

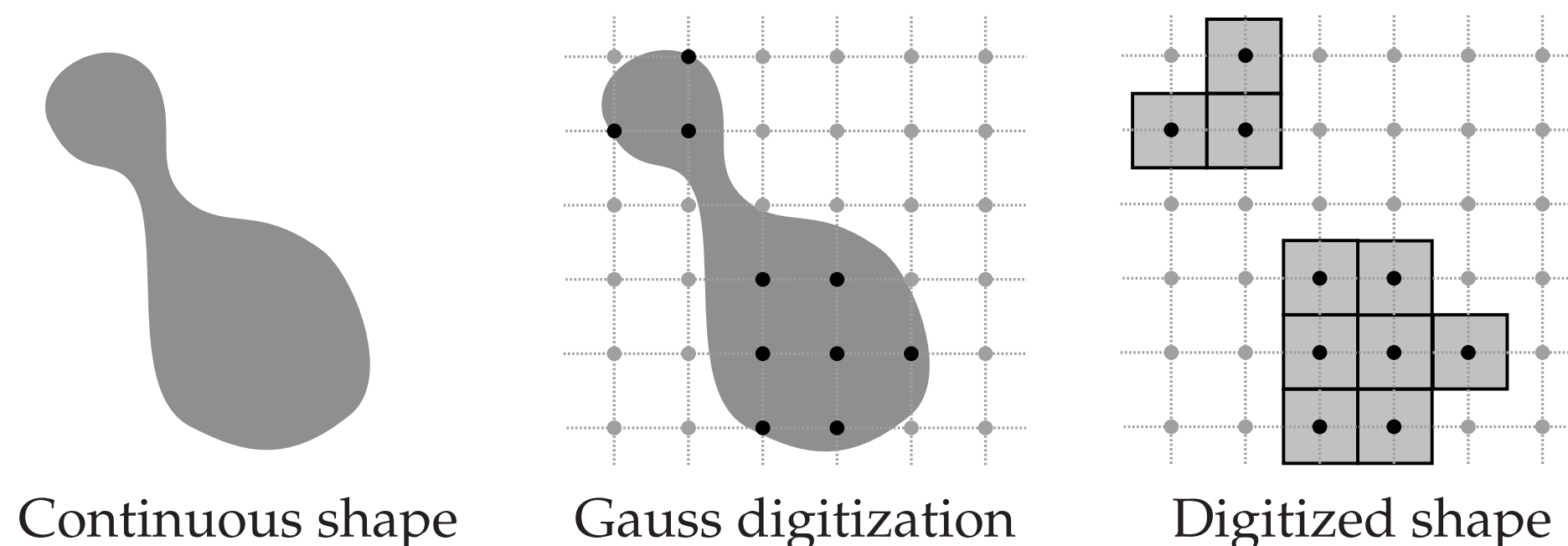
QUASI-REGULARITY VERIFICATION FOR 2D POLYGONAL OBJECTS BASED ON MEDIAL AXIS ANALYSIS

PHUC NGO – NICOLAS PASSAT – YUKIKO KENMOCHI
hoai-diem-phuc.ngo@loria.fr

1. MOTIVATIONS

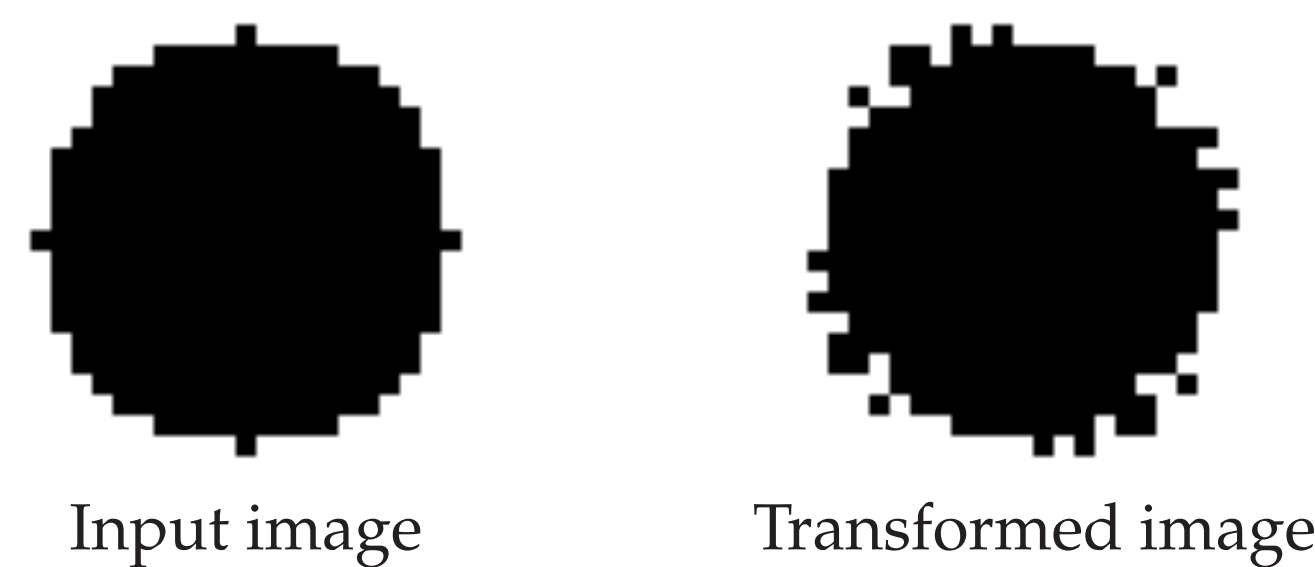
1. Topological preservation by digitization process

- ✓ Class of quasi-1-regular objects for connectivity preservation
- ✓ Exact method for verifying quasi-1-regularity of polygonal objects using medial axis



2. Rigid motion in discrete space of \mathbb{Z}^2 : topological and geometrical properties of the transformed objects

- ✓ framework of rigid motion using polygonal representation of digital object for the topological and geometrical preservation



2. BACKGROUND NOTIONS

- Gauss digitization:** Let $X \subset \mathbb{R}^n$, $n \geq 2$. The Gauss digitization is defined as the intersection of X with \mathbb{Z}^n : $X = X \cap \mathbb{Z}^d$.
- Well-composed sets [1]:** A digital set $X \subset \mathbb{Z}^2$ is *well-composed* if each 8-connected component of X and of its complement \bar{X} is also 4-connected.
- r -regularity [2]:** A finite and connected subset $X \subset \mathbb{R}^2$ is *r -regular* if for each boundary point of X , there exist two tangent open balls of radius r , lying entirely in X and its complement \bar{X} , respectively.
- Medial axis [3]:** Let $X \subset \mathbb{R}^2$ be a closed, bounded set whose boundary ∂X of X is a 1-manifold. The medial axis of X is defined as: $\mathcal{M}(X) = \{x \in X \mid \nexists y \in X, B(x, r(x)) \subset B(z, r(z))\}$, with $B(y, r(y)) \subseteq X$ the ball of center z and radius $r(z) \in \mathbb{R}_+$.

7. CONCLUSION

Contributions:

- Method for verifying quasi-1-regularity of polygonal objects using medial axis
- Framework of rigid motion in \mathbb{Z}^2 for preserving both topology and geometry

Perspectives:

- Polygonization methods for digital objects
- Repairing non-quasi-regular objects using medial axis
- Multi-resolution approaches for the limit case of quasi-1-regularity
- Extension of the proposed method in 3D

3. r -REGULARITY

Definition (in mathematical morphology): Let $X \subset \mathbb{R}^2$ be a bounded, simply connected object (i.e., connected without hole). X is *r -regular* if it satisfies the following four properties:

- $X \ominus B_r$ is non-empty and connected
- $\bar{X} \ominus B_r$ is connected
- $X \subseteq X \ominus B_r \oplus B_r$
- $\bar{X} \subseteq \bar{X} \ominus B_r \oplus B_r$

where $\bar{X} = \mathbb{R}^2 \setminus X$; \oplus, \ominus are the dilation and erosion operators; and $B_t \subset \mathbb{R}^2$ is the Euclidean ball of centre $0_{\mathbb{R}^2}$ and radius t .

Polygonal objects are not r -regular for any $r > 0$.

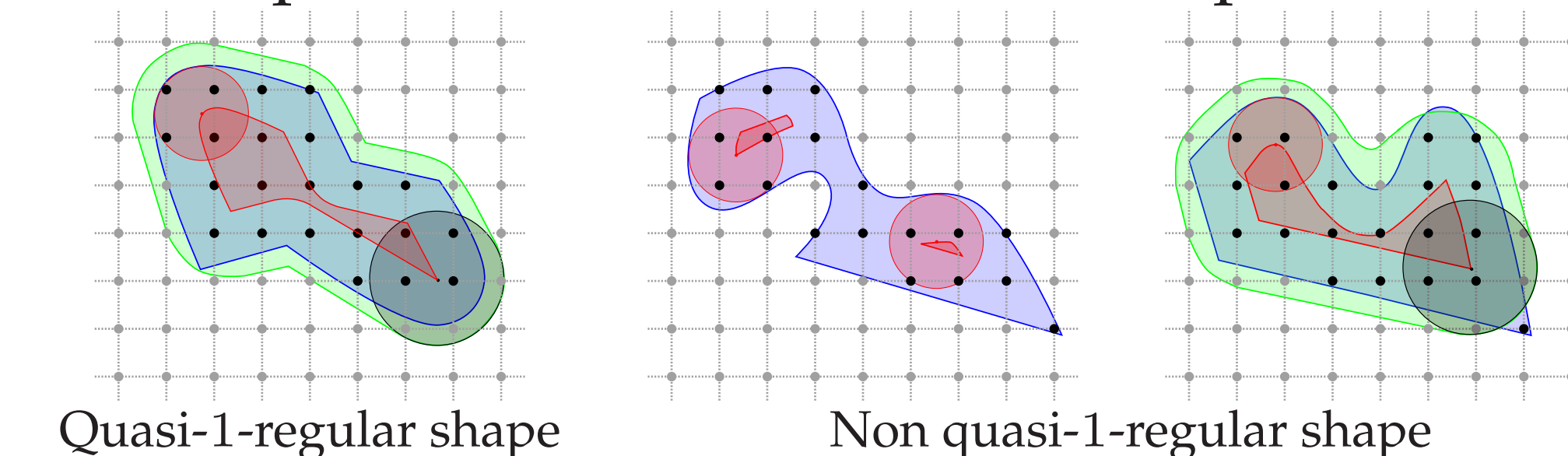
Proposition [2]: An r -regular set $X \subset \mathbb{R}^2$ has the same topological structure as its digitized version $X = X \cap \mathbb{Z}^2$ if $r \geq \frac{\sqrt{2}}{2}$.

4. QUASI- r -REGULARITY

Definition: Let $X \subset \mathbb{R}^2$ be a bounded, simply connected object (i.e., connected without hole). X is *r -quasi-regular* if it satisfies:

- $X \ominus B_r$ is non-empty and connected
- $\bar{X} \ominus B_r$ is connected
- $X \subseteq X \ominus B_r \oplus B_{r\sqrt{2}}$
- $\bar{X} \subseteq \bar{X} \ominus B_r \oplus B_{r\sqrt{2}}$

Proposition: Let $X \subset \mathbb{R}^2$ be a bounded, simply connected object. If X is quasi-1-regular, then $X = X \cap \mathbb{Z}^2$ and $\bar{X} = \bar{X} \cap \mathbb{Z}^2$ are both 4-connected. In particular, X is then well-composed.



5. VERIFICATION OF QUASI-1-REGULARITY

We define the level medial axis as:

$$\mathcal{M}_\lambda(X) = \{x \in \mathcal{M}(X) \mid r(x) \geq \lambda\}$$

$$\mathcal{M}_{\lambda_1}^{\lambda_2}(X) = \{x \in \mathcal{M}(X) \mid \lambda_1 \leq r(x) \leq \lambda_2\}$$

Let $Y \in \{X, \bar{X}\}$ and $M \subseteq \mathcal{M}_0^1(Y)$ a connected component of $\mathcal{M}_0^1(Y)$. M contains k points, noted z_i ($1 \leq i \leq k$) with $r(z_i) = 0$ and a point y with $r(y) = 1$.

Let $(\mathcal{P}) : \forall 1 \leq i \leq k, \|y - z_i\|_2 \leq \sqrt{2}$. We have

$$(\mathcal{P}) \Rightarrow \bigcup_{x \in M} B(x, r(x)) \subseteq Y \ominus B_1 \oplus B_{\sqrt{2}}$$

We denote $X \sim Y$ if X and Y are homotopy equivalent.

Proposition: Let $X \subset \mathbb{R}^2$ be a bounded, simply connected polygon. Let us suppose that $\mathcal{M}(X) \sim \mathcal{M}_1(X)$, $\mathcal{M}(\bar{X}) \sim \mathcal{M}_1(\bar{X})$ and that for each connected component of $\mathcal{M}_0^1(X)$ and $\mathcal{M}_0^1(\bar{X})$, property (\mathcal{P}) holds. Then, X is quasi-1-regular.

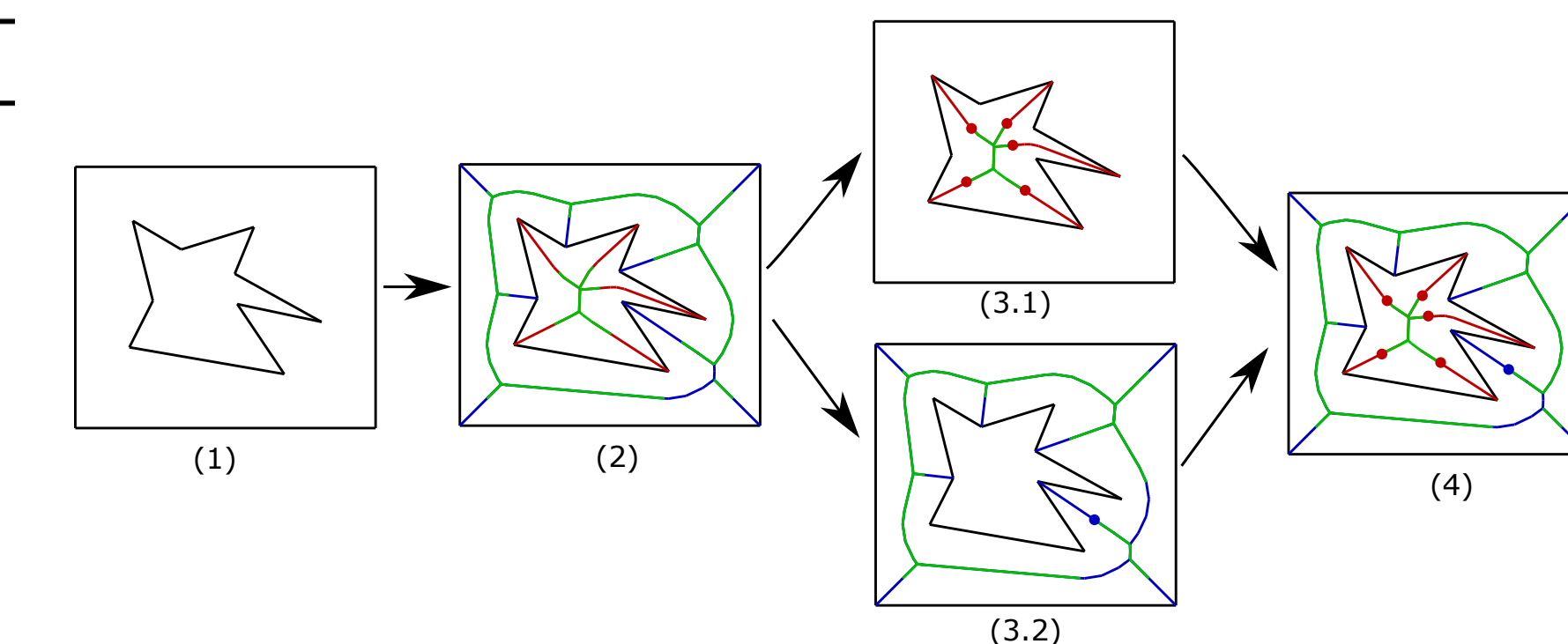
Algorithm 1: Quasi-regularity verification

Input: A simply connected polygonal object $X \subset \mathbb{R}^2$
Output: A Boolean indicating whether X is quasi-regular

```

1 for  $Y \in \{X, \bar{X}\}$  do
2   if not  $(\mathcal{M}(Y) \sim \mathcal{M}_1(Y))$  then return false
3   foreach connected component  $M \in \mathcal{M}_0^1(Y)$  do
4     Let  $y \in M$  s.t.  $r(y) = 1$ 
5     foreach  $z_i \in M$  s.t.  $r(z_i) = 0$  do
6       if  $\|y - z_i\|_2 > 2$  then return false
7 return true

```



Workflow of quasi-1-regular verification method

6. APPLICATION: RIGID MOTIONS ON \mathbb{Z}^2

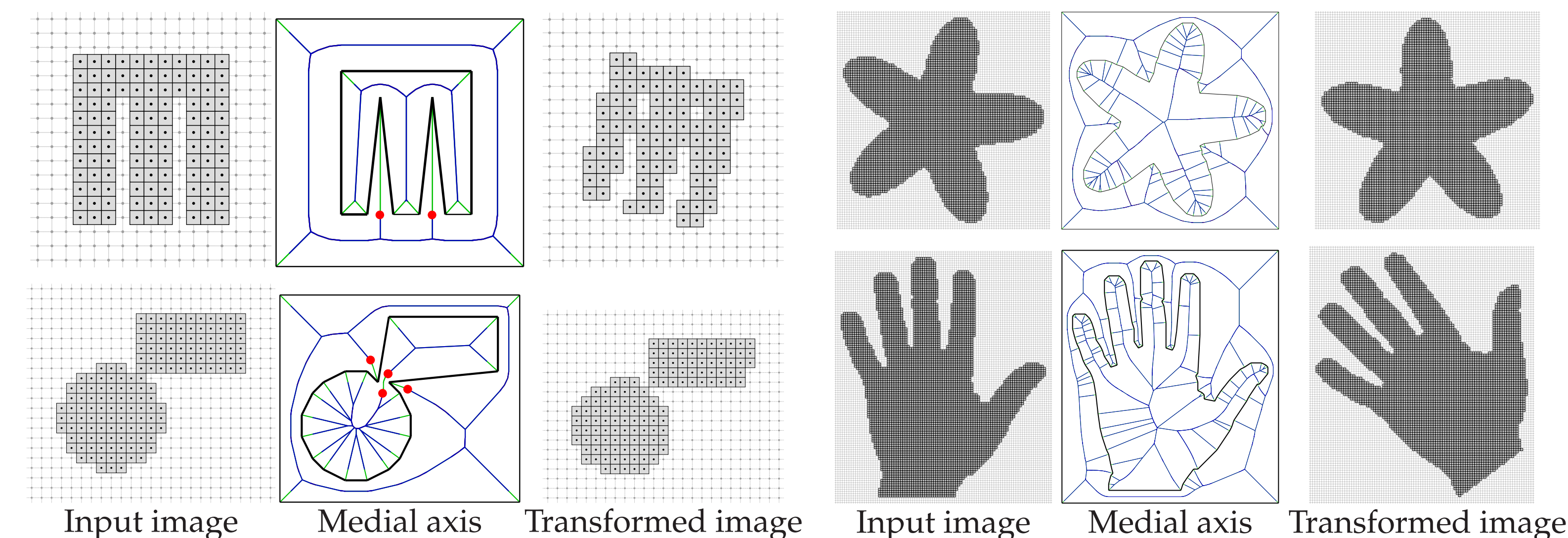
A rigid motion is defined for any $x = (x, y) \in \mathbb{R}^2$, as

$$\mathcal{T}(x) = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix}$$

where $a, b \in \mathbb{R}$ and $\theta \in [0, 2\pi[$.

Proposed method: Rigid motion on \mathbb{Z}^2 via polygonization:

- polygonal representation for the overall digital shape preservation by the transformation
- quasi-1-regularity for topology preservation
- exact calculation considering rational rigid motions



REFERENCES

- [1] L. J. Latecki, U. Eckhardt, and A. Rosenfeld. Well-composed sets. *Comput Vis Image Und*, 61(1):70–83, 1995.
- [2] T. Pavlidis. *Algorithms for graphics and image processing*. Berlin: Springer, and Rockville: Computer Science Press, 1982.
- [3] H. Blum. A transformation for extracting new descriptors of shape. In *Models for the Perception of Speech and Visual Form*, pages 362–380. MIT Press, Cambridge, 1967.