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# Social interactions issues in group decision-making

Ayeley Tchangani

**Abstract**—Decision-making is certainly the most widespread of all human activities, whether individual or by a group. Some decisions, especially individual decisions, are easy to make and do not require sophisticated algorithms to arrive at a solution. Others, on the other hand, and especially in the case of group decision-making, require the establishment of frameworks, rules or algorithms of varying degrees of sophistication to arrive at a satisfactory solution. In this process, the most difficult part is certainly the modeling and treatment of the relationships between the actors in the decision-making group. The objective of this paper is therefore to build a framework for modeling and analyzing interactions between decision-makers in a group on computational bases in the sense that these interactions will be characterized by numerical parameters. The constant concern in this work is to get as close as possible to human behavior by using bipolar analysis.

**Index Terms**—Group decision-making, Social interactions, Group coordination mechanisms, Bipolar analysis, BOCR analysis

## I. INTRODUCTION

Group decision making or collective choice is a sub-domain of decision analysis (a discipline comprising the philosophy, theory, methodology, and professional practice necessary to address important decisions in formal manner according to Wikipedia) that addresses decision making problems where a certain number of decision makers must select a subset (possibly reduced to a singleton) of alternatives from a large set of potential alternatives in order to achieve some collective as well as individual objectives, preferences, or desires. Such decision making problems are encountered in many practical situations such as: *management* (choosing a candidate to fill a job by a panel of experts, selecting or forming portfolio of projects to fund by a committee, selecting a short list of suppliers by members of a management board, etc.); *engineering* (choosing alternative design of an object (a car, a bridge, a road, a dam, etc.) to be realized by a panel of engineers and/or experts, choosing appropriate energy for a machine being developed, etc.); *economics* (setting up taxes by a government, forming portfolio of developing projects (transportation infrastructure, energy infrastructure, communication infrastructure, etc.), selecting appropriate manner of developing fund raising, etc.); *social* (choosing alternative social aid by a ruling committee, forming social projects (sanitary, housing, waste management, water infrastructure, etc.), portfolio of activities by a ruling committee, etc.); *politics* (electing representatives for a council, electing a president of a country, etc.); etc. Some concrete applications of this kind can be found in [1],

[6], and references therein for some real world applications even though in these references the problems are most of the time treated as a single decision maker problems. The existence of many decision makers necessitates to have a coordination mechanism (how to aggregate the view points of all decision makers) in order to reach a choice accepted by the group. The purpose of this paper is to address such coordination mechanism.

## II. SOCIAL CHOICE FRAMEWORK

Decision making is certainly the most pervasive human activity; indeed we spend a great proportion of our working day, hour, minute and even second, making decisions. Some decisions are made routinely and do not need models to support them whereas other decisions are so complex or important that sound decision support models are needed in order to avoid failure that may lead to very damageable or catastrophic consequences. These complex decisions share some features such as: multiplicity of objectives, multiplicity of attributes or criteria that characterize alternatives, uncertainty, multiplicity of actors, and so on. For these decision situations there is a need to have procedures or models that permit to capture all interactions and relationships between different elements of decision making process in order to reach an effective and efficient decision. Thus, a decision analysis problem is structured around the following important elements: decision makers, players, actors or stakeholders that are entities (persons, group of persons, organizations, etc.) that do have some interest or are engaged in decision analysis process; objectives (an objective in a decision analysis problem is something a decision maker cares about, wants to achieve, wants to optimize, wants to reach, etc.); alternatives (an alternative is a possibility opened to a decision maker that may permit him or her to realize his objectives); attributes or criteria (an attribute is a feature of an alternative that is used by a decision maker to evaluate this alternative with regard to pursued objectives).

A collective decision problem is a decision problem where a certain number (possibly reduced to one) of agents, stakeholders or decision makers must select, rank, classify, or sort alternatives from a large set or universe (discrete set)  $\mathcal{X} = \{x_1, x_2, \dots, x_m\}$ , of alternatives in order to satisfy some collective and/or individual objectives.

Classical approaches to coordinate social choice processes are dominated by electoral systems. An electoral system is a set of rules that determine how elections and referendums are conducted and how their results are determined; here is a brief description of some commonly used electoral rules to aggregate individual preferences; interested reader can

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consult for instance [2] for more details. Basically following approaches are highlighted in practice.

- **Ticking** one candidate at most (*majority rule*, first-past-the-post or *plurality voting* that is a system in which the candidate(s) with the highest amount of vote wins, with no requirement to get a majority of votes) or several candidates (*approval voting*, each decision maker or voter makes a list of alternatives that he/she wishes to support and the winner is the alternative that receives the greatest votes; this is equivalent for a decision maker to make bipolar list of alternatives namely alternatives that he/she approves and that he/she disapproves);
- **Rank-ordering** here, voters rank alternatives in a hierarchy on the ordinal scale (ordinal voting systems); one of the most representative here is the *Borda count* where ballots are counted by assigning a point value to each place in each voter's ranking of the candidates, and the choice with the largest number of points overall is elected.
- **Majority judgment**: each voter gives a rating on a qualitative scale for each candidate (e.g., "excellent", "very good", "good", etc.). For each candidate, the median score is then calculated, i.e. the best score such that half of the voters give the candidate a score at least as good; and the winner is the one who obtains the highest median score. To decide between two candidates with the same majority rating, two principles are used; victory criterion (to have the most voters assigning strictly more than the common majority mention and criterion of defeat (having the most voters strictly assigning less than the common majority mention). These two values are therefore calculated for each candidate to be separated and the larger of the 4 value determines the result, in order to satisfy the largest number of voters.

### III. MODELING SOCIAL INTERACTIONS

If coordination methods evoked in the previous section are easy to use and well suited for elections in group decision making, they may lead to some biases in some decision problems as voters will based their vote on only the positive aspects of alternative whereas it is well known that alternatives will convey positive aspects as well as negative aspects. Another issue is that these approaches disregard some social facts such as interactions and attitude toward members of a group. Indeed some human attributes make decision makers integrating some feelings that can be regrouped into two groups:

- positive feelings: *deference* (respectful consideration towards a person, and that brings to comply with his desires and his will), *trust* (firm expectation, assurance of a person relying on somebody or something), *empathy* (recognition and understanding of feelings and emotions of another individual), etc.
- negative feelings: *indifference* (state of the person who feels no pain, no pleasure, no fear, neither desire), *conflict* (encounter of opposing elements, opposing feel-

ings. Synonyms), *antipathy* (aversion, natural and not reasoned reluctance for someone), etc.

Positive feelings toward another decision maker may bring the corresponding decision maker to *altruism* (disposition to be interested and to devote oneself to others) whereas negative feelings will lead to *selfishness* (excessive attachment to oneself that make one searching exclusively his pleasure and his personal interest) toward the former one. Furthermore, in general, these feelings are not binary but rather a mix of positive and negative; in the same way the judgment of a decision maker toward an alternative will convey a mix of positive and negative aspect.

The attempt to integrate notions of altruism and selfishness has been addressed in the literature mainly in the context of satisficing game theory [7], [8]. In this theory interaction among agents is taken into account through the so called interdependent joint function or measures that can be represented as a praxeic network with  $2n + m$  possibles nodes if there are  $n$  agents and the effect of nature is considered through  $m$  possible states. From this interdependent function, joint selectability and rejectability measures are obtained by marginalization of this function and finally individual selectability and rejectability functions are derived by marginalizing the joint ones. This approach rise three observations or objections.

- Tractability: eliciting joint interdependent function may be tremendous when there are a great number of decision makers and a large possible states of the nature.
- By deriving individual satisficing measures by marginalizing joint satisfiability measures obtained from the marginalization of joint interdependent function, one is considering that individual preferences should emanate from global ones; this is not certainly the case in practice where one rather must aggregate individual preferences to reach global preference.
- Mutual influence or interaction among agents are not taken into account; indeed, by representing the joint interdependency function by a praxeic network that is a directed acyclic graph to capture influence between agents, one does not authorize mutual influence. For instance if the selectability of agent  $i$  is conditioned by that of agent  $j$ , the inverse will not be allowed whereas in practice this can happen.

In multi-actors decision analysis situation, there is always some conditionality in the sense that the preferences of a given actor may be conditioned to that of other ones [8]. When making decision, an actor may be influenced by its *social vicinity*, the influence one actor may have on another actor result of a sort of combination of positive and negative perception of later one. Particularly, influence does not always means positive perception (altruism, deference) of the influenced actor toward actors that influence him; it may represent repulsion, aggressivity of the influenced actor with regard to those influencing him; even for the same influencing actor, the influenced one may have a sort of combination of positive and negative perception. Given a

decision analysis situation and two decision makers  $i$  and  $j$ , there will be some decision aspects for which  $j$  will be in *concordance* with  $i$  and other aspects for which he will be in *discordance*. So, the global influence an actor  $j$  may have on another actor  $i$  will result from a combination of positive perception measured through a concordance degree and a negative perception measured through a discordance degree, highlighting the bipolarity concept [12]. In this paper we will rely on the notion of bipolarity (notion that consists, for an actor, in viewing or evaluating anything in two directions: a direction positively seen by the actor and a direction maker considered to impede his aspirations). Some indices such as *selectability degree* (the extent to which a decision maker considers selecting an alternative), *rejectability degree* (the extent to which a decision maker will avoid choosing an alternative) at the evaluation of alternatives level as well as other bipolar indices at social level will be introduced that permit in some extent to embed human attitude into the decision process.

#### IV. PROPOSED MECHANISM FOR SOCIAL CHOICE

To resolve a social choice problem, the approach proposed in this paper considers two levels: individual level and group level.

##### A. Individual level: bipolar analysis

At the individual level bipolar analysis framework can be used for the purpose of elicitation and evaluation of attributes that characterize an alternative towards achievement or not of an objective by that alternative. To this end one can rely on the notions of *supportability* and *rejectability* between pursued objectives and attributes characterizing alternatives in the sense that an attribute  $a(x)$  of the alternative  $x$  is supporting (respectively rejecting) a pursued objective  $o$  of the decision maker if the variations of  $a(x)$  and that of  $o$  are positively (respectively negative) correlated. By considering other inherent aspects of decision making such as uncertainty and/or internal/external aspects of an alternative, one can use well established methods such as BOCR (Benefits, Opportunities, Costs, Risks) analysis [9] [10] and/or SWOT (Strength, Weakness, Opportunities, Threats) analysis [9] to elicitate and evaluate attributes given an objective. At the end of this process and by aggregating separately positive aspect and negative aspect, each alternative  $x$  will be characterized by its group free selectability measure  $\mu_{S_i}^0(x)$  that conveys its positive aspects and its group free rejectability measure  $\mu_{R_i}^0(x)$  for decision making maker  $i$ ; these degrees should verify probability like conditions that is they are normalized to sum to 1 over the set of alternative  $\mathcal{X}$ .

##### B. Group level

To overcome the three issues raised in the previous section, we propose a novel approach which underlying philosophy relies on the ambiguous nature of human beings who are in general neither purely selfish nor purely altruist when making decisions that impact the preferences of other agents. Let define by  $\Omega(i)$  the social vicinity of actor  $i$  that is the set

of actors whose opinion matter for  $i$  or that do have some influence on  $i$ , be it negative, positive or both. For each actor  $j \in \Omega(i)$ , let us define the relative *concordance* degree  $\theta_{ij}^c$  or discordance degree  $\theta_{ij}^d$  (as  $\theta_{ij}^c + \theta_{ij}^d = 1$ ) to measure the extent to which the opinion of actor  $i$  is in concordance or in *discordance* with actor  $j$  compared to other actors of its vicinity  $\Omega(i)$ .

The satisfiability measures  $\mu_{S_i}^{\Omega(i)}(x)$  and  $\mu_{R_i}^{\Omega(i)}(x)$  of agent  $i$  taking only the view points of its vicinity are then given by equation (1)

$$\mu_{\times}^{\Omega(i)}(x) = \frac{\Phi_{\times}^{\Omega(i)}(x)}{\sum_{y \in \mathcal{X}} \{\Phi_{\times}^{\Omega(i)}(y)\}} \quad (1)$$

where  $\times$  stands for  $S_i$  or  $R_i$  with  $\Phi_{S_i}^{\Omega(i)}(x)$  and  $\Phi_{R_i}^{\Omega(i)}(x)$  given by equations (2) and (3)

$$\Phi_{S_i}^{\Omega(i)}(x) = \gamma_i C_{\theta_i^c}^{wcfm}(\mu_{S_{\Omega(i)}}(x)) + (1 - \gamma_i) C_{\theta_i^d}^{wcfm}(\mu_{R_{\Omega(i)}}(x)) \quad (2)$$

$$\Phi_{R_i}^{\Omega(i)}(x) = \gamma_i C_{\theta_i^c}^{wcfm}(\mu_{R_{\Omega(i)}}(x)) + (1 - \gamma_i) C_{\theta_i^d}^{wcfm}(\mu_{S_{\Omega(i)}}(x)) \quad (3)$$

where

- $\mu_{S_{\Omega(i)}} = [\mu_{S_1}^0(x) \ \mu_{S_2}^0(x) \ \dots \ \mu_{S_{|\Omega(i)|}}^0(x)]$  is a vector gathering network free selectability measures of agents in the vicinity of  $i$  and  $\mu_{R_{\Omega(i)}}(x)$  is defined similarly.
- $\theta_i^c = [\theta_{i1}^c \ \theta_{i2}^c \ \dots \ \theta_{i|\Omega(i)|}^c]$  is a vector gathering concordance degrees of agents in the vicinity of  $i$ ;  $\theta_i^d$  is defined similarly.
- $C_{\omega}^{wcfm}(Y)$  is the Choquet integral of  $Y$  associated to a weighted cardinal fuzzy measure with relative importance vector  $\omega$ , see [11].
- $0 \leq \gamma_i \leq 1$  is a degree that measures the altruist (or deference) attitude of actor  $i$  towards decision makers of its vicinity. Indeed if  $\gamma_i = 1$  then decision maker  $i$  does favor its vicinity.

Finally, real satisfiability measures  $\mu_{S_i}(x)$  and  $\mu_{R_i}(x)$  are obtained by making a trade-off between group free measures and deference measures through the selfishness degree  $0 \leq \delta_i \leq 1$  as given by (4) and (5)

$$\mu_{S_i}(x) = \delta_i \mu_{S_i}^0(x) + (1 - \delta_i) \mu_{S_i}^{\Omega(i)}(x) \quad (4)$$

$$\mu_{R_i}(x) = \delta_i \mu_{R_i}^0(x) + (1 - \delta_i) \mu_{R_i}^{\Omega(i)}(x) \quad (5)$$

##### C. Obtaining concordance and discordance degrees

Concordance degree  $\theta_{ij}^c$  or discordance degree  $\theta_{ij}^d$  (as  $\theta_{ij}^c + \theta_{ij}^d = 1$ ) of agent  $i$  with regard to its vicinity agent  $j \in \Omega(i)$  can be supplied by decision maker (in this case we referred to it as subjective concordance/discordance degree) or be deduced by the analyst from the prior selectability and rejectability measures, that we refer to as objective concordance/discordance degree assessment. In the case of this



later assessment process, one needs to dispose of sound measurement tool; to this end we can interpret the discordance degree  $\theta_{ij}^d$  as a measure of divergence of opinion of agent  $i$  with regard to agent  $j$  in the given context. One measure encountered that can be adapted for this purpose is the so called Kullback-Leibler divergence (also called relative entropy) [4] that measure how one probability distribution is different from a second one, reference probability distribution in mathematical statistics. For discrete probability distributions  $P$  and  $Q$  defined on the same probability space  $\mathcal{X}$ , the Kullback-Leibler divergence of  $P$  with regard to  $Q$  is defined to be (6)

$$D_{KL}(P//Q) = \sum_{x \in \mathcal{X}} P(x) \log \left( \frac{P(x)}{Q(x)} \right) \quad (6)$$

that always verifies  $D_{KL}(P//Q) \geq 0$ . So, let us define by  $D_{S,KL}^d(i//j)$  and  $D_{R,KL}^d(i//j)$  the divergence of opinion or preference degrees of agent  $i$  with agent  $j$  regarding selectability and rejectability respectively in the given context, therefore we define the discordance degree  $\theta_{ij}^d$  to be given by equation (7)

$$\theta_{ij}^d = \frac{D_{S,KL}^d(i//j) + D_{R,KL}^d(i//j)}{\sum_{l \in \Omega(i)} \{D_{S,KL}^d(i//j) + D_{R,KL}^d(i//l)\}} \quad (7)$$

with (8) and (9)

$$D_{S,KL}^d(i//j) = \sum_{x \in \mathcal{X}} \mu_{S_i}^0(x) \log \left( \frac{\mu_{S_i}(x)}{\mu_{S_j}^0(x)} \right) \quad (8)$$

$$D_{R,KL}^d(i//j) = \sum_{x \in \mathcal{X}} \mu_{R_i}^0(x) \log \left( \frac{\mu_{R_i}(x)}{\mu_{R_j}^0(x)} \right) \quad (9)$$

#### D. Measuring network influence on individual

Once the final selectability and rejectability measures have been obtained, it seems logical to try to determine the degree of influence of the network on the evolution of the distribution of these measures per individual. The distribution of final measures for a decision-maker with little network influence should not deviate significantly from the network free distribution; thus, the more a decision-maker is influenced by the network, the more the final and selfish distributions should deviate. Given the probabilistic structure of these distributions, we propose to use the Kullback-Leibler divergence to measure the degree of influence of the network on individual decision-makers. Note therefore  $I_S(i)$  and  $I_R(i)$  the degrees of influence of the network on the selectability and rejectability distributions of decision maker  $i$  respectively; they are therefore given by equations (10) and (11)

$$I_S(i) = \sum_{x \in \mathcal{X}} \mu_{S_i}(x) \log \left( \frac{\mu_{S_i}(x)}{\mu_{S_i}^0(x)} \right) \quad (10)$$

$$I_R(i) = \sum_{x \in \mathcal{X}} \mu_{R_i}(x) \log \left( \frac{\mu_{R_i}(x)}{\mu_{R_i}^0(x)} \right) \quad (11)$$

## V. SOLUTION PROCEDURE

From materials derived in the previous sections, different procedures can be used to arrive to a final decision. To this end, the group must agree over some earliest common parameters such as: *caution or boldness index* to use to manage the trade-off between selectability and rejectability of alternatives; we assume that the group agree to use a common value  $q$  for this index; *indicator* that must be used for final decision (selection or ranking), a function  $\pi$  over the alternatives set  $\mathcal{X}$  is defined through selectability measure  $\mu_S$  and rejectability measure  $\mu_R$  that is a non decreasing in  $\mu_S$  and non increasing in  $\mu_R$ ; most used of these functions are given by equations (12) and (13)

$$\pi : \mathcal{X} \rightarrow R, x \mapsto \pi(x) = \frac{\mu_S(x)}{\mu_R(x)} \quad (12)$$

$$\pi : \mathcal{X} \rightarrow R, x \mapsto \pi(x) = \mu_S(x) - q\mu_R(x) \quad (13)$$

Once the group agree over these materials, each decision maker  $i$  can determine its satisficing equilibrium set  $\mathcal{E}_S^i$  of alternatives at the common boldness index  $q$  using following procedure:

- determine its satisficing set  $\Sigma_q^i$  at the boldness index  $q$  as shown by equation (14)

$$\Sigma_q^i = \{x \in \mathcal{X} : \mu_{S_i}(x) \geq q\mu_{R_i}(x)\} \quad (14)$$

- determine its equilibrium set  $\mathcal{E}_{S,q}^i$  that are non dominated alternatives; an alternative  $x$  is dominated by another alternative  $y$  for decision maker  $i$  if one has  $\mu_{S_i}(x) < \mu_{S_i}(y)$  and  $\mu_{R_i}(x) \geq \mu_{R_i}(y)$  or  $\mu_{S_i}(x) \leq \mu_{S_i}(y)$  and  $\mu_{R_i}(x) > \mu_{R_i}(y)$ ; let us define by  $\mathcal{D}_i(x)$  the set of dominated alternatives of  $x$  for decision maker  $i$  then the equilibrium set  $\mathcal{E}^i$  is given by equation (15)

$$\mathcal{E}^i = \{x \in \mathcal{X} : \mathcal{D}_i(x) = \emptyset\}. \quad (15)$$

This set is always not empty and as by managing  $q$  one can render  $\Sigma_q^i$  not empty, the satisficing equilibrium  $\mathcal{E}_{S,q}^i$  is given by equation (16)

$$\mathcal{E}_{S,q}^i = \Sigma_q^i \cap \mathcal{E}^i. \quad (16)$$

Now that necessary materials for coordination, namely satisfiability measures  $\mu_{S_i}$  and  $\mu_{R_i}$  and satisficing equilibrium set  $\mathcal{E}_{S,q}^i$  are obtained, we will consider how to use them to reach a final decision by first adapting classical approaches (ticking, approval voting, ranking, Borda count), then deriving aggregation approaches mainly synergistic aggregation and probabilistic like aggregation approach, weighted minimum distance to ideal alternatives, to end up with consensus seeking approach.

#### A. Adapted classical approaches

1) *Ticking one alternative at most*: The alternative ticked by decision maker  $i$  is obviously that he considers to be the best in his satisficing equilibrium set  $\mathcal{E}_{S,q}^i$ ; let us denote by  $\tau_i(x)$  a binary variable indicating whether the alternative  $x$

is ticked by decision maker  $i$  or not; it is therefore given by equation (17)

$$\tau_i(x) = \arg \left\{ \max_{y \in \mathcal{E}_{S,q}^i} \{ \pi(y) \} \right\} \quad (17)$$

so that the winner  $x^*$  is given by equation (18)

$$x^* = \arg \left\{ \max_{x \in \mathcal{X}} \left\{ \sum_{i=1}^n \tau_i(x) \right\} \right\}. \quad (18)$$

2) *Approval voting*: The approved set of decision maker  $i$  is its satisfying equilibrium set  $\mathcal{E}_{S,q}^i$  so that the winner is given by equation (19)

$$x^* = \arg \left\{ \max_{x \in \mathcal{X}} \left\{ \sum_{i=1}^n 1_{\mathcal{E}_{S,q}^i}(x) \right\} \right\} \quad (19)$$

where  $1_{\mathcal{E}_{S,q}^i}$  is the indicator of the set  $\mathcal{E}_{S,q}^i$  defined as equation (20)

$$1_{\mathcal{E}_{S,q}^i}(x) = \begin{cases} 1 & \text{if } x \in \mathcal{E}_{S,q}^i \\ 0 & \text{otherwise} \end{cases} \quad (20)$$

3) *Rank-ordering (Borda's count approach)*: Each decision maker  $i$  ranks alternatives by following relations

$$x \succeq y \iff \pi(x) \geq \pi(y) \quad (21)$$

where  $\succeq$  stands for " $x$  is at least as good as  $y$ " for  $i$ ; then calculate Borda's score  $B_i(x)$  for alternative  $x$  in the ranking of  $i$  from the best  $B_i(x) = n$  to the last  $B_i(x) = 1$ ; the winner is given by equation (22)

$$x^* = \arg \left\{ \max_{x \in \mathcal{X}} \left\{ \sum_{i=1}^n B_i(x) \right\} \right\} \quad (22)$$

## B. Aggregation approaches

1) *Synergistic aggregation*: Let us consider that it is possible to assign relative weights to agents through the relative vector (23)

$$\omega = [\omega_1 \quad \omega_2 \quad \dots \quad \omega_n], \sum_{i=1}^n \omega_i = 1 \quad (23)$$

then one can define a global selectability and a global rejectability measures as given by equation (24)

$$\mu_{\times}(x) = \frac{\Theta_{i=1}^n(\mu_{S_i}(x))}{\sum_{y \in \mathcal{X}} \{ \Theta_{i=1}^n(\mu_{\times_i}(y)) \}} \quad (24)$$

where  $\times$  stands for  $S$  or  $R$  and  $\Theta_{i=1}^n(\cdot)$  is an aggregation operator and then use a single decision maker procedure. There are many aggregation operators in the literature going from weighted mean operator to more sophisticated operators such as fuzzy integrals [3]; in this approach, given the synergy between the preferences of decision makers, one may use a Choquet integral associated to a weighted cardinal fuzzy measure  $C_{\omega}^{wcfm}(\cdot)$  [10].

2) *Probabilistic aggregation approach*: If we interpret satisfiability measures  $\mu_{S_i}(x)$  and  $\mu_{R_i}(x)$  as the probabilities that agent  $i$  select or reject respectively the alternative  $x$  in the given context, and by considering independence between decision makers at this stage one can define group selectability and group rejectability measures  $\mu_S(x)$  and  $\mu_R(x)$  respectively to be the joint probability that each member of the group select (respectively reject) alternative  $x$  and therefore they are given by (25)

$$\mu_S(x) = \prod_{i=1}^n \{ \mu_{S_i}(x) \}; \mu_R(x) = \prod_{i=1}^n \{ \mu_{R_i}(x) \} \quad (25)$$

that can be used for final selection.

## C. Minimum distance to ideal alternatives

Let us define by  $x_i^*$  the selected alternative by agent  $i$  that we refer to as his ideal alternative then the final selected alternative  $x^*$  is obtained by minimizing the weighted distance to each ideal point as shown by equation (26)

$$x^* = \arg \left\{ \min_{x \in \mathcal{X}} \left\{ \sum_{i=1}^n (\omega_i d(x, x_i^*)) \right\} \right\} \quad (26)$$

where  $d(x, x_i^*)$  is the classical Minkowski distance between  $x$  and  $x_i^*$  in the plan  $(\mu_R, \mu_S)$  given by equation (27)

$$d(x, x_i^*) = (|\mu_{R_i}(x) - \mu_{R_i}(x_i^*)|^p + |\mu_{S_i}(x) - \mu_{S_i}(x_i^*)|^p)^{\frac{1}{p}} \quad (27)$$

with  $p = 1, 2, \dots, \infty$ ; most used are  $p = 1$  (Manhattan distance),  $p = 2$  (Euclidian distance), and  $p = \infty$  (Tchebychev distance, supremum).

## D. Consensus seeking approach

There is a consensus if the final reached decision could be reached by using individual decisions, that is the winner is a satisfying equilibrium of all decision makers; so to select the winner one proceeds as following:

- if equation (28)

$$\cap_{i=1}^n \mathcal{E}_{S,q}^i \neq \emptyset \quad (28)$$

is satisfied then there is a consensus and the winner  $x^*$  can be selected within this set by maximizing an indicator such as that of following equation (29)

$$x^* = \arg \left\{ \max_{x \in \cap_{i=1}^n \mathcal{E}_{S,q}^i} \left\{ \prod_{i=1}^n \left\{ \frac{\mu_{S_i}(x)}{\mu_{R_i}(x)} \right\} \right\} \right\} \quad (29)$$

- if equation (30)

$$\cap_{i=1}^n \mathcal{E}_{S,q}^i = \emptyset \quad (30)$$

then decision makers have to rework their  $\mathcal{E}_{S,q}^i$  by reducing the boldness index  $q$  until condition of equation (28) is reached and select the winner using equation (29).

## VI. ILLUSTRATIVE APPLICATION

### A. Description and modeling

To illustrate the potentiality of the established method, let us consider a real-world problem in the domain of waste management facility location. This application is adapted from [5] where the objective was to find the most plausible solution to a municipal solid waste management problem in a region of Central Finland. We suppose that the choice of this location is to be made by a group of 5 decision makers with social structure of Figure 1 where the arrow going from  $i$  to  $j$  means that  $j$  belongs to the vicinity of  $i$  that is  $j \in \Omega(i)$ .

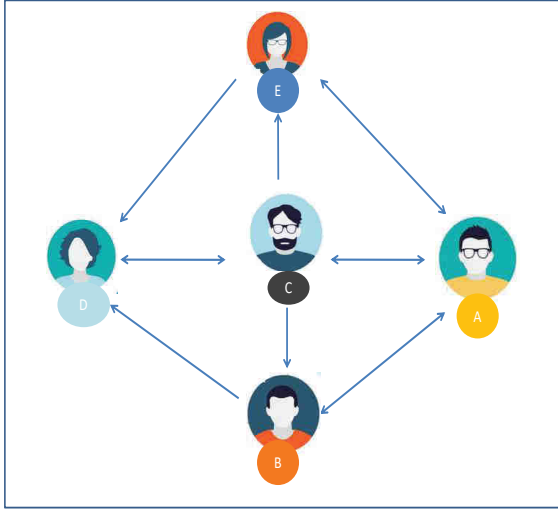


Fig. 1. Example of a social interaction framework

So that the vicinities of these decision makers (A, B, C, D, E) are given by equations (31) -(35)

$$\Omega(A) = \{B, C, E\} \quad (31)$$

$$\Omega(B) = \{A, D\} \quad (32)$$

$$\Omega(C) = \{A, B, D, E\} \quad (33)$$

$$\Omega(D) = \{C\} \quad (34)$$

$$\Omega(E) = \{A, D\} \quad (35)$$

Two main objectives have to be satisfied in the view of authorities, namely:

- $o_1$ : enhancement of the socioeconomic situation of the considered region;
- $o_2$ : respect of the environment.

A preliminary study has identified 11 alternatives (see [5] for the meaning of each alternative) and 8 attributes, meanings of which are described in the following points:

- $a_1$ : net cost per ton,
- $a_2$ : global effects,
- $a_3$ : local and regional health effects,
- $a_4$ : acidificative releases,
- $a_5$ : surface water dispersed releases,
- $a_6$ : technical reliability,

- $a_7$ : number of employees,
- $a_8$ : amount of recovered waste.

We consider that the principal goal is to select the most sustainable site that is the alternative with most social, economic and environmental scores. But here we merge economical and social objectives into one objective known as socioeconomic objective so that we have two objective functions  $o_1$  and  $o_2$  that are described below to be satisfied.

- $o_1$ : enhance the socioeconomic situation of the considered region;
- $o_2$ : respect the environment.

A bipolar analysis (see [10]) applied to attributes lead to following repartition of supportability and rejectability of the two objectives.

$$A_{o_1}^S = \{a_6, a_7, a_8\} \quad (36)$$

$$A_{o_1}^R = \{a_1, a_2, a_3, a_4, a_5\} \quad (37)$$

$$A_{o_2}^S = \{a_6, a_8\} \quad (38)$$

$$A_{o_2}^R = \{a_2, a_3, a_4, a_5\} \quad (39)$$

After normalizing (using linear Weitenorf procedure) data collected by [5] and aggregating these normalized data considering attributes to have equal importance in their category, we obtain results of Table I as evaluation of objectives.

Alternatives	Objective $o_1$		Objective $o_2$	
	$\Psi_S^{o_1}(u)$	$\Psi_R^{o_1}(u)$	$\Psi_S^{o_2}(u)$	$\Psi_R^{o_2}(u)$
<b>IA</b>	1.1133	2.1507	1.1133	2.1118
<b>IB1</b>	0.7813	1.8068	0.3369	1.6791
<b>IB2</b>	0.6926	1.8264	0.4703	1.6792
<b>IC1</b>	2.2030	3.1153	1.3697	2.4811
<b>IC2</b>	1.9444	3.0894	1.5000	2.4812
<b>IIA</b>	1.4467	2.1522	1.1133	2.1522
<b>IIB</b>	1.1370	1.4252	0.4703	1.2260
<b>IIC</b>	2.3889	2.8412	1.5000	2.2373
<b>IIIA</b>	0.2778	2.0220	0	1.7515
<b>IIIB</b>	0.7535	1.6437	0.0869	1.1502
<b>IIIC</b>	1.7775	2.9108	0.7775	1.9108

TABLE I  
APPLICATION DATA

Each decision maker has his own view with regard to the two objectives to satisfy that permits he to deduce his purely selfish satisfiability measures  $\mu_{S_i}^0(x)$  and  $\mu_{R_i}^0(x)$ ; to this end let us denote by  $\alpha_1^i$  the relative weight of objective  $o_1$  for decision maker  $i$  so that  $\mu_{S_i}^0(x)$  and  $\mu_{R_i}^0(x)$  are given by equations (40) and (41)

$$\mu_{S_i}^0(x) = \frac{\alpha_1^i \Psi_S^{o_1}(x) + (1 - \alpha_1^i) \Psi_S^{o_2}(x)}{\sum_y \{\alpha_1^i \Psi_S^{o_1}(y) + (1 - \alpha_1^i) \Psi_S^{o_2}(y)\}} \quad (40)$$

$$\mu_{R_i}^0(x) = \frac{\alpha_1^i \Psi_R^{o_1}(x) + (1 - \alpha_1^i) \Psi_R^{o_2}(x)}{\sum_y \{\alpha_1^i \Psi_R^{o_1}(y) + (1 - \alpha_1^i) \Psi_R^{o_2}(y)\}} \quad (41)$$

### B. Results

Let us consider that the numerical parameters characterizing decision group are given by Table II.



DM	$\alpha_1^i$	$\gamma_i$	$\delta_i$
A	0.9	0.5	0.5
B	0.5	0.5	0.5
C	0.1	0.5	0.5
D	0.3	0.5	0.5
E	0.7	0.5	0.5

TABLE II  
PARAMETERS CHARACTERIZING DECISION MAKERS

Using relations of equations (40) and (41), pure selfish selectability and rejectability degrees are obtained as Table III for each decision maker and then divergence degrees (equation (7)) are obtained as given by equations (42) - (46)

$$\theta_A^d = \begin{bmatrix} B & C & E \\ 0.1081 & 0.8701 & 0.0218 \end{bmatrix} \quad (42)$$

$$\theta_B^d = \begin{bmatrix} A & D \\ 0.6605 & 0.3395 \end{bmatrix} \quad (43)$$

$$\theta_C^d = \begin{bmatrix} A & B & D & E \\ 0.4460 & 0.1796 & 0.0637 & 0.3108 \end{bmatrix} \quad (44)$$

$$\theta_D^d = \begin{bmatrix} C \\ 1 \end{bmatrix} \quad (45)$$

$$\theta_E^d = \begin{bmatrix} A & D \\ 0.1111 & 0.8889 \end{bmatrix} \quad (46)$$

that are used to calculate altruist measures ( $\mu_{S_i}^{\Omega(i)}(x)/\mu_{R_i}^{\Omega(i)}(x)$ ) and ultimate ( $\mu_{S_i}(x)/\mu_{R_i}(x)$ ) selectability and rejectability degrees shown on Table IV and Table V respectively.

By considering same boldness index  $q = 1$  for all decision makers and from data of Table V, the satisficing equilibrium sets as defined by equation (16) of each decision makers are given by equations (47)-(51)

$$\mathcal{E}_{S,1}^A = \{IC1, IC2, IIA, IIB, IIC, IIIC\} \quad (47)$$

$$\mathcal{E}_{S,1}^B = \{IA, IIA, IIB, IIC, IIIC\} \quad (48)$$

$$\mathcal{E}_{S,1}^C = \{IA, IIA, IIC\} \quad (49)$$

$$\mathcal{E}_{S,1}^D = \{IA, IIA, IIB, IIC\} \quad (50)$$

$$\mathcal{E}_{S,1}^E = \{IIA, IIB, IIC, IIIC\} \quad (51)$$

Let us find the ultimate selected alternative using some proposed criteria, namely: ticking one alternative at most, approval voting, and consensus seeking.

1) *Ticking one alternative at most*: Let us suppose that each decision maker used the ratio ( $\mu_S/\mu_R$ ) between selectability measure and rejectability measure to choose the ticked alternative, then the parameter  $\tau_i(x)$  defined by equation (17) is obtained as  $\tau_A(IIB) = 1$  and  $\tau_B(IIC) = \tau_C(IIC) = \tau_D(IIC) = \tau_E(IIC) = 1$ . So only decision maker A should choose other alternative (IIB) than alternative IIC; alternative IIC is therefore selected alternative.

2) *Approval voting*: Let us denote by  $\nu(i)$  the approval score obtained by alternative  $i$ , so we have  $\nu(IC1) = \nu(IC2) = 1$ ,  $\nu(IIB) = 4$  and  $\nu(IIA) = \nu(IIC) = 5$ . Two alternatives have the same highest score so the choice must be made when using additional criterion such as

maximizing the ratio ( $\mu_S/\mu_R$ ) between selectability measure and rejectability measure; with this criteria the alternative IIC is selected.

3) *Consensus seeking*: as intersection of individual satisficing equilibrium sets is not empty and given by equation (52)

$$\mathcal{E}_{S,1}^A \cap \mathcal{E}_{S,1}^B \cap \mathcal{E}_{S,1}^C \cap \mathcal{E}_{S,1}^D \cap \mathcal{E}_{S,1}^E = \{IIA, IIC\} \neq \emptyset; \quad (52)$$

there is a consensus around alternatives IIA and IIC; the ultimate alternative to implement selected by using criteria of equation (29) is alternative IIC.

4) *Network influence degrees*: One can consider analyzing how network has influenced each of 5 decision makers by computing degrees  $I_S$  and  $I_R$  as defined by equations (10) and (11); these degrees are shown on Table VI. One can see from these degrees that decision maker C is the most influenced by the network and decision maker A is the least influenced one. One must keep in mind that the structure of Figure 1 is virtual, decision makers are not necessary aware of it; so such post decision analysis through these influence degrees can reveal some social behavior of members of the group; for instance decision maker D whose vicinity is reduced to C is more influenced than decisions makers A, B, and E (network propagation effect).

## VII. CONCLUSION

The problem of social interactions in group decision-making, issues rarely considered in the decision support literature, has been explored in this paper. The notions of social vicinity, social feelings (positive or negative) towards others, altruism or selfishness, etc. have been introduced and formalized. Finally, the decision-making process have two levels: at the local level, each decision-maker determines some parameters using formalized procedures, that will be used at the group level by aggregation (in the broadest sense) to arrive at the final solution. Many traditional ways of aggregating individual choices to arrive to a group choice such voting (ticking on alternative at most, approval voting, majority voting, Borda approach, etc.) as well as newer concept such as consensus seeking are compatible with the approach developed so far in this paper; furthermore some indices that can be used to measure how the social interactions influence individual behavior have been developed. An illustrative application based on real world problem shows the applicability of this approach.

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Alternatives	A		B		C		D		E	
	$\mu_S^0(x)$	$\mu_R^0(x)$	$\mu_S^0(x)$	$\mu_R^0(x)$	$\mu_S^0(x)$	$\mu_R^0(x)$	$\mu_S^0(x)$	$\mu_R^0(x)$	$\mu_S^0(x)$	$\mu_R^0(x)$
IA	0.0799	0.0874	0.0958	0.0930	0.1195	0.0995	0.1063	0.0961	0.0871	0.0901
IB1	0.0529	0.0730	0.0481	0.0760	0.0409	0.0795	0.0449	0.0777	0.0507	0.0745
IB2	0.0481	0.0737	0.0500	0.0765	0.0529	0.0796	0.0513	0.0780	0.0490	0.0751
IC1	0.1521	0.1242	0.1536	0.1221	0.1560	0.1196	0.1547	0.1209	0.1528	0.1232
IC2	0.1363	0.1233	0.1481	0.1215	0.1658	0.1195	0.1560	0.1205	0.1417	0.1224
IIA	0.1014	0.0876	0.1101	0.0939	0.1231	0.1012	0.1159	0.0974	0.1054	0.0906
IIB	0.0768	0.0572	0.0691	0.0578	0.0576	0.0586	0.0640	0.0582	0.0733	0.0575
IIC	0.1650	0.1132	0.1672	0.1108	0.1706	0.1080	0.1687	0.1094	0.1660	0.1120
IIIA	0.0179	0.0812	0.0119	0.0823	0.0030	0.0836	0.0080	0.0829	0.0152	0.0817
IIIB	0.0493	0.0649	0.0361	0.0609	0.0165	0.0564	0.0274	0.0588	0.0433	0.0630
IIIC	0.1204	0.1144	0.1099	0.1052	0.0942	0.0945	0.1029	0.1000	0.1156	0.1099

TABLE III  
SELFISH MEASURES

Alternatives	A		B		C		D		E	
	$\mu_{S_i}^{\Omega(i)}(x)$	$\mu_{R_i}^{\Omega(i)}(x)$	$\mu_{S_i}^{\Omega(i)}(x)$	$\mu_{R_i}^{\Omega(i)}(x)$	$\mu_{S_i}^{\Omega(i)}(x)$	$\mu_{R_i}^{\Omega(i)}(x)$	$\mu_{S_i}^{\Omega(i)}(x)$	$\mu_{R_i}^{\Omega(i)}(x)$	$\mu_{S_i}^{\Omega(i)}(x)$	$\mu_{R_i}^{\Omega(i)}(x)$
IA	0.0941	0.0936	0.0866	0.0852	0.0898	0.0858	0.1012	0.1012	0.0842	0.0875
IB1	0.0641	0.0586	0.0624	0.0634	0.0624	0.0639	0.0613	0.0613	0.0641	0.0617
IB2	0.0609	0.0601	0.0603	0.0604	0.0586	0.0603	0.0646	0.0646	0.0604	0.0602
IC1	0.1305	0.1392	0.1357	0.1353	0.1308	0.1334	0.1378	0.1378	0.1349	0.1360
IC2	0.1297	0.1367	0.1302	0.1285	0.1299	0.1265	0.1383	0.1383	0.1272	0.1315
IIA	0.1000	0.1036	0.0953	0.0950	0.0951	0.0958	0.1066	0.1066	0.0947	0.0956
IIB	0.0675	0.0593	0.0665	0.0676	0.0678	0.0671	0.0611	0.0611	0.0683	0.0657
IIC	0.1318	0.1481	0.1369	0.1363	0.1322	0.1345	0.1391	0.1391	0.1359	0.1373
IIIA	0.0506	0.0446	0.0493	0.0502	0.0505	0.0502	0.0454	0.0454	0.0509	0.0487
IIIB	0.0587	0.0489	0.0585	0.0597	0.0633	0.0614	0.0431	0.0431	0.0606	0.0576
IIIC	0.1120	0.1074	0.1183	0.1185	0.1195	0.1212	0.1015	0.1015	0.1187	0.1181

TABLE IV  
ALTRUIST MESAURES

Alternatives	A		B		C		D		E	
	$\mu_{S_i}(x)$	$\mu_{R_i}(x)$	$\mu_{S_i}(x)$	$\mu_{R_i}(x)$	$\mu_{S_i}(x)$	$\mu_{R_i}(x)$	$\mu_{S_i}(x)$	$\mu_{R_i}(x)$	$\mu_{S_i}(x)$	$\mu_{R_i}(x)$
IA	0.0870	0.0905	0.0912	0.0891	0.1046	0.0926	0.1038	0.0987	0.0856	0.0888
IB1	0.0585	0.0658	0.0553	0.0697	0.0517	0.0717	0.0531	0.0695	0.0574	0.0681
IB2	0.0545	0.0669	0.0551	0.0684	0.0557	0.0700	0.0580	0.0713	0.0547	0.0676
IC1	0.1413	0.1317	0.1447	0.1287	0.1434	0.1265	0.1462	0.1293	0.1439	0.1296
IC2	0.1330	0.1300	0.1392	0.1250	0.1479	0.1230	0.1471	0.1294	0.1344	0.1269
IIA	0.1007	0.0956	0.1027	0.0944	0.1091	0.0985	0.1113	0.1020	0.1000	0.0931
IIB	0.0721	0.0583	0.0678	0.0627	0.0627	0.0628	0.0626	0.0596	0.0708	0.0616
IIC	0.1484	0.1306	0.1521	0.1235	0.1514	0.1213	0.1539	0.1243	0.1510	0.1247
IIIA	0.0343	0.0629	0.0306	0.0663	0.0267	0.0669	0.0267	0.0642	0.0331	0.0652
IIIB	0.0540	0.0569	0.0473	0.0603	0.0399	0.0589	0.0352	0.0509	0.0519	0.0603
IIIC	0.1162	0.1109	0.1141	0.1118	0.1068	0.1079	0.1022	0.1008	0.1171	0.1140

TABLE V  
ULTIMATE MESAURES

	A	B	C	D	E
$I_S$	0.0086	0.0142	0.0536	0.0170	0.0107
$I_R$	0.0056	0.0038	0.0536	0.0053	0.0037
$I_S + I_R$	0.0142	0.0180	0.1072	0.0224	0.0144

TABLE VI  
NETWORK INFLUENCE DEGREES

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