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A family of synchrosqueezing transforms for multicomponent signal analysis

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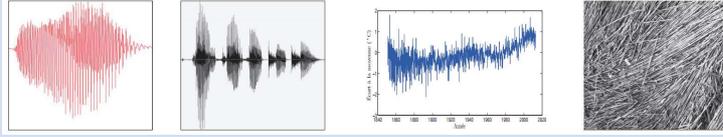
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Introduction

- Time-frequency analysis of multicomponent signals (MCS).
- MCS in real life:



- The **SynchroSqueezed Transform** (SST) [1] has two purposes:
 - **sharpen** the time-frequency (TF) representation given by Short-Time Fourier Transform (STFT)
 - **reconstruct** automatically the modes making up the signal.
- **The goal of this research:** put forward a **generalization of SST** using a new local estimate of instantaneous frequency (IF) \implies achieve a highly concentrated TF representation for a larger class of MCSs + reconstruct their modes with a high accuracy.

Multicomponent signal (MCS)

- **Superposition of AM - FM modes:** $f(t) = \sum_{k=1}^K f_k(t)$ with $f_k(t) = A_k(t)e^{i2\pi\phi_k(t)}$, for $K \in \mathbb{N}$, $A_k(t) > 0$, $\phi_k'(t) > 0$ and $\phi_{k+1}'(t) > \phi_k'(t)$ for $\forall t$.
- **Hypothesis:** all f_{k_s} are **well separated** in frequency, i.e. $|\phi_{k+1}'(t) - \phi_k'(t)| \geq 2\Delta$ for $\forall t$.

STFT

- Fourier transform (FT) of a signal $f \in L^1(\mathbb{R})$: $\hat{f}(\eta) = \int_{\mathbb{R}} f(t)e^{-i2\pi\eta t} dt$.
- A signal $f \in L^1(\mathbb{R})$ and a window $g \in \mathcal{S}(\mathbb{R})$:

$$V_f^g(t, \eta) = \int_{\mathbb{R}} f(\tau)g^*(\tau - t)e^{-2i\pi\eta(\tau - t)} d\tau.$$

Reassignment methods

- **Reassignment operators:**

- Local group delay (GD):

$$\hat{\tau}_f(t, \eta) = t - \frac{1}{2\pi} \partial_{\eta} \{ \arg(V_f^g(t, \eta)) \}$$

- Local instantaneous frequency:

$$\hat{\eta}_f(t, \eta) = \frac{1}{2\pi} \partial_t \{ \arg(V_f^g(t, \eta)) \}$$

- **Standard reassignment (RM):**

- **Oblique** mapping: $(t, \eta) \mapsto (\hat{\tau}_f, \hat{\eta}_f)$.
- Operator:

$$R_f^g(t, \omega) = \iint_{\mathbb{R}^2} |V_f^g(\tau, \eta)|^2 \times \delta(\omega - \hat{\omega}_f(\tau, \eta)) \delta(t - \hat{\tau}_f(\tau, \eta)) d\eta d\tau.$$

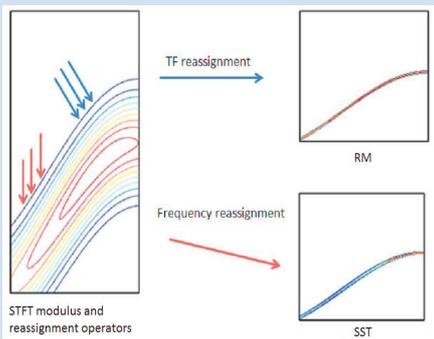
- Ideal TF representation of linear chirps.
- **Non reconstruction.**

- **SST:**

- **Vertical** mapping: $(t, \eta) \mapsto (t, \hat{\eta}_f)$.
- Operator:

$$T_f^g(t, \omega) = \frac{1}{g^*(0)} \int_0^{\infty} V_f^g(t, \eta) \delta(\omega - \hat{\omega}_f(t, \eta)) d\eta.$$

- Ideal TF representation of pure waves.
- **Reconstruction.**



Toward to high-order SST (SSTN)

- Let $f \in L^2(\mathbb{R})$, **frequency modulation operators** $\tilde{q}_{\eta, f}^{[p, N]}$ of $\phi^{(p)}(t)/(p-1)!$ for $p = 2, 3, 4$ and $N = 4$ are:

$$\tilde{q}_{\eta, f}^{[4, 4]} = G_4 \left(V_f^{t^{0 \dots 6} g}, V_f^{t^{0 \dots 3} g'} \right),$$

$$\tilde{q}_{\eta, f}^{[3, 4]} = G_3 \left(V_f^{t^{0 \dots 4} g}, V_f^{t^{0 \dots 2} g'} \right) - \tilde{q}_{\eta, f}^{[4, 4]} G_{3,4} \left(V_f^{t^{0 \dots 5} g} \right),$$

$$\tilde{q}_{\eta, f}^{[2, 4]} = G_2 \left(V_f^{t^{0 \dots 2} g}, V_f^{t^{0 \dots 1} g'} \right) - \tilde{q}_{\eta, f}^{[3, 4]} G_{2,3} \left(V_f^{t^{0 \dots 3} g} \right) - \tilde{q}_{\eta, f}^{[4, 4]} G_{2,4} \left(V_f^{t^{0 \dots 4} g} \right),$$

where $G_p \left(V_f^{t^{0 \dots m} g}, V_f^{t^{0 \dots n} g'} \right)$ is a function of $V_f^{t^l g}$ for $l = 0, \dots, m$ and $V_f^{t^l g'}$ for $l = 0, \dots, n$ while $G_{p,j} \left(V_f^{t^{0 \dots m} g} \right)$ is associated with coefficient $\tilde{q}_{\eta, f}^{[j, N]}$ in the computation of $\tilde{q}_{\eta, f}^{[p, N]}$ for $p \neq j$.

- **IF estimate of order 4** is:

$$\tilde{\omega}_{\eta, f}^{[4]}(t, \eta) = \tilde{\omega}_f(t, \eta) + \tilde{q}_{\eta, f}^{[2, 4]}(t, \eta) (-x_{2,1}(t, \eta)) + \tilde{q}_{\eta, f}^{[3, 4]}(t, \eta) (-x_{3,1}(t, \eta)) + \tilde{q}_{\eta, f}^{[4, 4]}(t, \eta) (-x_{4,1}(t, \eta)).$$

- **Exact IF estimate for a polynomial chirp of order 4.**

- $\tilde{\omega}_{\eta, f}^{[2]}$ is obtained by neglecting $\tilde{q}_{\eta, f}^{[3, 4]}$ and $\tilde{q}_{\eta, f}^{[4, 4]}$.

- **Synchrosqueezing operator** of order N (SSTN) is:

$$T_{N, f}^g(t, \omega) = \frac{1}{g^*(0)} \int_0^{\infty} V_f^g(t, \eta) \delta(\omega - \hat{\omega}_{\eta, f}^{[N]}(t, \eta)) d\eta.$$

Numerical results

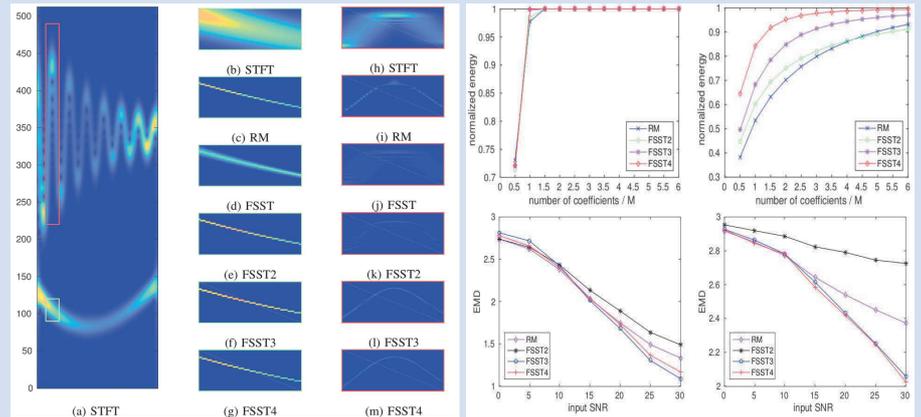
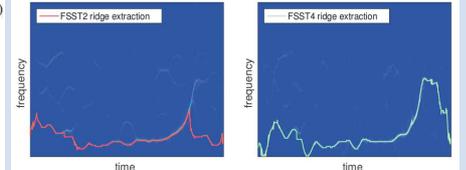


TABLE I
PERFORMANCE OF MODE RECONSTRUCTION (MEASURED BY SNR)

	FSST2	FSST3	FSST4
Mode f_1	17.8	25.7	28.8
Mode f_2	1.73	3.62	6.87
MCS f	3.57	5.57	8.82

TF representations of the gravitational-wave event GW150914



References

- [1] G. Thakur and H.-T. Wu, "Synchrosqueezing-based recovery of instantaneous frequency from nonuniform samples." *SIAM J. Math. Analysis*, vol. 43, no. 5, pp. 2078–2095, 2011.
- [2] D.-H. Pham and S. Meignen, "High-order synchrosqueezing transform for multicomponent signals analysis - with an application to gravitational-wave signal," *IEEE Transactions on Signal Processing*, vol. 65, no. 12, pp. 3168–3178, June 2017.

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Conclusion

- SSTN: a powerful tool for analysis of MCS containing very strongly modulated AM-FM modes.
- Combination of a sharp representation (like reassignment) and a reconstruction (like classical ridge analysis)
- An interesting application on gravitational-wave signal.

Current and future works

- Theoretical analysis of SSTN when applied to noisy signals and when the type of noise is non Gaussian.
- Extension to 2 or 3 dimensions (with monogenic SST).
- More applications for real-life signals (detection, monitoring, etc.).