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Testing the general theory of relativity using gravitational wave propagation from dark standard sirens

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ABSTRACT

Alternative theories of gravity predict modifications in the propagation of gravitational waves (GW) through space–time. One of the smoking-gun predictions of such theories is the change in the GW luminosity distance to GW sources as a function of redshift relative to the electromagnetic (EM) luminosity distance expected from EM probes. We propose a multimessenger test of the theory of general relativity from the propagation of GWs by combining EM and GW observations to resolve these issues from GW sources without EM counterparts (which are also referred to as dark standard sirens). By using the relation between the geometric distances accessible from baryon acoustic oscillation measurements, and luminosity distance measurements from the GW sources, we can measure any deviation from the general theory of relativity via the GW sources of unknown redshift that will be detectable by networks of GW detectors such as LIGO, Virgo, and KAGRA. Using this technique, the fiducial value of the frictional term can be measured to a precision $\Xi_0 = 0.98^{+0.04}_{-0.23}$ after marginalizing over redshift dependence, cosmological parameters, and GW bias parameters with ~ 3500 dark standard sirens of masses $30 M_\odot$ each distributed up to redshift $z = 0.5$. For a fixed redshift dependence, a value of $\Xi_0 = 0.99^{+0.02}_{-0.02}$ can be measured with a similar number of dark sirens. Application of our methodology to the far more numerous dark standard sirens detectable with next-generation GW detectors, such as LISA, Einstein Telescope and Cosmic Explorer, will allow achievement of higher accuracy than possible from use of bright standard sirens.

Key words: gravitational waves – large-scale structure of Universe.

1 INTRODUCTION

The general theory of relativity predicts a unique geodesic for both electromagnetic wave (EM) and gravitational wave (GW) signals. However, several alternative theories of gravity predict deviations from the general theory of relativity by having a difference in the speed of propagation between GW and EM signals, due to the non-zero mass of the graviton, the running of the effective Planck mass (also called the frictional term), and the anisotropic source term (Lombriser & Taylor 2016; Baker et al. 2017; Creminelli & Vernizzi 2017; Ezquiaga & Zumalacárregui 2017; Lombriser & Lima 2017; Sakstein & Jain 2017; Nishizawa 2018; Belgacem et al. 2018a,b, 2019; Mastrogiovanni, Steer & Barsuglia 2020b). Measurement of the GW signal from astrophysical sources such as binary neutron stars (BNS), neutron star–black holes (NS–BH), binary black holes (BBH) detectable from ground-based detectors such as LIGO (LIGO Scientific Collaboration et al. 2015; Tse et al. 2019; Acernese et al. 2019), Virgo (Acernese et al. 2014), and KAGRA (Akutsu

et al. 2019), and in the future from LIGO-India (Unnikrishnan 2013), Cosmic Explorer (Reitze et al. 2019), and Einstein Telescope (Punturo et al. 2010), and the supermassive BBHs detectable from space-based GW detectors such as LISA (Klein et al. 2016), brings a unique way to test alternative theories of gravity via the propagation of GW through space–time over cosmological scales. Key aspects of alternative theories of gravity can be tested via GW propagation, and cannot be tested via only EM observations. Our proposal provides a new window to test fundamental physics.

Measurement of the electromagnetic counterpart within about 1.7 s (Abbott et al. 2017a,b,c) from the BNS event GW170817 has enabled stringent constraints to be imposed on the speed of GW propagation and the mass of the graviton to high precision, and also constraints on the frictional term (Abbott et al. 2019c; Mastrogiovanni et al. 2020b). Recently, measurements of the plausible EM counterpart to the GW event GW190521 (Abbott et al. 2020b) by the Zwicky Transient Facility collaboration at redshift $z = 0.438$ (Graham et al. 2020) has enabled constraints on the frictional term, though significantly weaker due to the large error (~ 50 per cent) on the luminosity distance for GW190521 (Mastrogiovanni et al. 2020a). Several forecast proposals have proposed studying the frictional term

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via GW detectors such as LIGO-Virgo (Mastrogiovanni et al. 2020b), LISA (Belgacem et al. 2019; Baker & Harrison 2020), and Einstein Telescope (Belgacem et al. 2018b; D’Agostino & Nunes 2019; Hogg, Martinelli & Nesseris 2020) using the GW sources that have EM counterparts. As a result, all of these studies are limited to only sources from which EM counterparts are expected, such as BNS, NS–BH, and supermassive black holes (SMBHs) if there is a dedicated EM follow-up available. However, for most of the BNS, NS–BH, and SMBHs, EM counterparts will not be detectable, and also for sources such as stellar mass BBHs, EM counterparts are unlikely if the baryonic matter is not present in their environment. For all such GW sources without EM counterparts, testing alternative theories of gravity is not possible in the current framework. Along with testing the propagation of GW signals, it is also possible to test other aspects of alternative theories of gravity (Abbott et al. 2019b, 2020a) from the GW sources detectable from the network of LIGO–Virgo detectors (Abbott et al. 2019a).¹

Here, we propose a new method that makes it possible to explore alternative theories of gravity from redshift-unknown GW sources (called dark sirens). Our method relies on exploiting the three-dimensional spatial clustering of the GW sources with galaxy redshift surveys, as we previously proposed for measuring the expansion history (H_0, w_0, w_a) and GW bias parameters ($b_{\text{GW}}(z) = b_{\text{GW}}(1+z)^\alpha$) (Mukherjee & Wandelt 2018; Mukherjee et al. 2020a; Mukherjee, Wandelt & Silk 2020c). Angular clustering to measure expansion history was proposed by Oguri (2016) and Bera et al. (2020). In this method, we propose to perform a cross-correlation of galaxy surveys with dark sirens to find the host redshifts of the dark sirens. Along with using the cross-correlation signal to find the host redshift, in our method, we propose to use the baryonic acoustic oscillation (BAO) scale measured from the galaxy power spectrum to infer the angular diameter distance for the sources at that redshift. For metric theories of gravity, the angular diameter distance is uniquely related to the EM luminosity distance ($d_A(z) = d_l^{\text{EM}}(z)/(1+z)^2$). As a result, by combining the geometric distance measurements from BAO, and inferring the redshifts of the dark sirens using cross-correlation with galaxy surveys, we can measure the redshift dependence of the frictional term from the GW luminosity distance.

The paper is organized as follows. In Section 2, we discuss the basic formalism of the method. In Section 3, we discuss the statistical framework for implementation of the cross-correlation method. In Sections 4 and 5, we discuss the set-up of the mock samples and forecasts for measurement of the frictional term from the LIGO–Virgo–KAGRA detectors. In Section 7, we conclude with the main findings of this method and discuss future prospects.

2 FORMALISM: RELATION BETWEEN THE GEOMETRIC DISTANCE AND THE GW LUMINOSITY DISTANCE

GW propagation in space–time can be written according to the general theory of relativity as

$$h''_I + 2\mathcal{H}h'_I + c^2k^2h_I = 0, \quad (1)$$

where h_I denotes the GW strain with polarization states $I \in \{+, \times\}$ where the prime indicates the derivative with respect to the conformal time η , and \mathcal{H} is the Hubble parameter in comoving coordinates.

However, in alternative theories of gravity, GW propagation can be written as

$$h''_I + 2(1 - \gamma(z))\mathcal{H}h'_I + (c_{\text{GW}}^2k^2 + m_{\text{GW}}^2a^2)h_I = a^2\Pi_I, \quad (2)$$

where $\gamma(z)$ is the frictional term, c_{GW} is the speed of GW propagation, m_{GW} is the graviton mass, and Π_I is the anisotropic stress term. Comparing equations (1) and (2) indicates that for the general theory of relativity, $\gamma(z) = 0$, $c_{\text{GW}} = c$, $m_{\text{GW}} = 0$, and $\Pi_I = 0$. Recent measurements from GW170817 have obtained strong constraints on $c_{\text{GW}} = c$, and $m_{\text{GW}} = 0$ (Abbott et al. 2017c). In the absence of the anisotropic source term $\Pi_I = 0$, the effect of the frictional term $\gamma(z)$ leads to a modified luminosity distance to the GW source situated at redshift z by the relation

$$d_l^{\text{GW}}(z) = \exp\left(-\int dz' \frac{\gamma(z')}{1+z'}\right) d_l^{\text{EM}}(z), \quad (3)$$

where $d_l^{\text{EM}}(z)$ is the luminosity distance according to the propagation of electromagnetic waves and which is related to the expansion history $H(z)$ by the relation $d_l^{\text{EM}}(z) = c(1+z) \int_0^z \frac{dz'}{H(z')}$, where $H(z) = H_0 \sqrt{\Omega_m(1+z)^3 + (1 - \Omega_m)}$ is related to cosmological parameters such as the Hubble constant H_0 , and the matter density Ω_m for the flat Lambda cold dark matter (LCDM) cosmological model. The above equation (equation 3) shows that the modification in the luminosity distance for the GW can be larger or smaller than the EM luminosity distance for different theories of gravity. The parameter $\gamma(z)$ is degenerate with the electromagnetic (EM) luminosity distance $d_l^{\text{EM}}(z)$, and hence with other cosmological parameters. An independent probe of the EM luminosity distance from electromagnetic observables can be useful for breaking the degeneracy between the cosmological parameters and the parameters related to alternative theories of gravity $\gamma(z)$.

One of the independent measures to the EM luminosity distance $d_l^{\text{EM}}(z)$ for a metric theory is through its relation with the geometric distance (the angular diameter distance) $d_A(z) = d_l^{\text{EM}}(z)/(1+z)^2$, according to Etherington’s reciprocity theorem (Etherington 2007) or the distance duality relation. This relation is valid for EM probes if photon number is conserved, and photons propagate along null geodesics. The distance duality relation has been tested from several observations (Holanda, Lima & Ribeiro 2010; Holanda, Gonçalves & Alcaniz 2012; Liao et al. 2016) and will also be tested more stringently in the future (Liao et al. 2016; Arjona et al. 2020; Martinelli et al. 2020; Renzi et al. 2020). The angular diameter distance to any redshift z is related to the BAO scale θ_{BAO} in the matter correlation function by (Peebles & Yu 1970; Bond & Efstathiou 1984; Hu & Sugiyama 1996; Eisenstein & Hu 1998, 1997)

$$\theta_{\text{BAO}}(z) = \frac{r_s}{(1+z)d_A(z)}, \quad (4)$$

where, $r_s = \int_{z_d}^{\infty} dz' c_s(z')/H(z')$ is the sound horizon where z_d denotes the drag redshift. As a result, we can relate the BAO scale with the EM luminosity distance d_l^{EM} by the relation

$$d_l^{\text{EM}}(z) = \frac{(1+z)r_s}{\theta_{\text{BAO}}(z)}. \quad (5)$$

Using equation (5) in equation (3), we can write

$$d_l^{\text{GW}}(z) = \exp\left(-\int dz' \frac{\gamma(z')}{1+z'}\right) \frac{(1+z)r_s}{\theta_{\text{BAO}}(z)}. \quad (6)$$

This is the key equation of this paper. In this expression, the measurement of the term $\frac{r_s}{\theta_{\text{BAO}}(z)}$ comes from EM probes such as large-scale structure galaxy redshift surveys (Eisenstein et al. 2005; Dawson et al. 2013; Alam et al. 2017) and cosmic microwave

¹The catalogue of the GW sources is available in this link <https://dcc.ligo.org/P2000061/public>.

background (CMB) (Spergel et al. 2003, 2007; Komatsu et al. 2011; Hinshaw et al. 2013; Planck Collaboration XIII 2016; Aghanim et al. 2020), and the measurement of d_l^{GW} arises from the GW strain. We can write the above equation as

$$d_l^{\text{GW}}(z)\theta_{\text{BAO}}(z) = \exp\left(-\int dz' \frac{\gamma(z')}{1+z'}\right)(1+z)r_s. \quad (7)$$

This relation shows that the product of the BAO angular scale $\theta_{\text{BAO}}(z)$ and the luminosity distance $d_l^{\text{GW}}(z)$ can measure the frictional term $\gamma(z)$ as a function of redshift. So, the concordance between the EM geometric probes and the GW luminosity probes allows a way to test the theory of gravity. If the general theory of relativity is the correct theory of gravity, then the product between $\theta_{\text{BAO}}(z)$ and $d_l^{\text{GW}}(z)$ should vary with redshift as $(1+z)$. Any deviation from this scaling can be a signature of alternative theories of gravity. The quantities θ_{BAO} and d_l^{GW} are measured from large-scale structure and GW data, and the value of r_s depends on recombination physics and the sound speed in the baryon–photon fluid at the time of decoupling at redshift $z \approx 1100$ (Silk 1968; Peebles & Yu 1970; Sunyaev & Zeldovich 1970; Hu, Sugiyama & Silk 1997). As a result, this relation is nearly model-independent, and can be written directly in terms of observables such as $d_l^{\text{GW}}(z)$, and $\theta_{\text{BAO}}(z)$.

The BAO scale $\theta_{\text{BAO}}(z)$ can be inferred from the correlation function of the large-scale structure density field $\xi(r)$, and r_s can be constrained from the CMB observations. As a result, the right-hand side of equation (6) can be measured independently from large-scale structure observations and from CMB data. The inference of $\theta_{\text{BAO}}(z)$ from large-scale structure observations at redshift z , and the measurement of the GW luminosity distance from GW sources situated at the same redshift z , will make it possible to reconstruct the frictional term $\gamma(z)$ as a function of redshift. The value of the sound horizon r_s is obtained from CMB measurements. Currently, the measurement of the CMB temperature and the polarization field from CMB experiments provides a measurement of the value of $r_s \approx 147$ Mpc (Alam et al. 2017).

However, for this method to work, we need to infer the redshifts of the galaxies and also the redshifts of GW sources. The redshifts of the galaxies can be identified from photometric or spectroscopic surveys. Several ongoing/upcoming missions such as eBOSS (Alam et al. 2020), Dark Energy Survey (DES) (Collaboration et al. 2016), Dark Energy Spectroscopic Instrument (Aghamousa et al. 2016), *Euclid* (Refregier et al. 2010), Nancy Grace Roman Telescope (Green et al. 2012; Spergel et al. 2013; Dore et al. 2018a), Vera Rubin Observatory (LSST Science Collaboration et al. 2009), Spectro-Photometer for the History of the Universe, Epoch of Reionization, and Ices Explorer (SPHEREx) (Dore et al. 2018b), Subaru Prime Focus Spectrograph (Takada et al. 2014) will be covering nearly the full-sky up to redshift $z = 3.0$. The redshifts to the GW sources having EM counterparts (bright standard sirens) can be measured by identifying the host galaxy and the corresponding redshift from the catalogue. For GW sources without EM counterparts (dark standard sirens), host galaxy identification is not possible. Dark standard sirens are expected to be detectable from larger luminosity distances (so up to high redshift) and hence should be more numerous due to the volume factor. Also, the redshift dependence of the frictional term can be measured from the sources that are distributed up to high redshift. So, it is important to be able to use dark standard sirens to reconstruct the frictional term up to high redshift. We discuss below a framework that can be used to infer the redshift using cross-correlation with redshift-known galaxies.

In this paper, we will consider a parametric form of the modification of the GW luminosity distance in terms of Ξ_0 and n (Belgacem

et al. 2018a,b, 2019)

$$\frac{d_l^{\text{GW}}(z)}{d_l^{\text{EM}}(z)} = \Xi_0 + \frac{1 - \Xi_0}{(1+z)^n}. \quad (8)$$

In terms of these parameters Ξ_0 and n , $\gamma(z)$ can be expressed by the relation

$$\gamma(z) = \frac{n(1 - \Xi_0)}{1 - \Xi_0 + \Xi_0(1+z)^n}. \quad (9)$$

$\Xi_0 = 1$ and $n = 0$ represents the fiducial value of these two parameters for the general theory of relativity. Our method can capture any functional form of $\gamma(z)$, and is not only restricted to this particular form.

3 FRAMEWORK FOR TESTING GR FROM DARK STANDARD SIRENS

To measure the frictional term using GW sources, it is important to infer the redshifts to the GW sources. For GW sources without EM counterparts, one cannot identify the host galaxy and its redshift. So we propose to use the cross-correlation with galaxy surveys to find the host redshift of the GW sources, as proposed by Mukherjee & Wandelt (2018), Mukherjee et al. (2020a, c). The autopower spectrum $P_{XX}^{ij}(k, z)$ and cross-power spectrum $P_{XY}^{ij}(k, z)$ between GW sources and galaxy samples can be written in terms of the matter power spectrum $P_m(k, z)$ by the relation (Mukherjee et al. 2020a, c)

$$\begin{aligned} P_{gg}^{ss}(\vec{k}, z) &= b_g^2(k, z)(1 + \beta_g \mu_k^2)^2 P_m(k, z), \\ P_{g\text{GW}}^{sr}(\vec{k}, z) &= b_g(k, z)b_{\text{GW}}(k, z)(1 + \beta_g \mu_k^2) P_m(k, z), \\ P_{\text{GWGW}}^{rr}(\vec{k}, z) &= b_{\text{GW}}^2(k, z) P_m(k, z), \end{aligned} \quad (10)$$

where $b_g(k, z)$ is the galaxy bias, $b_{\text{GW}}(k, z)$ is the GW bias parameter, $\beta_g = fb_g$ which is related to the growth function D by the relation $f \equiv \frac{d \ln D}{d \ln a}$ (Peebles 1980). The term $\mu_k \equiv \cos(\hat{n} \cdot \hat{k})$ denotes the angle between the line of sight \hat{n} and the Fourier modes \hat{k} .

The autocorrelation between the galaxy samples at each tomographic redshift bins $P_{gg}^{ss}(\vec{k}, z)$ is a measure of the BAO scale $\hat{\theta}_{\text{BAO}}$. The convenient approach for this is to use the angular correlation function

$$\xi(\theta_{12}, \bar{z}) = \int dz_1 \Phi(z_1) \int dz_2 \Phi(z_2) \int \frac{dk k^2}{2\pi^2} b_g^2 P_m(k, \bar{z}) j_0(kx) \quad (11)$$

where $\Phi(z)$ is the selection function for the galaxies, $j_0(kr)$ is the zeroth-order Bessel function, $\bar{z} = (z_1 + z_2)/2$ is the mean redshift, and $x = \sqrt{r(z_1)^2 + r(z_2)^2 - 2r(z_1)r(z_2)\cos(\theta_{12})}$ is the comoving distance between a galaxy pair denoted in terms of the angular separation between the galaxies θ_{12} . The BAO scale can be obtained by fitting the angular correlation function with a power-law and a Gaussian model given by (Xu et al. 2012; Carvalho et al. 2016; Alam et al. 2017)

$$\xi(\theta_{12}, \bar{z}) = A_1 + A_2 \theta_{12}^k + A_3 \exp\left(-\frac{(\theta_{12} - \theta_{\text{FIT}}(\bar{z}))^2}{\sigma_\theta^2}\right), \quad (12)$$

where A_i are the coefficients, σ_θ is the width of the BAO feature, and θ_{FIT} is related to the θ_{BAO} by the relation

$$\theta_{\text{BAO}}(\bar{z}) = \theta_{\text{FIT}}(\bar{z}) + \epsilon(\bar{z}, \Delta z) \theta_E(\bar{z}, \Delta z = 0), \quad (13)$$

where the second term arises due to the finite bin width $\Delta z = z_2 - z_1$ and the correction term $\epsilon \equiv 1 - \theta_E(\bar{z}, \Delta z)/\theta_E(\bar{z}, \Delta z = 0)$ is related to the change in the peak position $\theta_E(\bar{z}, \Delta z = 0)$ in the limit of zero binwidth $\Delta z = 0$. One can also use the linear point measurement to measure the BAO scale (Anselmi et al. 2018).

As proposed in previous work (Mukherjee & Wandelt 2018; Mukherjee et al. 2020a), the cross-correlation power spectrum between GW sources and galaxy surveys $P_{\text{gGW}}^{sr}(\vec{k}, z)$ can be used to infer the host redshift shell of the GW sources. A similar approach to use the clustering signature to measure the Hubble constant has been performed (Bera et al. 2020) which depends on the method used (Oguri 2016). Using Bayes' theorem, we can write the joint estimation of the cosmological parameters $\Theta_c \in \{H_0, \Omega_m, w_0, w_a\}$, non-GR parameters $\Theta_{\text{GR}} \in \{\Xi_0, n\}$, and bias parameters $\Theta_n \in \{b_{\text{GW}}, \alpha\}$ as

$$\begin{aligned} \mathcal{P}(\Theta_{\text{GR}}, \Theta_c, \Theta_n | \vec{\vartheta}_{\text{GW}}, \vec{d}_{\text{g}}) &\propto \Pi(\Theta_c) \Pi(\Theta_{\text{GR}}) \\ &\times \Pi(\Theta_n) \int \int dr_s dz \left[\prod_{i=1}^{N_{\text{GW}}} \mathcal{L}(\vec{\vartheta}_{\text{GW}} | P_{\text{gg}}^{ss}(\vec{k}, z), \Theta_n, \vec{d}_{\text{g}}(z)) \right. \\ &\times \mathcal{P}(\vec{d}_{\text{g}} | P_{\text{gg}}^{ss}(\vec{k}, z)) \mathcal{P}(\{d_{\text{g}}^i\}_{\text{GW}} | z, \Theta_{\text{GR}}, \Theta_c, r_s, \{\theta^i, \phi^i\}_{\text{GW}}) \\ &\left. \times \mathcal{P}(\{\hat{\theta}_{\text{BAO}}\} | \Theta_c, z) \Pi(z) \Pi(r_s) \right], \end{aligned}$$

where each term can be written as follows: $\Pi(X)$ denotes the prior on a quantity X , $\mathcal{P}(\{\hat{\theta}_{\text{BAO}}\} | \Theta_c, z)$ is the posterior on the BAO peak position given the cosmological parameters and the redshift, $\mathcal{P}(\{d_{\text{g}}^i\}_{\text{GW}} | z, \Theta_{\text{GR}}, \Theta_c, r_s, \{\theta^i, \phi^i\}_{\text{GW}})$ is the posterior on the luminosity distance given the cosmological parameters, GR parameters, redshift, sound horizon, and sky localization of the GW sources, $\mathcal{P}(\vec{d}_{\text{g}} | P_{\text{gg}}^{ss}(\vec{k}, z))$ denotes the posterior on the galaxy power spectrum, and $\mathcal{L}(\vec{\vartheta}_{\text{GW}} | P_{\text{gg}}^{ss}(\vec{k}, z), \Theta_n, \vec{d}_{\text{g}}(z))$ denotes the likelihood to estimate the source redshift of the GW sources, and can be written as

$$\begin{aligned} \mathcal{L}(\vec{\vartheta}_{\text{GW}} | P_{\text{gg}}^{ss}(\vec{k}, z), \Theta_n, \vec{d}_{\text{g}}(z)) &\propto \exp\left(-\frac{V_s}{4\pi^2} \int k^2 dk \right. \\ &\times \left. \int d\mu_k \frac{(\hat{P}(\vec{k}, \Delta\Omega_{\text{GW}}) - \tilde{P}(k, z))^2}{2(P_{\text{gg}}^{ss}(\vec{k}, z) + n_{\text{g}}(z)^{-1})(P_{\text{GW}}^{rr}(\vec{k}, z) + n_{\text{GW}}(z)^{-1})}\right), \end{aligned} \quad (14)$$

where $\hat{P}(\vec{k}, z) = \delta_{\text{g}}(\vec{k}, z) \delta_{\text{GW}}^*(\vec{k}, \Delta\Omega_{\text{GW}})$, $\tilde{P}(k, z) = b_{\text{g}}(k, z) b_{\text{GW}}(k, z) (1 + \beta_{\text{g}} \mu_k^2) P_{\text{m}}(k, z) e^{-\frac{k^2}{k_{\text{eff}}^2}}$, $n_{\text{GW}}(z) = N_{\text{GW}}(d_{\text{g}}^i(z)) / V_s$ denotes the number density of GW sources in terms of GW sources in the luminosity distance bin $N_{\text{GW}}(d_{\text{g}}^i(z))$, $\Delta\Omega_{\text{GW}}$ is the sky localization error, and V_s is the overlapping sky volume accessible to both GW and galaxy surveys.

4 APPLYING THE METHOD TO GW AND GALAXY MOCK SAMPLES

Galaxy samples: In this analysis, we generate the mock sample galaxies using the `nbodykit` (Hand et al. 2018) with box size with 1350 Mpc h^{-1} in the direction perpendicular to the line of sight, and the line-of-sight direction is considered to be redshift z . The galaxy samples considered in this analysis are distributed up to redshift $z = 1$. The mock sample is generated with a matter power spectrum $P_{\text{m}}(k)$ and using the cosmological parameters according to Planck-2015 (Planck Collaboration XIII 2016). We include redshift space distortions in the mock sample according to the method prescribed in Hand et al. 2(018). For the mock samples, we consider the galaxy bias parameter $b_{\text{g}} = 1.6$ (Anderson et al. 2012; Alam et al. 2017; Desjacques, Jeong & Schmidt 2018). The contribution from weak lensing is negligible for sources below redshift $z = 1$, and we have not included the contribution from weak lensing in this analysis.

GW samples: GW mock samples are produced which follow the same underlying galaxy distribution over the redshift range $z = 0.1 - 1.0$. For the mock samples, we consider equal mass GW sources with each mass $30 M_{\odot}$ with two different sky localization error 10 and 100 deg^2 , as expected from the network of the LIGO–Virgo–KAGRA detectors (Chan 2018). The uncertainty in the luminosity distance error is calculated using the matched filtering technique (Sathyaprakash & Dhurandhar 1991; Cutler & Flanagan 1994; Balasubramanian, Sathyaprakash & Dhurandhar 1996; Nissanke et al. 2010) up to $f_{\text{max}} = f_{\text{merg}}$ which can be written in terms of the symmetric mass ratio $\eta = m_1 m_2 / (m_1 + m_2)^2$ and the total mass $M = m_1 + m_2$ as $f_{\text{merg}} = c^3 (a_1 \eta^2 + a_2 \eta + a_3) / \pi G M$, where $a_1 = 0.29740$, $a_2 = 0.044810$, $a_3 = 0.095560$ (Ajith et al. 2008).

$$\rho^2 \equiv 4 \int_0^{f_{\text{max}}} df \frac{|h(f)|^2}{S_n(f)}, \quad (15)$$

where ρ denotes the matched-filtering signal-to-noise ratio, $S_n(f)$ is the noise power spectrum for the LIGO design sensitivity (Abbott et al. 2016)² and the strain of the GW signal $h(f)$ can be written in terms of the redshifted chirp mass $\mathcal{M}_z = (1+z)\mathcal{M}_c$, inclination angle with respect to the orbital angular momentum $\hat{L} \cdot \hat{n}$ (which is denoted by the function $\mathcal{I}_{\pm}(\hat{L} \cdot \hat{n})$), and luminosity distance to the source d_L by the relation (Hawking & Israel 1987; Cutler & Flanagan 1994; Poisson & Will 1995; Ajith et al. 2008; Maggiore 2008)

$$h_{\pm}(f) = \sqrt{\frac{5}{96}} \frac{G^{5/6} \mathcal{M}_z^2 (f_z \mathcal{M}_z)^{-7/6}}{c^{3/2} \pi^{2/3} d_L} \mathcal{I}_{\pm}(\hat{L} \cdot \hat{n}). \quad (16)$$

For these simulations, we have considered our fiducial model to be the Λ CDM model of cosmology with parameter values according to Planck-2018 (Aghanim et al. 2020), consistent with the Planck-2015 results (Planck Collaboration XIII 2016), and the fiducial frictional terms ($\Xi_0 = 1$, and $n = 0$) according to the general theory of relativity.

5 FORECAST FOR THE NETWORK OF LIGO–VIRGO–KAGRA DETECTORS

In this analysis, we consider the combination of cosmological parameters (H_0, w_0), GW bias parameter, and its redshift dependence which is parametrized as a power-law $b_{\text{GW}}(z) = b_{\text{GW}}(1+z)^{\alpha}$ with two parameters b_{GW} and α , and the parameters related to the redshift dependence of the frictional term which can be parametrized in terms of Ξ_0 , and n (using equation 8). The dependence of the parameters Ξ_0 and n on redshift makes it essential for joint estimation of both the parameters in order to not keep the redshift dependence at a fixed value, unlike several previous analyses (Belgacem et al. 2018b; Baker & Harrison 2020). The cross-correlation measurement of the GW sources with galaxies requires us to marginalize over the redshift-dependent GW bias parameter which depends on the GW source merger rate and population. We show the joint estimation for two cases, (i) five parameter model ($H_0, w_0, b_{\text{GW}}, \alpha, \Xi_0$) with the value of n kept fixed at 1.5 from our setup, and (ii) six parameter model ($H_0, w_0, b_{\text{GW}}, \alpha, \Xi_0, n$). For the forecast study, the error on the BAO scale θ_{BAO} is considered according to current large-scale structure surveys (Alam et al. 2017). The priors on the parameters are considered flat as follows: $\Pi(H_0) \in [20, 150] \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Pi(w_0) \in [-3, -0.1]$, $\Pi(b_{\text{GW}}) \in [0, 6]$, $\Pi(\alpha) \in [-4, 4]$, $\Pi(\Xi_0) \in [0, 1.5]$, and $n \in [0, 5]$. The value

²The noise power spectrum is available on the webpage <https://dcc-lho.ligo.org/LIGO-T2000012/public>.

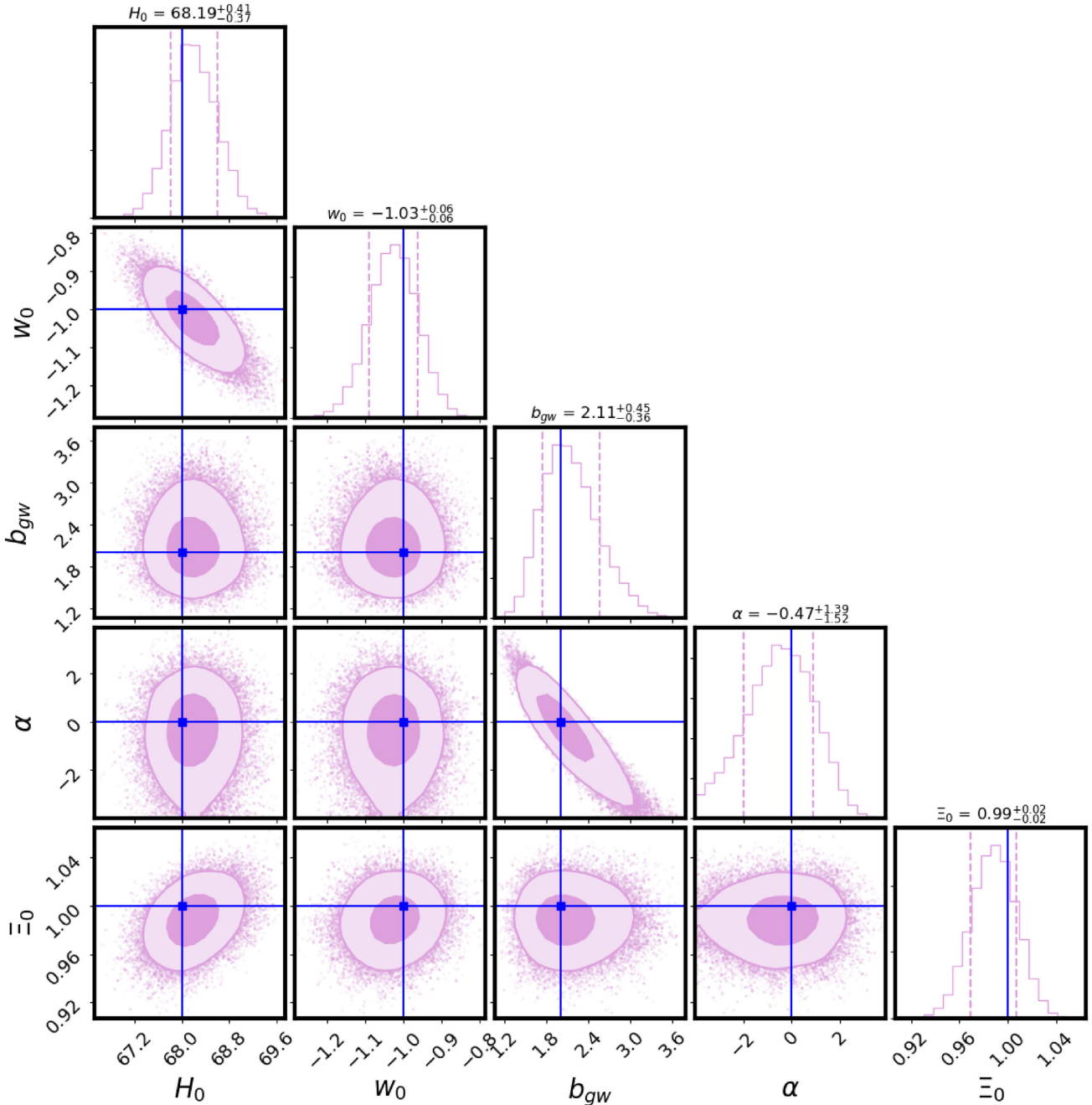


Figure 1. The joint estimation of the cosmological parameters H_0 , w_0 , GW bias parameters b_{GW} , α , and non-GR parameter Ξ_0 with the fixed value of $n = 1.5$. The number of GW sources is considered to be ~ 3500 and detectable up to redshift $z = 0.5$ with the aLIGO design sensitivity and with a sky localization error $\Delta\Omega_{\text{GW}} = 100 \text{ deg}^2$. The fiducial values used in the mock samples are shown by the blue solid line.

of r_s is accordingly taken to be fixed at the value considered by the eBOSS analysis (Alam et al. 2017).

We obtain results for the five parameters (H_0 , w_0 , b_{GW} , α , Ξ_0) keeping the redshift dependence fixed at a value $n = 1.5$. The results are not sensitive to the choice of the value of n . We find similar unbiased estimations also for other values such as $n = 0$, and $n = 2.5$. Previous analyses were carried out with a fixed value of $n = 2.5$ (Belgacem et al. 2018b). The measurability of such a scenario from the dark standard sirens detectable from the LVK network of detectors is shown in Fig. 1 for $N_{\text{GW}} = 3502$ (~ 3500) BBHs distributed up to redshift $z = 0.5$ with sky localization errors $\Delta\Omega_{\text{GW}}$

$= 100 \text{ deg}^2$. The use of the BAO scale for every redshift improves the constraining power of the cosmological parameters H_0 , w_0 . As a result, measurement of the non-GR parameter Ξ_0 is possible with about 2 per cent accuracy. A similar accuracy [about a factor of 2 better, $\Delta\Xi_0 = 0.008$ (Belgacem et al. 2018b)] is only possible for 1000 BNS sources with EM counterparts from the Einstein Telescope. Recent work has shown that LVK detectors can obtain only a 10 per cent measurement of the frictional term with BNS (Baker & Harrison 2020) for a fixed redshift dependence, which is a factor of 5 weaker than constraints possible with our method using ~ 3500 BBHs. With the feasibility of using dark standard sirens

proposed by our method, we can measure the modification in the GW luminosity distance up to high redshift ($z = 0.5$) with numerous sources detectable due to the larger accessible volume, and from sources such as BBHs which are intrinsically louder than the BNS events due to the mass dependence. Assuming that the BBH merger rate will be governed by the Madau–Dickinson star formation rate (Madau & Dickinson 2014), we expect to be able to measure about 250–600 GW sources per year from the advanced LIGO design sensitivity depending upon the mass distribution of the GW sources (Fishbach, Holz & Farr 2018). So, within a time-scale of about 6–14 yr of observation time of LVK detectors, we will be able to measure the Ξ_0 parameter with 2 per cent accuracy. Hence, on a time-scale shorter than previously expected, this method will enable us to test the frictional term from GW observations. These joint studies show that the three-dimensional cross-correlation technique makes it possible to reduce the degeneracy between the GW bias parameter (which is related to the GW merger rates, and its population) with the parameters related to cosmology and theories of gravity. This arises in particular from the three-dimensional correlation function that takes into account the shape of the correlation function (Mukherjee et al. 2020a). The shape of the correlation function is not affected by the bias parameter but is affected by the cosmological parameters. As a result, the inference of the clustering redshift becomes more robust through the three-dimensional correlation function, as previously shown by Mukherjee et al. (2020a). For cases with sky localization error $\Delta\Omega_{\text{GW}} = 10 \text{ deg}^2$, the error bar on the GW bias parameters improves by about a factor of 2 due to the better estimate of the three-dimensional clustering signal. However, the error bars on the cosmological parameters and the non-GR parameters do not change significantly. This is because the improvement in the error bars associated with the redshift estimation is marginal in comparison to the error bars associated with the GW luminosity distances.

In Fig. 2, we show the forecast for the joint estimation of the six parameters (H_0 , w_0 , b_{GW} , α , Ξ_0 , n) for sky localization error 100 deg^2 for ~ 3500 GW sources distributed up to redshift $z = 0.5$. The plot shows the existence of degeneracy between the non-GR parameters Ξ_0 and n . This indicates that keeping a fixed value of n or assuming a particular form of the redshift dependence will significantly underestimate the error bar. In comparison to the case with a fixed value of n , the posterior on Ξ_0 becomes non-Gaussian as can be seen from Fig. 2. The values of $\Xi_0 \geq 1$ can be constrained with about 4 per cent accuracy (a factor of 2 degradation in comparison to the fixed n case), and the values of $\Xi_0 < 1$ can be constrained with about 25 per cent accuracy, instead of 2 per cent possible from the case with fixed redshift dependence. The large error in the GW luminosity distance at higher redshifts leads to poor constraints on the parameter n , and hence causes a broad tail in the posterior distribution of this parameter. Our results also show the existence of degeneracy between the non-GR parameters and the GW bias parameters and other cosmological parameters. The inclusion of BAO helps in breaking the degeneracy between the cosmological parameters (H_0 , w_0) and the non-GR parameters (Ξ_0 , n). Our method can be used to jointly infer the cosmological parameters, bias parameters, and the non-GR parameters by using the dark standard sirens. In Fig. 3, we show the uncertainty on the Ξ_0 parameter with the change in the number of objects from 1500 to 10 000 for sources distributed up to redshift $z = 0.5$ with sky localization error $\Delta\Omega_{\text{GW}} = 100 \text{ deg}^2$. The error on the frictional term scales roughly with the increase in the number of objects by $1/\sqrt{N_{\text{GW}}}$. The improvement in the measurement gets saturated (even with an increase in the number of GW sources), due to the presence of the error on the BAO scale.

More precise measurement of the BAO scale will lead to further improvement in the estimation of the non-GR parameters.

6 APPLICABILITY TO THE DARK STANDARD SIRENS DETECTABLE FROM LISA AND COSMIC EXPLORER/EINSTEIN TELESCOPE

The future space-based GW detector Laser Interferometer Space Antenna (LISA) (Amaro-Seoane et al. 2017) and ground-based GW detectors such as the Einstein Telescope (Punturo et al. 2010) and Cosmic Explorer (Reitze et al. 2019) are going to detect numerous GW sources beyond redshift $z = 1$. However, several of these sources are not going to have EM counterparts, and hence the current techniques are limited to only those sources for which EM counterpart detection is possible. Though BNSs, NS–BHs, and SMBHs are expected to have electromagnetic counterparts, their detection from an EM follow-up telescope can be challenging due to fading EM counterparts, incomplete galaxy catalogs, large sky localization errors, and unavailability of EM telescopes with cadence. Along with the bright standard sirens, there are also going to be numerous dark standard sirens up to high redshift for which EM counterparts cannot be measured.

The method proposed in this paper can be used for the dark standard sirens detectable from LISA and Einstein Telescope/Cosmic Explorer by exploring the cross-correlation of the spatial position of the GW sources with the multifrequency EM data which can be detected from Euclid (Refregier et al. 2010), Vera Rubin Observatory (LSST Science Collaboration et al. 2009), SPHEREx (Dore et al. 2018b), Nancy Grace Roman Telescope (Green et al. 2012; Spergel et al. 2013; Dore et al. 2018a) and SKA (Maartens et al. 2015). The spatial cross-correlation of GW sources with galaxies provides the clustering redshift to the GW sources, and by combining the measurement of the BAO scale from autocorrelation of the galaxies detectable from these surveys, we will be able to measure the frictional term as a function of redshift. In future work (Mukherjee et al., in preparation), we will study the feasibility of this method for LISA, the Einstein Telescope, and Cosmic Explorer in synergy with the large-scale structure probes. Due to the availability of a large number of GW sources with better luminosity distance measurements from future GW detectors, the error budget from the GW sector is going to be reduced. Also with the availability of greater numbers of galaxies from future large-scale structure surveys, it will be possible to measure the BAO scale to an accuracy ≤ 1 per cent in the future (LSST Science Collaboration et al. 2009; Zhan, Knox & Tyson 2009). The major source of error on the non-GR parameter Ξ_0 will be the error associated with the GW sources. With spectroscopic measurements of galaxy redshifts, the relative error $\Delta\Xi_0 \equiv \sigma_{\Xi_0}/\Xi_0$ (σ_{Ξ_0} denotes 1σ error bar on the non-GR parameter Ξ_0) will approximately scale with the relative error on the luminosity distance $\Delta d_l \equiv \sigma_{d_l}/d_l$ and relative error $\Delta_{\text{BAO}} \equiv \sigma_{\theta_{\text{BAO}}}/\theta_{\text{BAO}}$ on the measurement of the BAO-scale by

$$\Delta\Xi_0^2 \sim \left[\sum_{z_{\text{bin}}} \frac{1}{\frac{\Delta_{d_l}^2(z)}{N_{\text{GW}}(z)} + \Delta_{\text{BAO}}^2(z)} \right]^{-1}, \quad (17)$$

where z_{bin} denotes the number of redshift bins and $N_{\text{GW}}(z)$ denotes the number of GW sources at the redshift z . So, we expect to be able to provide more stringent constraints using this method for dark standard sirens than accessible from only those sources with EM counterparts in future, a large number of sources having better luminosity distance measurements than provided by current

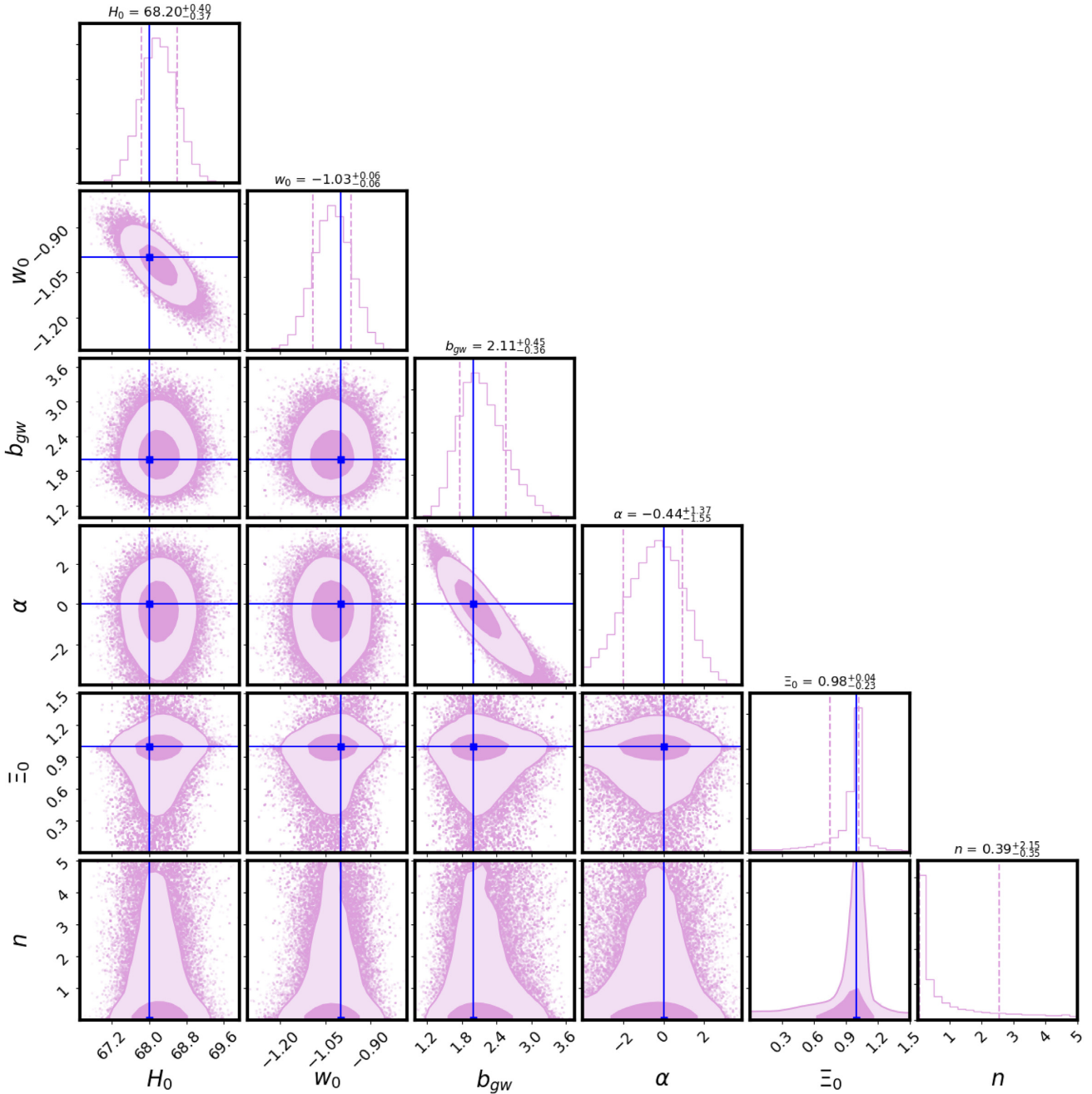


Figure 2. The joint estimation of the cosmological parameters H_0 , w_0 , GW bias parameters b_{GW} , α , and non-GR parameters Ξ_0 , n with ~ 3500 GW sources detectable up to redshift $z = 0.5$ at the aLIGO design sensitivity with a sky localization error $\Omega_{\text{GW}} = 100 \text{ deg}^2$. The fiducial values used in the mock samples are shown by the blue solid line.

detectors. With the availability of a large number of GW sources, the contribution to the total error on Ξ_0 associated with the GW measurement will become subdominant relative to uncertainties arising from the measurement of the BAO scale. As a result, accurate and precise measurement of the Ξ_0 parameter will be limited by the error associated with the BAO scale, when the number of GW sources will be $N_{\text{GW}}(z) \gg \Delta_{\text{d}}^2(z)/\Delta_{\text{BAO}}^2(z)$. With future detectors such as LISA, Cosmic Explorer, and Einstein Telescope, testing of the theory of gravity will be possible not only from the frictional terms but also from the lensing of GW (Mukherjee, Wandelt & Silk 2020b; Mukherjee et al. 2020c). In future work, we will show the

constraints on different theories of gravity by combining both of these aspects.

7 CONCLUSION

GW propagation in space–time provides a unique way to test theories of gravity by measuring the frictional term. The non-zero value of the frictional term leads to modification in the luminosity distance to the GW source from the canonical expression which is probed by the EM observable. The success of this avenue to test the theory of gravity depends on two quantities (i) accurate redshift identification to the

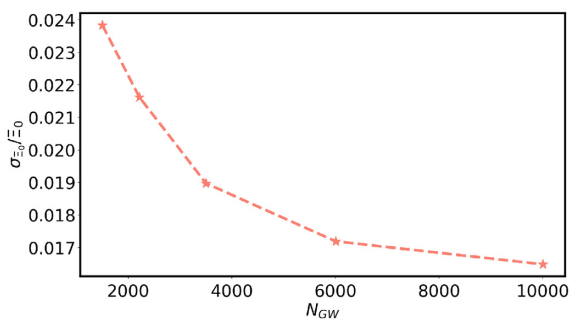


Figure 3. We show the change in the error bar σ_{Ξ_0} on the non-GR parameter Ξ_0 with the change in the total number of GW sources N_{GW} for sky localization error 100 deg^2 after marginalizing over cosmological parameters (H_0, w_0) , GW bias parameters (b_{GW}, α) , and fixed value of n .

GW sources, (ii) identifying the EM distances to this redshift. In all the existing methods, EM counterparts are essential to measuring the frictional term. As a result, all these methods are limited to only GW sources such as BNSs, NS–BH, and supermassive BBHs, for which EM counterparts are likely to occur. Also, due to the limitations of observing the EM follow-ups, EM counterparts to all such GW sources are not going to be detectable, even though they exist. Also, most sources, such as the stellar mass BBHs which are currently detectable from the LVK detectors and are more numerous than the GW sources with EM counterparts, cannot be used to measure the frictional term in the currently existing method. So, the current methods only rely on sources at low redshift $z \leq 0.1$ to measure the friction term from the LVK detectors. However, the variation of the luminosity distance for redshift $z > 0.1$ is significant in several theories of gravity (Belgacem et al. 2018a, 2019). The current method fails to use the high redshift sources from the LVK detectors.

In this paper, we propose a method that avoids the existing hurdle of using only bright GW sources to measure the frictional term. We show that by exploiting the three-dimensional clustering scale of the GW sources with galaxies, one can use dark standard sirens up to high redshift to measure the redshift dependence of the frictional term. Along with the usage of the three-dimensional clustering between GW sources and galaxies, we propose to use the BAO scale determined from the galaxy correlation function which provides an independent measure of the geometric distance at any redshift as probed by the EM observations. By combining BAO measurements from the galaxies and luminosity distance measurements from GW sources, and by using the cross-correlation between galaxies and GW sources, we show that one can make a joint measurement of the cosmological parameters, non-GR parameters, and also the GW merger rates and population denoted by the GW bias parameters $b_{GW}(z)$.

We argue that with ~ 3500 dark standard sirens detectable from the LVK detectors up to redshift $z = 0.5$, one will be able to measure the frictional term with an accuracy of about 4 per cent for $\Xi_0 \geq 1$, and with about 25 per cent accuracy for $\Xi_0 < 1$, after marginalizing over the redshift dependence for BBHs of masses $30 M_\odot$ each. However, when a fixed redshift dependence of the frictional term is assumed, then the error on the parameter Ξ_0 improves to 2 per cent for both ($\Xi_0 < 1$, and $\Xi_0 \geq 1$) for the same BBH masses and redshift distribution. These results show that measurement of the frictional term is possible from the dark sirens detectable with the LVK detectors, and goes beyond the sensitivity of currently existing methods. Moreover, the accuracy possible by our method with ~ 3500 GW sources of mass $30 M_\odot$ is nearly comparable with 1000 GW

binaries with EM counterparts detectable from future GW detectors such as the Einstein Telescope (Belgacem et al. 2018b), which may not be operational until 2040 or later. As a result, by using the cross-correlation technique proposed in this work, we can achieve a similar uncertainty on the frictional term from the currently ongoing GW detectors, and hence in a time-scale shorter than possible from ET sources with EM counterparts. This technique will also be useful for the dark standard sirens detectable from the future space-based detector LISA and ground-based detectors Einstein Telescope and Cosmic Explorer.

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DATA AVAILABILITY

The data underlying this article will be shared on a reasonable request to the corresponding author.

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