



Comprehensive study of the pentad bending triad region of germane: Positions, strengths, widths and shifts of lines in the $2\nu 2$, $\nu 2 + \nu 4$ and $2\nu 4$ bands of 70 GeH_4 , 72 GeH_4 , 73 GeH_4 , 74 GeH_4 , 76 GeH_4

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Comprehensive study of the pentad bending triad region of germane:
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 $2\nu_4$ bands of $^{70}\text{GeH}_4$; $^{72}\text{GeH}_4$; $^{73}\text{GeH}_4$; $^{74}\text{GeH}_4$; $^{76}\text{GeH}_4$

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Highlights

- Positions, strengths, widths and shifts of lines are recorded for the pentad bending triad of ${}^M\text{GeH}_4$ (M = 70, 72, 73, 74, 76)
- Effective dipole moment parameters of the pentad bending triad of MGeH₄ are determined
- List of 164220 transitions is presented

Comprehensive study of the pentad bending triad region of germane: Positions, strengths, widths and shifts of lines in the $2\nu_2$, $\nu_2 + \nu_4$ and $2\nu_4$ bands of $^{70}\text{GeH}_4$, $^{72}\text{GeH}_4$, $^{73}\text{GeH}_4$, $^{74}\text{GeH}_4$, $^{76}\text{GeH}_4$

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Abstract

The high resolution infrared spectra of GeH_4 in its natural abundance were recorded with a Bruker IFS125 HR Fourier transform infrared spectrometer at an optical resolution of 0.003 cm^{-1} and analyzed in the region of $1400\text{--}2000 \text{ cm}^{-1}$ where the first bending overtone $2\nu_2$, $2\nu_4$ and combinational $\nu_2 + \nu_4$ bands are located. Ro-vibrational line positions and energies of the $^{70}\text{GeH}_4$ species were analysed for the first time and line positions and energies of the $^{72}\text{GeH}_4$ and $^{74}\text{GeH}_4$ species were improved considerably in comparison with the preceding studies. The numbers of $2316/2406/873/3007/2257$ transitions with $J^{\max} = 21/23/18/22/19$ of the $2\nu_2$ (A_1- and $E-$ type sub-bands), $2\nu_4$ (A_1- , $E-$ and F_2- type sub-bands) and $\nu_2 + \nu_4$ (F_1- and F_2- type sub-bands) bands of the $^{70}\text{GeH}_4$, $^{72}\text{GeH}_4$, $^{73}\text{GeH}_4$, $^{74}\text{GeH}_4$, and $^{76}\text{GeH}_4$ molecules were used in the joint weighted fit of experimentally assigned transitions with the Hamiltonian model which takes the resonance interactions between the seven, $(0200, A_1)$, $(0200, E)$, $(0101, F_1)$, $(0101, F_2)$, $(0002, A_1)$, $(0002, E)$ and $(0002, F_2)$, vibrational states into account. From our preceding studies 5563 hot band transitions were also taken into account. As the result of a joint fit, a set of 129 fitted parameters was obtained which reproduce the initial 16422 experimental (including "hot") transitions of five isotopologues with the $d_{\text{rms}} = 3.26 \times 10^{-4} \text{ cm}^{-1}$. A line strength analysis of the 1697 experimentally recorded transitions of all species was made by the fit of their line shapes with the Hartmann-Tran profile and 13 effective dipole moment parameters were obtained from the weighted fit which reproduce the initial experimental line strengths with the $d_{\text{rms}} = 3.4\%$. Self-broadening coefficients of 993 lines and self-shift coefficients of 674 lines were determined from the multi-spectrum analysis of these lines.

1 Introduction

This work is a continuation of the recent high-resolution study of different isotopologues of the germane molecule [1] – [3]. Knowledge of spectroscopic characteristics of different isotopologues of germane is important in many fields of science and

technology, such as high purity semi-conductor manufacturing, [4] – [5], chemical physics and astrophysics, [6] – [11], etc. One of the important problems of chemical physics is the precise determination of intramolecular multidimensional potential and dipole moment surfaces, which can be used in numerous applied investigations. This problem can be solved by semi-empirical methods, or on the basis of *ab initio* calculations. In both cases, knowledge of high accurate spectroscopic information not only about the main species (in our case, $^{74}\text{GeH}_4$), but also about all possible isotopologues is very important. So one can state that high accuracy of spectroscopic information about characteristics of spectral lines (both line positions, and line strengths and widths) of different isotopologues of a molecule is essential and timely.

Germane in a natural isotopic composition produces complex infrared spectra. Not less than five stable isotopologues exist in proper abundances with mass numbers of 70 (20.55 %), 72 (36.74 %), 73 (7.67 %), 74 (27.37 %) and 76 (7.67 %). Additional complexity of the germane spectra arises from strong Coriolis interactions between the ν_2/ν_4 and ν_1/ν_3 pairs of its fundamentals. Germane is a spherical top molecule, and, as a consequence, it has no permanent dipole moment, and the possibilities of the ground state combination differences (GSCD) method (which was efficiently used by the authors in the study of molecules of other types, see, *e.g.*, Refs. [12] – [16]) are very limited as applied to germane.

In the present work we focus on the study of positions, strengths, line widths and pressure shifts of different isotopologues of germane in the region of their $2\nu_2$, $\nu_2 + \nu_4$ and ν_4 bands. Earlier high resolution spectra of germane have been studied in many laboratory investigations ([17]–[31] and references therein; see also recent quantitative study of line positions and strengths of the Dijon group, [32], with the resulting line lists included in the VAMDC/GeCaSDa database [33] and to be also inserted in the next HITRAN update, as a new molecule for this database). As to the spectral region considered in the present study, line positions in the $2\nu_2$, $\nu_2 + \nu_4$

and $2\nu_4$ bands were discussed earlier in [1] – [3] where specially enriched $^{76}\text{GeH}_4$ [1], $^{73}\text{GeH}_4$ [2] and $^{72}\text{GeH}_4$ [3] species have been analysed. In this case, line positions of the discussed absorption bands of $^{70}\text{GeH}_4$ and $^{74}\text{GeH}_4$ were not discussed earlier (in [1] the ro-vibrational structure of the $(0101, F_1)$ and $(0101, F_2)$ vibrational states was obtained from "hot" transitions). To our knowledge, experimental line strengths, widths and line shifts of germane in the region of its pentad bending triad were not discussed earlier at all for all isotopologues.

2 Experimental

The used GeH_4 sample gas was purchased from Linde with a declared purity of 99.999%. This indicated purity was verified by three freeze-pump-thaw cycles. As germane is a highly flammable, potentially pyrophoric/self-igniting and highly toxic gas [34], precautions have to be taken. So before inlet into the optical cell, a limited gas amount was pre-filled in a 1000 ml bowl flask within a fume cupboard. The closed filling system for the optical measuring cells was flushed with nitrogen immediately after spectra recording, also compare [35].

For the present work, seven spectra of mono-germane in natural composition, GeH_4 , have been recorded in our Braunschweig infrared laboratory in the spectral range of $600 - 4800 \text{ cm}^{-1}$ (mostly $1400 - 2700 \text{ cm}^{-1}$) with a Bruker IFS125HR Fourier transform infrared spectrometer (Zürich prototype ZP2001 [36], see Table 1 for detailed conditions. The first spectrum has been recorded with a stainless-steel White cell with a base length of 1 m at the optical pathlength of $24.0524 \pm 0.012 \text{ m}$ and further six spectra were recorded with a one-path stainless-steel cell at the optical pathlength of $230.5 \pm \text{mm}$, all in combination with a MCT313 semiconductor detector. Further, a globar radiation source and a KBr beamsplitter were used at 0.003 cm^{-1} optical resolution and self-apodization (Boxcar). Between 300 and 1400 scans have been averaged, see Table 1.

For line strength determination, the accurate sample pressure is needed, so we used a cascade of three temperature compensating Pfeiffer capacitance sensor gauges in ceramic technology (CMR 361, CMR 362, and CMR 363), which are resistant against aggressive gaseous media and independent on the type of gas. Here, we used the first two with the ranges of 0.1 – 1100 hPa and 0.01 – 110 hPa. The manufacturer claims an accuracy on the measured value of 0.2 % so that we estimate the total errors of the sample pressures to be in the range of $\pm 0.5\%$ during the whole time spans of the presented experiments. The temperature was in the range of 23.4°C to 23.8°C with a deviation within a single measurement of up to $\pm 0.7^\circ\text{C}$, compare Table 1. It was monitored with a PT100 resistance thermometer, Ahlborn Almemo 2590, which has a stated accuracy of $\pm 0.03\%$ on the measured value.

The line strength S can be derived from the area of a single absorption line A_{Line} using the Beer–Lambert law, the pressure P of the pure gaseous species (GeH_4), the temperature T and the optical pathlength L :

$$S = \frac{k_B T}{PL} A_{\text{Line}}, \quad (1)$$

where

$$A_{\text{Line}} = \frac{1}{\log(e)} \int \log \left(\frac{I_0(\nu)}{I(\nu)} \right) d\nu. \quad (2)$$

The line intensities can be obtained by direct integration of the measured effective line absorbance, which can be fitted well by a Voigt profile or a Hartmann–Tran line profile (as in the present work; compare line strength analysis in the following Sections). At 1750 cm^{-1} and room temperature the Doppler broadening for $^{74}\text{GeH}_4$ (the most abundant isotopologue of mono germane) is 0.0024 cm^{-1} and the instrumental line width is 0.0020 cm^{-1} at the adjusted optical resolution of 0.003 cm^{-1} in combination with boxcar apodisation. The full pressure widths for the seven spectra in the order of Table 1 are 0.00045 (0.0032), 0.0036 (0.0048), 0.0072 (0.0079), 0.0108

(0.0113), 0.0162 (0.0165), 0.0216 (0.0218), 0.0270 (0.0272) in cm^{-1} . The numbers in brackets are the total line widths which are approximated here by the root sum square of the convolution of Doppler, pressure and instrumental line widths which is in accordance with the experimental results. One can see that from the second pressure value of 20 hPa on, the pressure width is dominating the total line width increasingly with the pressure.

3 Assignment of transitions of the $^{70}\text{GeH}_4$, $^{72}\text{GeH}_4$ and $^{74}\text{GeH}_4$ species

As the first step of the present analysis, we performed an assignment of transitions of the $^{70}\text{GeH}_4$, $^{72}\text{GeH}_4$ and $^{74}\text{GeH}_4$ species in the studied spectra. For the convenience of the reader two overview spectra I and IIa which have been recorded under different experimental conditions, are shown in the upper traces of Figs. 1 and 2. For illustration of the quality of the recorded spectra, small parts of the high resolution spectrum IIa (for the stronger $\nu_2 + \nu_4$ band) and I (for the weaker $2\nu_2$ and $2\nu_4$ bands) are shown in Figs. 3 – 6 where clusters belonging to different isotopic species are marked.

For the assignment of transitions, the ground state parameters from Ref. [37] were used. As the result of the analysis, we assigned 3007 transitions with the value $J^{\max} = 22$ to the $^{74}\text{GeH}_4$ species, 2406 transitions with the value $J^{\max} = 23$ to the $^{72}\text{GeH}_4$ species, and 2316 transitions with the value $J^{\max} = 21$ to the $^{70}\text{GeH}_4$ species (for details, see statistical information in Table 2). The full list of assigned transitions is presented as the Supplementary data 1 to this paper.

4 Joint ro-vibrational analysis of the pentad bending triad of five, $^{70}\text{GeH}_4$, $^{72}\text{GeH}_4$, $^{73}\text{GeH}_4$, $^{74}\text{GeH}_4$, and $^{76}\text{GeH}_4$ isotopologues of germane

The information on 6873 transitions obtained in the present study and discussed in Section 3 was added to the known information about the pentad bending triad and "hot" transitions of $^{72}\text{GeH}_4$, $^{73}\text{GeH}_4$, $^{74}\text{GeH}_4$, and $^{76}\text{GeH}_4$ from Refs. [1] – [3] (for details, see statistical information in Table 2), and both used then in the joint weighted fit of parameters of the effective rotational Hamiltonian,

$$\begin{aligned} H^{vib.-rot.} &= \sum_{v\gamma, v'\gamma'} \sum_{n\Gamma} [(|v\gamma\rangle \otimes \langle v'\gamma'|)^{n\Gamma} \otimes H_{v\gamma, v'\gamma'}^{n\Gamma}]^{A_1} \\ &\equiv \sum_{v\gamma, v'\gamma'} \sum_{n\Gamma} \sum_{\Omega K} [(|v\gamma\rangle \otimes \langle v'\gamma'|)^{n\Gamma} \otimes R^{\Omega(K, n\Gamma)}]^{A_1} Y_{v\gamma, v'\gamma'}^{\Omega(K, n\Gamma)}. \end{aligned} \quad (3)$$

Here $|v\gamma\rangle$ are symmetrized vibrational functions; $Y_{v\gamma, v'\gamma'}^{\Omega(K, n\Gamma)}$ are spectroscopic parameters; and \otimes denotes a tensorial product. In this case, when $v = v'$ and $\gamma = \gamma'$, the parameters $Y_{v\gamma, v'\gamma'}^{\Omega(K, n\Gamma)}$ describe the rotational structure of the vibrational state $(v\gamma)$. If $\gamma = \gamma'$, but $v \neq v'$, the parameters $Y_{v\gamma, v'\gamma'}^{\Omega(K, n\Gamma)}$ describe Fermi-type interactions. If $\gamma \neq \gamma'$ (v and v' are arbitrary), the parameters $Y_{v\gamma, v'\gamma'}^{\Omega(K, n\Gamma)}$ describe Coriolis-type interactions. The operators $R^{\Omega(K, n\Gamma)}$ in Eq. (3) are determined as

$$R^{\Omega(K, n\Gamma)}_{\sigma} = \sum_m {}^K G_{n\Gamma\sigma}^m R_m^{\Omega(K)}, \quad (4)$$

where $R_m^{\Omega(K)}$ are the so-called irreducible rotational operators of the $SO(3)$ symmetry group, [38] – [41]; and the reduction matrix elements ${}^K G_{n\Gamma\sigma}^m$ are determined by the specific point symmetry group. Speaking about molecules of the T_d symmetry group, the numerical representation of the ${}^K G_{n\Gamma\sigma}^m$ values can be found in [42, 43, 44].

The following strategy was used in the fit of the Hamiltonian parameters. The values of all 16422 transitions (including "hot" transitions) obtained in the present study (see Section 3) and taken from [1] – [3] were used as the initial experimental data in the fit procedure. High order centrifugal distortion, tetrahedral splitting

and resonance interaction parameters which have been obtained in [1] – [3] from the large amount of experimental data for the $^{76}\text{GeH}_4$, $^{73}\text{GeH}_4$ and $^{72}\text{GeH}_4$ isotopologues were constrained to the values of corresponding parameters from [1] – [3] for all five isotopologues and were not varied during the fit. Such parameters are presented in Table 3 without confidence intervals (in Table 3, the latter are shown in parenthesis for the fitted parameters). All the other parameters were fitted by the usual procedure with the computer code SPHETOM of Tomsk Polytechnic University. After that (for the convenience of readers because they are more familiar with the Dijon STDS computer code [45] than with the SPHETOM code), values of all obtained parameters were transformed (re-fitted with the Dijon STDS code) to the values of corresponding parameters of the Dijon STDS format (about connection between the SPHETOM and Dijon STDS parameters, see, e.g., Ref. [46]). The final values of parameters of the triad bending states are presented (just in the Dijon STDS format) in Table 3 together with their 1σ statistical confidence intervals which are shown in parenthesis. For the convenience of the reader, parameters being necessary for calculation of the ground state and of the bending dyad are shown in Table 4 (parameters both from Table 3 and Table 4 can be directly used in the Dijon STDS code). From comparison of the parameters of the same label for different isotopologues one can make the conclusion that the obtained results are physically suitable. In particular, for any fitted parameter the condition is fulfilled that the value of a parameter, as a function of the mass of the Ge-nuclei, practically linearly depends on the value of this mass (this condition is one of the main conclusions of the isotopic substitution theory, [47], for the GeH_4 -type molecules). The correctness of the result is confirmed by the good reproduction of the initial experimental line positions of both the "cold" and "hot" transitions by the set of 129 fitted parameters from Table 3: the general d_{rms} value for the whole set of 16422 transitions (both "cold" and "hot") is $3.26 \times 10^{-4} \text{ cm}^{-1}$ (for more detailed statistical information on the different bands of different isotopologues, see Table 2). For the "cold" bands of the $^{70}\text{GeH}_4$, $^{72}\text{GeH}_4$ and $^{74}\text{GeH}_4$ species which have been analysed in the present study for the first time, Figs. 7a – 7c show the (observed – calculated) line positions

and fit statistics.

5 Line strengths: Brief theoretical background

The theory of the line strength analysis of spherical top molecules on the basis of the Irreducible Tensorial Sets theory (ITST) was derived in [48] (see also [49]). In this section we very briefly present some basic relations necessary for the further analysis.

In the absence of external fields the intensity of the vibration–rotation line, due to transitions from the $|A\rangle$ to $|B\rangle$ state, is determined by the expression (see., e. g., [50]):

$$S_{\nu_0} = \frac{8\pi^3\nu_0}{4\pi\epsilon_0 3hc} \left[1 - \exp\left(-\frac{h\nu_0}{k_B T}\right) \right] N \frac{g_A}{Z(T)} \exp\left(-\frac{E_A}{k_B T}\right) \mathcal{R}_A^B, \quad (5)$$

where $\nu_0 = (E_B - E_A)/hc$ is the wavenumber of the transition, and E_B and E_A are the upper and the lower ro-vibrational energies of the transition; g_A is the nuclear spin statistical weight (for the GeH₄ molecule $g_A = 5, 5, 2, 3$, and 3 for the ro-vibrational states of the A_1 , A_2 , E , F_1 , and F_2 symmetry, respectively, Ref. [51]). The value $Z(T)$ is the partition function. In the present study we used $Z(296.75) = 1724.46$ for the ⁷⁴GeH₄ species which has been calculated with the formula from Ref. [52]. In general case, for the other temperatures, the partition function $Z(T)$ can be easily determined by the formula:

$$\begin{aligned} Q(T) &= Q_v(T) \times Q_r(T) = \sum_v \exp\left(-\frac{hcE_v}{kT}\right) \\ &\times \left(\frac{27}{4}\right) \pi^{1/2} \left(\frac{kT}{hcB_{gr.}}\right)^{3/2} \exp\left(\frac{hcB_{gr.}}{4kT}\right), \end{aligned} \quad (6)$$

which is reproduced from Ref. [52] and valid for the molecules of the XH₄ (T_d symmetry) type with an accuracy no worse than 1 % (in Ref. [52] the notation $Q(T)$ is used for the partition function instead of $Z(T)$). For the calculation of the partition function $Z(T)$, the value of $B_{gr.} = 2.695864734 \text{ cm}^{-1}$ has been taken from Ref. [37]. Analogously, partition functions were obtained for the four other species of germane.

The according results are shown in the last line of Table 5.

The value $\mathcal{R}_A^B = |\langle A | \mu'_Z | B \rangle|^2$ in Eq. (5) is the matrix element of the operator

$$\mu'_Z = G^+ P_Z G \quad (7)$$

on the functions $|A\rangle$ and $|B\rangle$ of the lower and upper ro-vibrational states. The value P_Z is the Z -component of the dipole moment of a molecule (see, e.g., [53], [54]) and, depending on the instantaneous distances between nuclei, it can be written as

$$P_Z = \sum_{\alpha} k_{Z\alpha} \left(\mu_{\alpha}^e + \sum_{\lambda} \mu_{\alpha}^{\lambda} q_{\lambda} + \sum_{\lambda, \nu \geq \lambda} \mu_{\alpha}^{\lambda\nu} q_{\lambda} q_{\nu} + \dots \right). \quad (8)$$

Here $k_{Z\alpha}$ are elements of the direction cosines matrix, Ref. [55]; μ_{α}^e are the components of the equilibrium (permanent) dipole moment of the molecule in the molecular-fixed coordinate system; q_{λ} are dimensionless normal vibrational coordinates, Refs. [56]–[57], of a molecule; and μ_{α}^{λ} , $\mu_{\alpha}^{\lambda\nu}$, ... are the parameters which describe the dependence of dipole moment components μ_{α} on the normal vibrational coordinates. The first terms (μ_{α}^e in Eq. (8)) are responsible for the pure rotational transitions; the second terms (which are proportional to the first order of vibrational coordinates) are responsible for the appearance of the fundamental band transitions in absorption (in more general case, of the bands which correspond to a change of only one vibrational quantum number by unit), etc. The operator G which is presented in Eq. (7) is an unitary operator known from the theory of effective operators (see, *e.g.*, Refs. [56]–[67]). If one takes the results of the mentioned articles into account, then it is possible to show that for an arbitrary polyatomic molecule Eq. (7) can be transformed to the following expression

$$\mu'_Z = \sum_v |0\rangle^v \mu_Z \langle v|, \quad (9)$$

where the values ${}^v\mu_Z$ depend only on the operators $k_{Z\alpha}$ and J_{α} and do not depend on the vibrational operators; $|0\rangle$ and $\langle v|$ are the vibrational function of the lower (in our case, of the ground) and upper vibrational states. In this case, the operators

${}^v\mu_Z$ have the following form:

$${}^v\mu_Z = \sum_j {}^v\mu_j {}^vA_j, \quad (10)$$

and ${}^v\mu_j$ and vA_j are called as the effective dipole moment parameters of a specific vibrational band, $\langle v| \leftarrow |0\rangle$, and symmetrized rotational operators.

It is well known (see, e.g., [40, 41], [68]–[70]), that the ro-vibrational spectroscopic study of the XY₄ (T_d) molecules is most efficient if one uses a special mathematical formalism of the irreducible tensorial sets theory (ITST), [71]–[73]. The biggest advantage of the ITST use consists in the possibility to replace the calculation of numerous matrix elements of different operators by combinations of simple and well-known values, such as Clebsch–Gordan coefficients, 3Γ and 6Γ symbols of the SO(3) and of point symmetry (in our case, T_d –symmetry) groups and of so-called G-reduction matrixes. For the T_d –symmetry molecules all of these coefficients are well known (see, e.g., Refs. [55], [69], [74]–[75]), and they can be easily used for calculation of the matrix elements \mathcal{R}_A^B in Eq. (5) on the basis of formulas, Eqs. (6) – (10). All the above formulas should be adapted to ITST notations. In particular, it is possible to show that the formula, Eq. (9), can be transformed to the following expression [49]:

$$(\mu'_z)^\Gamma = \sum_{\substack{v_1 l_1 \Gamma_1 \\ v_2 l_2 \Gamma_2}} \left([|v_1 l_1 \Gamma_1\rangle \otimes \langle v_2 l_2 \Gamma_2|]^{(\Gamma \otimes \Gamma_r)} \otimes R^{\Omega K(\tilde{K}, n\Gamma_r)} \right)^\Gamma P_{v_1 l_1 \Gamma_1, v_2 l_2 \Gamma_2}^{\Omega K(\tilde{K}, n\Gamma_r)}. \quad (11)$$

In Eq. (11), the $|v_1 l_1 \Gamma_1\rangle$ and $\langle v_2 l_2 \Gamma_2|$ are the symmetrized vibrational functions (v and l are vibrational quantum numbers, Γ is the symmetry of the function); $P_{v_1 l_1 \Gamma_1, v_2 l_2 \Gamma_2}^{\Omega K(\tilde{K}, n\Gamma_r)}$ are the parameters of the effective dipole moment operator for the band $(v_2 l_2 \Gamma_2) \leftarrow (v_1 l_1 \Gamma_1)$; $R^{\Omega K(\tilde{K}, n\Gamma_r)}$ are the symmetrized rotational operators (for more details, see [48], [49]); and the symbol \otimes denotes the direct product, [72].

Finally, for calculation of the matrix elements \mathcal{R}_A^B in Eq. (5), it is necessary to have ro-vibrational functions $|A\rangle$ and $|B\rangle$. For the XY₄ (T_d) molecules, it is clearly logical to take functions which are symmetrized on the basis of ITST. Such ro-vibrational

functions can be taken in the following form:

$$|A\rangle_s^C = |vl\Gamma; Jn\gamma^r; CS\rangle = [|vl\Gamma\rangle \otimes |Jn\gamma^r\rangle]_s^C, \quad (12)$$

where $|Jn\gamma^r\rangle$ is the rotational function of the γ^r symmetry and C is the symmetry of a function of the ro-vibrational state.

In the further analysis, Eqs. (11) and (12) were used for calculation of the matrix elements $\langle v_1 l_1 \Gamma_1; J_1 n_1 \gamma_1^r; C_1 s_1 | (\mu'_z)^\Gamma | v_2 l_2 \Gamma_2; J_2 n_2 \gamma_2^r; C_2 s_2 \rangle$ which are necessary for determination of the values \mathcal{R}_A^B in Eq. (5).

6 Line strengths: Analysis of the $2\nu_2/\nu_2 + \nu_4/2\nu_4$ bending triad of germane: The $^{70}\text{GeH}_4$, $^{72}\text{GeH}_4$, $^{73}\text{GeH}_4$, $^{74}\text{GeH}_4$ and $^{76}\text{GeH}_4$ species

For the line strength analysis of the $\nu_2 + \nu_4(F_2)$ and $\nu_2 + \nu_4(F_1)$ bands, spectrum IIa (see Fig. 2) was used. The line strength analysis of the considerably weaker $2\nu_2$ and $2\nu_4$ bands was made with spectrum I (see Fig. 1). Spectrum I was also used for the strength analysis of transitions of the $\nu_2 + \nu_4$ band which correspond to high values of the quantum number J . The 1697 unblended nonsaturated and not too weak ro-vibrational lines have been chosen in these two experimental spectra (see column 5 of the Supplementary data 2): 382, 511, 116, 556 and 132 for the $^{70}\text{GeH}_4$, $^{72}\text{GeH}_4$, $^{73}\text{GeH}_4$, $^{74}\text{GeH}_4$ and $^{76}\text{GeH}_4$ species, respectively (line positions in column 3 of the Supplementary data 2 are reproduced from Refs. [1, 2] and from the Supplementary data 1 of the present paper). Integrated intensities of these lines were obtained from the fit of their line shapes. A box-car apodization line shape function, [36], [76], and the Hartmann-Tran profile of individual ro-vibrational transition, Refs. [77] – [78], which have been recommended recently for description of isolated high-resolution spectroscopic transitions, Refs. [79], were used in the fit. The Hartmann-Tran profile function is based on the binary collision model, Ref. [80], has the following

form, Ref. [79]:

$$F_{HTP}(\nu) = \frac{1}{\pi} \operatorname{Re} \left\{ \frac{A(\nu)}{1 - [\nu_{vc} - \eta(C_0 - 3C_2/2)]A(\nu) + \left(\frac{\eta C_2}{\nu_{a0}^2}\right) B(\nu)} \right\} \quad (13)$$

and generalizes many of the precursor profile models (see, *e.g.*, Voigt, Refs. [81] – [83]; Rautian, Ref. [84]; speed-dependent Voigt, Refs. [85] – [86]; speed-dependent Rautian, Ref. [87], *etc.*, being one of the speed depended models, see, *e.g.*, Refs. [88] – [94]), the Hartmann – Tran profile allows one to take into account collisional narrowing for isolated spectral lines). In Eq. (13) it is denoted (with notations reproduced from [79]):

$$A(\nu) = \frac{\sqrt{\pi}c}{\nu_0 \nu_{a0}} [\omega(iZ_-) - \omega(iZ_+)] \quad (14)$$

and

$$B(\nu) = \frac{\nu_{a0}^2}{\tilde{C}_2} \left[-1 + \frac{\sqrt{\pi}}{2\sqrt{Y}} (1 - Z_-^2) \omega(iZ_-) - \frac{\sqrt{\pi}}{2\sqrt{Y}} (1 - Z_+^2) \omega(iZ_+) \right], \quad (15)$$

where

$$\omega(z) = \frac{i}{\pi} \int_{-\infty}^{+\infty} \frac{e^{-t^2}}{z-t} dt = e^{-z^2} \operatorname{erfc}(-iz), \quad (16)$$

and "erfc" in Eq. (16) is the Gauss error function, [95]–[96]. In the above expressions

$$Z_{\pm} = \sqrt{X+Y} \pm \sqrt{Y}, \quad (17)$$

$$X = \frac{-i(\nu_0 - \nu) + \tilde{C}_0}{\tilde{C}_2}, Y = \left(\frac{\nu_0 \nu_{a0}}{2c \tilde{C}_2} \right)^2, \quad (18)$$

$$\tilde{C}_0 = (1 - \eta) \left(C_0 - \frac{3C_2}{2} \right) + \nu_{vc}, \quad (19)$$

and

$$\tilde{C}_2 = (1 - \eta) C_2, \quad (20)$$

where $C_n = \Gamma_n + i\Delta_n$ ($n = 0, 2$). To illustrate the quality of the analysis, Fig. 8 shows examples of the lineshape fits from which the line intensities have been determined.

For comparison, the results of the line shape fits for the $R(7, A_2)$ transition of all five isotopologues are shown. The value $\tau/(P \cdot L)$ (in units of $\text{cm}^{-1}\text{atm}^{-1}$) in Fig. 8 is given vs wavenumber ν and

$$\tau(\nu) = S_{\nu_0}^N \cdot F^{HT}(\nu - \nu_0) \cdot N \cdot L; \quad (21)$$

L is the optical path length; $F^{HT}(\nu - \nu_0)$ is the Hartmann–Tran profile function; and $S_{\nu_0} = (S_{\nu_0}^N \cdot N)$ is the individual line intensity which is determined in accordance with the formula, Eq. (5). The values which are necessary for the calculations have been taken in accordance with the discussion in Section 5. In this case, it was taken into account that, as the analysis showed, the studied sample contained also about 0.001% of CO₂, 0.17% of N₂O and 0.49% of H₂O.

At the second step of the analysis, the 556 experimental line intensities of the ⁷⁴GeH₄ species mentioned above were used in the weighted fit of the effective dipole moment parameters (see Eq. (11)) and seven effective dipole moment parameters were obtained (parameters obtained from the fit are shown in column 7 of Table 6 together with their 1σ statistical confidence intervals which are given in parenthesis). These seven parameters, obtained from the fit, reproduce the 556 initial experimental line strengths of ⁷⁴GeH₄ used in the analysis with the $d_{rms} = 3.42\%$ (for more details see statistical information in the bottom part of Table 5). The d_{rms} value was determined in accordance with the formula

$$\left\{ \frac{1}{n} \sum_i \left(100 \times \frac{S_{\nu_i}^{Nexp.} - S_{\nu_i}^{Ncalc.}}{S_{\nu_i}^{Nexp.}} \right)^2 \right\}^{1/2} \equiv \left\{ \frac{1}{n} \sum_i \delta_i^2 \right\}^{1/2}, \quad (22)$$

where n is the number of individual line strengths used in the fit.

Columns 4 and 5 of the Supplementary data 2 show experimental values of line strengths (in $\text{cm}^{-2}\cdot\text{atm}^{-1}$) and values δ_i (in per cent) of differences between the experimental line strengths and line strengths calculated with the parameters from column 7 of Table 6. For the convenience of the reader, a small part of the Supplementary data 2 is shown in Table 5. To illustrate the quality of the result, Fig. 9b

presents the fit residuals for line intensities as a function of the quantum number J (also see simulation of the spectra in the corresponding tracks of Figs. 1–6).

As the last step, the weighted fit of the effective dipole moment parameters of the four other isotopologues was made. In this case, the biggest part of the parameters of the $^{70}\text{GeH}_4$, $^{72}\text{GeH}_4$, $^{73}\text{GeH}_4$ and $^{76}\text{GeH}_4$ isotopologues was constrained to the values of corresponding parameters of the $^{74}\text{GeH}_4$ and was not varied in the fit procedure. Values of the $P_{(0000,A_1)(0002,F_2)}^{(00A_1)}$ parameters of the $^{73}\text{GeH}_4$ and $^{76}\text{GeH}_4$ isotopologues have been estimated from interpolation/extrapolation of the values of the $P_{(0000,A_1)(0002,F_2)}^{(00A_1)}$ parameters for the three other isotopologues. These two values also were not varied in the fit. The obtained values of fitted parameters are shown in columns 4, 5, 6 and 8 of Table 6. One can see a good correlation between values of different isotopologues. The 2, 2, 1 and 1 parameters obtained from the fit reproduce experimental strengths of 382, 511, 116 and 132 lines of $^{70}\text{GeH}_4$, $^{72}\text{GeH}_4$, $^{73}\text{GeH}_4$ and $^{76}\text{GeH}_4$ with the d_{rms} equal to 3.46, 3.49, 3.34 and 3.36 per cent, respectively. As for the $^{74}\text{GeH}_4$ molecule, the results of the line strengths calculation, as well as δ_i differences for the $^{70}\text{GeH}_4$, $^{72}\text{GeH}_4$, $^{73}\text{GeH}_4$ and $^{76}\text{GeH}_4$ species are shown in columns 4 and 5 of the Supplementary data 2. As a confirmation of the correctness of the results, see also Figs. 9a, 9c, 9d and 9e where the fit residuals for line intensities as functions of the quantum number J are presented, and the bottom tracks of Figs. 1 – 6.

7 Line shape analysis of the $2\nu_2/\nu_2 + \nu_4/2\nu_4$ pentad bending triad of germane

For the line width analysis (full width at half maximum, FWHM), we exploited six spectra, IIa–IIf recorded at different pressures from 20 hPa up to 150 hPa (see Table 1 for details). Lines of the $\nu_2 + \nu_4(F_2)$ and $\nu_2 + \nu_4(F_1)$ sub–bands of the $\nu_2 + \nu_4$ band of all five isotopologues were analysed using a multi–spectrum fit with the Hartmann–Tran profile (see Fig. 10 where the same lines as in Fig. 8 are shown) in

accordance with the formula

$$\gamma = \gamma_{self} \times P \quad (23)$$

for the self-broadening γ_{self} -coefficient and

$$\delta_{\nu_0} = \delta_{self} \times P \quad (24)$$

for the shift δ_{self} -coefficients. The obtained values of the γ_{self} -coefficients (for 993 lines) and δ_{self} -coefficients (for 676 lines) are shown in columns 6 and 7 of the Supplementary data 2. For illustration of the quality of the performed analysis, the upper part of Fig. 10 presents the pressure dependence of the half-width for the same $R(7, A_2)$ transition of the $\nu_2 + \nu_4(F_2)$ band (spectra IIa–IIf) which are shown in Fig. 8. The bottom part of Fig. 10 illustrates the correctness of the determined self-broadening γ_{self} and shifts δ_{self} coefficients.

8 Conclusion

For the first time we present a comprehensive quantitative ro-vibrational study of line positions, strengths, widths and shifts in the high-resolution spectra of the pentad bending triad of all five stable germane isotopologues. We performed the joint ro-vibrational analysis of 16422 line positions in 35 "cold" and 23 "hot" bands of the $^{70}\text{GeH}_4$, $^{72}\text{GeH}_4$, $^{73}\text{GeH}_4$, $^{74}\text{GeH}_4$, $^{76}\text{GeH}_4$ isotopologues and obtained a set of 129 effective spectroscopic parameters (upper vibrational energies, rotational, centrifugal distortion, tetrahedral and resonance interaction parameters) from the weighted fit which reproduce the initial 16422 experimental line positions with the $d_{rms} = 3.26 \times 10^{-4} \text{ cm}^{-1}$. Line strengths of 1697 individual ro-vibrational lines belonging to the thirty five different bands of all five isotopologues were obtained from the fit of their experimental line shapes with the Hartmann-Tran profile. A set of 13 effective dipole moment parameters obtained from the weighted fit allows one to reproduce 1697 experimental line strength with the $d_{rms} = 3.4\%$. Self-broadening and self-shifts coefficients are obtained for the same 1697 lines of the pentad bending

triad of germane. The present data will be soon included in the VAMDC/GeCaSDa database [33].

9 Acknowledgments

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10 Figure captions

Fig. 1. Experimental survey spectrum I of germane in the region of the first dyad (upper trace 1a). Experimental conditions: absorption path length is 24 m; room temperature $(23.6 \pm 0.1)^\circ\text{C}$; number of scans is 1400; sample pressure is 250 Pa. Traces (1c)–(1g) show the simulated spectra of separate germane isotopologues; trace (1b) is the global simulated spectrum as a sum of the simulated (1c)–(1g) spectra. Lines belonging to the $\nu_2 + \nu_4$, $2\nu_4$ and $2\nu_2$ bands are marked by the black, orange and red color.

Fig. 2. Experimental survey spectrum IIa of germane in the region of the first dyad $\nu_2 + \nu_4$ band (upper trace). Experimental conditions: absorption path length is 0.23 m; room temperature $(23.8 \pm 0.3)^\circ\text{C}$; number of scans is 400; sample pressure is 20 hPa. The lower trace shows the corresponding simulated spectrum.

Fig. 3. Small part of the high resolution experimental spectrum IIa of germane in the region of the $\nu_2 + \nu_4$ band (upper trace). The lower trace shows the corresponding simulated spectrum. The cluster $P(9, \Gamma)$ ($\Gamma = A_2, F_2, F_1$ and A_1) of the $\nu_2 + \nu_4(F_2)$ sub-band of all five ${}^M\text{GeH}_4$ isotopologues is marked.

Fig. 4. Small part of the high resolution experimental spectrum IIa of germane in the region of the $\nu_2 + \nu_4$ band (upper trace). The lower trace shows the corresponding simulated spectrum. The cluster $R(8, \Gamma)$ ($\Gamma = E, F_1$ and A_1) of the $\nu_2 + \nu_4(F_2)$ sub-band of all five ${}^M\text{GeH}_4$ isotopologues is marked.

Fig. 5. Small part of the high resolution experimental spectrum I of germane in the region of the $2\nu_4$ band (upper trace). The lower traces (5c)–(5g) show the corresponding simulated spectra for the individual isotopologues. The cluster $P(12, \Gamma)$ ($\Gamma = E, F_1$ and A_1) of the $2\nu_4(F_2)$ sub-band of all five ${}^M\text{GeH}_4$ isotopologues is marked. Trace (5b) is the global simulated spectrum as the sum of the simulated

(5c)–(5g) spectra .

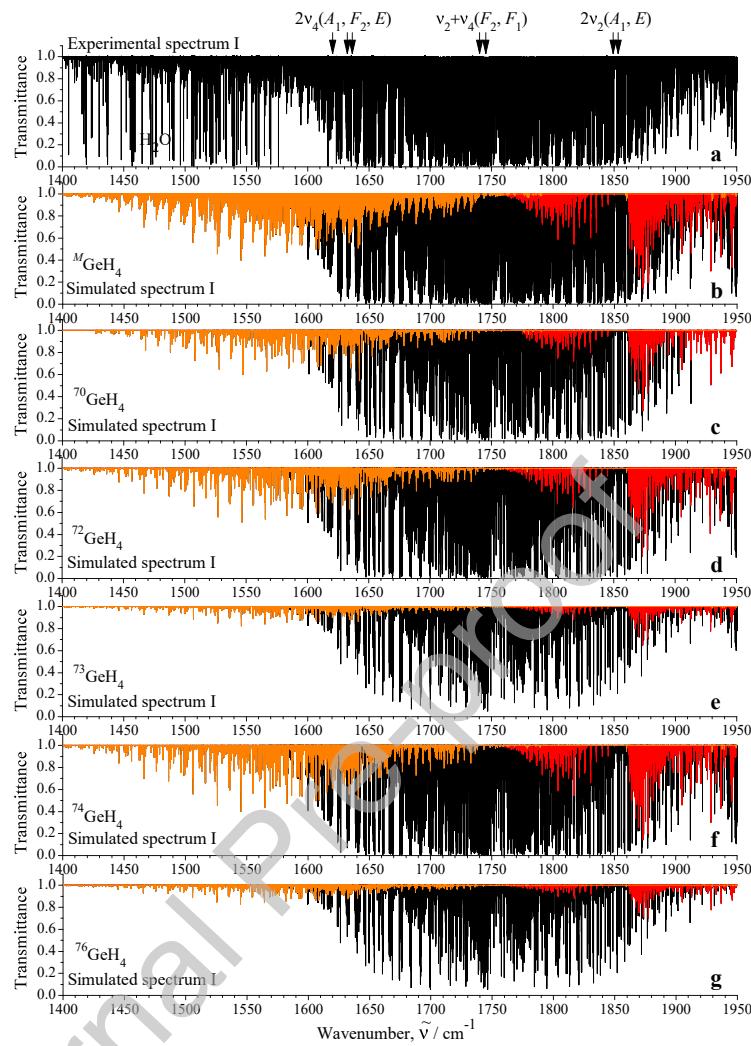
Fig. 6. Small part of the high resolution experimental spectrum I of germane in the region of the $2\nu_2$ band (upper trace). Lower traces (5c)–(5g) show the corresponding simulated spectra for the individual isotopologues. The clusters $Q(9, \Gamma)$ ($\Gamma = F_2, F_1$ and A_1) and $Q(10, \Gamma)$ ($\Gamma = A_2, F_2$ and E) of the $2\nu_2(E)$ sub-band of all five ${}^M\text{GeH}_4$ isotopologues is marked. Trace (5b) is the global simulated spectrum as the sum of simulated (5c)–(5g) spectra .

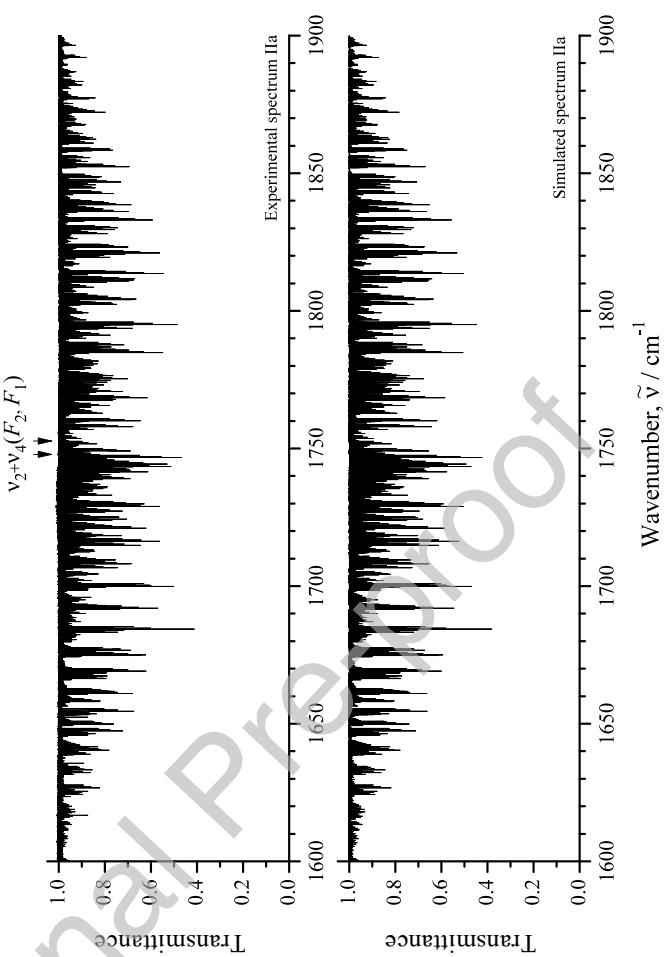
Fig. 7. Observed minus calculated line positions and fit statistics for "cold" bands studied in the present paper of the ${}^{74}\text{GeH}_4$, ${}^{72}\text{GeH}_4$ and ${}^{70}\text{GeH}_4$ species.

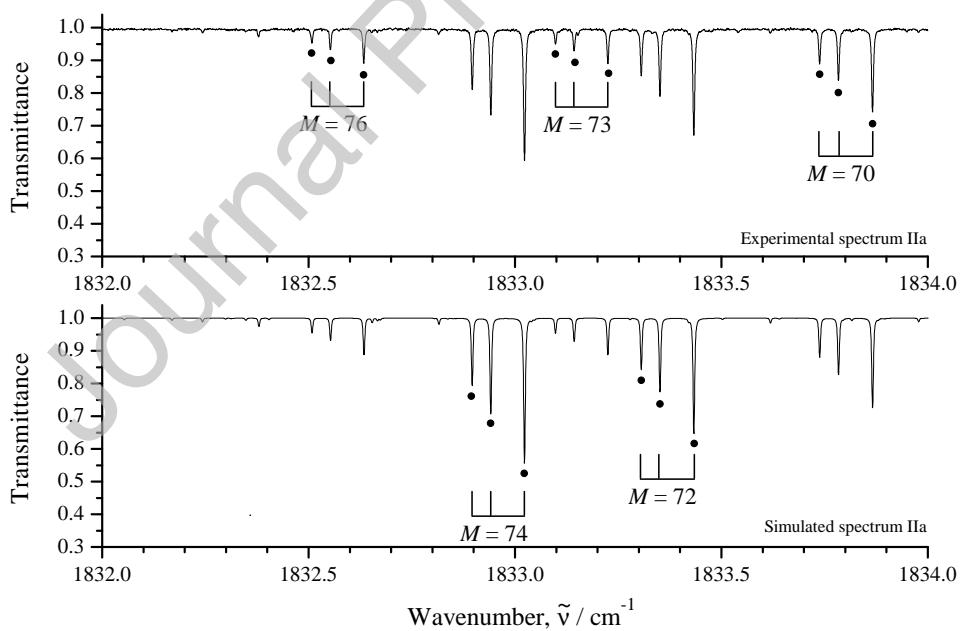
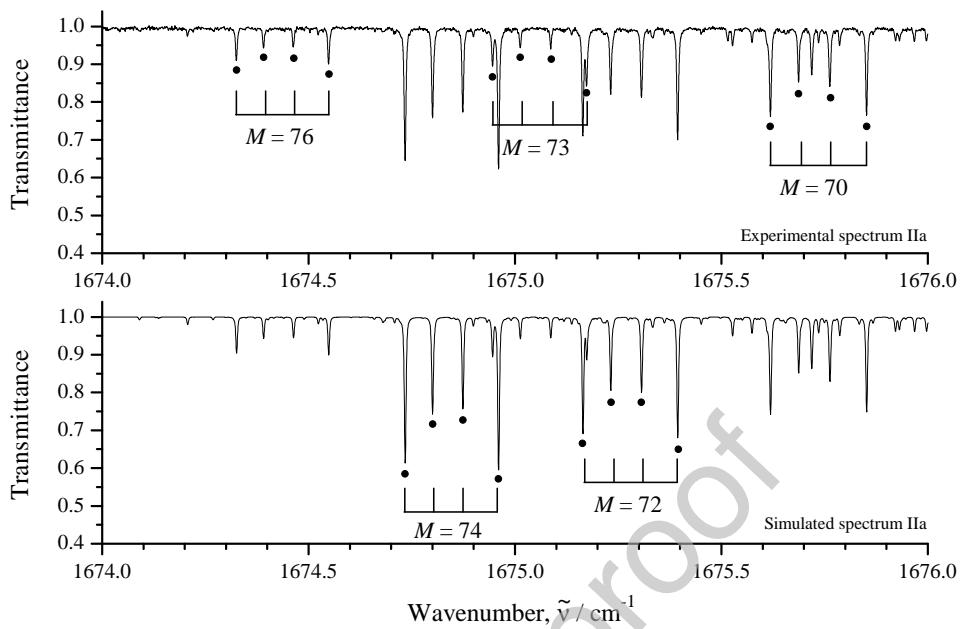
Fig. 8. Experimental line shape of the $R(7, A_2)$ transition of the $\nu_2 + \nu_4(F_2)$ band for five different isotopologues of germane (for experimental conditions, see Section 2 and Table 1). The fit of the experimental line shapes was made with the Hartmann–Tran profile of individual lines. The solid and dashed lines correspond to the experimental and calculated value of $\tau(\tilde{\nu})$. The bottom part of the figures shows the (exp.–calc.) residuals.

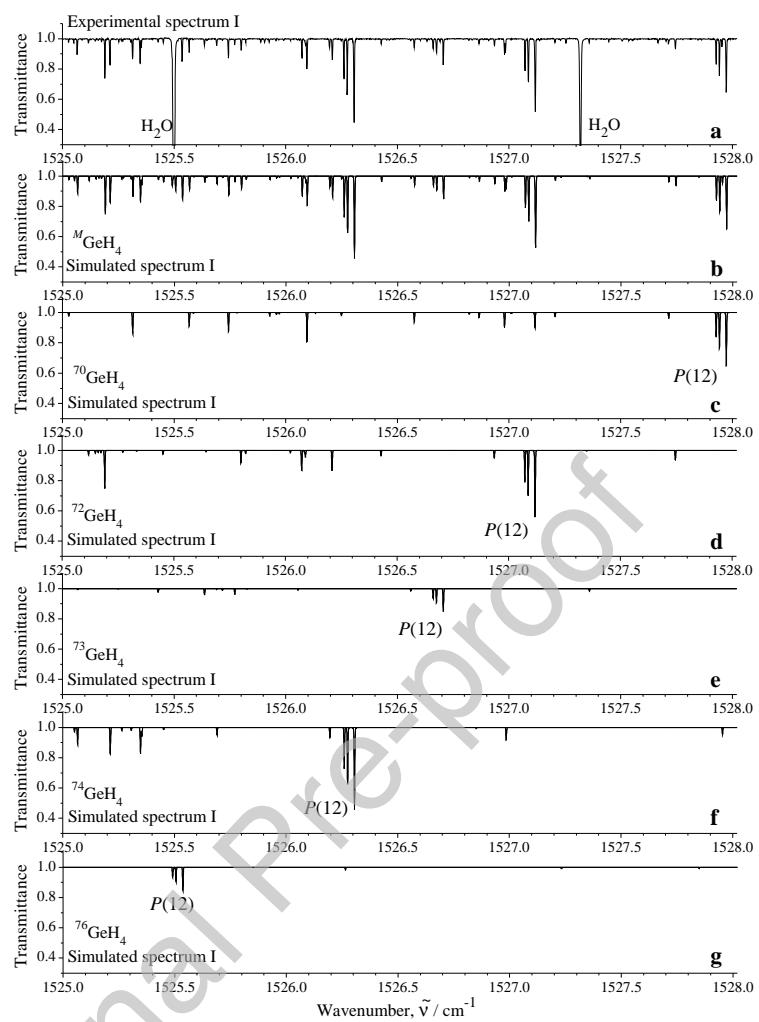
Fig. 9. Observed minus calculated line intensities (in per cent) including fit statistics for the $2\nu_2$, $\nu_2 + \nu_4$ and $2\nu_4$ bands of the five stable isotopologues of germane.

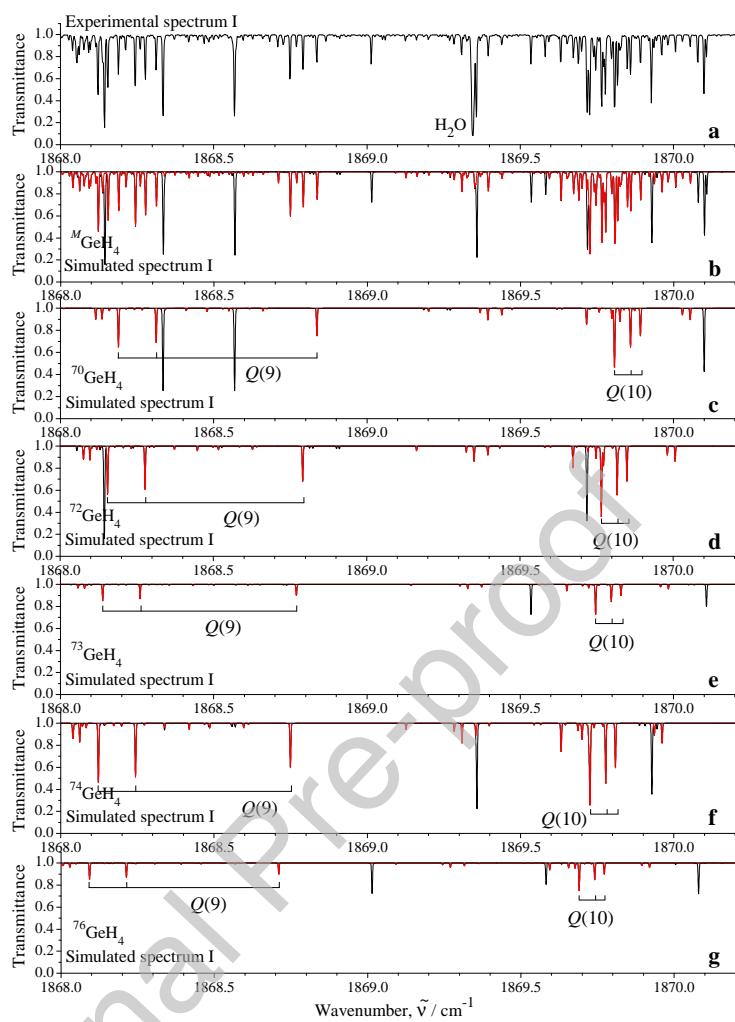
Fig. 10. Upper part: Examples of FTIR line spectra IIa–IIf recorded at different pressures for the $R(7, A_2)$ transition of the $\nu_2 + \nu_4(F_2)$ band for five different isotopologues of germane (for experimental conditions, see Section 2 and Table 1). Bottom part: measured self–broadening and maximum line–shift coefficients at room temperature vs pressure.

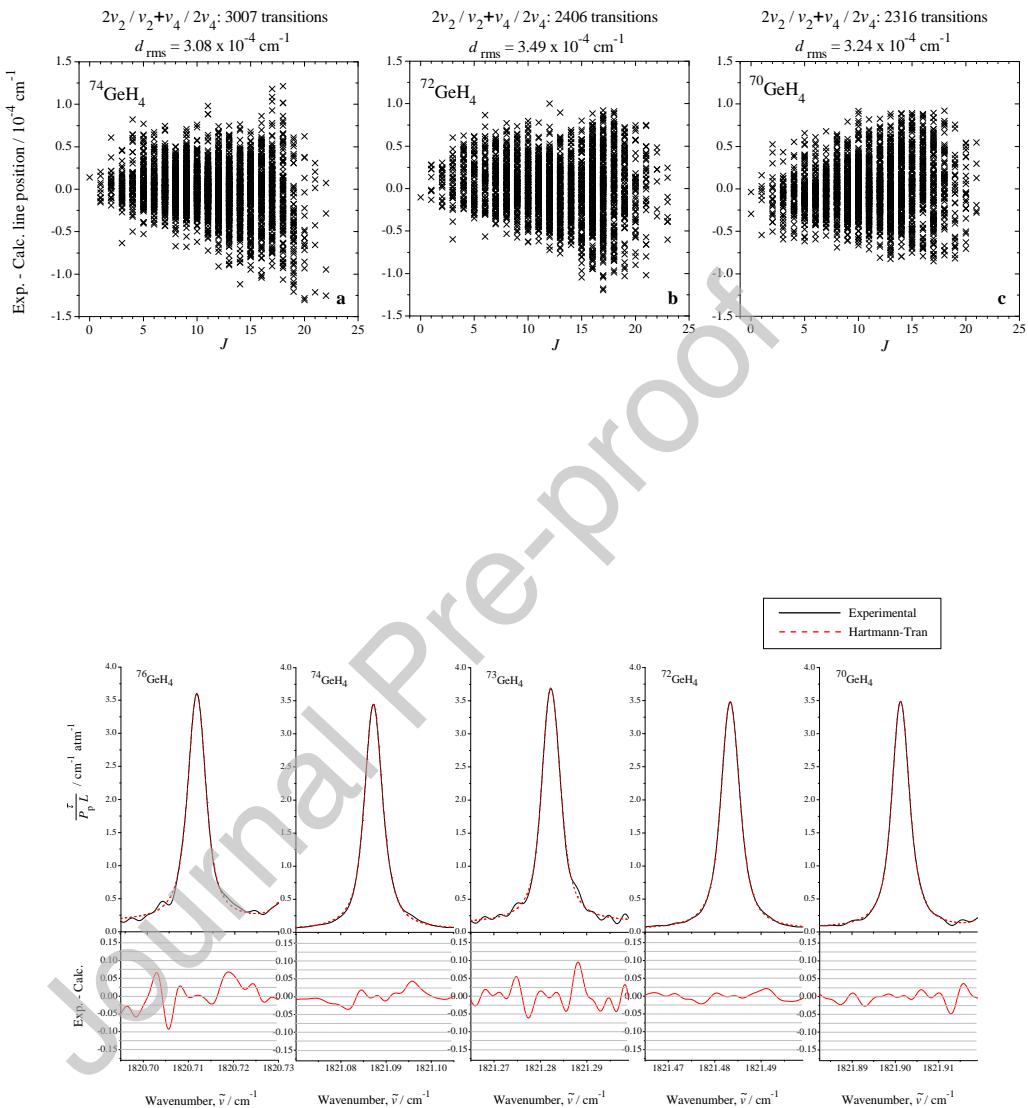












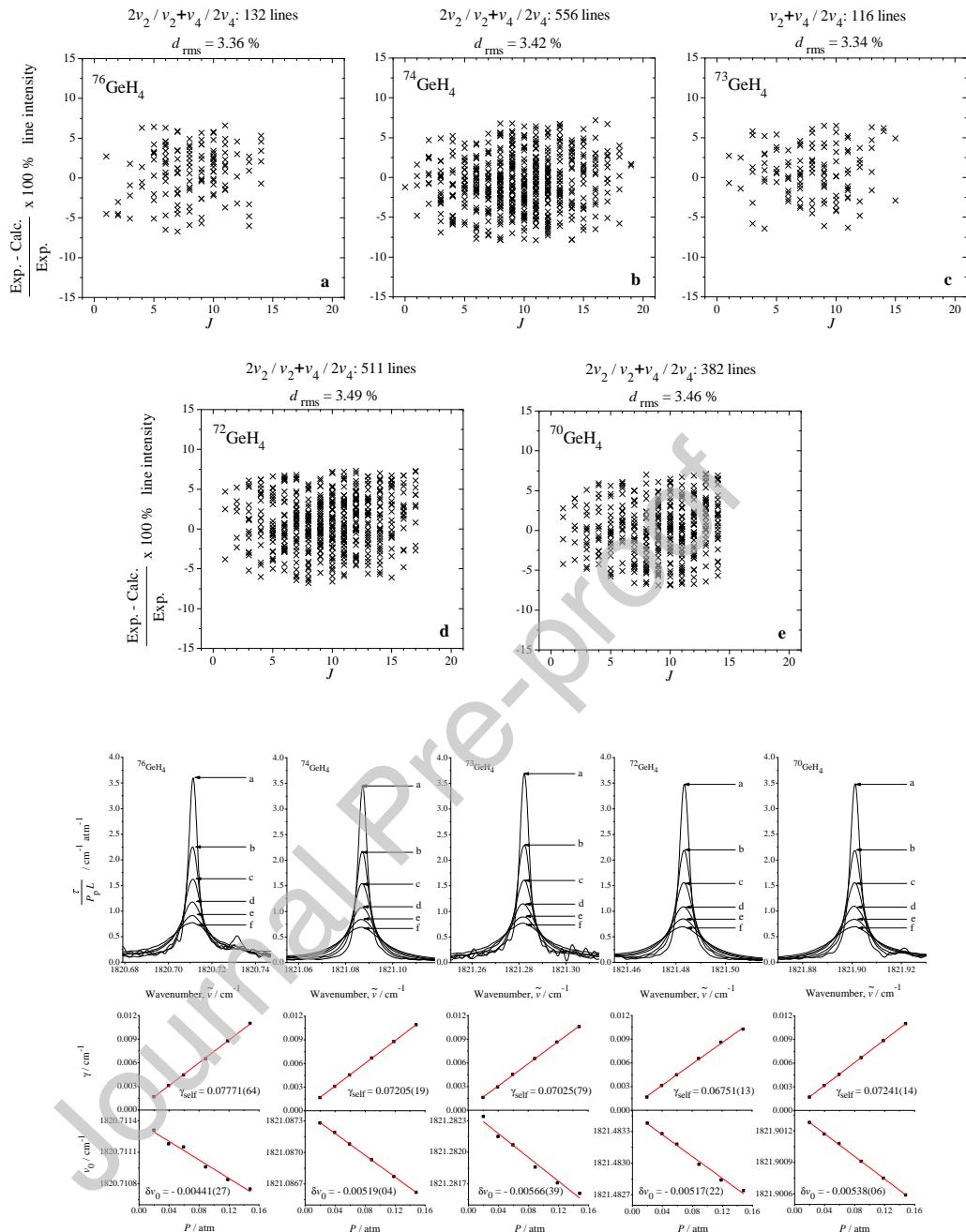


Table 1. Experimental setup of the infrared spectra of ${}^9\text{GeH}_4$ ($M=70, 72, 73, 74, 76$).

Spectrum	Region /cm ⁻¹	Resolution /cm ⁻¹	No. of scans	Source	Detector	Beam-splitter	Opt. path-length/m	Temp. /°C	Pressure /Pa	Calibr. gas
I	600 - 4800	0.003	1400	Globar	MCT313	KBr	24	23.6 ± 0.1	250	$\text{H}_2\text{O}, \text{N}_2\text{O}$
Ia	1400 - 2700	0.003	400	Globar	MCT313	KBr	0.23	23.8 ± 0.3	2000	H_2O
Ib	1400 - 2700	0.003	300	Globar	MCT313	KBr	0.23	23.7 ± 0.3	4000	H_2O
Ic	1400 - 2700	0.003	520	Globar	MCT313	KBr	0.23	23.6 ± 0.3	6000	H_2O
Id	1400 - 2700	0.003	600	Globar	MCT313	KBr	0.23	23.5 ± 0.7	9000	H_2O
Ie	1400 - 2700	0.003	680	Globar	MCT313	KBr	0.23	23.4 ± 0.7	12000	H_2O
If	1400 - 2700	0.003	600	Globar	MCT313	KBr	0.23	23.5 ± 0.7	15000	H_2O

Table 2. Statistical information for the $2v_2(A_1, E)$, $v_2 + v_4(F_1, F_2)$ and $2v_4(A_1, E, F_2)$ bands of $^{24}\text{GeH}_4$ ($M = 70, 72, 73, 74, 76$).

Band	Energy ^a /cm ⁻¹	δ^{exp}	N_{1^+} ^b	N_1 ^c	m_1 ^d	m_2 ^d	m_3 ^d	
1	2	3	4	5	6	7	8	9
$^{76}\text{GeH}_4$								
$2v_4(A_1) - v_4(F_2)$	1627.4950	11	76	29	74.9	11.9	9.2	[1]
$2v_4(F_2) - v_4(F_2)$	1628.1775	10	230	101	79.4	8.2	4.2	[1]
$2v_4(E) - v_4(F_2)$	1630.2570	15	100	189	54.6	24.8	20.6	[1]
$2v_4(E) - v_4(F_1)$	1642.1422	13	52	101	52.2	17.7	12.7	[1]
$v_2 + v_4(F_2) - v_4(F_2)$	1642.1422	13	28	43	35.7	28.6	35.7	[1]
$v_2 + v_4(F_2) - v_4(F_1)$	1642.1422	16	119	56.2	28.1	15.7	[1]	
$v_2 + v_4(F_2) - v_4(F_1)$	1642.1422	15	230	101	52.2	17.7	12.7	[1]
$v_2 + v_4(F_2) - v_4(F_2)$	1748.3962	19	793	401	59.9	26.5	13.6	[1]
$v_2 + v_4(F_1) - v_4(F_2)$	1748.3962	14	45	50	34.5	18.3	13.3	[1]
$v_2 + v_4(F_1) - v_4(F_1)$	1748.3962	15	179	59.6	30.1	10.3	[1]	
$2v_2(A_1) - v_2(E)$	1752.5031	16	259	289	100.0	34.0	20.0	[1]
$2v_2(A_1) - v_2(E)$	1857.2721	18	131	64	73.1	15.9	11.0	[1]
$2v_2(E) - v_2(E)$	1860.6673	17	34	55.6	32.1	12.3	[1]	
$2v_2(E) - v_2(E)$	1860.6673	19	503	257	80.8	12.7	6.5	[1]
Total N_{1^+} (cold bands)		2254						
Total N_{1^+} (hot bands)		994						[1]
Total N_1		1272						[1]
d_{exp} (cold bands)	2.64×10^{-4} cm ⁻¹							[1]
d_{exp} (hot bands)	2.91×10^{-4} cm ⁻¹							[1]
$^{74}\text{GeH}_4$								
$2v_4(A_1)$	1628.2541	11	17	7	53.0	47.0	0.0	tw
$2v_4(F_2)$	1640.4120	20	230	101	52.2	17.7	12.7	tw
$2v_4(E) - v_4(F_2)$	1642.9985	17	82	48	75.6	14.6	9.8	tw
$v_2 + v_4(F_2) - v_4(F_2)$	1643.3059	14	50	50	34.5	18.3	13.3	tw
$v_2 + v_4(F_2) - v_4(F_1)$	1748.7773	15	159	156	71.3	21.5	7.2	[1]
$v_2 + v_4(F_2) - v_4(F_1)$	1748.7768	22	1171	455	60.9	23.7	15.4	tw
$v_2 + v_4(F_1) - v_4(F_2)$	1748.7768	14	45	50	34.5	18.3	13.3	[1]
$v_2 + v_4(F_1) - v_4(F_1)$	1752.8865	13	150	129	58.8	28.1	13.1	[1]
$v_2 + v_4(F_1) - v_4(F_1)$	1752.8864	16	105	105	58.8	28.1	7.7	[1]
$2v_2(A_1)$	1857.2650	16	77	42	42.9	28.6	28.5	tw
$2v_2(E)$	1860.6593	21	362	200	43.7	28.5	27.8	tw
Total N_{1^+} (cold bands)		3607						
Total N_{1^+} (hot bands)		99						[1]
Total N_1		1459						tw
d_{exp} (cold bands)	3.08×10^{-4} cm ⁻¹							[1]
d_{exp} (hot bands)	2.9×10^{-4} cm ⁻¹							[1]
$^{73}\text{GeH}_4$								
$2v_4(A_1) - v_4(F_2)$	1628.6480	19	268	128	66.3	18.8	11.2	[2]
$2v_4(F_2) - v_4(F_2)$	1640.4120	22	665	445	70.0	19.7	10.3	[2]
$2v_4(E) - v_4(F_2)$	1643.3059	20	171	105	52.2	17.7	12.7	[2]
$v_2 + v_4(F_2) - v_4(F_1)$	1643.3059	19	399	63.8	25.8	10.5	[2]	
$v_2 + v_4(F_2) - v_4(F_2)$	1748.9748	13	333	133	40.0	12.5	8.5	[2]
$v_2 + v_4(F_2) - v_4(F_2)$	1748.9748	18	465	419	74.2	16.2	9.6	[2]
$v_2 + v_4(F_1) - v_4(F_2)$	1748.9748	19	354	40	67.8	22.4	10.6	[2]
$v_2 + v_4(F_1) - v_4(F_2)$	1748.9748	17	69	40	67.8	22.4	10.6	[2]
$v_2 + v_4(F_1) - v_4(F_1)$	1753.0649	16	294	337	70.1	25.7	4.9	[2]
$v_2 + v_4(F_1) - v_4(F_1)$	1753.0649	14	8	7	4.0	0.0	0.0	[2]
$2v_2(A_1) - v_2(E)$	1857.2607	19	12	41.7	41.7	41.7	16.6	[2]
$2v_2(E) - v_2(E)$	1860.6552	14	114	94	52.7	28.5	18.2	[2]
Total N_{1^+} (cold bands)		873						
Total N_{1^+} (hot bands)		2119						
Total N_1		1535						
d_{exp} (cold bands)	2.55×10^{-4} cm ⁻¹							
d_{exp} (hot bands)	3.00×10^{-4} cm ⁻¹							
$^{72}\text{GeH}_4$								
$2v_4(A_1) - v_4(F_2)$	1629.0544	20	299	453	67.1	22.1	10.5	[3]
$2v_4(F_2) - v_4(F_2)$	1640.8186	20	131	406	47.4	31.3	21.3	tw
$2v_4(E) - v_4(F_2)$	1641.6643	22	578	507	55.4	26.8	17.8	tw
$2v_4(E) - v_4(F_2)$	1643.7775	21	138	134	63.0	31.9	25.0	tw
$v_2 + v_4(F_2) - v_4(F_2)$	1746.7157	17	146	146	65.8	22.6	11.6	[3]
$v_2 + v_4(F_2) - v_4(F_2)$	1746.7157	16	164	386	73.8	20.5	8.8	[3]
$v_2 + v_4(F_2) - v_4(F_2)$	1746.7157	23	733	515	64.9	23.5	17.6	tw
$v_2 + v_4(F_1) - v_4(F_2)$	1746.7157	16	183	337	78.7	16.4	4.9	[3]
$v_2 + v_4(F_1) - v_4(F_2)$	1746.7157	17	62	54	34.4	13.4	10.3	[3]
$v_2 + v_4(F_1) - v_4(F_2)$	1753.2898	15	5	40	40.0	60.0	0.0	[3]
$v_2 + v_4(F_1) - v_4(F_2)$	1753.2898	17	67	40	32.7	24.2	12.2	[3]
$2v_2(A_1) - v_2(E)$	1857.2676	17	106	46.2	32.7	21.1	[3]	[3]
$2v_2(E) - v_2(E)$	1860.6515	20	393	254	41.7	28.5	29.8	tw
Total N_{1^+} (cold bands)		2406						tw
Total N_{1^+} (hot bands)		2551						[3]
Total N_1		2030						tw
d_{exp} (cold bands)	3.00×10^{-4} cm ⁻¹							[3]
d_{exp} (hot bands)	3.11×10^{-4} cm ⁻¹							[3]
$^{70}\text{GeH}_4$								
$2v_4(A_1)$	1620.8004	16	35	23	57.2	25.7	17.1	tw
$2v_4(F_2)$	1641.6643	18	409	201	50.4	29.6	20.0	tw
$2v_4(E)$	1643.7775	19	111	74	52.2	23.9	11.2	tw
$v_2 + v_4(F_2)$	1749.6524	21	839	418	53.8	23.9	22.3	tw
$v_2 + v_4(F_1)$	1753.7155	21	794	329	67.4	21.7	10.9	tw
$v_2 + v_4(F_1)$	1753.7155	23	53	4	3.7	20.0	20.0	tw
$2v_2(A_1) - v_2(E)$	1860.6437	20	120	91	37.5	28.4	65.9	tw
Total N_{1^+} (cold bands)		2316						tw
Total N_{1^+} (hot bands)		1228						tw
Total N_1		2544						tw
d_{exp} (cold bands)	3.24×10^{-4} cm ⁻¹							tw

^a Value of the energy difference.^b N_{1^+} is the number of observed transitions.^c N_1 is the number of obtained upper-state energies.^d Here $m_i = n_1 N_{1^+} \times 100\%$ ($i = 1, 2, 3$); n_1 , n_2 , and n_3 are the numbers of transitions for which the differences $\delta = \nu^{\text{exp}} - \nu^{\text{calc}}$ satisfy the conditions $\delta \leq z \leq 10^{-4}$ cm⁻¹, 2×10^{-4} cm⁻¹ $< \delta \leq 4 \times 10^{-4}$ cm⁻¹, and $\delta > 4 \times 10^{-4}$ cm⁻¹.

a	e	3	ec	r	sc	ic	aram	ers	$\frac{\Omega(K, n\Gamma)}{v}$	$\frac{\Gamma}{v' \cdot v^r \cdot r}$	e	se	in	era	c	n	ence	in	er	s	ra			
a es in aren ces are s is ca c n ence in er va arame ers resen e i c n ence in er va s er e c n ns raine																								
1	2	3	4	70	e	4	72	e	4	73	e	4	74	e	4	7	e	4	70	e	4			
(0200, A_1)	(0200, A_1)	0(0, A_1)	2.6	369	2.6	369	2.6	369	2.6	369	2.6	369	2.6	369	2.6	369	2.6	369	2.6	369	2.6	369		
(0200, A_1)	(0200,)	2(0, A_1) 10^3	0.256300(53)	0.256050(67)	0.255925(4)	0.255 00(69)	0.25550(52)																	
(0200,)	(0200,)	0(0, A_1)	0.409419(71)	0.409 73(4)	0.410100(69)	0.40 000()	0.4107 1(62)																	
(0200,)	(0200,)	2(2, A_1) 10^3	0.3393	0.3393	0.3393	0.3393	0.3393	0.3393	0.3393	0.3393	0.3393	0.3393	0.3393	0.3393	0.3393	0.3393	0.3393	0.3393	0.3393	0.3393	0.3393			
(0200,)	(0200,)	3(3, A_2) 10	0.26045	0.26045	0.26045	0.26045	0.26045	0.26045	0.26045	0.26045	0.26045	0.26045	0.26045	0.26045	0.26045	0.26045	0.26045	0.26045	0.26045	0.26045	0.26045			
(0200, A_1)	(0101, F_2)	2(2, F_2) 10^3	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11		
(0101, F_1)	(0101, F_1)	1(1, F_1)	0.295675(5)	0.295651(3)	0.29567024()	0.29567437(42)	0.29567263(52)																	
(0101, F_1)	(0101, F_1)	2(2, F_1) 10^3	0.21017	0.21017	0.21017	0.21017	0.21017	0.21017	0.21017	0.21017	0.21017	0.21017	0.21017	0.21017	0.21017	0.21017	0.21017	0.21017	0.21017	0.21017	0.21017	0.21017		
(0200,)	(0101, F_2)	3(3, F_2) 10	0.4234	0.4234	0.4234	0.4234	0.4234	0.4234	0.4234	0.4234	0.4234	0.4234	0.4234	0.4234	0.4234	0.4234	0.4234	0.4234	0.4234	0.4234	0.4234	0.4234		
(0200,)	(0101, F_2)	3(1, F_1) 10	0.03145	0.03145	0.03145	0.03145	0.03145	0.03145	0.03145	0.03145	0.03145	0.03145	0.03145	0.03145	0.03145	0.03145	0.03145	0.03145	0.03145	0.03145	0.03145	0.03145		
(0200,)	(0101, F_2)	3(1, F_1) 10	0.4543	0.4543	0.4543	0.4543	0.4543	0.4543	0.4543	0.4543	0.4543	0.4543	0.4543	0.4543	0.4543	0.4543	0.4543	0.4543	0.4543	0.4543	0.4543	0.4543		
(0200, A_1)	(0002, A_1)	0(0, A_1)	5.46 35()	5.4613(16)	5.45797()	5.456 19(19)	5.45261(14)																	
(0200,)	(0002, A_1)	0(0, A_1)	0.195659(35)	0.195666(51)	0.195672(11)	0.170639(39)	0.1717(14)																	
(0200,)	(0002, A_1)	2(0, A_1) 10^3	0.03027(41)	0.03027(41)	0.03027(41)	0.03027(41)	0.03027(41)	0.03027(41)	0.03027(41)	0.03027(41)	0.03027(41)	0.03027(41)	0.03027(41)	0.03027(41)	0.03027(41)	0.03027(41)	0.03027(41)	0.03027(41)	0.03027(41)	0.03027(41)	0.03027(41)	0.03027(41)		
(0200,)	(0002, F_2)	2(3, F_2) 10^3	0.2390 46(47)	0.2390 46(47)	0.2390 46(47)	0.2390 46(47)	0.2390 46(47)	0.2390 46(47)	0.2390 46(47)	0.2390 46(47)	0.2390 46(47)	0.2390 46(47)	0.2390 46(47)	0.2390 46(47)	0.2390 46(47)	0.2390 46(47)	0.2390 46(47)	0.2390 46(47)	0.2390 46(47)	0.2390 46(47)	0.2390 46(47)	0.2390 46(47)		
(0101, F_1)	(0101, F_1)	3(1, F_1) 10	0.3016	0.3016	0.3016	0.3016	0.3016	0.3016	0.3016	0.3016	0.3016	0.3016	0.3016	0.3016	0.3016	0.3016	0.3016	0.3016	0.3016	0.3016	0.3016	0.3016		
(0101, F_1)	(0101, F_1)	4(1, F_1)	0.2699	0.2699	0.2699	0.2699	0.2699	0.2699	0.2699	0.2699	0.2699	0.2699	0.2699	0.2699	0.2699	0.2699	0.2699	0.2699	0.2699	0.2699	0.2699	0.2699		
(0101, F_1)	(0101, F_1)	5(1, F_1)	0.0510521	0.0510521	0.0510521	0.0510521	0.0510521	0.0510521	0.0510521	0.0510521	0.0510521	0.0510521	0.0510521	0.0510521	0.0510521	0.0510521	0.0510521	0.0510521	0.0510521	0.0510521	0.0510521	0.0510521		
(0101, F_1)	(0101, F_1)	6(1, F_1)	0.5059	0.5059	0.5059	0.5059	0.5059	0.5059	0.5059	0.5059	0.5059	0.5059	0.5059	0.5059	0.5059	0.5059	0.5059	0.5059	0.5059	0.5059	0.5059	0.5059		
(0101, F_1)	(0101, F_1)	7(1, F_1)	0.7694	0.7694	0.7694	0.7694	0.7694	0.7694	0.7694	0.7694	0.7694	0.7694	0.7694	0.7694	0.7694	0.7694	0.7694	0.7694	0.7694	0.7694	0.7694	0.7694		
(0101, F_1)	(0101, F_1)	8(1, F_1)	0.056470	0.056470	0.056470	0.056470	0.056470	0.056470	0.056470	0.056470	0.056470	0.056470	0.056470	0.056470	0.056470	0.056470	0.056470	0.056470	0.056470	0.056470	0.056470	0.056470		
(0101, F_1)	(0101, F_1)	9(1, F_1)	0.0565 91(33)	0.0565 91(33)	0.0564319(20)	0.0564319(20)	0.0563944(60)	0.0563944(60)	0.0562747(25)															
(0101, F_1)	(0101, F_1)	10(1, F_1)	0.0565 91(33)	0.0565 91(33)	0.0564319(20)	0.0564319(20)	0.0563944(60)	0.0563944(60)	0.0562747(25)															
(0101, F_1)	(0101, F_1)	11(1, F_1)	0.5 1 1	0.5 1 1	0.5 1 1	0.5 1 1	0.5 1 1	0.5 1 1	0.5 1 1	0.5 1 1	0.5 1 1	0.5 1 1	0.5 1 1	0.5 1 1	0.5 1 1	0.5 1 1	0.5 1 1	0.5 1 1	0.5 1 1	0.5 1 1	0.5 1 1	0.5 1 1		
(0101, F_2)	(0101, F_2)	0(0, A_1)	2.015121(63)	2.0143(14)	2.014017(2)	2.014017(52)	2.014759(52)	2.013554(10)																
(0101, F_2)	(0101, F_2)	1(1, F_1)	0.05499 2()	0.054 642(13)	0.054792(2)	0.054792(56)	0.0547456(67)	0.0546425(9)																
(0101, F_2)	(0101, F_2)	2(0, A_1) 10^3	0.37 9()	0.37 74(61)	0.37 622()	0.37 622()	0.37 622()	0.37 622()	0.37 622()	0.37 622()	0.37 622()	0.37 622()	0.37 622()	0.37 622()	0.37 622()	0.37 622()	0.37 622()	0.37 622()	0.37 622()	0.37 622()	0.37 622()	0.37 622()		
(0101, F_2)	(0101, F_2)	3(2, F_2) 10^3	0.3357	0.3357	0.3357	0.3357	0.3357	0.3357	0.3357	0.3357	0.3357	0.3357	0.3357	0.3357	0.3357	0.3357	0.3357	0.3357	0.3357	0.3357	0.3357	0.3357		
(0101, F_2)	(0101, F_2)	4(2, F_2) 10^3	0.515	0.515	0.515	0.515	0.515	0.515	0.515	0.515	0.515	0.515	0.515	0.515	0.515	0.515	0.515	0.515	0.515	0.515	0.515	0.515		
(0101, F_2)	(0101, F_2)	5(3, F_2) 10^4	0.1474	0.1474	0.1474	0.1474	0.1474	0.1474	0.1474	0.1474	0.1474	0.1474	0.1474	0.1474	0.1474	0.1474	0.1474	0.1474	0.1474	0.1474	0.1474	0.1474		
(0101, F_2)	(0101, F_2)	6(3, F_2) 10^4	0.0565 10()	0.0565 10()	0.0564 77(1)	0.0564 77(1)	0.0564 77(1)	0.0564 77(1)	0.0564 77(1)	0.0564 77(1)	0.0564 77(1)	0.0564 77(1)	0.0564 77(1)	0.0564 77(1)	0.0564 77(1)	0.0564 77(1)	0.0564 77(1)	0.0564 77(1)	0.0564 77(1)	0.0564 77(1)	0.0564 77(1)	0.0564 77(1)	0.0564 77(1)	
(0101, F_2)	(0101, F_2)	7(3, F_2) 10^4	0.14725	0.14725	0.14725	0.14725	0.14725	0.14725	0.14725	0.14725	0.14725	0.14725	0.14725	0.14725	0.14725	0.14725	0.14725	0.14725	0.14725	0.14725	0.14725	0.14725		
(0002, A_1)	(0002, A_1)	0(0, A_1)	13.057692(23)	13.04 400(27)	13.04 3753(11)	13.03 0565(19)	13.02 0925(15)	13.02 0925(15)	13.02 0925(15)	13.02 0925(15)	13.02 0925(15)	13.02 0925(15)	13.02 0925(15)	13.02 0925(15)	13.02 0925(15)	13.02 0925(15)	13.02 0925(15)	13.02 0925(15)	13.02 0925(15)	13.02 0925(15)	13.02 0925(15)	13.02 0925(15)	13.02 0925(15)	
(0002, A_1)	(0002, A_1)	1(2, A_1) 10^3	0.1675	0.1675	0.1675	0.1675	0.1675	0.1675	0.1675	0.1675	0.1675	0.1675	0.1675	0.1675	0.1675	0.1675	0.1675	0.1675	0.1675	0.1675	0.1675	0.1675	0.1675	
(0002, A_1)	(0002, A_1)	2(2, A_1) 10^3	0.20973	0.20973	0.20973	0.20973	0.20973	0.20973	0.20973	0.20973	0.20973	0.20973	0.20973	0.20973	0.20973	0.20973	0.20973	0.20973	0.20973	0.20973	0.20973	0.20973		
(0002, A_1)	(0002, A_1)	3(3, A_1) 10^4	0.295192	0.295192	0.295192	0.295192	0.295192	0.295192	0.295192	0.295192	0.295192	0.295192	0.295192	0.295192	0.295192	0.295192	0.295192	0.295192	0.295192	0.295192	0.295192	0.295192		
(0002, A_1)	(0002, A_1)	4(3, A_1) 10^4	0.09 70()	0.09 70()	0.09 70()</td																			

	a	e	4	ec	r	sc	ic	aram	ers	$\frac{\Omega(K)}{e} \frac{n\Gamma}{e' n'\Gamma}$	e	r	n	an	(0100)	(0001) vi	ra	in	s	a	es	e	4	(in cm ⁻¹) ^{a)}				
(v, γ)				(v', γ')							70	e	4	72	e	4	73	e	4	74	e	4	7	e	4			
	1				2			3			4			5			6			7								
(0000, A ₁)	(0000, A ₁)	2(0, A ₁)	2 695	5536	2 695	5944	2 695	629	2 695	64734	2 695	70305																
(0000, A ₁)	(0, A ₁)10 ⁴	0 33416	2	0 33416	2	0 33416	2	0 33416	2	0 33416	2	0 33416	2															
(0000, A ₁)	4(4, A ₁)10	0 1547079		0 1547079		0 1547079		0 1547079		0 1547079		0 1547079																
(0000, A ₁)	6(4, A ₁)10	0 1547079		0 1547079		0 1547079		0 1547079		0 1547079		0 1547079																
(0000, A ₁)	8(4, A ₁)10 ¹⁰	0 51075		0 51075		0 51075		0 51075		0 51075		0 51075																
(0000, A ₁)	6(6, A ₁)10 ¹⁰	0 1563		0 1563		0 1563		0 1563		0 1563		0 1563																
(0100,)	0(0, A ₁)	929 901299		929 905235		929 9070622		929 9093056		929 9130275																		
(0100,)	2(2,)10 ³	0 107	392	0 107	392	0 107	392	0 107	7 1	0 107	7 1	0 107	7 1															
(0100,)	3(3, A ₂)10 ⁴	0 2265992		0 2265992		0 2265992		0 2265992		0 2261		0 2261																
(0100,)	4(0, A ₁)10	0 4052		0 4052		0 4052		0 4052		0 4052		0 4052																
(0100,)	4(2,)10	0 31077		0 31077		0 31077		0 31077		0 31077		0 31077																
(0100,)	4(4, A ₁)10 ⁷	0 134		0 134		0 134		0 134		0 134		0 134																
(0100,)	1(3, F ₁)10 ³	0 125	3	0 125	3	0 125	3	0 125	3	0 125	3	0 125	3															
(0100,)	1(1, F ₁)	4 51245	6	4 5091962		4 507623		4 506094		4 503062																		
(0001, F ₂)	2(2, F ₂)	0 0212915	6	0 021291422		0 021290795		0 021290795		0 021290795		0 021290795																
(0001, F ₂)	3(1, F ₁)10 ³	0 1179753		0 1179753		0 1179753		0 1179753		0 1179267		0 1179267																
(0001, F ₂)	3(3, F ₂)10 ⁴	0 136921		0 136921		0 136921		0 136921		0 13 096		0 13 096																
(0001, F ₂)	4(2, F ₂)10	0 212431		0 212431		0 212431		0 212431		0 2122		0 2122																
(0001, F ₂)	4(4, F ₁)10	0 1 552		0 1 552		0 1 552		0 1 552		0 1 552		0 1 552																
(0001, F ₂)	4(4, F ₂)10	0 209	79	0 209	79	0 209	79	0 209	79	0 209	79	0 209	79															
(0001, F ₂)	5(1, F ₁)10	0 2903	3	0 2903	3	0 2903	3	0 2903	3	0 2903	3	0 2903	3															
(0001, F ₂)	5(3, F ₁)10	0 13677		0 13677		0 13677		0 13677		0 13677		0 13677																
(0001, F ₂)	5(3, F ₂)10	0 5	5	0 5	5	0 5	5	0 5	5	0 5	5	0 5	5															
(0001, F ₂)	0(0, A ₁)	21 5443402		21 1167767		20 9110	61	20 711	50	20 3270025																		
(0001, F ₂)	1(1, F ₁)	6 3712225		6 37 46454		6 3 19626		6 3 534	5	6 391 6203																		
(0001, F ₂)	2(0, A ₁)10	0 10	2241	0 1075775		0 1075775		0 1075775		0 10672	96	0 1060455																
(0001, F ₂)	2(2,)10 ³	0 152390		0 152390		0 1509974		0 1504	33	0 149	34	0 14 49411																
(0001, F ₂)	2(2, F ₂)	0 01070173		0 010699349		0 010695954		0 010695954		0 010695954		0 010695954																
(0001, F ₂)	3(1, F ₁)10 ⁴	0 704	13	0 704	13	0 704	13	0 704	13	0 704	13	0 704	13															
(0001, F ₂)	3(3, F ₁)10 ⁴	0 4 0174		0 4 0174		0 4 0174		0 4 0174		0 479437		0 479437																
(0001, F ₂)	4(0, A ₁)10	0 370472		0 370472		0 3653		0 3653		0 3653		0 3653																
(0001, F ₂)	4(2, F ₁)10	0 3519		0 3519		0 3519		0 3519		0 3519		0 3519																
(0001, F ₂)	4(4, A ₁)10 ⁷	0 6407		0 6407		0 6407		0 6407		0 6407		0 6407																
(0001, F ₂)	5(1, F ₁)10	0 25953		0 25953		0 25953		0 25953		0 25953		0 25953																
(0001, F ₂)	5(3, F ₁)10	0 16967		0 16967		0 16967		0 16967		0 16967		0 16967																
(0001, F ₂)	6(0, A ₁)10 ¹⁰	0 4276		0 4276		0 4276		0 4276		0 4276		0 4276																

^{a)} e r ce r m e 37

Table 5. Part of experimental line strengths, widths and shifts in the $\nu_2 + \nu_4$ bands of ${}^M\text{GeH}_4$ ($M = 70, 72, 73, 74, 76$)^{a)}.

J	γ	n	J'	γ'	n'	ν_{Exp} cm^{-1}	Int_{Exp} $\text{cm}^{-2}\text{atm}^{-1}$	δ in %	γ_{self} $\text{cm}^{-1}\text{atm}^{-1}$	δ_{self} $\text{cm}^{-1}\text{atm}^{-1}$	Band	M
1	2	3	4	5	6	7	8	9				
9	A_2	10	8	A_1	1	1832.6341	0.2191E-01	4.1	6.93(17)	-6.06(49)	$\nu_2 + \nu_4(F_2)$	76
9	E	19	8	E	1	1832.8964	0.8282E-02	-0.8	7.17(04)	-5.32(16)	$\nu_2 + \nu_4(F_2)$	74
9	F_2	28	8	F_1	1	1832.9415	0.1236E-01	-1.7	7.03(03)	-6.08(18)	$\nu_2 + \nu_4(F_2)$	74
9	A_2	10	8	A_1	1	1833.0235	0.2117E-01	0.2	7.09(01)	-5.14(17)	$\nu_2 + \nu_4(F_2)$	74
9	A_2	10	8	A_1	1	1833.2250	0.2149E-01	1.5	7.20(07)	-7.91(60)	$\nu_2 + \nu_4(F_2)$	73
9	E	19	8	E	1	1833.3060	0.8213E-02	-2.0	7.47(05)	-6.05(45)	$\nu_2 + \nu_4(F_2)$	72
9	A_2	10	8	A_1	1	1833.4334	0.2127E-01	0.3	9.24(93)	-7.81(44)	$\nu_2 + \nu_4(F_2)$	72
9	F_2	28	8	F_1	1	1833.7836	0.1259E-01	-0.6	7.41(06)	-3.95(42)	$\nu_2 + \nu_4(F_2)$	70
9	A_2	10	8	A_1	1	1833.8660	0.2167E-01	1.8	7.26(03)	-7.05(13)	$\nu_2 + \nu_4(F_2)$	70
15	F_1	37	14	F_1	3	1834.2135	0.1241E-02	1.5	6.79(28)	-5.64(49)	$\nu_2 + \nu_4(F_1)$	74
10	A_1	11	9	A_2	1	1835.1132	0.1401E-02	-7.1	9.15(22)	-7.11(60)	$\nu_2 + \nu_4(F_2)$	73
10	A_1	11	9	A_2	1	1835.7684	0.1501E-01	2.0	7.99(14)	-6.60(46)	$\nu_2 + \nu_4(F_2)$	76
10	A_1	11	9	A_2	1	1836.1254	0.1491E-01	0.8	7.37(02)	-4.01(22)	$\nu_2 + \nu_4(F_2)$	74
10	A_1	11	9	A_2	1	1836.3107	0.1545E-01	4.1	7.32(08)	-6.05(32)	$\nu_2 + \nu_4(F_2)$	73
10	F_1	30	9	F_2	2	1836.6374	0.8229E-02	-0.8	7.55(04)	-3.87(34)	$\nu_2 + \nu_4(F_2)$	74
10	F_2	30	9	F_1	3	1836.7787	0.9119E-02	5.5	8.22(27)	-4.46(51)	$\nu_2 + \nu_4(F_2)$	76
10	F_1	30	9	F_2	2	1836.8241	0.7981E-02	-4.2	6.27(14)	-7.11(60)	$\nu_2 + \nu_4(F_2)$	73
10	A_1	11	9	A_2	1	1836.8986	0.1522E-01	2.1	7.30(11)	-4.50(13)	$\nu_2 + \nu_4(F_2)$	70
10	F_1	30	9	F_2	2	1837.0167	0.8462E-02	1.4	6.78(04)	-4.38(39)	$\nu_2 + \nu_4(F_2)$	72
10	F_2	30	9	F_1	3	1837.1421	0.8424E-02	-2.9	7.44(03)	-4.66(22)	$\nu_2 + \nu_4(F_2)$	74
10	F_2	30	9	F_1	3	1837.5254	0.8595E-02	-1.3	7.48(04)	-3.85(26)	$\nu_2 + \nu_4(F_2)$	72
10	F_2	30	9	F_1	3	1837.9292	0.8788E-02	0.5	7.57(06)	-2.60(19)	$\nu_2 + \nu_4(F_2)$	70
10	A_2	10	9	A_1	1	1838.1854	0.1620E-01	4.6	6.80(08)	-6.05(24)	$\nu_2 + \nu_4(F_2)$	76
10	A_2	10	9	A_1	1	1838.5572	0.1550E-01	-0.2	7.52(03)	-4.73(20)	$\nu_2 + \nu_4(F_2)$	74
10	A_2	10	9	A_1	1	1838.7503	0.1513E-01	-2.8	7.30(06)	-5.46(54)	$\nu_2 + \nu_4(F_2)$	73
10	F_2	31	9	F_1	2	1838.8620	0.9477E-02	-2.0	6.62(05)	-8.06(29)	$\nu_2 + \nu_4(F_2)$	76
10	A_2	10	9	A_1	1	1838.9493	0.1539E-01	-1.3	7.39(04)	-4.55(29)	$\nu_2 + \nu_4(F_2)$	72
10	F_2	31	9	F_1	2	1839.2378	0.9619E-02	-1.0	7.22(01)	-5.99(15)	$\nu_2 + \nu_4(F_2)$	74
10	A_2	10	9	A_1	1	1839.3627	0.1560E-01	-0.3	7.66(03)	-2.32(32)	$\nu_2 + \nu_4(F_2)$	70
10	E	21	9	E	1	1839.4373	0.6454E-02	-2.3	6.77(03)	-9.85(77)	$\nu_2 + \nu_4(F_2)$	74
10	E	21	9	E	1	1839.8349	0.6931E-02	4.4	6.89(05)	-4.12(36)	$\nu_2 + \nu_4(F_2)$	72
10	E	21	9	E	1	1840.2541	0.6309E-02	-5.3	7.93(30)	-\dots	$\nu_2 + \nu_4(F_2)$	70
16	A_1	14	15	A_2	2	1841.2260	0.1812E-02	1.7	6.34(16)	-6.24(24)	$\nu_2 + \nu_4(F_2)$	74
10	F_2	32	9	F_1	1	1842.3102	0.1088E-01	2.4	6.54(11)	-2.65(16)	$\nu_2 + \nu_4(F_2)$	76
10	F_1	31	9	F_2	1	1842.6406	0.1057E-01	-0.6	6.94(04)	-5.90(19)	$\nu_2 + \nu_4(F_2)$	74
10	F_2	32	9	F_1	1	1842.7012	0.1052E-01	-1.4	7.01(04)	-5.17(11)	$\nu_2 + \nu_4(F_2)$	74
17	F_1	40	16	F_2	3	1842.7930	0.5620E-03	-0.7	\dots	\dots	$\nu_2 + \nu_4(F_2)$	74
10	F_1	31	9	F_2	1	1842.8437	0.1126E-01	5.4	5.90(11)	-11.27(66)	$\nu_2 + \nu_4(F_2)$	73
10	F_2	32	9	F_1	1	1842.9049	0.1091E-01	2.0	6.08(07)	-5.92(24)	$\nu_2 + \nu_4(F_2)$	73
10	F_1	31	9	F_2	1	1843.0529	0.1042E-01	-2.4	7.05(05)	-7.01(19)	$\nu_2 + \nu_4(F_2)$	72
10	F_2	32	9	F_1	1	1843.1144	0.1056E-01	-1.4	6.85(03)	-6.30(42)	$\nu_2 + \nu_4(F_2)$	72
10	F_1	31	9	F_2	1	1843.4879	0.1058E-01	-1.2	8.20(09)	-2.88(31)	$\nu_2 + \nu_4(F_2)$	70
10	F_2	32	9	F_1	1	1843.5497	0.1074E-01	-0.1	7.53(09)	-7.67(27)	$\nu_2 + \nu_4(F_2)$	70
11	F_1	33	10	F_2	3	1844.6698	0.7060E-02	1.0	7.30(05)	-5.54(23)	$\nu_2 + \nu_4(F_2)$	74
11	F_1	33	10	F_2	3	1844.8541	0.6822E-02	-2.6	6.74(30)	-4.74(28)	$\nu_2 + \nu_4(F_2)$	73
11	E	22	10	E	2	1845.4026	0.4588E-02	2.1	7.27(06)	-4.42(34)	$\nu_2 + \nu_4(F_2)$	74
11	E	22	10	E	2	1845.7808	0.4617E-02	2.2	7.08(10)	-4.78(43)	$\nu_2 + \nu_4(F_2)$	72
11	F_2	33	10	F_1	2	1845.8618	0.6884E-02	-0.9	7.84(01)	-4.11(14)	$\nu_2 + \nu_4(F_2)$	74
11	F_2	33	10	F_1	2	1846.0496	0.6863E-02	-1.4	\dots	\dots	$\nu_2 + \nu_4(F_2)$	73
11	F_2	33	10	F_1	2	1846.2436	0.7021E-02	0.6	8.06(13)	-7.38(29)	$\nu_2 + \nu_4(F_2)$	72
11	A_2	12	10	A_1	1	1846.4633	0.1309E-01	4.7	7.29(28)	-5.85(36)	$\nu_2 + \nu_4(F_2)$	76
11	A_2	12	10	A_1	1	1846.8316	0.1264E-01	0.8	7.25(02)	-5.56(31)	$\nu_2 + \nu_4(F_2)$	74
11	A_2	12	10	A_1	1	1847.0230	0.1296E-01	3.1	6.81(06)	-6.03(41)	$\nu_2 + \nu_4(F_2)$	73

^{a)} Temperature is 296.75 K.

Table 6. Spectroscopic parameters $P_{(v_l\Gamma_l)(v_u\Gamma_u)}^{(\Omega K \Gamma)}$ of the effective dipole moment parameters of the $\nu_2 + \nu_4$, $2\nu_2$ and $2\nu_4$ bands of the ${}^M\text{GeH}_4$ isotopologues (in D)^{a)}

$(v_l, \Gamma_l)/(v_u, \Gamma_u)$	Γ_{ab}^b	(Ω, K, Γ)	$M = 70$	$M = 72$	$M = 73$	$M = 74$	$M = 76$
1	2	3	4	5	6	7	8
$(0000, A_1)/(0200, E)$	E	$(1, 1, F_1) \times 10^4$	-0.5200	-0.5200	-0.5200	-0.5200(32)	-0.5200
$(0000, A_1)/(0101, F_1)$	F_1	$(1, 1, F_1) \times 10^4$	-0.6932	-0.6932	-0.6932	-0.6932(38)	-0.6932
$(0000, A_1)/(0101, F_2)$	F_2	$(0, 0, A_1) \times 10^3$	0.175352(55)	0.175202(48)	0.175114(51)	0.175010(24)	0.174847(84)
$(0000, A_1)/(0101, F_2)$	F_2	$(1, 1, F_1) \times 10^3$	0.21711	0.21711	0.21711	0.21711(38)	0.21711
$(0000, A_1)/(0002, E)$	E	$(1, 1, F_1) \times 10^4$	-0.5219	-0.5219	-0.5219	-0.5219(25)	-0.5219
$(0000, A_1)/(0002, F_2)$	F_2	$(0, 0, A_1) \times 10^2$	-0.36664(25)	-0.36410(23)	-0.36290 ^{c)}	-0.36161(64)	-0.35950 ^{c)}
$(0000, A_1)/(0002, F_2)$	F_2	$(1, 1, F_1) \times 10^4$	0.9064	0.9064	0.9064	0.9064(56)	0.9064
$N^{\text{tr}}/J^{\text{max}}$			382/14	511/17	116/15	556/19	132/14
d_{rms}			3.46%	3.49%	3.34%	3.42%	3.36%
$Z(296.75)$			1724.08	1724.26	1724.37	1724.46	1724.64

^{a)} Values in parentheses are 1σ statistical confidence intervals. Parameters presented without confidence intervals have been constrained to the values of corresponding parameters of the ${}^{74}\text{GeH}_4$ isotopologue.

^{b)} Γ_{ab} is a symmetry of vibrational subband.

^{c)} Parameter was constrained to the theoretically estimated values (see text for details).

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Declaration of Competing Interest

There are no conflicts to declare.