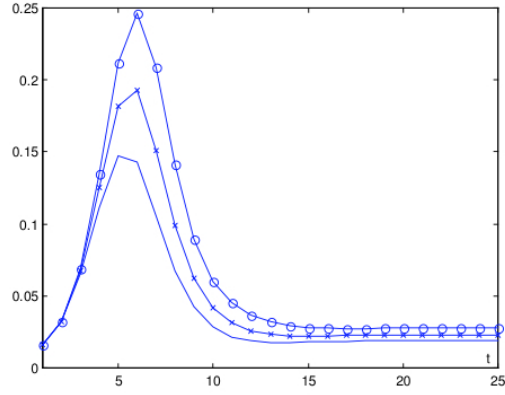
	max /min	t_{\max} / t_{\min}	Δ	SS
Y	187.0906	5	1.4068	132.9943
C	48.8720	5	1.2481	39.1573
I	110.6696	5	1.2113	91.3671
Z	159.5416	5	1.2223	130.5244
H	110.6696	6	1.2113	91.3671
H_f	68.3028	5	1.5914	42.9190
H_p	46.4955 (local max)	8	0.9597	48.4480
X/F	2.4500	4	2.4500	1
R	monotonous degrowth			19.7753
s	0.6900 (min) 0.7019 (local max)	7 13		0.7
m	monotonous degrowth			0.4697
E	monotonous growth			0.7626
v	1.3359 (min) 1.4442 (local max)	9 15		1.4286
q	monotonous growth			4.5893
k	monotonous growth			17.6447
y	monotonous growth			8.8187
x	monotonous degrowth			0.4998

Figure 6: Impact of pollution persistence (BS2: — ; V7: -x- ; V8: -o-)

(a) production Y_t



(b) loss rate d_t



6 Conclusion

This paper examines the path of an economy constrained by bio-physical limits, due to (i) limited natural resource availability and (ii) compliance with the postulate of strong sustainability (which requires that the productivity of these resources is limited). In doing so, it is at the heart of an important and very topical issue: the possibility of pursuing economic growth in a finite (bio-physical) world.

Based on the assumptions of Ecological Economics, the model distinguishes between (i) human (labour, physical and human capital) and natural (renewable and non-renewable natural resources) production inputs and (ii) a primary sector that operates natural capital and a final sector that produces a general purpose good. As part of the simulations, it uses the concepts of ecological footprint and biocapacity developed by the Global Footprint Network and is calibrated on the basis of global data. Two versions of the model are developed: (i) a basic version that ignores pollution and its impacts and (ii) a version that integrates them.

The simulations obtained with the basic model show that absolute economic quantities (final production, consumption, stocks of human and natural factors) have a four-step trajectory: (i) a growth phase that slows down, (ii) a reversal phase, (iii) a degrowth phase, and finally (iv) a quasi-stationary phase leading to a stationary state. Ecological footprint behaves in a similar way. In this respect, these results are similar to those of Meadows et al. (2004).

The first phase is characterized by a slowdown in growth, due to the continued increase of the operating costs of natural capital. This slowdown in growth echoes that of the world economy in recent decades, and more particularly that of industrialized countries, which has revived the debate around their entry into "secular stagnation". However, there is a significant difference in the causes advanced. The protagonists of this debate (a.o. Gordon (2015), Summers (2016)) explain the slowdown through internal economic mechanisms (induced by a demand or supply problem), with resource constraints playing no decisive role. In the present model, on the contrary, the tightening of these constraints leads to a structural change in the economy whereby the share of human factors allocated to the primary sector increases continuously to the detriment of that allocated to the final sector. It is this structural change that is the primary cause of the slowdown and halt in economic growth.

This result is at odds with certain contributions (such as López et al., 2007) where structural change allows an economy under environmental constraints to bypass them and continue to grow. The reason for this is that these contributions are based on the concept of low sustainability, whereas the present paper postulates that of strong sustainability.

In addition, growth is accompanied by an increase in the operating costs of natural capital, which in turn results in an increase of primary sector selling price at the beginning of the trajectory. This result echoes the trend increase in commodity prices observed since the beginning of the century (Hallam (2018), King (2015)), in contrast to the secular decline observed during the 20th century. However, caution should be used because, given the high variability of these prices, an upward phase of a new cycle cannot be excluded.

The persistent increase in operating costs of natural capital ultimately results in a reversal of economic activity. This is characterized by the interesting fact that absolute quantities (production, stocks of human factors, ecological footprint) reach their respective maxima at different periods. The first variable to reach a maximum is the ecological overshoot (the ratio between ecological footprint and biocapacity), over a period of about 50 years. The value achieved is about 2.6, which roughly corresponds to the generalization of the current average European overshoot to the whole of humanity. Because biocapacity is growing, the ecological footprint is maximum only about a decade later, i.e. at a time horizon comparable to that given by Dale et al. (2012) for energy production.

While the ecological footprint turns around, technical progress temporarily allows GDP and human factors to continue to grow. GDP reaches its peak at the end of the century, significantly later than in Meadows et al. (2012) where the peak is reached between 2020 and 2060 depending on the scenarios. This is because these authors' complex model incorporates other negative factors for the economy than increasing operating costs (such as pollution), factors that are ignored by

our basic model.

Economic growth has resulted in a level of exploitation of natural capital that is significantly in excess of what it can sustain over the long term. It is therefore inevitable that the economy will enter a third phase characterized by degrowth. The same goes for ecological footprint. However, the intertemporal smoothing of consumption desired by households does not allow a sudden adjustment towards a sustainable level of activity. As a result of this inertia phenomenon and although the economy has entered a downward trend, the natural capital stock continues to decline for a few more decades. As the latter decreases faster than the ecological footprint, the exploitation rate of natural capital continues to increase, so do operating costs, and structural change continues to benefit the primary sector at the expense of the final sector. To the question (asked in the introduction) of whether structural change is reversible depending on whether the economy grows or degrows, the answer is therefore negative.

The decrease in activity and ecological footprint ultimately reconciles the latter with biocapacity. As a result, the rarefaction process of natural capital in place since the beginning of the path slows down and then stops. The economy is no longer forced to divert more and more human factors to the primary sector and structural change is also halting. The last phase, i.e. the convergence of the economy towards the steady state, is beginning.

But there is no return to a situation similar to that at the beginning of the trajectory. Compared to the initial state, the steady state is characterized by (i) a much lower natural capital stock and a much higher human factors stock and (ii) a much more favorable sharing of human factors for the primary sector. This last result supports the intuitions of different energy specialists mentioned in the introduction (a.o. van den Bergh, 2013) regarding a similar potential displacement of human factors in the context of the energy transition, while extending them to a framework that is not limited to energy.

In addition, various comparative dynamics exercises were carried out with the basic model in order to test the sensitivity of the results to certain assumptions. The first exercise shows that doubling the technical progress potential at the beginning of the trajectory results in a higher peak of GDP, a more predatory economy and a more pronounced structural change in favor of the primary sector, without however changing the general shape of the economy's path. The same applies if technical progress is endogenized on the basis of a learning-by-doing process. While the results presented above are based on an increase in biocapacity (as a continuation of the past), the third exercise assumes a constant biocapacity. This results in a less pronounced and advanced peak of GDP over a period of about fifteen years, as well as a lower stationary level. But again, the overall shape of the trajectory is preserved.

The simulations carried out with the pollution model show that, whatever the absolute quantity considered, we find the growth-reversal-degrowth-stabilization phases observed in the absence of pollution. However, the peak of GDP is less pronounced than in the absence of pollution, which is explained by the fact that pollution reduces the profitability of the final sector and therefore has a deleterious effect on activity. The peak occurs during the last quarter of this century and is advanced compared to the pollution-free situation. It still remains after the 2020-2060 interval of Meadows et al. (2004) mentioned above. Now an exercise (done with the basic model and mentioned in the previous paragraph) which assumed a lower biocapacity also led to the peak being advanced. It can therefore be suspected that if the pollution were to have negative impacts on biocapacity, the peak time would be even more advanced and would be closer to the interval proposed by these authors.

In addition, it appears that damages are mainly increasing around the peak of GDP, whether measured in absolute or relative terms. In the case where pollution is the least persistent, damages reach a maximum of almost 15% of GDP, a figure similar to that of DARA (2012), which estimates that the overall costs of pollution could exceed 10% of world GDP by the end of the century. For comparison purposes, let us mention that Stern (2006) provides for climate change alone a range of damage from 0.3 to 5.9% in 2100 depending on the scenarios and impacts selected.

The model developed in this article is likely to be developed in several directions in order to evaluate the results obtained in richer frameworks. Several avenues of research deserve to be highlighted. The first would be to study pollution with impacts on biocapacity, in order to

verify the intuition (expressed above) that the economic degrowth phase could be advanced and/or exacerbated. Introducing the possibility of recycling some of this pollution would be another interesting line of research as it would increase the resources of the economy and affect its trajectory. A third development would be to explicitly consider labour and productive capital alongside natural capital, which would enrich the analysis of factor mobility between sectors. In this perspective and given that the model is very long-term, endogenizing the population and in particular the demographic transition would be a useful extension. Finally, generalizing the analysis to any (non-unitary) human factors depreciation rate would allow to refine the analysis of the dynamics.

7 Bibliographie

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8 Appendix

8.1 Derivation of (3)

(1), (2) \Rightarrow

$$\begin{aligned} C_t &= v_t H_t - I_t \\ I_t &= H_{t+1} - [1 - \delta] H_t \end{aligned}$$

Then these 3 equations \Rightarrow

$$C_t = v_t H_t + [1 - \delta] H_t - H_{t+1} \quad (41)$$

The households' objective becomes:

$$\max_{\{H_{t+1}\}_{t=1, \dots, T}} \Theta = \sum_{t=1}^T \beta^t \frac{C_t^{1-\sigma}}{1-\sigma}$$

where C_t is determined by (41).

The first order optimality condition implies :

$$\begin{aligned} \frac{\partial \Theta}{\partial H_t} &= \frac{\beta^{t-1}}{C_{t-1}^\sigma} \frac{\partial C_{t-1}}{\partial H_t} + \frac{\beta^t}{C_t^\sigma} \frac{\partial C_t}{\partial H_t} = -\frac{\beta^{t-1}}{C_{t-1}^\sigma} 1 + \frac{\beta^t}{C_t^\sigma} [v_t + 1 - \delta] = 0 \\ &\Rightarrow \left[\frac{C_t}{C_{t-1}} \right]^\sigma = \beta [v_t + 1 - \delta] \end{aligned}$$

which corresponds well to (3).

In infinite horizon, the final condition is written:

$$\lim_{t \rightarrow +\infty} \beta^t \frac{H_{t+1}}{C_t^\sigma} = 0 \quad (42)$$

8.2 Resolution of the steady state

At the steady state, (3) \Rightarrow

$$v = \frac{1}{\beta} - 1 + \delta \quad (43)$$

The HFs unit income v can be directly computed from the exogenous parameters β, δ .

(10) and (18) $\Rightarrow \alpha x^{1+\rho} \bar{A} = v \Rightarrow$

$$x = \left[\frac{v}{\alpha \bar{A}} \right]^{\frac{1}{1+\rho}} \quad (44)$$

(8), (18) and (17) $\Rightarrow \alpha \left[\frac{Y}{A H_f} \right]^\rho + [1 - \alpha] \left[\frac{Y}{X} \right]^\rho = 1 \Rightarrow \alpha x^\rho + [1 - \alpha] y^\rho = 1 \Rightarrow$

$$y = \left[\frac{1 - \alpha x^\rho}{1 - \alpha} \right]^{\frac{1}{\rho}} \quad (45)$$

The productivity of the quantities extracted from the NC stock is an inverse function of the productivity of the HFs, and therefore also of HFs unit income. Note that it is necessary that $y > 0$, which implies $1 > \alpha x^\rho$. Then (44) $\Rightarrow \alpha \left[\frac{v}{\alpha \bar{A}} \right]^{\frac{\rho}{1+\rho}} < 1 \Rightarrow \alpha^{1-\frac{\rho}{1+\rho}} < \left[\frac{\bar{A}}{v} \right]^{\frac{\rho}{1+\rho}} \Rightarrow \alpha^{\frac{1}{1+\rho}} < \left[\frac{\bar{A}}{v} \right]^{\frac{\rho}{1+\rho}} \Rightarrow \alpha < \left[\frac{\bar{A}}{\frac{1}{\beta} - 1 + \delta} \right]^\rho$. It follows the existence condition:

$$C1 : \alpha^{\frac{1}{\rho}} \left[\frac{1}{\beta} - 1 + \delta \right] < \bar{A} \quad (46)$$

(11) $\Rightarrow [1 - \alpha] y^{1+\rho} = q$, which with (45) \Rightarrow

$$q = [1 - \alpha] \left[\frac{1 - \alpha x^\rho}{1 - \alpha} \right]^{\frac{1+\rho}{\rho}} \quad (47)$$

Then, given (7), $B(E) = q/v$, which with (47) \Rightarrow

$$B(E) = \frac{1 - \alpha}{v} \left[\frac{1 - \alpha x^\rho}{1 - \alpha} \right]^{\frac{1+\rho}{\rho}} \quad (48)$$

The price of the primary sector output is negatively correlated with the productivity of the HFs and therefore also with their unit income. By inverting B , we obtain the operating rate of NC:

$$E = B^{-1} \left(\frac{1 - \alpha}{v} \left[\frac{1 - \alpha x^\rho}{1 - \alpha} \right]^{\frac{1+\rho}{\rho}} \right) \quad (49)$$

(4) implies at the steady state:

$$X = F \quad (50)$$

The constancy of the NC stock implies equality between taking and renewal flows.

(17), (45) and (50) $\Rightarrow Y = yX \Rightarrow$

$$Y = \left[\frac{1 - \alpha x^\rho}{1 - \alpha} \right]^{\frac{1}{\rho}} F \quad (51)$$

Stationary production is a negative function of x (and therefore of v) and a positive linear function of F .

(22) \Rightarrow

$$m = \alpha x^\rho \quad (52)$$

Condition C1 ensures that $m < 1$.

(1) and (12) $\Rightarrow K = \frac{Y}{v} \Rightarrow$

$$K = \frac{yF}{v} \quad (53)$$

In addition, (2) implies at the stationary state that $I = \delta K = \frac{\delta Y}{v} \Rightarrow$

$$C = \left[1 - \frac{\delta}{v} \right] Y \quad (54)$$

According to (43), it follows that the stationary savings rate is written:

$$s = \frac{\delta}{\frac{1}{\beta} - 1 + \delta} \quad (55)$$

The fact that $0 < \beta, \delta < 1$ ensures that $0 < s < 1$.

Finally, given (6), (49) and (50), the natural capital stock at the stationary state is written:

$$R = \frac{F}{B^{-1} \left(\frac{1 - \alpha}{v} \left[\frac{1 - \alpha x^\rho}{1 - \alpha} \right]^{\frac{1+\rho}{\rho}} \right)} \quad (56)$$

Remarks

The above results assume that technical progress in the use of HFs is limited. What if it is if $t \rightarrow \bar{A} = +\infty$? (43) shows that the unit income of the HFs is not affected. Given (44) and (45), $x = 0$ and $y = [1 - \alpha]^{-\frac{1}{\rho}}$. This last equality, (50) and (17) imply at the steady state:

$$Y = \frac{F}{[1 - \alpha]^{\frac{1}{\rho}}} \quad (57)$$

The NC stock being limited, X is necessarily a finite quantity. In order for production to be unbounded, the denominator must tend towards 0. This is impossible here because, under the assumption of strong sustainability, it was precisely imposed that the productivity of X was bounded (which is guaranteed by the conditions $0 < \rho, 0 < \alpha < 1$).

The fact that operating costs also tend towards 0 does not change anything, because the function $B(E) = \lim_{t \rightarrow +\infty} B_t(E)$ is not involved in (57). In conclusion, even if the productivity of HFs tends towards infinity in *both* sectors, the economy can only tend towards a stationary state at best. Given (52), the fact that $x \rightarrow 0 \Rightarrow m \rightarrow 0$. All the HFs are asymptotically concentrated in the primary sector. In addition, in the case where $B_t(E)$ is given by (30) with $\lim_{t \rightarrow +\infty} b_t = 0$, the stationary exploitation rate of NC is unitary. Indeed, (48) with $x = 0$ implies that $B(E)$ is asymptotically positive and finite, which is only possible if $E_t \rightarrow 1$. Then, (4), (6) imply $R = F = X$ at the steady state.

The previous results assume that the renewal flow of NC F_t tends towards a given and exogenous limit F . What if F_t depends on the resource stock, as assumed in some models with renewable resources? If $F_t = f(R_t)$ where f is a known function, then (4) implies at the steady state that $X = F = f(R)$. Combined with the identity $E = X/R$, this equation leads to:

$$ER = f(R) \quad (58)$$

It is easy to verify that the stationary equations (43), (44), (45), (47), (49) allowing to calculate v, x, y, q, E remain valid whether F is endogenous or not. It follows from this that the previous equation allows in principle to compute R .

8.3 Decomposition of growth

Total differentiation of (8) w.r.t. K_{ft}, A_t, X_t leads to :

$$dY_t = \alpha \left[\frac{Y_t}{A_t K_{ft}} \right]^{1+\rho} [A_t dK_{ft} + K_{ft} dA_t] + [1 - \alpha] \left[\frac{Y_t}{X_t} \right]^{1+\rho} dX_t$$

After dividing the two members by Y_t , we get :

$$\frac{dY_t}{Y_t} = \alpha \left[\frac{Y_t}{A_t K_{ft}} \right]^{\rho} \left[\frac{dK_{ft}}{K_{ft}} + \frac{dA_t}{A_t} \right] + [1 - \alpha] \left[\frac{Y_t}{X_t} \right]^{\rho} \frac{dX_t}{X_t} \quad (59)$$

Given (5), the variation of the ecological footprint can in turn be decomposed: $\frac{dX_t}{X_t} = \frac{dK_{pt}}{K_{pt}} - \frac{dB_t}{B_t}$. Given (14), $\frac{dK_{ft}}{K_{ft}} = \frac{dK_t}{K_t} + \frac{dm_t}{m_t}$ and $\frac{dK_{pt}}{K_{pt}} = \frac{dK_t}{K_t} - \frac{dm_t}{1-m_t}$. After substituting these equalities in (59) and taking into account (18) and (17), we get:

$$\frac{dY_t}{Y_t} = \alpha x_t^{\rho} \left[\frac{dK_t}{K_t} + \frac{dm_t}{m_t} + \frac{dA_t}{A_t} \right] + [1 - \alpha] y_t^{\rho} \left[\frac{dK_t}{K_t} - \frac{m_t}{1-m_t} \frac{dm_t}{m_t} - \frac{dB_t}{B_t} \right]$$

After regrouping according to the different variations, the previous equation becomes:

$$\begin{aligned} \frac{dY_t}{Y_t} &= [\alpha x_t^{\rho} + [1 - \alpha] y_t^{\rho}] \frac{dK_t}{K_t} + \alpha x_t^{\rho} \frac{dA_t}{A_t} + \left[\alpha x_t^{\rho} - [1 - \alpha] y_t^{\rho} \frac{m_t}{1-m_t} \right] \frac{dm_t}{m_t} - [1 - \alpha] y_t^{\rho} \frac{dB_t}{B_t} \\ &= [\alpha x_t^{\rho} + [1 - \alpha] y_t^{\rho}] \frac{dK_t}{K_t} + \alpha x_t^{\rho} \frac{dA_t}{A_t} + \frac{\alpha x_t^{\rho} [1 - m_t] - [1 - \alpha] y_t^{\rho} m_t}{1 - m_t} \frac{dm_t}{m_t} - [1 - \alpha] y_t^{\rho} \frac{dB_t}{B_t} \\ &= \frac{dK_t}{K_t} + \alpha x_t^{\rho} \frac{dA_t}{A_t} - [1 - \alpha] y_t^{\rho} \frac{dB_t}{B_t} \end{aligned} \quad (60)$$

the third equality resulting from (19) and (22).

In the particular case where operating costs are defined by (30), $dB_t = \frac{E_t db_t}{1-E_t} + \frac{b_t dE_t}{[1-E_t]^2} \Rightarrow \frac{dB_t}{B_t} = \frac{db_t}{b_t} + \frac{dE_t}{[1-E_t]E_t}$. Formula (60) becomes:

$$\frac{dY_t}{Y_t} = \frac{dK_t}{K_t} + \alpha x_t^\rho \frac{dA_t}{A_t} - [1 - \alpha] y_t^\rho \left[\frac{db_t}{b_t} + \frac{dE_t}{[1 - E_t] E_t} \right] \quad (61)$$

If we define the rate of change of variable z_t at period t as $g_{zt} = \ln\left(\frac{z_{t+1}}{z_t}\right)$, the previous equation leads in first approximation to (33).

8.4 Calibration of the model

The values of the parameters $\alpha, \beta, \nu, \sigma$ are chosen from the literature or on the basis of reasonable values commonly accepted:

- intertemporal substitution elasticity σ : a recent literature review by Thimme (2017) argues for values below 1 in the case of models that assume a representative agent that consumes a single, nondurable consumer good, which is the case here. At the same time, his article suggests values closer to 1 than 0, so the reference value chosen for σ is .75.
- discount factor β : in order not to reduce the weight of the future too much, the reference value chosen is $\beta = .7$. This parameter is linked to the annual discount rate r according to the formula $\beta = [1 + r]^{-\phi}$, where ϕ refers to the length of the period. As $\phi \approx 15$ years, $\beta = .8$ corresponds to a rate r around 2.4%.
- substitution elasticity between natural and human inputs ν : the most documented case in the literature is probably that of energy. Based on industrial data for 12 OECD countries and on the basis of a nested CES function, van der Werf (2008) estimates the elasticity of substitution between energy and the capital-labour couple to be between .17 and .65 at the level of industries and between .17 and .61 at the level of countries. He tests the hypothesis of unit elasticity (the Cobb-Douglas case) and rejects it for all the industries and countries considered. Based on the same production function, Stern and Kander (2012) estimate values of ν around 2/3. On the basis of previous estimates, the reference value chosen for ν is .5. Since the parameter ρ appearing in the production function (8) is linked to ν by the relationship $\rho = 1/\nu - 1$, $\rho = 1$.
- distribution parameter α : again, the energy literature will be used as a reference. According to the variants studied, Stern and Kander (2012) propose values of α around .8 and Heun et al. (2017) values between .7 and 1. This parameter is calibrated on the basis of the initial distribution of HFs between the two sectors (see below), while checking that it belongs to the interval [.7, 1].

The other exogenous parameters and sequences are chosen (i) so as to respect the existence conditions of the ES and (ii) so that the paths generated by the model have certain desirable properties.

- Formula (31) governing technical progress involves several parameters. Remember that the sequence $A_t, t \geq 1$ describes technical progress affecting all HFs and not just work. The baseline is characterized by a technical progress potential at the beginning of the trajectory of 100%, i.e. $\bar{A}/A_0 = 2$. g_A is estimated on the basis of data on total factor productivity gains produced by the Conference Board (Conference Board Total Economy Database, Regional Aggregates, 1990-2018). \bar{A} is calibrated so that the economy' growth rate in $t = 1$ extends the trend observed for this rate over previous periods.
- The initial value of production Y_1 is the average world GDP over the period 2000-2014 in trillion US \$ of 2010 (source: World Bank).
- The initial values of X_1 and F_1 are respectively the average global ecological footprint and biocapacity over the period 2000-2014, expressed in billions of global hectares (source: Global Footprint Network). The future sequence $F_t, t \geq 1$ is obtained by extending the past trend of global biocapacity produced by the Global Footprint Network (a trend that shows that it has increased at a decreasing rate itself). The resulting asymptotic value F is almost 30% higher than F_1 .

- Dale et al. (2012) provide a ratio H_f/H_p in 2010 equal to 16. Unfortunately, their final sector is limited to the energy sector. Therefore, for this ratio in $t = 1$, we assume a lower range of values equal to $[3, 5]$. The baseline is characterized by the median value $f = H_{f1}/H_{p1} = 4$. It is then possible to estimate α from the initial values of Y_1 and X_1 , f and ρ . Indeed, (19) $\Rightarrow \alpha = 1 - \frac{f}{[1+f]y_1^\rho}$ where $y_1 = Y_1/X_1$. We obtain $\alpha = .94$ which is well within the desired range (see above).

The model with pollution

- For the purposes of simulations, the formula (40) defining the pollutant stock becomes:

$$S_t = \frac{1}{\theta} \sum_{\tau=1}^{\theta} [\theta + 1 - \tau] X_{t-\tau} \quad (62)$$

which assumes that pollutant emissions (supposed to be proportional to the ecological footprint) over a period of time t contribute to the pollution stock during the periods $t + 1, \dots, t + \theta$, while degrading linearly following a natural purification process. The advantage of the latter assumption is that the initial pollution stock can be rebuilt on the basis of a limited number of past periods. Now the Global Footprint Network only provides past ecological footprints up to 1961.

- The damage function defined by (39) becomes:

$$d_t = \pi S_t^\zeta \quad (63)$$

where π and ζ are two positive parameters.

A study by the NGO DARA (DARA, 2012) provides two global estimates of d_t for the years 2010 and 2030, equal to 1.6% and 3.2% respectively. In the simulations, these damage values are used for the periods $t = 1$ and $t = 2$. On the basis of (62) and the ecological footprint numbers produced by the Global Footprint Network, the stock values S_t for the same periods are estimated. (63) then provides a system of two equations with two unknowns to calibrate π and ζ .