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## Investigation of snow sintering at microstructural scale from micro-penetration tests

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### Abstract

Snow sintering is investigated at microscopic and macroscopic scales with high-resolution cone penetration tests. In a cold room at  $-10^{\circ}\text{C}$ , vertical profiles of penetration force were measured periodically during 24 hours using the SnowMicroPenetrometer in four snow samples, which differed only by their grain sizes. We estimated the evolution of snow micromechanical properties, namely the bond rupture force, the deflection at rupture and the number of ruptures per penetration increment, by applying a statistical analysis to penetration profiles. The upper part of the profiles is transient due to the progressive formation of a compaction zone in front of the cone tip. In order to explicitly account for this process in the statistical analysis, we used a non-homogeneous Poisson shot noise model which considers a depth dependency of the rupture occurrence rate. On simulated transient profiles, this analysis is shown to provide accurate estimates of the micromechanical properties. On our experimental data, the method effectively revealed that the vertical heterogeneity of penetration force was essentially due to variations of the rupture rate. Conversely, the time evolution of the macroscopic force was mainly due to microstructural bond strengthening. Both macroscopic force and bond rupture force followed a power law with an average exponent of 0.27 and 0.29, respectively. On our samples, a higher exponent for larger grains was observed on the microscopic bond force, while no trend with grain size was visible in the exponent characterizing the macroscopic force evolution.

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microstructure

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## 1. Introduction

Snow on Earth exists at a relative high homologous temperature. Once deposited on the ground, snow thus remains very active thermodynamically and its structure changes continuously. One process of this metamorphism is sintering, i.e. the creation and growth of bonds between snow particles [1]. The sintering has long been recognized to significantly affect the evolution of snow mechanical properties, generally in the form of a progressive strengthening of the material [2, 3], and manifests over a wide range of time scales: from sub-second sintering (e.g. for cornice formation [4]) to hourly time scales (e.g. for fracture healing [5]), to weekly-monthly time scales (e.g. for seasonal snowpack evolution [6]), and up to centuries (e.g. for the formation of glaciers and ice sheets, [7]). Sintering thus plays an important role, for instance, in avalanche formation [5, 8], in snow management practices such as grooming in ski resorts [9], or in building an igloo.

Sintering is driven by the reduction of surface energy and, depending on environmental conditions, can involve different mechanisms: viscous flow, plastic flow, evaporation, condensation, volume diffusion, and surface diffusion [e.g. 10]. The first studies to identify the driving mechanisms of sintering were conducted in metallurgic powders [e.g. 11, 10]. Kuczynski [10] decomposed the neck growth between metallic particles into different processes and modeled the neck size evolution as a power-law:

$$\left(\frac{r_b}{r_g}\right)^n = \frac{F(T)}{r_g^{n-m}} t \quad (1)$$

where  $r_g$  denotes the particle radius,  $r_b$  is the neck radius, and  $F(T)$  is a temperature dependent parameter. The exponents  $n$  and  $m$  depend on the active sintering process: ( $n = 2$ ,  $m = 1$ ) for viscous or plastic flow, ( $n = 3$ ,  $m = 2$ ) for evaporation and condensation, ( $n = 5$ ,  $m = 3$ ) for volume diffusion and ( $n = 7$ ,  $m = 4$ ) for surface diffusion. This formalism was then used to understand the neck growth between ice spheres [e.g. 12, 13, 14]. Hobbs and Mason [14] considered that Kuczynski's model was not directly applicable to ice, particularly for vapour transport. Applied to ice, they found values of  $n = 5$  and  $m = 3$  in Eq. 1 for the sintering process corresponding to vapour transport. In

27 their experiments on ice spheres of diameters between 50 and 700  $\mu\text{m}$  and at temperatures  
 28 between  $-3$  and  $-20^\circ\text{C}$ , they also found that the dominant mechanism for ice sintering is  
 29 evaporation-condensation and that the diffusion process plays only a minor role. Maeno  
 30 and Ebinuma [15] revisited the different mechanisms contributing to ice neck growth and  
 31 concluded that vapour transport from a surface source is the major mechanism in most  
 32 natural and laboratory conditions, while surface diffusion dominates only for small neck  
 33 radii and either high ( $> 0.95$ ) or low ( $< 0.85$ ) homologous temperatures. Chen and Baker  
 34 [16] showed on high resolution images that the evolution of a neck between two ice spheres  
 35 is primarily due to the direction and rate of water vapour transport. In addition to grain  
 36 size and temperature dependence, sintering mechanisms in snow were also shown to be  
 37 controlled by contact pressure and grain shape [e.g. 17, 1].

38 While previous studies have provided valuable insight into the physics of snow sin-  
 39 tering in simplified geometries, namely ice spheres, quantitative data on natural snow  
 40 sintering remained rather scarce, as recalled by [18, 19]. Indeed, many studies have in-  
 41 vestigated the general evolution of the snow microstructure with time under different  
 42 temperature and loading conditions [e.g. 20, 21, 22, 23] but only a few ones have focused  
 43 on the specific evolution of bond size without any other changes of the microstructure  
 44 (e.g. grain shape evolution, settlement). Ramseier and Sander [24] measured the influence  
 45 of temperature on the strengthening of natural snow with mechanical compression tests.  
 46 They observed faster strengthening in compression for higher temperature. Similarly,  
 47 Matsushita et al. [25] observed a more active sintering with higher snow temperatures  
 48 and higher normal loads caused by overlying snow. Montmollin [26] observed a regen-  
 49 eration of destroyed bonds under shear deformation for low deformation rates, which  
 50 highlights the competition between bond re-welding by sintering and bond failure during  
 51 deformation. Herwijnen and Miller [19] used a SnowMicroPenetrometer (SMP, [27]) to  
 52 measure sintering through the time-evolution of the penetration force in different snow  
 53 types (for times up to 5 – 8 hours). Podolskiy et al. [28] developed mechanical shear  
 54 tests to investigate the effect of isothermal sintering on interface strengthening at various  
 55 normal pressures. These latter authors [19, 28] proposed to represent the time-evolution  
 56 of macroscopic force or strength as a power-law:

$$F(t) = F(t = t_0) \left( \frac{t}{t_0} \right)^{\alpha_F} \quad (2)$$

57 with  $F$  the force or strength,  $t$  the time and  $\alpha_F$  a sintering exponent. Typical sintering  
58 exponents for seasonal snow are reported in the range between 0.07 and 0.36, as reviewed  
59 by Podolskiy et al. [28]. In their experiments, Podolskiy et al. [28] measured an average  
60 exponent of  $0.21 \pm 0.08$ , and observed that higher pressures applied during sintering tend  
61 to increase this exponent. Herwijnen and Miller [19] derived a mean sintering exponent of  
62  $0.18 \pm 0.05$ , and observed lower sintering rates for low-density snow (consisting mostly of  
63 dendritic grain morphologies) compared to higher density snow. They hypothesize that  
64 this relation is the result of two competing processes: (1) bond creation and growth and  
65 (2) bond vanishing due to the metamorphism of dendritic forms into compact rounded  
66 particles. In parallel, they simulated the microscopic bond-to-grain ratio for simplified  
67 spherical ice grains and observed a power-law growth with the same average exponent as  
68 that derived from the experiments. Based on this agreement, they suggest that the mean  
69 macroscopic penetration resistance closely relates to the microscopic bond-to-grain ratio  
70 in snow.

71 In spite of the advances described above, direct and concurrent measurements of  
72 sintering exponents at both microscopic and macroscopic scales in snow, are still lacking.  
73 Accordingly, the precise relation between bond growth, at the microscopic scale, and  
74 increase in mechanical strength or resistance at the macroscopic scale, largely remains to  
75 be elucidated. In this study, we use micro-cone penetration tests ( $\mu$ CPT) to investigate  
76 snow sintering at both macroscopic and microscopic scales. **Here, the microscopic scale**  
77 **corresponds to the grain scale of about tenths of millimeters and the macroscopic scale**  
78 **corresponds to the sample scale of about a few centimeters.**  $\mu$ CPT measurements were  
79 conducted with a modified SMP on thin snow samples during 24 hours after snow sieving  
80 in a cold room at  $-10^\circ\text{C}$ . In parallel to the macroscopic penetration force, micromechanical  
81 properties, including the bond rupture force, are estimated through an extension of the  
82 shot noise model introduced for snow by Löwe and Herwijnen [29]. In the first section, we  
83 present the experimental setup and models required to derive micromechanical properties  
84 from the penetration profiles. The models are first evaluated on simulated penetration  
85 profiles. The penetration profiles measured on the samples evolving with sintering are  
86 then analyzed at the macroscopic and microscopic scales. Finally, the outcomes in terms  
87 of  $\mu$ CPT processing techniques and knowledge on snow sintering are discussed.

Table 1: Physical properties of the snow samples used in this study.

Sample name	Snow type	Sieve size (mm)	Grain size (mm)	SSA (m <sup>2</sup> kg <sup>-1</sup> )	Density (kg m <sup>-3</sup> )
SSA18a	RGlR	0.8	0.7	17.9±0.3	500±2
SSA18b	RGlR	0.8	0.7	17.9±0.3	500±2
SSA14	RGlR	1.0	0.9	14.0±0.3	496±2
SSA10	RGlR	1.6	1.5	10.4±0.1	484±3

## 88 2. Material and Methods

### 89 2.1. Snow samples

90 We prepared four snow samples with a snow type characterized as large rounded  
91 grains (RGlR, Fierz et al. [30]). The samples were prepared at a controlled temperature  
92 of  $-10\pm 0.5^\circ\text{C}$  by sieving about 35 mm height of snow into rectangular boxes of size  
93 300 x 400 mm. In order to get samples with different grain sizes, the rounded grains were  
94 sieved using various sieve sizes (Tab. 1). Two samples with a sieve size of 0.8 mm (samples  
95 SSA18a and SSA18b) one sample with a sieve size of 1.0 mm (sample SSA14), and one  
96 sample with a sieve size of 1.6 mm (sample SSA10) were prepared. The samples showed  
97 densities of about  $500\text{ kg m}^{-3}$ , grain sizes between 0.7 and 1.5 mm and specific surface  
98 areas (SSA) ranging between 10 and  $18\text{ m}^2\text{ kg}^{-1}$ . Density was measured by weighing  
99  $50\text{ cm}^3$  sub-samples extracted with a cutter. SSA was measured using an optical method  
100 (DUFISSS, Gallet et al. [31]) at different heights and different horizontal locations to  
101 characterize spatial variability. The vertical variability of SSA was found of the same  
102 order as the horizontal one (typically, the standard deviations were less than  $1\text{ m}^2\text{ kg}^{-1}$ ).  
103 Grain size was estimated with microscope images (magnification by a factor of 32) of the  
104 grains.

### 105 2.2. Micro-penetration tests in sintering snow

106 In the following 24 hours after the sieving, a total of 31 micro-cone penetration tests  
107 ( $\mu\text{CPT}$ ) (4 to 5 measurements for 7 different times) were conducted in each sample.  
108 This high number of measurements enables to take into account the spatial variability of  
109 the sample due to the sieving procedure [e.g. 19]. To avoid interference in neighboring

110 penetration tests, we kept a distance of 40 mm between the measurements. To avoid  
111 effects of the lateral walls, no measurement were performed closer than 40 mm to the  
112 walls.

113 The  $\mu$ CPTs in this study were conducted with a modified version of the SnowMi-  
114 croPenetrometer (SMP version 4, [27]). The SMP is a high-resolution penetrometer,  
115 which consists of a conical tip with a  $60^\circ$  apex angle and a maximum diameter of 5 mm,  
116 driven into the snow by a motor with a constant speed of  $20 \text{ mm s}^{-1}$ . The depth and  
117 force sensors record measurement points at a sampling frequency of 5 kHz, i.e. every  
118  $4 \mu\text{m}$  penetration increment. The SMP measures the snow resisting force applied to the  
119 cone tip only and not to the rod. The modification of the SMP for this study consisted in  
120 lengthening the apex by 40 mm (Fig. 1). This lengthening of the apex was motivated to  
121 prevent the rod of the SMP to penetrate in the sample, and thus influence the measured  
122 force through the formation of a rod-induced compaction zone (CZ). LeBaron et al. [32]  
123 measured the CZs around a split-axis SMP probe that has the same geometry as the  
124 SMP (tip and rod), and found significantly larger CZs than in the study of Herwijnen  
125 [33], who analyzed of the CZ around the SMP tip without the rod. Given the size of our  
126 snow samples, we wanted to avoid any potential impact of the rod, and account only for  
127 the CZ due to the tip. Note that side-by-side measurements performed with the original  
128 and modified SMP versions showed essentially identical penetration profiles, as long as  
129 the rod of the original SMP does not touch snow (Appendix A).

130 As a typical example, Figure 2 shows  $\mu$ CPT profiles measured after a rest time of  
131 24 hours on sample SSA18a. Single profiles present high frequency fluctuations and large  
132 vertical variations (Fig. 2a). Even though the sieving procedure may induce a limited  
133 vertical heterogeneity of the snow sample [e.g. 19], vertical profiles of SSA did not reveal  
134 any significant vertical variations (standard deviations were less than  $1 \text{ m}^2 \text{ kg}^{-1}$ ). Based  
135 on the study of Podolskiy et al. [28], we can also argue that pressure exerted by overburden  
136 snow is unlikely to explain this vertical heterogeneity. The overburden pressure at a depth  
137 of 35 mm was estimated about 0.2 kPa, and these authors showed no significant increase  
138 of the sintering rate for normal loads lower than 0.5 kPa.

139 We thus argue that the vertical heterogeneity in the  $\mu$ CPT profiles is mainly caused  
140 by the interaction between the snow and the  $\mu$ CPT tip. The increase of the penetration

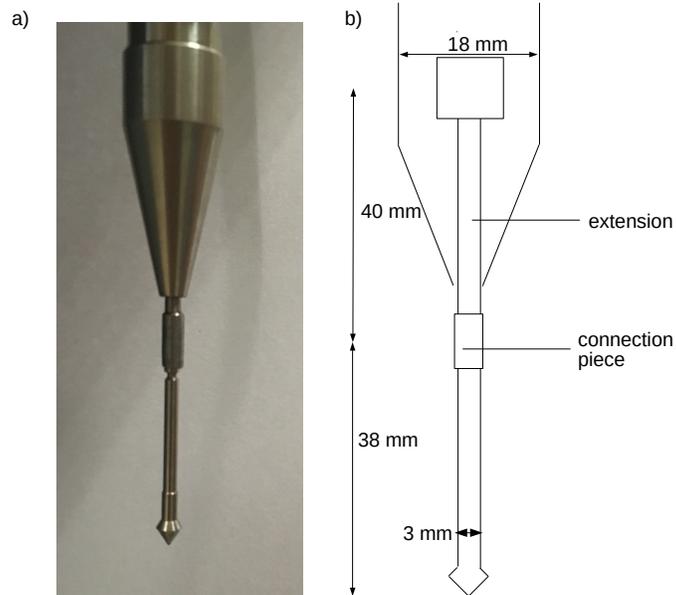


Figure 1: Experimental setup: (a) photo and (b) scheme of the modified tip.

141 force up to a depth of about 5 mm is due to the progressive penetration of the conic  
 142 apex into the sample. The increase of force for depths between 5 and 25 mm is then  
 143 probably caused by the progressive formation of the CZ in front of the tip. According  
 144 to Herwijnen [33], the full development of the tip-induced CZ is reached after 40 mm  
 145 penetration depth, and therefore probably not reached in our experiments. Finally, the  
 146 decrease of force for depths larger than 25 mm is presumably due to the influence of the  
 147 rigid bottom of the sample box. Penetration profiles shown by Herwijnen and Miller [19]  
 148 also exhibited a maximum force a few centimeter above the sample holder bottom. In the  
 149 following analyses, to avoid this artifact caused by sample bottom, we will only account  
 150 for the first 25 mm of the penetration profiles.

151 Figure 2b shows four smoothed  $\mu$ CPT profiles measured in the same box and at the  
 152 same time. Slight differences between the profiles are visible. This horizontal heterogene-  
 153 ity in the samples remains comprised between 10 and 15% (relative horizontal standard  
 154 deviation), and is probably due to the sieving.

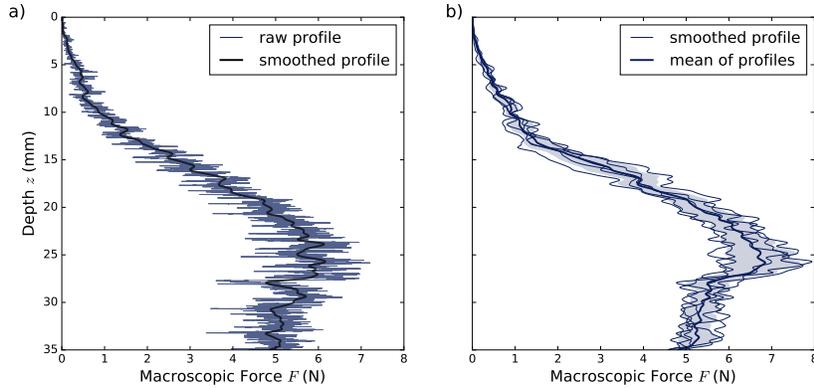


Figure 2: Examples of  $\mu$ CPT profiles measured on sample SSA18a after a rest time of 24 h: (a) one raw profile and the corresponding smoothed profile (black), (b) four smoothed  $\mu$ CPT profiles measured at different positions. The profiles were smoothed with a Gaussian filter with a standard deviation of 0.2 mm. The mean of the four smoothed profiles is also shown (bold line), and its standard deviation represented as the shaded area.

### 155 2.3. Micromechanical analysis methods

156 Several statistical models were proposed to interpret SMP profiles in terms of microme-  
 157 chanical property proxies. Johnson and Schneebeli [34] assumed that the penetration  
 158 force profile results from the superposition of spatially uncorrelated ruptures. Marshall  
 159 and Johnson [35] extended this theory by using Monte-Carlo simulations to take into  
 160 account simultaneous rupture events and inverted the signal. Löwe and Herwijnen [29]  
 161 adapted the model into the formalism of a homogeneous Poisson point process (HPP),  
 162 such that individual ruptures can overlap and are randomly distributed.

163 All these models interpret a SMP profile  $F(z)$ , i.e. the macroscopic penetration force  
 164  $F$  as a function of depth  $z$ , as the superposition of spatially uncorrelated rupture events  
 165 (Fig. 3). Each single event corresponds to the rupture of one bond behaving in an elastic-  
 166 brittle manner. These events are assumed to be identical, but to occur at **random** depths.  
 167 They are described by two microstructural properties, namely the deflection length  $\delta$  and  
 168 the rupture force  $f$ , and express as  $f_z(z) = f/\delta \times \theta(z) \times \theta(\delta - z)$  with  $\theta$  the Heaviside  
 169 function (Fig. 3 inset). The number of events occurring per penetration increment is  
 170 described by a Poisson distribution with an intensity  $\lambda_z$ . The convolution of the single  
 171 event function  $f_z$  and a random sampling of the number of events results in a simulated

172 profile  $F(z)$  (Fig. 3).

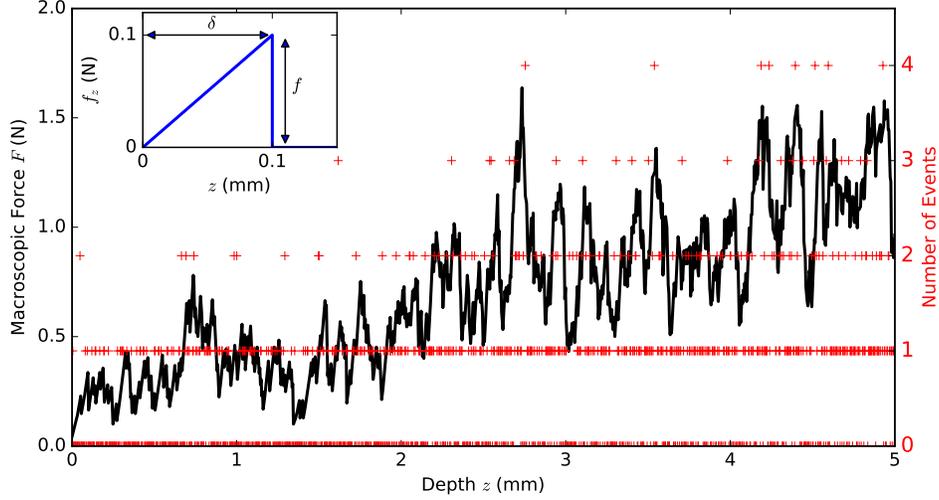


Figure 3: Simulated force penetration profile obtained as the superposition of uniform elastic brittle events (inset) whose number of occurrence follows a homogeneous Poisson distribution. Here,  $\delta=0.1$  mm,  $f=0.1$  N, and  $\lambda_z$  is linear increasing with depth:  $\lambda_z(z) = a_\lambda z + b_\lambda$  with  $a_\lambda = 40 \text{ mm}^{-2}$  and  $b_\lambda = 50 \text{ mm}^{-1}$ .

### 173 Homogeneous Poisson process (HPP)

174 Assuming that the intensity  $\lambda_z$  is constant over depth and equal to  $\lambda$ , Löwe and  
 175 Herwijnen [29] derived analytical expressions directly linking the stochastic cumulants  
 176 and correlation function of the SMP profile  $F$  to the micromechanical properties  $\delta$ ,  $f$  and  
 177  $\lambda$ . In particular, they obtained the following relations:

$$\kappa_n(z) = \frac{f^n \delta \lambda}{n+1} \quad \text{and} \quad C(z, z+r, |r| < \delta) = f^2 \delta \lambda \left( \frac{1}{3} - \frac{1}{2} \frac{|r|}{\delta} + \frac{1}{6} \frac{|r|^3}{\delta^3} \right) \quad (3)$$

178 where  $\kappa_n$  is the cumulant of order  $n$  (e.g.  $\kappa_1$  is the mean,  $\kappa_2$  is the variance) and  $C$   
 179 is the two-point correlation function. This process with a constant intensity  $\lambda_z(z) = \lambda$ , is  
 180 called henceforth homogeneous Poisson process (HPP). Note that the formalism of the  
 181 Poisson processes implies constant event characteristics, i.e.  $\delta$  and  $f$  are constant (or in a  
 182 constant distribution). The assumption of a constant intensity also implies stationarity of  
 183  $F(z)$ , and thus enables to compute the stochastic cumulants and correlation function as  
 184 “depth” cumulants and correlation function (ergodicity). In practice however, measured  
 185 penetration profiles are rarely stationary with depth even for homogeneous snow layers

186 [19]. This difficulty is overcome by splitting the profile in smaller windows where the  
 187 assumption of stationarity remains valid [e.g. 36]. It is observed on simulated HPPs that  
 188 better results are obtained with larger windows [29]. Therefore, a balance between large  
 189 windows that are stochastically representative, and small windows on which the profile  
 190 can be considered stationary, needs to be found. Typical window sizes  $\Delta z$  of 1 to 5 mm  
 191 are reported in the literature [e.g. 29, 36, 37]. Note that, in practice, the correlation  
 192 function  $C$  is also generally computed on detrended profiles (i.e. the mean and linear  
 193 trend are subtracted on each window) [e.g., 29, although not shown therein].

#### 194 **Non-homogeneous Poisson process (NHPP)**

195 As shown in Fig. 2, the penetration force measured in our experiments displays strong  
 196 changes with depth. The profiles were limited to a depth of 35 mm due to the modified  
 197 design of the SMP tip. As explained in Sect. 2.2, the snow structure exhibited a rather  
 198 homogeneous vertical profile. We can thus reasonably assume that the microstructural  
 199 properties  $\delta$  and  $f$  are constant over the entire depth, while only the number of events,  
 200 thus the intensity  $\lambda_z$ , varies with depth. This depth variation of  $\lambda_z$  can be related to  
 201 the **build-up** of the compaction zone in front of the tip. We thus propose to analyze our  
 202 measured  $\mu$ CPT profiles as a non-homogeneous Poisson process (NHPP), with a variable  
 203 intensity  $\lambda_z$ , instead of a homogeneous Poisson process (HPP). **Note that the NHPP**  
 204 **equals to the HPP if  $\lambda_z$  is constant with depth.**

205 We describe here the main steps of the mathematical developments. Details can be  
 206 found in Appendix B. Let us consider  $F(z)$  as a NHPP characterized by an elastic brittle  
 207 event  $f_z$ , whose properties  $f$  and  $\delta$  are assumed to be constant with depth and a number of  
 208 events in a Poisson distribution of intensity  $\lambda_z(z)$ . Assuming that the relative variations  
 209 of  $\lambda_z(z)$  over an interval of length  $\delta$  are negligible, it can be shown that:

$$\kappa_n(z) = \frac{f^n \delta \lambda_z(z)}{n+1} \quad \text{and} \quad C(z, z+r, |r| < \delta) = f^2 \delta \lambda_z(z) \left( \frac{1}{3} - \frac{1}{2} \frac{|r|}{\delta} + \frac{1}{6} \frac{|r|^3}{\delta^3} \right). \quad (4)$$

210 Equation (4) is identical to Eq. (3) obtained by Löwe and Herwijnen [29], but with  $\lambda_z(z)$   
 211 instead of a constant intensity  $\lambda$ . However,  $F$  cannot anymore be assumed stationary,  
 212 and therefore its cumulants and correlation function cannot be computed directly from a  
 213 single force profile  $F(z)$ . We thus define  $\tilde{F}$  as:

$$\tilde{F} = \frac{F - \kappa_1(F)}{\kappa_1(F)^{1/2}} \quad (5)$$

214 From Eq. (4), it can be shown that  $\tilde{F}$  is stationary and we obtain:

$$f = \frac{3 \overline{\tilde{F}^2}}{2}, \quad \delta = -\frac{3 C(0)}{2 C'(0)}, \quad \lambda_z(z) = \frac{4 \kappa_1(F)}{3\delta \overline{\tilde{F}^2}} \quad (6)$$

215 where  $\bar{\bullet}$  denotes the mean over depth. The cumulant  $\kappa_1(F)$  appearing in Eq. (6) is the  
 216 stochastic mean of the NHPP. To compute this quantity from our data, we approximate  
 217  $\kappa_1(F)$  by the “depth” mean  $\bar{F}$  calculated on a running window of width  $\Delta z=3$  mm and  
 218 on a single profile. Since  $\kappa_1(F)$  is a first order cumulant, this approximation is relatively  
 219 robust, and even exact if the intensity  $\lambda_z$  is linear over intervals of width  $\Delta z$ .

### 220 3. Results

#### 221 3.1. Evaluation of the $\mu$ CPT analysis method

222 In this section, we compare the proposed NHPP analysis and the original HPP analysis  
 223 both on simulated penetration profiles with known properties, and on the  $\mu$ CPT profiles  
 224 measured in this study.

##### 225 3.1.1. Evaluation on simulated profiles

226 First, we evaluated the models on profiles produced by the simulation of a non homo-  
 227 geneous Poisson process. A linearly evolving intensity  $\lambda_z(z) = a_\lambda z + b_\lambda$  was considered.  
 228 The simulated profiles have a length of 25 mm and a resolution of 4  $\mu$ m, with prescribed  
 229 parameters (see legend of Fig. 4) similar to the micromechanical properties of our samples.  
 230 To analyze the influence of the different parameters ( $f$ ,  $\delta$ , and  $a_\lambda$ ), we performed a sensi-  
 231 tivity analysis by varying one parameter within a certain range while keeping the others  
 232 constant. For each set of parameters, 500 independent profiles were simulated. Since  
 233 the intensity  $\lambda_z(z)$  varies with depth, we defined  $\bar{\lambda}$  as its mean over the whole depth to  
 234 provide a single scalar in the sensitivity analysis. The value  $\bar{\lambda}$  was varied by varying  $a_\lambda$   
 235 in the range  $[0, 300]$   $\text{mm}^{-2}$ , while  $b_\lambda$  was kept constant at 50  $\text{mm}^{-1}$ . Moreover, three  
 236 different analysis window sizes  $\Delta z$  of 1, 3 and 10 mm were tested.

237 Both methods provide similar estimates of the deflection at rupture  $\delta$  with a root mean  
 238 square error (RMSE) of 0.59 mm for the HPP and 0.54 mm for the NHPP (Figs. 4a,b).  
 239 Note that large values of  $\delta$  tend to be underestimated by the two analyses. Nonetheless,  
 240 this parameter is correctly estimated if the analysis window is sufficiently large ( $\Delta z =$

241 10 mm), especially with the NHPP. Overall, estimates of the microscopic rupture force  
 242  $f$  are more accurate with the NHPP method with a RMSE of 0.263 N for the HPP and  
 243 0.051 N for the NHPP (Figs. 4c,d). As expected, for profiles with an almost constant  
 244 intensity ( $a_\lambda=1 \text{ mm}^{-2}$ ), the two models provide similar results: a slight underestimation  
 245 of  $f$ . For profiles with a significant evolution of  $\lambda_z(z)$  ( $a_\lambda=10$  or  $40 \text{ mm}^{-2}$ ), the HPP  
 246 provides accurate estimates of  $f$  only for small window sizes, while the NHPP provides  
 247 accurate estimates for all window sizes. Lastly, the NHPP produces in general a better  
 248 estimation of the mean intensity  $\bar{\lambda}$  (RMSE of  $1305 \text{ mm}^{-1}$ ) than the HPP (RMSE of  
 249  $2885 \text{ mm}^{-1}$ ) (Figs. 4e,f). The HPP provides correct estimates of  $\bar{\lambda}$  only for large windows  
 250 and very small values of  $a_\lambda$ , while the NHPP provides correct estimates for all tests. Note  
 251 that the NHPP tends to slightly overestimate the mean intensity, especially for samples  
 252 with a large  $\delta$  and small window sizes.

253 We can thus conclude that, on non-stationary profiles with an intensity linearly in-  
 254 creasing with depth, the NHPP analysis performs better than the HPP analysis in re-  
 255 covering the micromechanical properties. Larger analysis window sizes generally improve  
 256 the results, especially when the deflection at rupture  $\delta$  is large. As expected, on almost  
 257 stationary profiles both methods show similar results.

### 258 3.1.2. Comparison on measured profiles

259 In this section, we compare the two analysis methods on real  $\mu\text{CPT}$  profiles. Figure 5  
 260 shows the micromechanical properties estimated on the  $\mu\text{CPT}$  profiles measured after a  
 261 rest time of 24 hours as derived from the HPP and NHPP models.

262 As explained in Section 2.3, with the HPP model,  $\delta$ ,  $f$  and  $\lambda$  are assumed constant  
 263 over small windows of size  $\Delta z=3 \text{ mm}$  and were computed according to Eqs. (11) and  
 264 (12) in [29]. With the NHPP model,  $\delta$  and  $f$  are assumed constant over the whole profile,  
 265 while  $\lambda_z$  can vary with  $z$ , and were computed according to Eq. (6). Therefore, the vertical  
 266 resolutions of the computed micromechanical properties are, by construction, different for  
 267 the two models.

268 The HPP analysis shows non-monotonic variations of  $f$  with depth (Fig. 5a). It  
 269 indicates an increase of  $\delta$  with depth, especially for sample SSA10 characterized by large  
 270 grains (Fig. 5b). Lastly, an overall increase of  $\lambda$  with depth is observed, with a slight  
 271 decrease between 10 and 15 mm depth (Fig. 5c). This decrease might be due to the

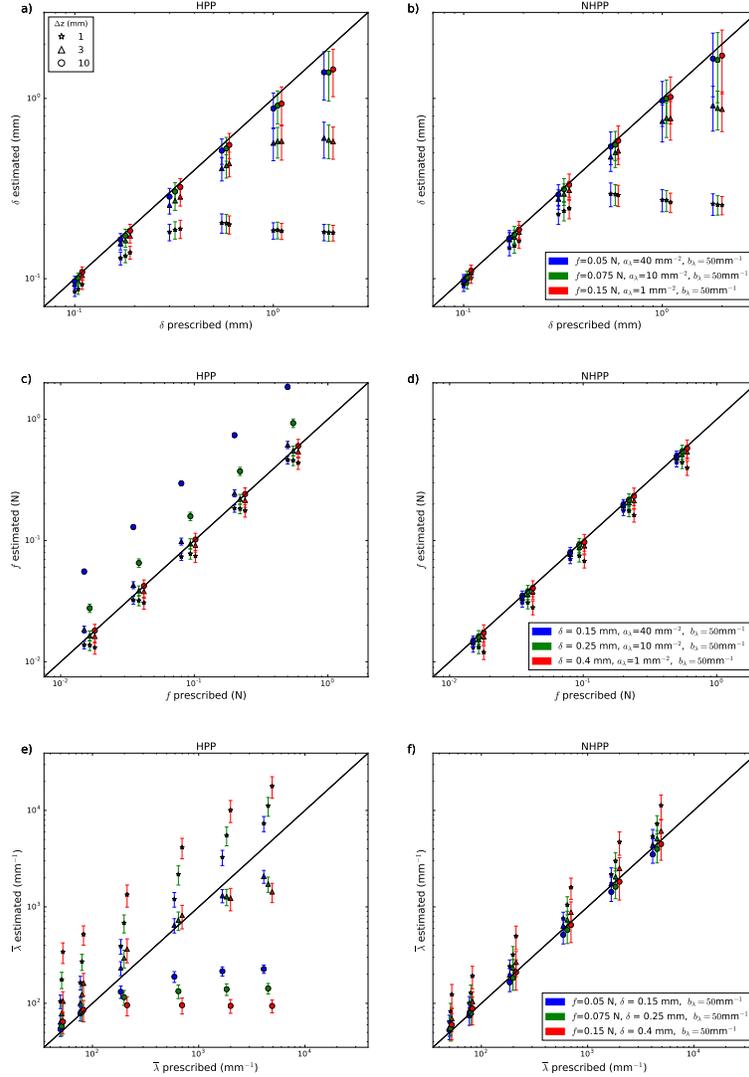


Figure 4: Comparison of estimated and prescribed micromechanical properties  $\delta$ ,  $f$ ,  $\bar{\lambda}$  on simulated penetration profiles. The results of the HPP analysis are shown in (a, c, e), while the results of the NHPP analysis are shown in (b, d, f). In each plot, the values of the constant micromechanical properties, chosen to be similar to those of the samples presented in this study (SSA10 in red, SSA14 in green and SSA18a,b in blue), are indicated in legend. The three different symbols denote the different window sizes  $\Delta z$ . Error bars correspond to the standard deviation over the 500 independent profile realizations.

272 rapid increase of the macroscopic force  $F$  with  $z$  in this depth interval (Fig. 2), for which  
 273 the HPP analysis is not well-designed. Globally, the three parameters follow the overall

274 increasing trend with depth exhibited in the  $\mu$ CPT force profiles. The HPP analysis  
 275 distinguishes the different samples on  $\delta$  and  $\lambda$ , but computes a similar value of  $f$  for all  
 276 samples **and almost all depths**.

277 The NHPP analysis distinguishes the different samples on all properties: the largest  
 278 grains exhibit the largest values of  $f$  and  $\delta$ , and the smallest values of  $\lambda$ . In particular,  
 279 a noticeable difference between the two analyses is visible on the computed values of  $f$   
 280 (Figs. 5a,d): unlike the HPP analysis, the NHPP analysis shows a high dependency of  $f$   
 281 on the different samples. The values of  $\lambda_z$  are found to strictly increase with  $z$ , which is  
 282 consistent with the progressive formation of the compaction zone around the  $\mu$ CPT tip  
 283 (Fig. 5f). **The differences between the two methods become smaller for depths between**  
 284 **20 and 25 mm**.

285 Note that, here, no reference profile of the micromechanical properties is available  
 286 to definitely evaluate the two models. However, the monotonic increase of intensity  
 287 with depth, and the the increase of  $f$  with grain size, which are only visible in the  
 288 NHPP estimates, seem more consistent with our knowledge of the sample properties (see  
 289 Sect. 2.1) and of the progressive formation of the compaction zone [33]. **Indeed, Eq. (1)**  
 290 **indicates a higher bond size for larger grains and thus an increase of  $f$  with grain size can**  
 291 **be expected**.

### 292 3.2. Evolution of the macroscopic force with time

293 **The macroscopic force  $F$  clearly increases with time due to sintering (see Fig. 6 or**  
 294 **Fig. C.11).** Moreover, the relative increase of  $F$  with time also appears to be enhanced  
 295 with depth. For instance,  $F$  increases by a factor of about 1.5 over 24 hours at a depth  
 296 of 5 mm, while it increases by a factor of about 3 at a depth of 20 mm.

297 In line with previous studies [e.g. 19, 28], the time evolution of the depth-averaged  
 298 macroscopic force  $\bar{F}$  (Fig. 7), was approximated by fitting power laws of the form:

$$\bar{F}(t) = F_{1h} \left( \frac{t}{t_{1h}} \right)^{\alpha_F}, \quad (7)$$

299 with  $t_{1h} = 1$  hour and  $F_{1h}$  is the mean macroscopic force value after 1 hour of sintering.  
 300  $\bar{F}(t)$  is computed as the mean between 0 and 25 mm depth, to exclude any potential  
 301 influence of the bottom of the box (see Sect. 2.1). The start time  $t = 0$  is defined as the  
 302 middle of the sieving procedure, which took between 5 and 10 minutes. Hence, the error

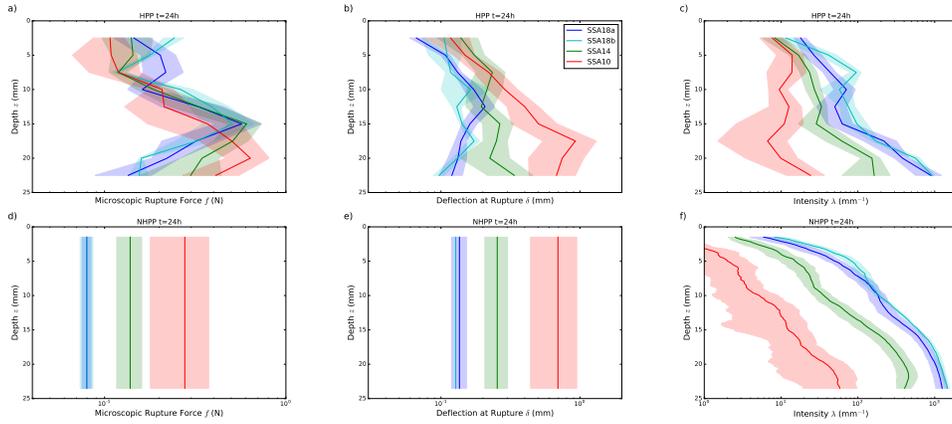


Figure 5: Estimated micromechanical properties derived from the  $\mu$ CPT profiles measured on the samples after a rest time of 24 hours: (a, c) microscopic rupture force  $f$ , (b, d) deflection at rupture  $\delta$ , and (e, f) intensity  $\lambda_z$ , as computed by the HPP (a, b, c) and NHPP (d, e, f) models. The shaded area around the curves represents the standard deviation of the values obtained for the different profiles measured at one sampling time.

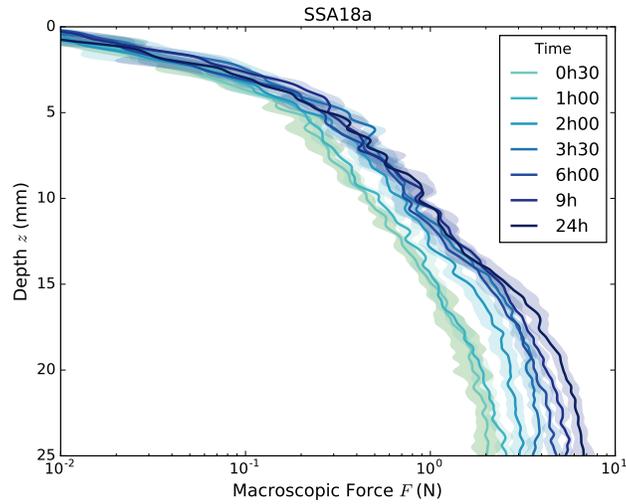


Figure 6: Vertical profile of the macroscopic force  $F$  for different sintering times, measured on sample SSA18a.

303 on sintering time is estimated to be half of the sieving time. The fit was performed with  
 304 an orthogonal distance linear regression (ODR) on the logarithm of the data using scipy

python package [38]. The function finds the maximum likelihood and gives the estimated properties with their standard error, i.e. the error at one sigma interval. We accounted for the time error in the ODR fit and used all the 4 to 5 profiles measured at any given time. We chose a Pearson p-value of 1% to analyze the significance of the fit.

Figure 7 shows that the mean macroscopic force exhibits a clear power law trend. Only little deviations from the power law are observed, and the confidence intervals on the fitted parameters are rather small. The pre-factor  $F_{1h}$  ranges between 0.36 N and 1.22 N depending on the considered sample, and increases with specific surface area. Note that the values of  $F_{1h}$  for samples SSA18a and SSA18b, which are composed of similar grains, are very close but do not exactly coincide (see indicated errors). The exponent  $\alpha_F$  is about 0.3 for all samples. Hence, it appears that the initial value of the macroscopic force  $F_{1h}$  depends on sample properties, while the power law exponent  $\alpha_F$  is independent of the sample.

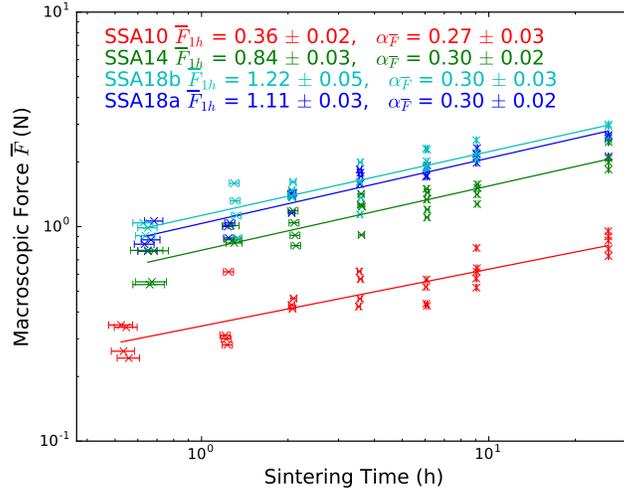


Figure 7: Evolution of the depth-averaged macroscopic force  $\bar{F}$  with time for the different snow samples. The power law fits are shown by the solid lines, and the corresponding parameters are indicated in legend. The horizontal error bars represent the time error (around 10 minutes) due to non-instantaneous sample preparation.

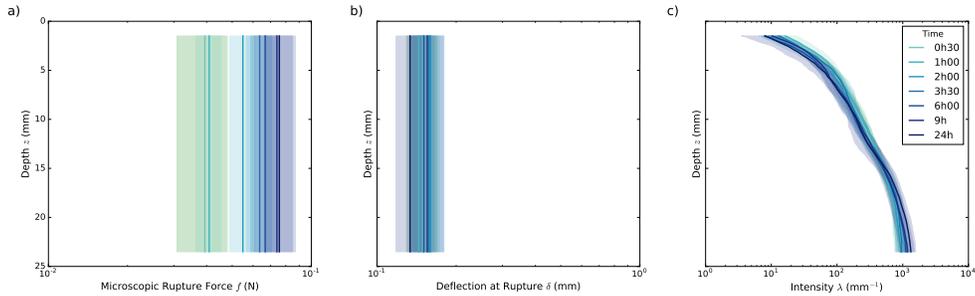


Figure 8: Vertical profiles of the micromechanical properties estimated by the NHPP model on sample SSA18a for different sintering times. The shaded area around the curves represents the standard deviation obtained for the different profiles at one sampling time.

### 318 3.3. Evolution of the micromechanical properties with time

319 Figure 8 shows the evolution with time, for sample SSA18a, of the micromechanical  
 320 properties estimated by the NHPP analysis. Similar plots for the other samples are visible  
 321 in Appendix C, Figs. C.12, C.13, C.14. The deflection  $\delta$  and intensity  $\lambda_z$  do not show  
 322 obvious changes with time (Figs. 8b, c). On the contrary, for the microscopic rupture  
 323 force  $f$ , a clear increase with time is observed (Fig. 8a). Hence, according to the NHPP  
 324 analysis, the evolution of the macroscopic force  $F$  (Fig. 6) with time is mainly due to the  
 325 evolution of the microscopic rupture force, whereas its evolution with depth is explained  
 326 by the vertical profile of the intensity. We recall that the NHPP model assumes that the  
 327 microscopic rupture force is constant over depth, but not that intensity is constant over  
 328 time.

329 As in Sect. 3.2, the time evolution of the micromechanical properties was approximated  
 330 by fitting a power law :  $X(t) = X_{1h} (t/t_{1h})^{\alpha_X}$ , where  $X$  is either  $f$ ,  $\delta$  or  $\bar{\lambda}$  (Figure 9).  
 331 Again, a Pearson p-value of 1 % is used to assess the significance of the fits. The lines  
 332 in Fig. 9 are dotted if the fit is not significant. Note that we consider here the averaged  
 333 intensity over depth. Figures 9b,c confirm that the deflection and average intensity do  
 334 not significantly change with time. The initial values of  $\delta$  and  $\bar{\lambda}$  are however dependent  
 335 on sample properties. On the contrary, the microscopic rupture force  $f$  exhibits a clear  
 336 power law trend. Again, only little deviations from the power law are observed, and the  
 337 confidence intervals on the fit parameters are rather small. The pre-factor  $f_{1h}$  ranges from  
 338 0.05 N to 0.09 N and decreases when specific surface area increases. The value of  $f_{1h}$

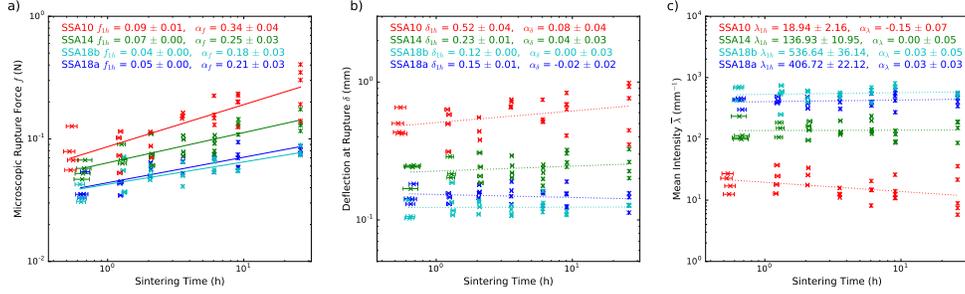


Figure 9: Time evolution of the micromechanical properties estimated by the NHPP model. The significant (resp. non-significant) power law fits are represented by the solid (resp. dotted) lines, with corresponding parameter indicated in legend. The horizontal error bars represent the time error (around 10 minutes) due to non instantaneous sample preparation.

339 for samples SSA18a and SSA18b, which were composed of similar snow grains, are very  
 340 close. The exponent  $\alpha_f$  ranges from 0.18 to 0.34, and appears to decrease with increasing  
 341 specific surface area (or decreasing grain size). Hence, both the initial value and power  
 342 law exponent of the microscopic rupture force are dependent on grain size.

#### 343 4. Discussion

344 Due to a limited sample height of 35 mm, the presented  $\mu$ CPT profiles exhibit a  
 345 non-stationary regime due to the progressive build-up of a compaction zone in front  
 346 of the cone tip. To properly account for this feature and to estimate micromechanical  
 347 properties from the  $\mu$ CPT profiles, we proposed a non-homogeneous Poisson shot noise  
 348 model (NHPP). This approach follows up on the ideas of Löwe and Herwijnen [29], who  
 349 described the fluctuating penetration force as a homogeneous Poisson shot noise process  
 350 (HPP) with a single event described as an elastic-brittle rupture. We showed, under the  
 351 assumption that the relative variations of intensity over an interval of size  $\delta$  are small,  
 352 that the analytical expressions for the cumulants and the correlation of the macroscopic  
 353 force given by Löwe and Herwijnen [29] (Eq. 6 and 10 there) can be extended to a  
 354 non homogeneous process, i.e. with an intensity  $\lambda_z$  varying with depth. This extension  
 355 amounts to separate two spatial scales: the scale of the bond rupture characterized by  
 356  $\delta$  and  $f$ , and the scale of the compaction zone that governs the evolution of  $\lambda_z$ . Using  
 357 simulated force profiles with a linear evolving intensity (and properties close to those

358 of the snow samples used in this study), we showed that the NHPP model was able to  
359 retrieve accurate micromechanical properties (Fig. 4). In particular, on simulated profiles  
360 characterized by large variations of  $\lambda_z$  with depth, which mimic our measurements, the  
361 NHPP was more accurate than the HPP whose underlying assumptions are violated. On  
362 this type of profile, no window size, small enough to satisfy the assumption of constant  
363 parameters and large enough compared to the event spatial extent  $\delta$ , can be found to  
364 correctly apply the HPP. The NHPP development was thus required to provide a robust  
365 estimates of the micromechanical parameters from the presented measurements. The  
366 main approximation of the NHPP model is the calculation of the first cumulant  $\kappa_1(F)$ .  
367 We approximate  $\kappa_1(F)$  by its running mean over a window size of  $\Delta_z=3$  mm. Therefore,  
368 we assumed that the profile can be approximated by a linear evolution over such window  
369 sizes. A window of 3 mm was chosen as the best compromise between a sufficiently large  
370 window to get a better estimation of the parameter, and a sufficiently small window to  
371 assume the signal to evolve linearly over it. Comparing the HPP and NHPP on measured  
372 profiles showed clear differences between the estimated parameters by the two models  
373 (Fig. 5). Nevertheless, these differences become less pronounced for depths larger than  
374 20 mm. In particular, the models estimate similar values of  $\delta$ . The two models predict  
375 a similar evolution of  $f$  and  $\lambda$  with SSA but the estimated values are different for these  
376 depths. Thus, it would be interesting to compare the NHPP and HPP on thicker snow-  
377 samples, where the CZ can fully develop. In such a case, the NHPP and HPP would  
378 probably lead to similar estimates. Indeed, the NHPP is a generalization of the HPP,  
379 as it also includes zero-slope evolution of the intensity with depth. On a profile with  
380 no significant evolution of lambda, the NHPP would not provide any advantage over the  
381 standard HPP method but would perform as well as this method. Besides, it would  
382 be interesting to apply the NHPP on a whole profile of a snowpack with different snow  
383 layers. In particular, it may be still possible that a new compaction zone develops at each  
384 transition between a soft layer and a harder one, and not only at the snowpack surface.

385 We measured numerous  $\mu$ CPT profiles in quasi-homogeneous snow samples created in  
386 the laboratory and let to sinter during 24 hours at  $-10^\circ\text{C}$ . The NHPP model was applied to  
387 these measurements. The derived micromechanical properties consistently distinguished  
388 the four snow samples prepared with different sieve sizes and varying mainly by grain size

389 (or SSA) and slightly by density. The largest values of microscopic rupture force  $f$  and  
 390 deflection  $\delta$ , and lowest values of intensity  $\lambda_z$ , were obtained for the largest grains and  
 391 slightly lower densities than average (Fig. 5). Furthermore, only the microscopic rupture  
 392 force was found to evolve significantly with time, while the parameters  $\lambda_z$  and  $\delta$  remained  
 393 essentially constant with time (Fig. 8). These results can be interpreted as a progressive  
 394 growth of the bonds without any other structural changes in the snow.

395 The microstructural analysis thus exhibits a partition between the variations with  
 396 depth (borne only by the intensity) and with time (borne only by the microscopic rupture  
 397 force) of the macroscopic force profile. This observation is consistent with our expecta-  
 398 tion that the evolution over one day of the snow microstructure, excepted bond growth,  
 399 would remain limited for samples composed of large rounded grains with a density around  
 400  $500 \text{ kg m}^{-3}$  and a SSA between 10 and  $18 \text{ m}^2 \text{ kg}^{-1}$ . Hence,  $\mu\text{CPT}$  profiles with an ap-  
 401 propriate statistical analysis indeed appear to provide relevant proxies of bond evolution  
 402 during sintering. This point corroborates the finding of Herwijnen and Miller [19] who  
 403 suggested that the SMP profile relates to bond scale processes, based on an agreement  
 404 between the SMP mean force evolution and a physical bond-to-grain evolution model.

405 In previous studies, snow sintering was investigated at different scales, from the bond  
 406 scale by recording the evolution of the bond size [e.g. 13, 14, 16] to the snowpack scale  
 407 by recording the evolution of macroscopic mechanical properties such as shear or com-  
 408 pression strength [e.g. 24, 26]. With the  $\mu\text{CPT}$  and the NHPP analysis, we were able  
 409 to characterize the sintering process at two scales, through simultaneous records of the  
 410 macroscopic penetration force  $F$  (Fig. 7) and of the microscopic rupture force  $f$  (Fig. 9a),  
 411 a direct proxy for the bond size. Both  $F$  and  $f$  were shown to obey power laws with  
 412 time, with similar exponents, namely  $\alpha_F \approx 0.29$  (Fig. 7) and  $\alpha_f \approx 0.27$  (Fig. 9a) (average  
 413 values). Hence, the time-evolution of the macroscopic force  $F$  appears closely related to  
 414 the evolution of the microscopic force  $f$ . Recall also that the intensity profile  $\lambda_z$  remains  
 415 essentially constant over time, meaning that the progressive build-up of the tip-induced  
 416 compaction zone is unaffected by sintering, at least in the considered dataset.

417 The obtained values of  $\alpha_F$  are in agreement with previous studies at the macroscopic  
 418 scale: Herwijnen [33] observed exponents between 0.2 and 0.3 for rounded grains with  
 419 densities between about  $350$  to  $400 \text{ kg m}^{-3}$ . To compare the values of  $\alpha_f$  to sintering

420 exponents involved in the evolution of bond size (a geometrical property), we can assume  
 421 that  $f$  is proportional to  $\sigma_{ice} \times r_b^2$  with  $\sigma_{ice}$  the strength of ice and  $r_b$  the bond radius. This  
 422 assumption yields the relation  $\alpha_f = 2 \alpha_{rb}$  between the sintering exponents of  $f$  and bond  
 423 radius  $r_b$ . According to Hobbs and Mason [14], the evolution of the bond-to-grain ratio for  
 424 ice can be theoretically described by Eq.1 with  $n = 5$  and  $m = 3$ , which yields  $\alpha_{rb} = 1/5$ .  
 425 From experimental data, Hobbs and Mason [14] estimated sintering exponents of bond  
 426 size between 0.16 and 0.24 at a temperature of  $-10^\circ\text{C}$ . Kingery [12] measured exponents  
 427 between 0.15 and 0.18 at temperatures of about  $-9.5^\circ\text{C}$  and Kuroiwa [13] got an exponent  
 428 of 0.2 at the same temperature. The sintering exponents of microscopic rupture force that  
 429 we measured (between 0.18 and 0.34) are effectively higher than the ones theoretically  
 430 obtained for bond size, but not by a factor of 2. Note, however, that all these studies  
 431 considered ice spheres, while we worked with irregular grain shapes. Besides, Kuroiwa [13]  
 432 and Hobbs and Mason [14] worked with much smaller ice spheres than ours, only Kingery  
 433 [12] used ice spheres with a diameter between 0.6 and 1 mm. Moreover, deviations to the  
 434 relation  $\alpha_f = 2 \alpha_{rb}$  can be expected if the stress distribution in a bond between grains  
 435 is not homogeneous. The measured values of  $\alpha_F$  do not change with SSA in the range  
 436 between 10 and 18  $\text{m}^2 \text{kg}^{-1}$  and densities between 480 and 500  $\text{kg m}^{-3}$ . In contrast, the  
 437 values of  $\alpha_f$  were found to slightly increases with grain size, as observed by Hobbs and  
 438 Mason [14]. Lastly, it is also noteworthy that a different behaviour with grain size was  
 439 observed for the two pre-factors  $F_{1h}$  and  $f_{1h}$ :  $F_{1h}$  decreases, while  $f_{1h}$  increases, with  
 440 increasing grain size. The behaviour of  $f_{1h}$  is consistent with classical sintering models  
 441 predicting that bond radius should increase with grain size according to  $r_b \propto r_g^{m/n}$  (see  
 442 Eq. 1) [10].

## 443 5. Conclusions

444 We investigated snow sintering at  $-10^\circ\text{C}$  by measuring numerous  $\mu\text{CPT}$  profiles con-  
 445 ducted with a modified version of the SMP on samples characterized by different grain  
 446 sizes during 24 hours. To analyze the fluctuating macroscopic force in terms of microme-  
 447 chanical properties, we extended the work of Löwe and Herwijnen [29] based on an homo-  
 448 geneous Poisson shot noise model. To this end, we also considered the penetration force as  
 449 the result of the contribution of independent elastic-brittle failure events, but we relaxed

450 the assumption that the number of failure occurrences per penetration increment, i.e. the  
451 intensity, is constant over a certain analysis window. This non-homogeneous Poisson shot  
452 noise model is able to characterize the snow micromechanical properties of a homogeneous  
453 snow layer even if the cone penetration test is in a transient state, due to the progressive  
454 formation of a compaction zone in front of the cone tip. On simulated profiles with pre-  
455 scribed spatial variations of the intensity, the model provided accurate estimates of the  
456 micromechanical properties. On the measured  $\mu$ CPT profiles, the model decomposed the  
457 evolution of the macroscopic force with depth and time as a constant vertical intensity  
458 profile and a time evolution of the microscopic rupture force. This partition is consistent  
459 with the absence of evolution of the grain themselves (e.g. shape) in the tested samples  
460 and bond growth with sintering. The power law exponents for the macroscopic and mi-  
461 croscopic rupture forces ( $\alpha_F \approx 0.29$  and  $\alpha_f \approx 0.27$ , respectively) were very close and in  
462 line with previous studies. In addition, the analysis of  $\alpha_f$  revealed a higher sintering rate  
463 for the largest grains and the pre-factor  $f_{1h}$  of the power law was shown to increase with  
464 grain size, which is in line with the theory of bond growth. However, the limited number  
465 (four) of tested samples did not enable us to provide quantitative relations between grain  
466 size and sintering rate. Our analysis nevertheless showed that micro-cone penetration  
467 tests, combined with an appropriate analysis method enables a fast and simple character-  
468 ization of the snow structure. It would be thus possible to investigate the snow evolution  
469 with metamorphism on snow samples spanning a wider range of microstructural patterns.

470 The HPP and NHPP models could be evaluated with micro-tomographical measure-  
471 ments, as mentioned by Löwe and Herwijnen [29]. Indeed, micro-tomography can be used  
472 to provide direct estimates of microstructural properties derived from  $\mu$ CPT. In particu-  
473 lar, the contact force  $f$  could be related to the bond size derived from tomographic data  
474 using the concept of the minimum cut surface [39]. We are currently working on combined  
475  $\mu$ CPT and micro tomographical measurements. This data includes high-resolution mea-  
476 surements of the snow microstructure before and after a  $\mu$ CPT. It could provide a direct  
477 evaluation of the HPP and NHPP models and could give new insights on the deformation  
478 mechanisms occurring at the cone tip.

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## 484 Appendix A. Evaluation of modified SMP

485 In this appendix, we compare the measured force signal of the original SMP and  
486 the modified version used in our study. For simplicity, we denote the measured profiles  
487 as followed:  $\mu$ CPT profiles are measured with the modified SMP and SMP profiles are  
488 measured with the original setup. Fig. A.10 shows two couples of SMP and  $\mu$ CPT profiles,  
489 which were measured within a distance of about 5 cm in one snow sample. The snow  
490 sample was made of large rounded grains, with a density of about  $480 \pm 20 \text{ kg m}^{-3}$  and  
491 a SSA of  $14.38 \pm 0.9 \text{ m}^2 \text{ kg}^{-1}$ . The maximal depth of the SMP profiles is the height of  
492 the snow sample. Small differences in the sample height are due to the sieving procedure.  
493 Comparing the first 35 mm of the profiles, we notice a higher force for the SMP profiles,  
494 than for the  $\mu$ CPT profiles. A clear difference between the profiles can be observed for  
495 depth larger than about 5 mm of penetration, where the rod touches in the snow. Above,  
496 no obvious difference is visible.

497 A more quantitative analysis was done by computing the mean and standard deviation  
498 on the first 5 and 25 mm of the profile. At a depth of about 5 mm, the SMP rod reaches  
499 the snow surface and its CZ could influence the measured signal. To get an information of  
500 the fluctuations of the force profile, we computed the mean of the macroscopic force, its  
501 standard deviation, and its correlation length ( $C(0)/C'(0)$ , with the two point correlation  
502 function  $C$ , [29]) over the two depth intervals. The results of these computations are  
503 shown in Tab. A.2. For the computed properties over the first 5 mm, we observe a good  
504 agreement between the SMP and  $\mu$ CPT profiles. In contrast to this, the analysis over  
505 25 mm depth shows obvious differences between both measuring methods. Here, a clear  
506 influence due to the indentation of the SMP rod is observed. These results show that  
507 the extension seems to have no influence on the force sensor, as there were no remarkable  
508 differences in the signals for the depths where the rod did not touch the snow. In contrast,

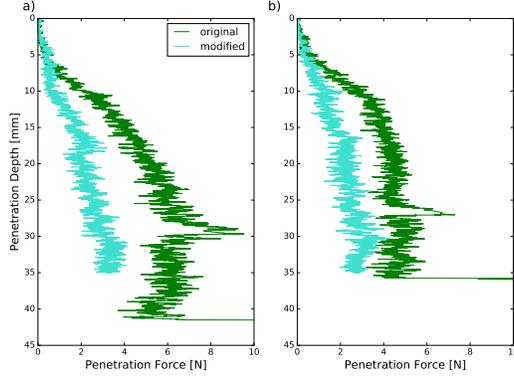


Figure A.10: Penetration profiles measured with the original setup (SMP) and the modified setup ( $\mu$ CPT). The comparison was repeated for two couples of profiles (a and b). In one couple, the profiles were measured close to each other.

Table A.2: Statistical calculations on the profiles of the SMP and  $\mu$ CPT measurements. The calculations were done over two different windows, the first 5 and the first 25 mm of the profile.

window (mm)	mean				standard deviation				C(0)/C'(0)			
	SMP		$\mu$ CPT		SMP		$\mu$ CPT		SMP		$\mu$ CPT	
5	0.13	0.22	0.17	0.16	0.13	0.18	0.17	0.15	0.067	0.078	0.077	0.094
25	2.94	2.82	1.37	1.38	2.14	1.71	0.88	0.89	0.15	0.38	0.10	0.11

509 large differences in the measured profile were observed when the rod indented the snow,  
 510 **which might be enhanced by the limited depth of our samples.**

## 511 Appendix B. Mathematical developments

512 In this appendix, we prove Eq. 4 and explain why  $\tilde{F}$  can be considered as stationary.

513 The general equation for the cumulant  $\kappa_n$  of order  $n$  of a signal, which can be described  
 514 by a non-homogeneous shot noise process, is given by Campbell's theorem [e.g. 40]:

$$\kappa_n(z) = \int_{-\infty}^{\infty} \lambda_z(\tau) f_z^n(z - \tau) d\tau \quad (\text{B.1})$$

515 Furthermore, the auto-correlation  $C$  of the signal is given by [e.g. 40]:

$$C(z, z + u) = \int_{-\infty}^{\infty} \lambda_z(\tau) f_z(z - \tau) f_z(z + u - \tau) d\tau \quad (\text{B.2})$$

516 We now express  $\lambda_z(z)$  a polynomial function  $\sum_{m=0}^M a_m z^m$ . Inserting the expression  
 517  $f_z(z) = f/\delta \times \theta(z) \times \theta(\delta - z)$ , where  $\theta$  is the Heaviside function, into Eqs. B.1 and B.2,  
 518 we obtain

$$\kappa_n(z) = f^n \delta \sum_{l=0}^M \left[ \frac{\delta^l (-1)^l}{(l+n+1)!} \frac{d^l}{dz^l} \left( \sum_{m=0}^M a_m z^m \right) \right], n = 1, 2 \quad (\text{B.3})$$

$$C(z, z+u) = \frac{f^2}{\delta^2} \sum_{l=0}^M \left[ (-1)^l \left( \frac{u(\delta-u)^{l+2}}{(l+2)!} + \frac{(\delta-u)^{l+3}}{(l+3)!} \right) \frac{d^l}{dz^l} \left( \sum_{m=0}^M a_m z^m \right) \right] \quad (\text{B.4})$$

519 If the variations of the intensity over an interval of the size of  $\delta$  are small compared  
 520 to the mean intensity, all the terms with  $l \geq 1$  in the above summation can be neglected.  
 521 Equation B.4 then simplifies to:

$$\kappa_n(z) = \frac{f^n \delta \lambda_z(z)}{n+1} \quad \text{and} \quad C(z, z+r, |r| < \delta) = f^2 \delta \lambda_z(z) \left( \frac{1}{3} - \frac{1}{2} \frac{|r|}{\delta} + \frac{1}{6} \frac{|r|^3}{\delta^3} \right). \quad (\text{B.5})$$

522 Computing the cumulants and the correlation function of  $\tilde{F} = (F - \kappa_1(F))/\kappa_1(F)^{1/2}$   
 523 with Eq. B.5, it appears that they are independent of  $z$ , which indicates that  $\tilde{F}$  is sta-  
 524 tionary.

### 525 Appendix C. Time evolution of the macroscopic and microscopic property 526 profiles

527 This appendix provides the equivalents of Figs. 6 and 8 (corresponding to sample  
 528 SSA18a above) for samples SSA18b, SSA14 and SSA10.

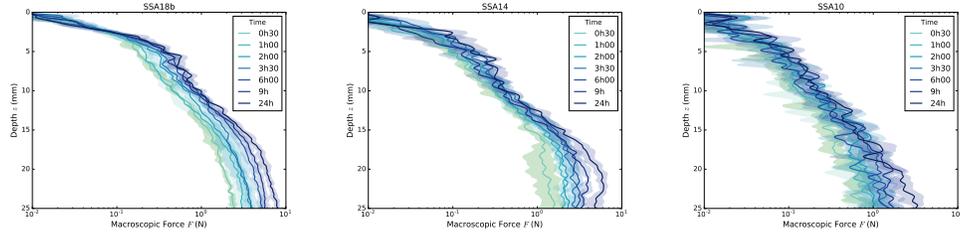


Figure C.11: Evolution of the vertical profiles of the macroscopic force with time, measured on the samples SSA18b, SSA14 and SSA10.

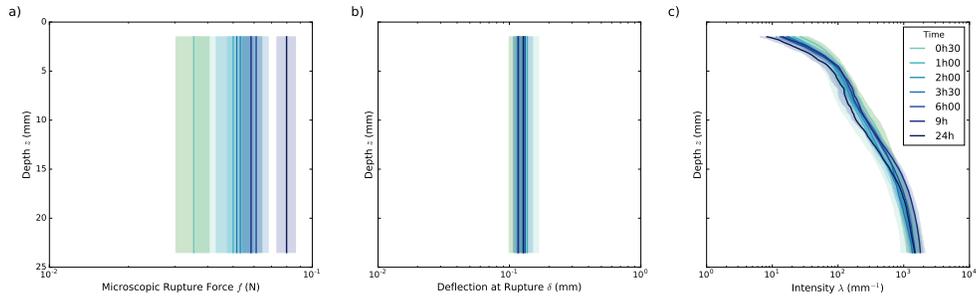


Figure C.12: Time evolution of the profiles of the micromechanical properties estimated by the NHPP model on sample SSA18b. The shaded area around the curves represents the standard deviation obtained for the different profiles at one sampling time.

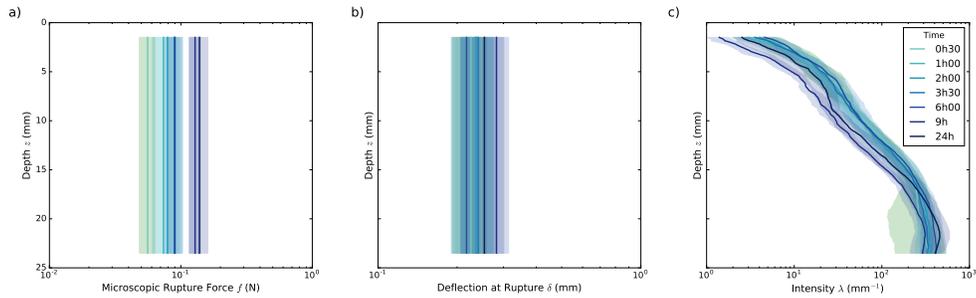


Figure C.13: Time evolution of the profiles of the micromechanical properties estimated by the NHPP model on sample SSA14. The shaded area around the curves represents the standard deviation obtained for the different profiles at one sampling time.

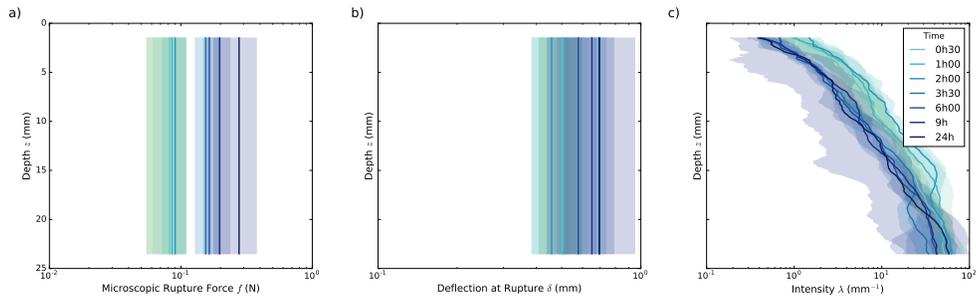


Figure C.14: Time evolution of the profiles of the micromechanical properties estimated by the NHPP model on sample SSA10. The shaded area around the curves represents the standard deviation obtained for the different profiles at one sampling time.

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